Un fluide pas si idéal

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Can we tell directly from the data, without any theoretical prejudice, whether or not thermal equilibrium is achieved in heavy-ion collisions at RHIC?

- Why ideal fluids?
- The time scale of transverse flow
- Model-independent predictions for ideal fluids
  Borghini JYO nucl-th/0506045
- Deviations from ideal fluid behaviour
  Bhalerao Blaizot Borghini JYO, in preparation
- Revisiting hydro

Why ideal fluids? (1/2)
Physical quantities

Particles are produced, and then interact ("final state" interactions).
If there are many interactions,

- Particle ratios equilibrate (chemical equilibrium)
- Momentum distributions become locally isotropic (kinetic equilibrium)
- Collective expansion (flow)

Microscopic parameters

- λ = mean free path between two collisions
- v_{thermal} = average velocity of particles

Macroscopic parameters

- L = system size
- v_{fluid} = fluid velocity

Micro and macro are connected : kinetic theory

- c_s = sound velocity $\sim v_{\text{thermal}}$
- η = viscosity $\sim \lambda\, v_{\text{thermal}}$

Why ideal fluids? (2/2)
Types of flow

Thermal equilibrium or not?

Knudsen number $Kn = \lambda/L$

- $Kn \gg 1$: Ballistic (free-streaming) limit
- $Kn \ll 1$: Thermalization : Hydro (fluid) limit

Viscous or Ideal ?

Reynolds number $Re = Lv_{\rm fluid}/\eta$

- $Re \gg 1$: Ideal (non-viscous) fluid
- $Re \leq 1$: Viscous fluid

Compressible or Incompressible ?

Mach number $Ma = v_{\rm fluid}/c_s$

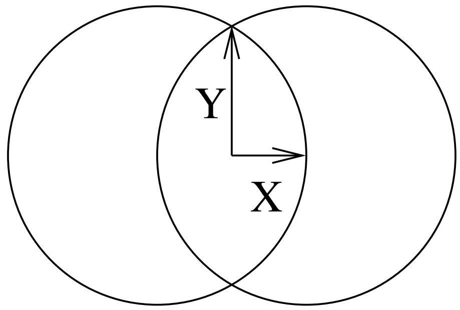
- $Ma \ll 1$: Incompressible fluid
- $Ma > 1$: Compressible (supersonic) fluid

An important relation

$$Kn \times Re = \frac{\lambda\, v_{\rm fluid}}{\eta} \sim \frac{v_{\rm fluid}}{c_s} = Ma$$

Compressible fluid: Thermalized means Ideal:
Viscosity $\equiv$ departure from equilibrium

Length & time scales (1/2)



- X, Y = transverse sizes
- ct = longitudinal size at time t

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

scales with the initial eccentricity

$$\epsilon \equiv \frac{Y^2 - X^2}{Y^2 + X^2}$$

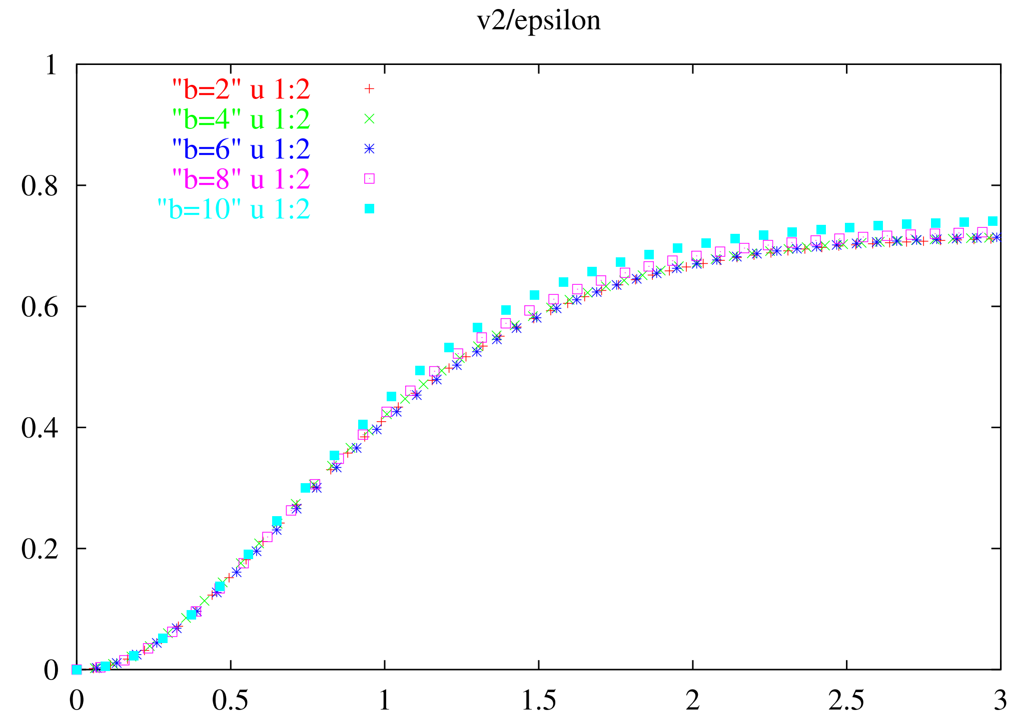
Flow generated by gradients: $1/X$, $1/Y$ are relevant: natural scale for transverse size

$$\frac{1}{R} \equiv \sqrt{\frac{1}{X^2} + \frac{1}{Y^2}}$$

The time scale for elliptic (more generally, transverse) flow is R/c_s where $c_s \equiv$ sound velocity ($\sim c/\sqrt{3}$ in QGP).

Length & time scales (1/2)

v_2/ϵ versus $c_s t/R$: perfect scaling !



Au-Au at 200 GeV

b (fm)	ϵ	R (fm)
0	0	2.07
2	0.033	2.02
4	0.115	1.89
6	0.215	1.68
8	0.315	1.45
10	0.398	1.22
12	0.433	1.04

Predictions for ideal fluids (1/8) general picture

- Ideal fluid dynamics describes the evolution if, at time $t \sim R$, the mean free path is much smaller than R
- The fluid expands (size increases: this is important) $\rightarrow$ density decreases $\rightarrow$ mean free path increases
- At some point, the mean free path becomes again of the same order as the size of the system: ideal fluid dynamics no longer applies. This is the freeze-out point.
- "Sudden" freeze-out: abrupt transition from ideal hydro to free streaming (other options are available in the literature)
- If the mean free path varies smoothly with the temperature, consistency requires that the freeze-out temperature be much smaller than the temperature at time $t = R$: the ideal-fluid limit is the limit where the freeze-out goes to zero
- At RHIC, blast-wave fits generally give $T \simeq 100$ MeV. This is not really small: viscous effects are likely to be important.

Predictions for ideal fluids (2/8)
general picture

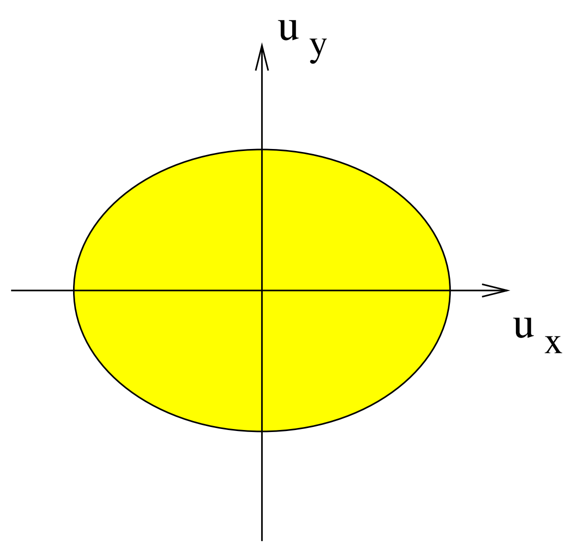
Neglecting quantum statistics, the fluid emits particles with Boltzmann distributions:

$$\frac{dN}{d^3x d^3p} \propto \exp\left(-\frac{p_\mu u^\mu(x)}{T}\right)$$

where $p^\mu u_\mu$ is the energy of the particle in the rest frame of the fluid, and

$$u^\mu = \begin{pmatrix} \sqrt{1+\vec{u}^2} \\ \vec{u} \end{pmatrix},$$

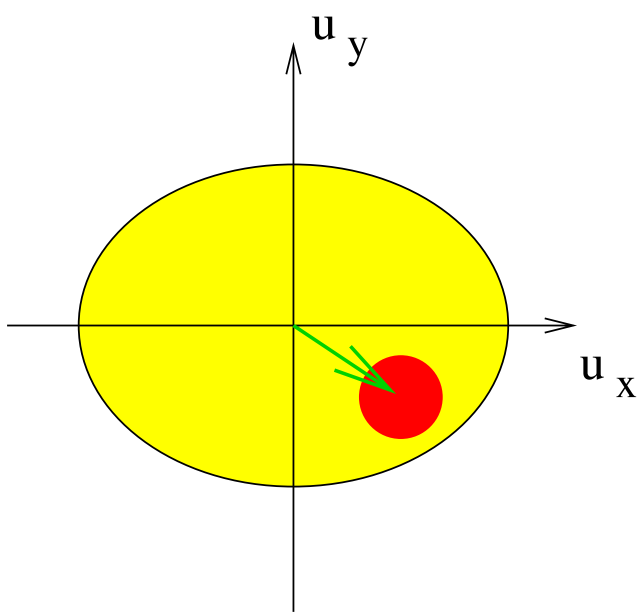
The domain for the transverse components of $\vec{u}$ is typically



The general idea: if T is small, the dominant contribution for a given $\vec{p}$ comes from the regions where $p_\mu u^\mu$ is minimum: saddle-point approximation of momentum distributions.

Predictions for ideal fluids (3/8)
"slow" particles

$p^\mu u_\mu$ is the energy of the particle in the rest frame of the fluid:
For a given p^μ, its minimum value of $p^\mu u_\mu$ is the particle mass m; it is reached when the fluid velocity equals the particle velocity.
This holds for particles with $p_t/m < u_{\rm max}$ ("slow" particles).

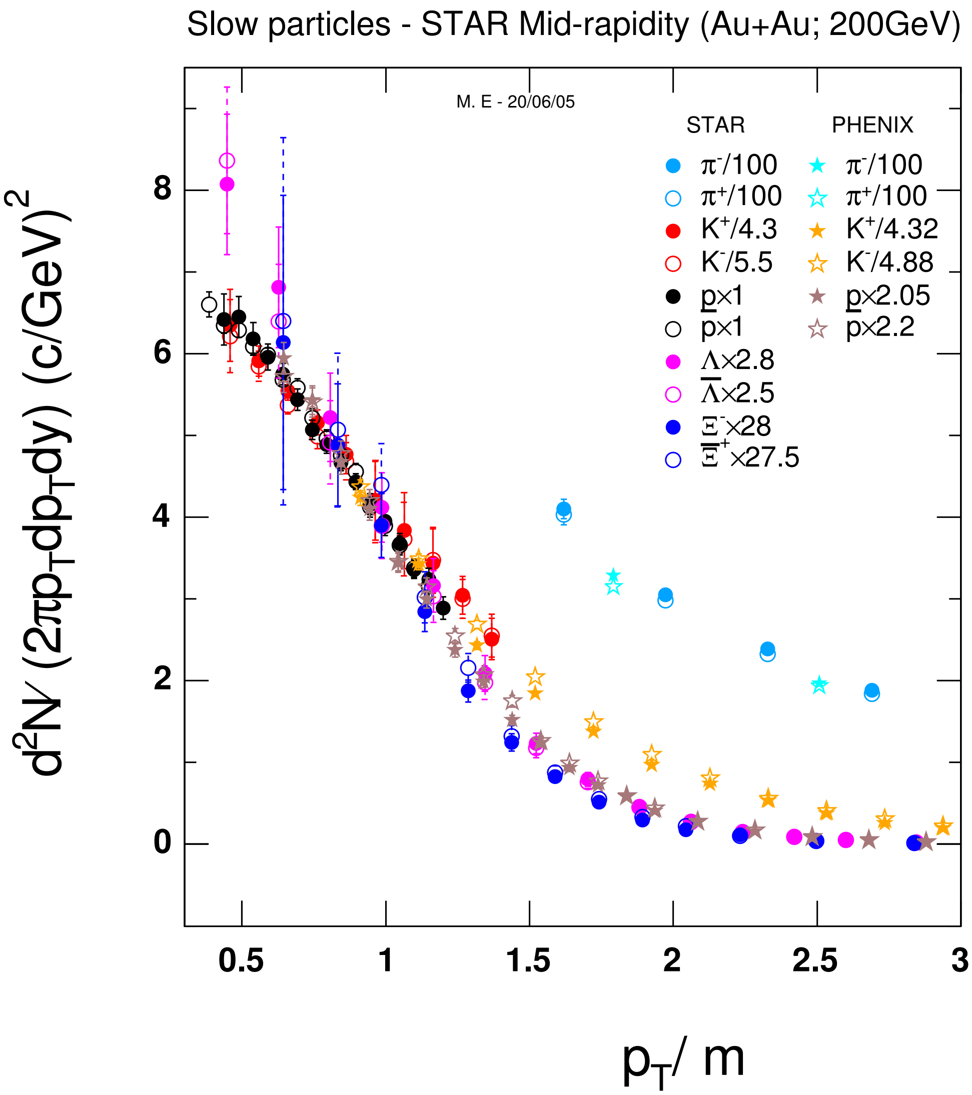


The width of the circle is of order $\sqrt{T/m}$. It is small if

$$T \ll m,$$

i.e., for kaons and heavier particles.
Momentum spectra and anisotropic flow of identified particles are the same for different particles when plotted versus particle velocities.

Predictions for ideal fluids (4/8)
slow particles at RHIC

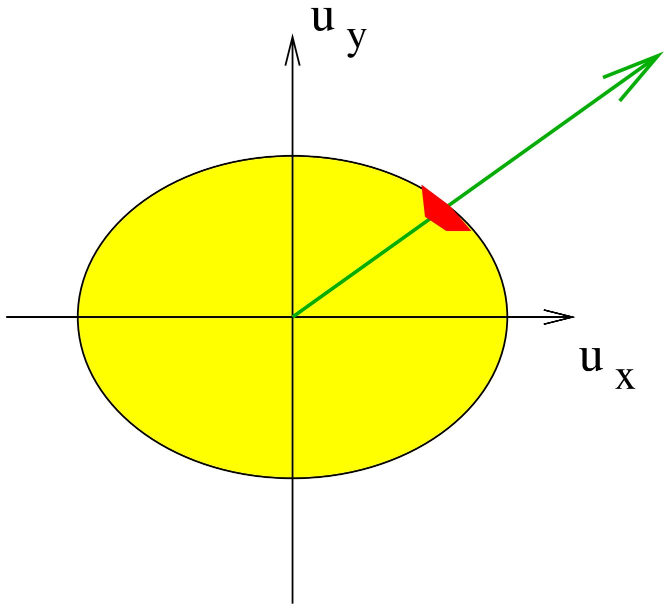


(Magali Estienne, private communication)

Approximate scaling is observed for kaons and heavier particles up to $p_t/m \simeq 1.2$, which suggests $u_{\max} \simeq 1.2$.

Predictions for ideal fluids (5/8)
fast particles

If a particle goes faster than the fluid ($p_t/m > u_{\max}$) the minimum of $p_\mu u^\mu$ is larger than m. It occurs when fluid and particle velocities are parallel, and when $u = u_{\max}$.



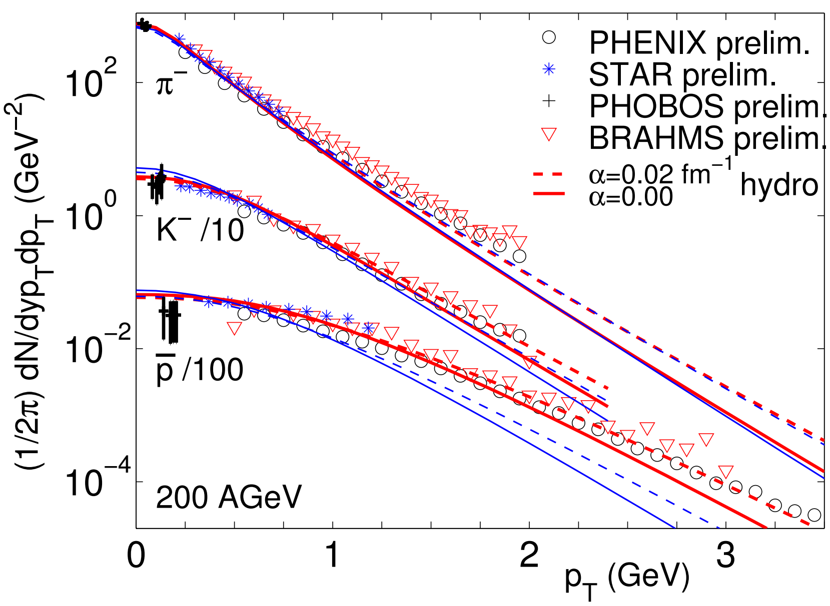
Particles that go faster than the fluid ($p_t/m > u_{\max}$) are all produced in the same region! Even simpler and more predictive than slow particles.

Predictions for ideal fluids (6/8) spectra of fast particles

Momentum distribution for fast particles:

$$\frac{dN}{d^2p_t dy} \propto \exp\left(-\frac{m_t u^0_{\max} - p_t u_{\max}}{T}\right),$$

where $u^0_{\max} \equiv \sqrt{1 + u^2_{\max}}$.
The slope is $(p_t/m_t)u^0_{\max} - u_{\max}$, smaller for heavier particles at a given p_t.



(Review by Kolb and Heinz)

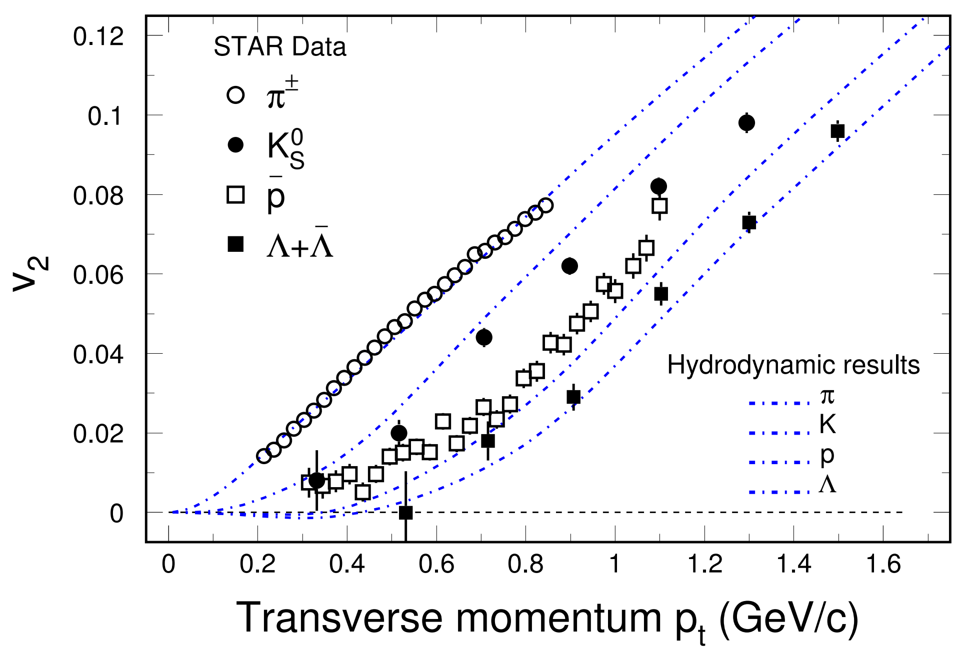
Predictions for ideal fluids (7/8)
elliptic flow of fast particles

$$u_{\max}(\phi) = u_{\max}\left(1 + 2V_2 \cos 2\phi\right)$$

where $V_2 \simeq 4\%$ for mid-central Au-Au at RHIC. To first order in V_2, this gives

$$v_2(p_t) = \frac{V_2 u_{\max}}{T}(p_t - m_t v_{\max})$$

where $v_{\max} = u_{\max}/u^0_{\max}$ is the max. fluid velocity. General result from hydro: $v_2(p_t)$ smaller for heavier particles.



(STAR, nucl-ex/0409033)

Predictions for ideal fluids (8/8) hexadecupole flow

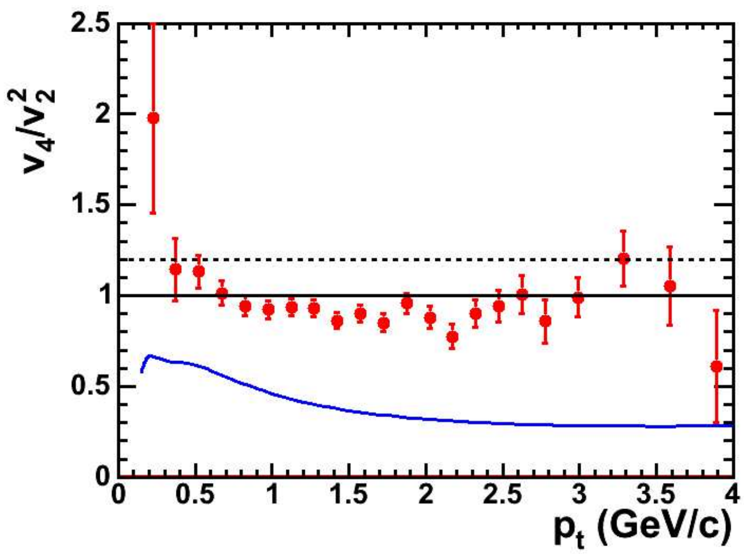
For small T, the ϕ dependence is dominated by the V_2 term, so that the ϕ distribution is essentially

$$\frac{dN}{d\phi} \propto \exp(A\cos 2\phi)$$

This gives

$$v_4(p_t) = \frac{v_2(p_t)^2}{2}$$

Comparing with numerical hydro and data:



(A.M. Poskanzer, private communication)

STAR, nucl-ex/0310029
Peter Kolb, nucl-th/0306081

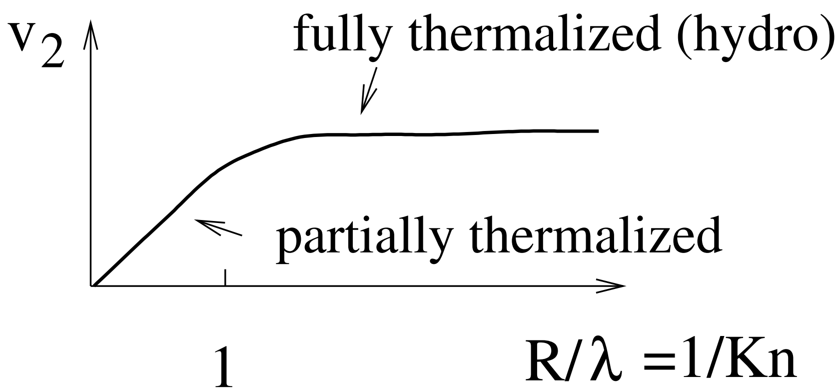
This is a first hint that the ideal fluid picture is not very good at RHIC...

Deviations from ideal fluid (1/6)
The general idea

v_2 is a good probe of equilibration because

- It is independent of the system size for a given shape and density in the ideal fluid limit.
- It vanishes if there is no final state interaction.

So that one naturally expects



Where are we standing on this curve at RHIC?
Easy in principle: vary R, keep λ (i.e., density) constant.
Problem: we can't choose size, shape, density as we like.
Solution: we can vary the shape and plot v_2/ϵ instead of v_2. But first try to work at constant density.

Deviations from ideal fluid (2/6)
Density

The density at time R/c_s is given by

$$n(R/c_s) = \frac{c_s}{RS}\frac{dN}{dy}.$$

where S=transverse surface.

Au-Au@RHIC

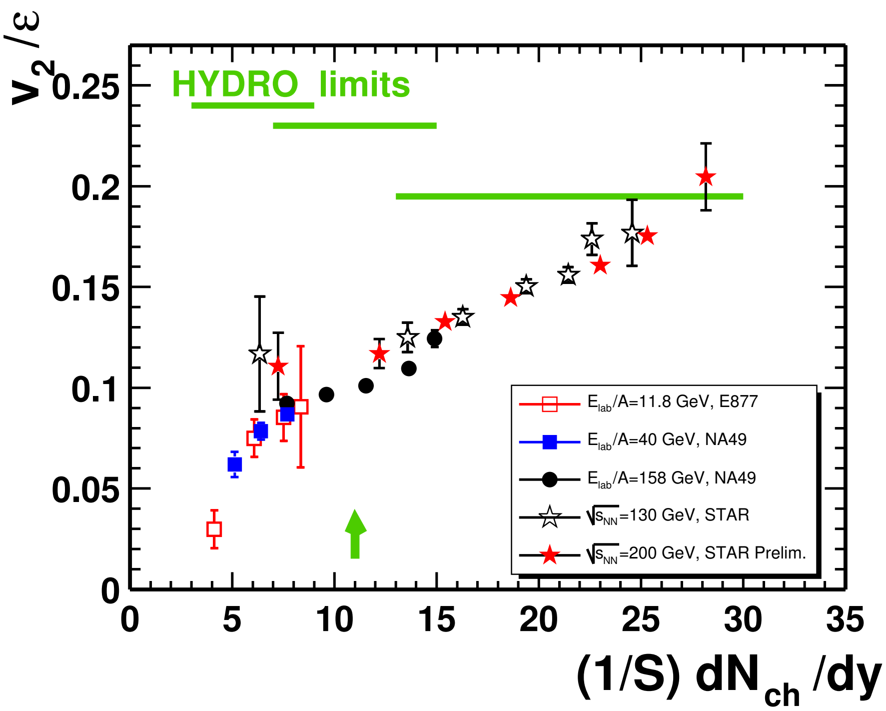
b	R (fm)	$\frac{dN}{dy}$	$n\left(\frac{R}{c_s}\right)$ (fm^{-3})
0	2.07	1050	5.4
2	2.02	975	5.4
4	1.89	790	5.5
6	1.68	562	5.3
8	1.45	344	4.9
10	1.22	167	3.8

Good news! The density varies little for b between 0 and 8 fm, while the size varies by 40%.

One can plot v_2/ϵ as a function of R, or, equivalently, $(1/S)(dN/dy)$.

Deviations from ideal fluid (3/6)
RHIC data

RHIC data:



NA49 nucl-ex/0303001

SPS are also shown on the plot but the density is lower at SPS that at RHIC.

No indication of saturation of v_2, i.e., of hydro behaviour, from data alone.

Deviations from ideal fluid (4/6)
Predictions for Cu-Cu

The matching between central SPS and peripheral RHIC suggests that we can even compare systems with different densities, and do the approximation $\lambda = 1/\sigma n$, with σ constant.
Then we directly obtain the Knudsen number as

$$\frac{1}{Kn} = \sigma n R = \sigma c_s \frac{1}{S}\frac{dN}{dy}$$

Several ways of varying Kn: one can vary

- Centrality dependence
- Beam energy
- Lighter systems
- Rapidity dependence

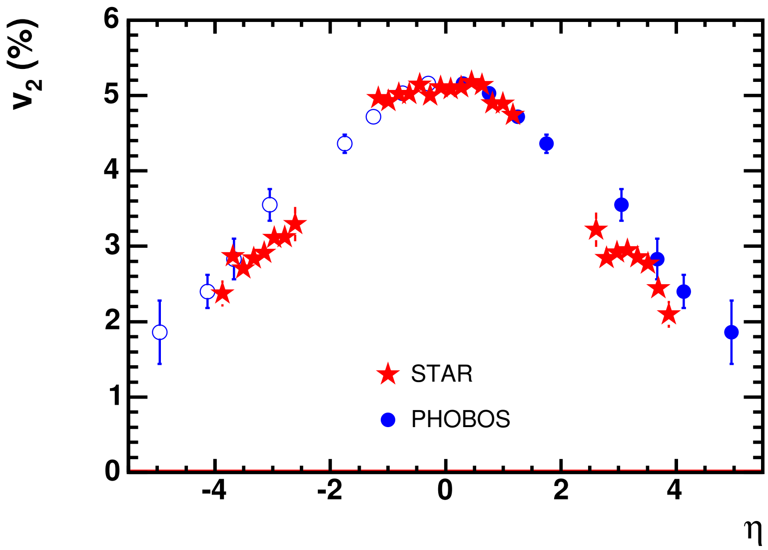
Compare Au-Au at b=8 fm with Cu-Cu at b=5.5 fm (similar centrality).
If hydro holds, v_2 should scale like ϵ:

$$v_2(Cu) = 0.69\ v_2(Au)$$

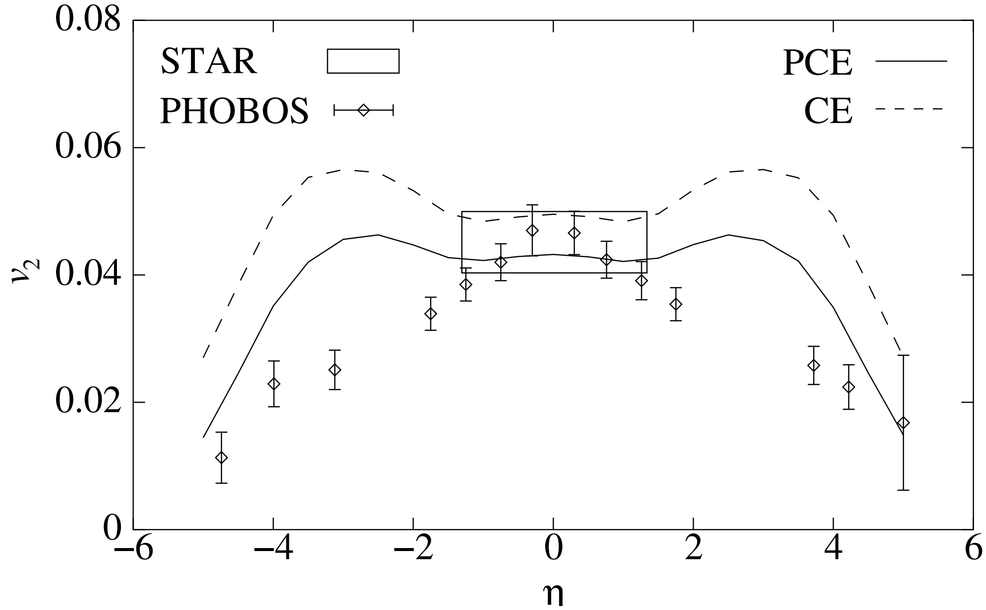
In the low density limit, v_2/ϵ should scale like $(1/S)dN/dy$, i.e.,

$$v_2(Cu) = 0.34\ v_2(Au).$$

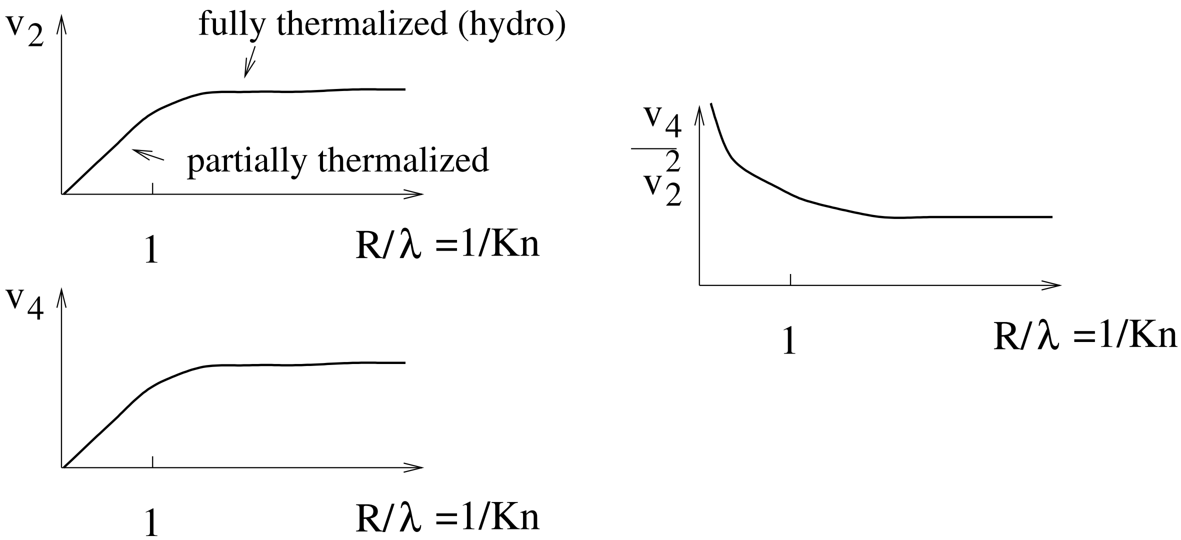
Deviations from ideal fluid (5/6)
Rapidity dependence of v_2



Hydro predictions (from Kolb and Heinz)



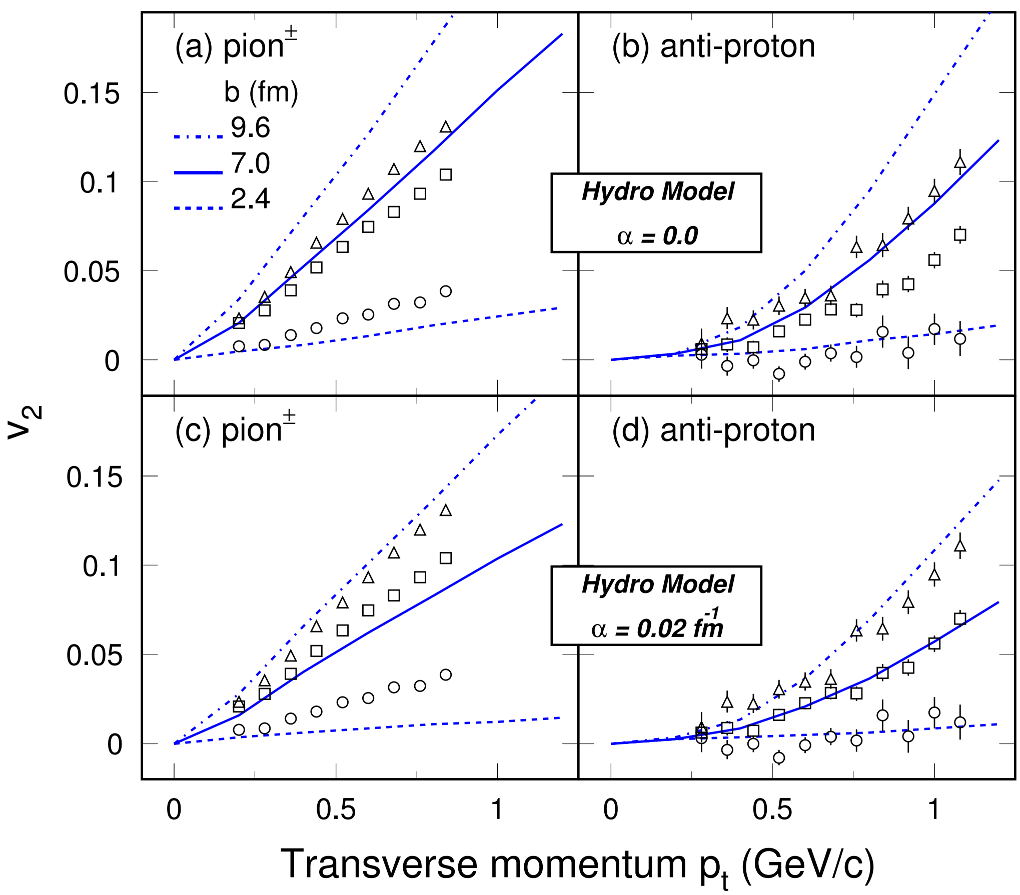
Deviations from ideal fluid (6/6)
Hexadecupole flow



Partial thermalization naturally explains an increase of $v_4/(v_2)^2$ relative to hydro, as seen at RHIC.

Revisiting hydro (1/3)

The standard statement is that v_2 at RHIC "reaches the hydro limit", but there may be a problem with the hydro limit...



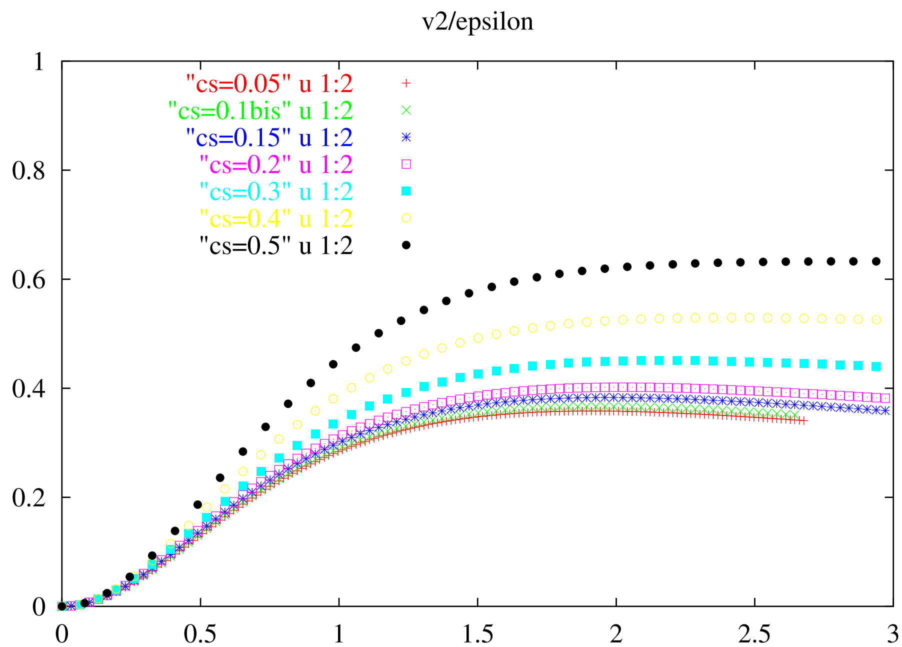
STAR, nucl-ex/0409033

Data above "hydro limit" for central collisions !!

Revisiting hydro (2/3)

Can we increase the “hydro limit” on v_2?

Yes, by changing the speed of sound



Revisiting hydro (3/3)

But in hydro, the speed of sound is constrained by p_t spectra which require a soft equation of state. The energy per particle is too high with a hard equation of state

But all relies on the assumption that the energy per particle is related to the density, i.e., that chemical equilibrium is maintained (even recent papers by Hirano assume chemical equilibrium initially)

The only experimental evidence for chemical equilibrium so far is from particle ratios, but this is a rather weak evidence since there is also such "equilibrium" in $e^+ - e^-$.

If there is no chemical equilibrium, energy per particle and density are independent variables, as in ordinary thermodynamics.

There is no constraint on the equation of state from p_t spectra.

Conclusions

- Hydro is rather successful at RHIC
- But transverse thermalization at RHIC is probably not complete
- It is therefore unlikely that full (i.e. longitudinal+transverse+che equilibration occurs
- More work must be done in order to reach a quantitative understanding of RHIC data
- Better agreement with hydro is expected at LHC: in particular, $v_4/(v_2)^2$ should be smaller at LHC than at RHIC.
- There is still room for a significant increase of v_2 at LHC.