The thermal width of the Higgs boson in hot QCD matter

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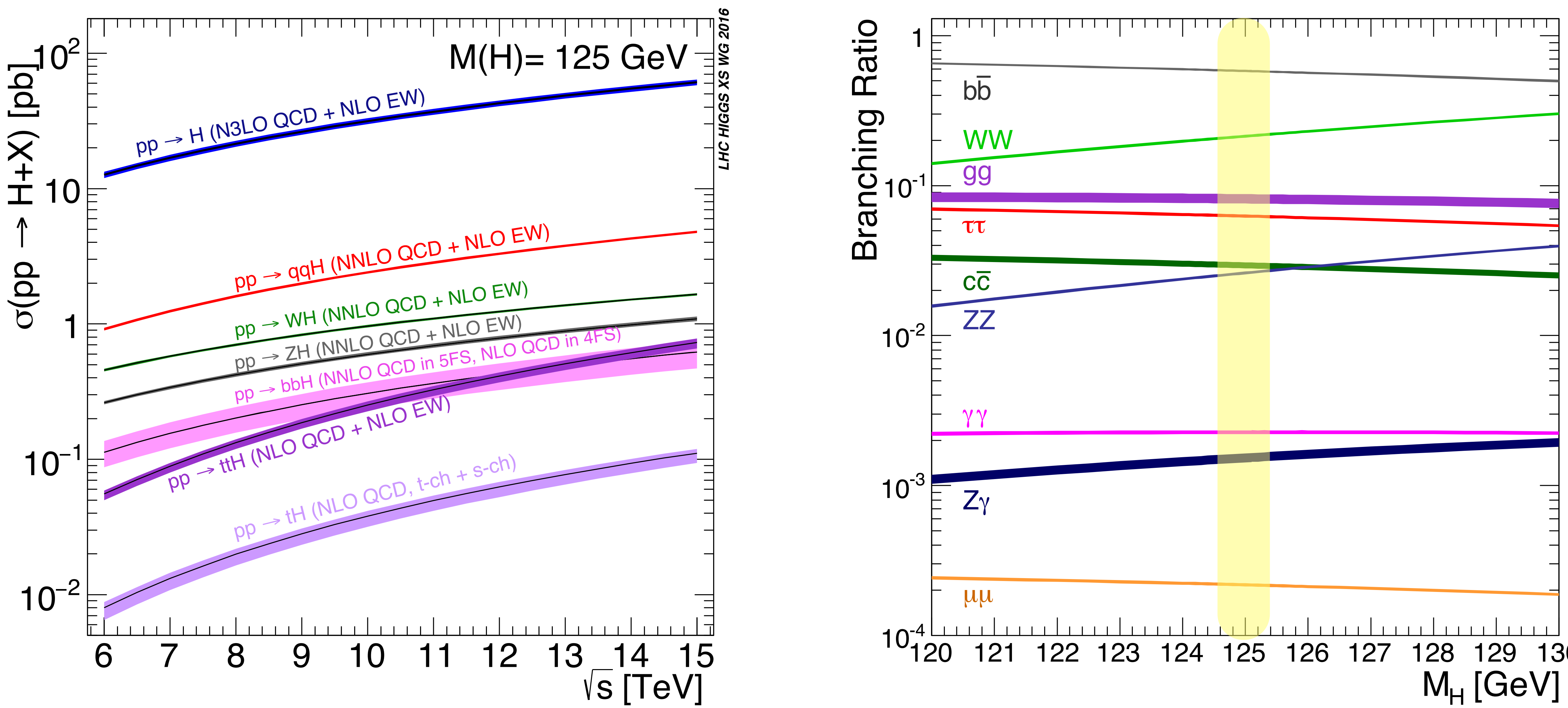
IPN Orsay, February 13th 2020

Outline

- The Higgs boson: production and decays
- Higgs decays in the QCD medium: are modifications to be expected?
- Thermal width of the Higgs boson and the Operator Product Expansion (OPE)
- Comparison with diagrammatic approach

JG Wiedemann **PRD99** (2019)

The Higgs boson



PDG 2018

- Total width (theory) $\Gamma_H \approx 4$ MeV (exp. $\Gamma_H < 130$ MeV) . Very different from the top quark

The Higgs boson in HICs

- The LHC does not have the luminosity to see Higgses in HICs
- This will be different at the FCC

process	PbPb(pp) in nb(pb)		
	5.5 TeV	11 TeV	39.4 TeV
GF	480(10.2)	1556(35.2)	9580(235)
VBF	15.3(0.316)	65.6(1.40)	421(10.02)
ZH	10.2(0.230)	28.1(0.687)	147(3.97)
W^+H	8.38(0.162)	21.8(0.716)	94.2(3.19)
W^-H	9.22(0.143)	23.4(0.435)	99.5(2.34)

- Claim: Higgs production and decay unaffected by medium, while background is ⇒ increased significance in the bbbar channel!

5σ treshold luminosity

lumi.(pb^{-1})	strong	medium	mild	vacuum
LHC	16(5.9)	27(9.8)	26(9.3)	48(17)
HE-LHC	11(4.0)	20(7.2)	20(7.2)	34(12)
FCC-hh	8.0(2.9)	13(4.7)	14(5.0)	22(8.0)

Berger Gao Jueid Zhang **PRL122** (2018)

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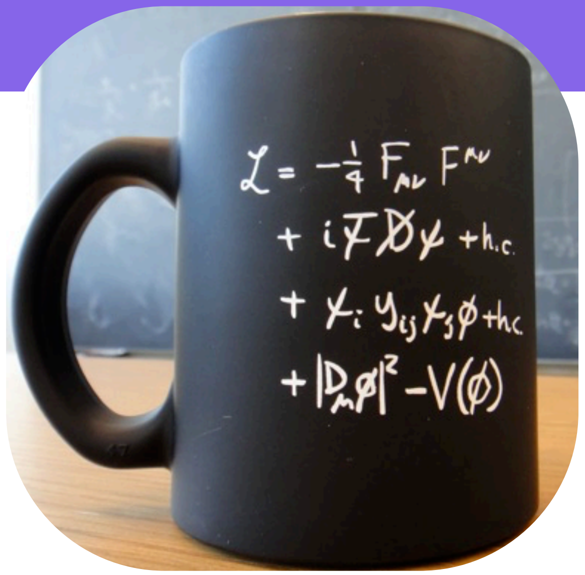
- Claim: Higgs production and decay unaffected by medium, while background is $\Rightarrow$ increased significance!
- Are we sure about this? Calculations of $Hp_1 \to p_2 p_3$ (ex. $Hg \to gg$) cross sections folded over thermal partons seem to show a large

D'Enterria Loizides (2018)

Higgs decays to partons



- The Higgs couples to quarks (directly)

$$\mathcal{L}_{Hq} = -\mathcal{O}_q \frac{H}{v} \qquad \mathcal{O}_q \equiv m_q \bar{\psi}_q \psi_q$$

and gluons (via top loop mostly)

$$\mathcal{L}_{Hg} = -C_{Hg} \mathcal{O}_g \frac{H}{v} \qquad \mathcal{O}_g \equiv -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} \qquad C_{Hg} = \frac{\alpha_s}{3\pi} + \mathcal{O}(\alpha_s^2)$$

in the heavy top limit ($M_t \gg M_H$, more effective than one would expect) Inami Kubota Okada (1983)

- The decay widths are given by the spectral functions of the O

$$\Gamma_{H\to q\bar{q}} = \frac{1}{v^2} \frac{1}{2k^0} \rho_{\mathcal{O}_q}(K) \qquad \Gamma_{H\to gg} = \frac{\alpha_s^2}{(3\pi)^2 v^2} \frac{1}{2k^0} \rho_{\mathcal{O}_g}(K)$$

$$\rho_J(K) \equiv \int d^4x\, e^{iK\cdot X} \langle [J(X), J(0)] \rangle$$

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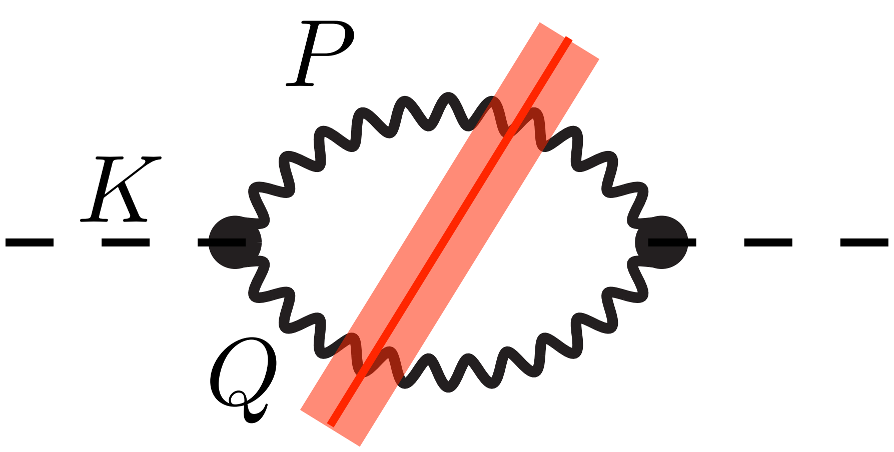

$$\rho_{\mathrm{LO}}^{\mathrm{vac}} \sim \int_{\mathbf{p}}\int_{\mathbf{q}} \left|\mathcal{M}_{H\to p_1p_2}\right|^2 (2\pi)^4\delta^4(K-P-Q)$$

$$\int_{\mathbf{p}} = \int \frac{d^3p}{(2\pi)^3 2p}$$

Higgs decays to partons

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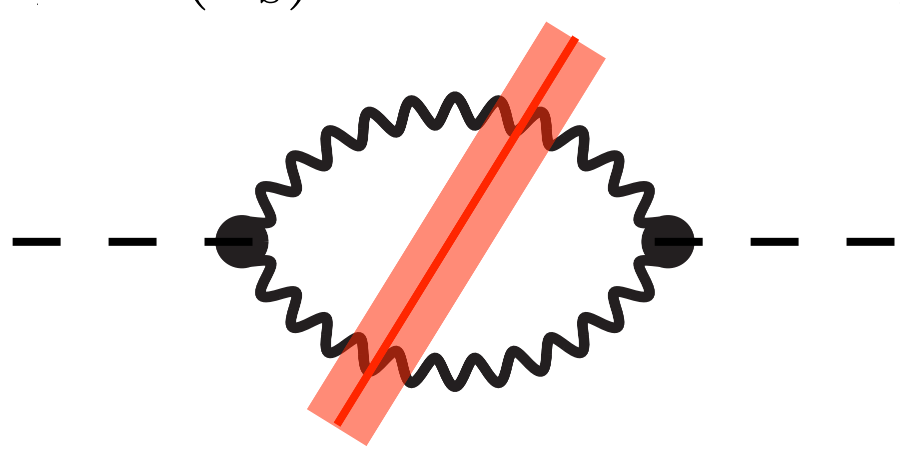
- In vacuum the spfs can only be a function of K^2=M^2. By dimensional analysis $\rho_{Og}\propto M^4$, $\rho_{Og}\propto m_q^2 M_H^2$

$$\Gamma^{\rm vac}_{H\to q\bar{q}} = \frac{d_F m_q^2 M_H}{8\pi v^2} + \mathcal{O}(\alpha_s) \qquad \Gamma^{\rm vac}_{H\to gg} = \frac{\alpha_s^2 M_H^3}{72\pi^3 v^2} + \mathcal{O}(\alpha_s^3)$$



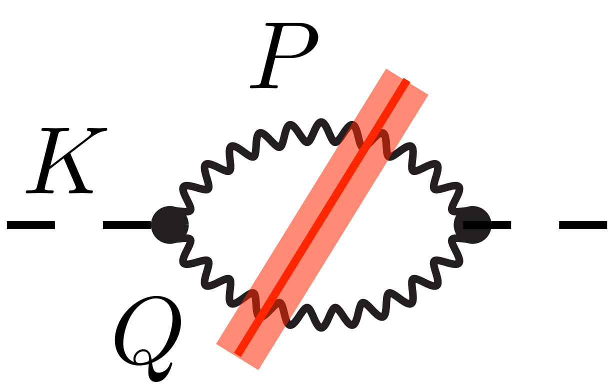
Thermal corrections to the width

- This formalism remains valid at finite temperature

$$\Gamma_{H\to q\bar{q}} = \frac{1}{v^2}\frac{1}{2k^0}\rho_{\mathcal{O}_q}(K) \qquad \Gamma_{H\to gg} = \frac{\alpha_{\rm s}^2}{(3\pi)^2 v^2}\frac{1}{2k^0}\rho_{\mathcal{O}_g}(K)$$

$$\rho_J(K) \equiv \int d^4x\, e^{iK\cdot X}\,\langle [J(X), J(0)]\rangle \qquad \mathcal{O}_{\color{red}g} \equiv -\frac{1}{4}F^a_{\mu\nu}F^{a\,\mu\nu} \qquad \mathcal{O}_{\color{blue}q} \equiv m_q\bar{\psi}_q\psi_q$$

- Now the spfs can be a function of k^0 and **k**. Naively then



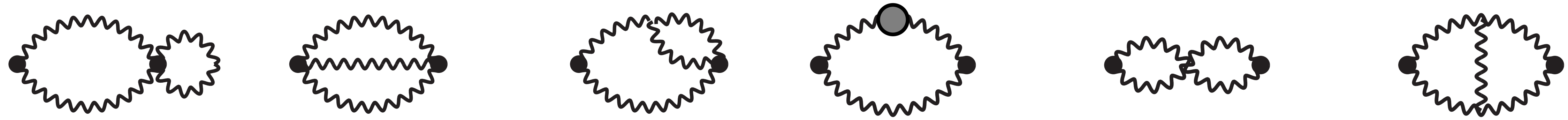
$$\rho^{\rm th}_{\rm LO} \sim \int_{\mathbf{p}}\int_{\mathbf{q}} |\mathcal{M}_{H\to p_1p_2}|^2\,(2\pi)^4\delta^4(K-P-Q)(1\pm f(p))(1\pm f(q))$$

$$\pm f(p) = (\exp(p/T)\mp 1)^{-1}$$

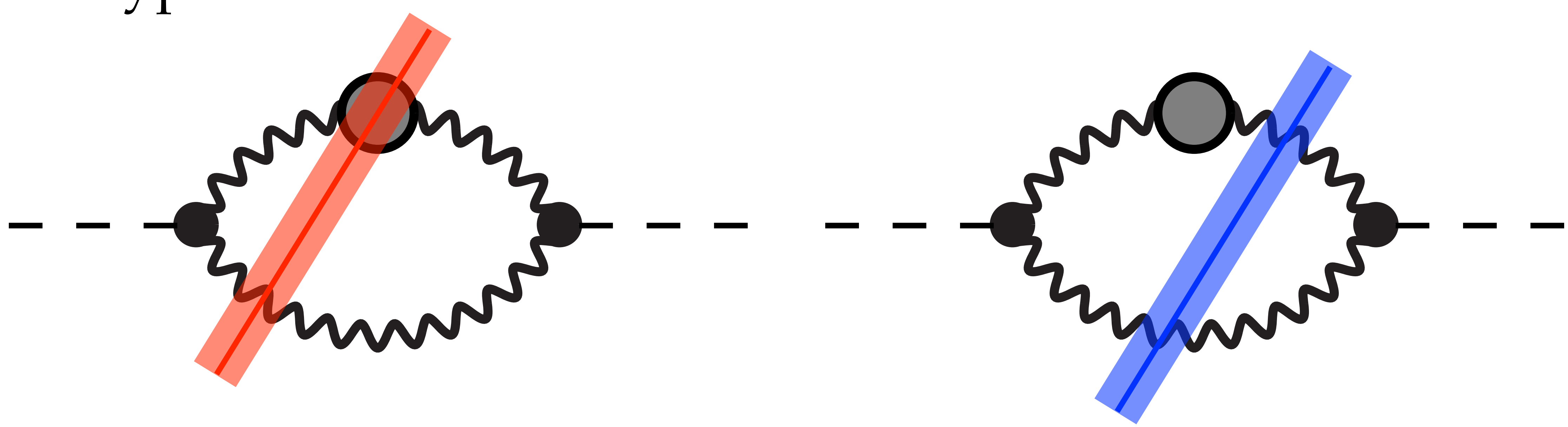
- Kinematics forces $p,q \geq M_H/2 \gg T$ (= if Higgs at rest in the plasma). Then the thermal modifications are exponentially suppressed

Thermal corrections to the width

- To find a possible non-vanishing thermal correction to the width we need to go beyond LO in perturbation theory



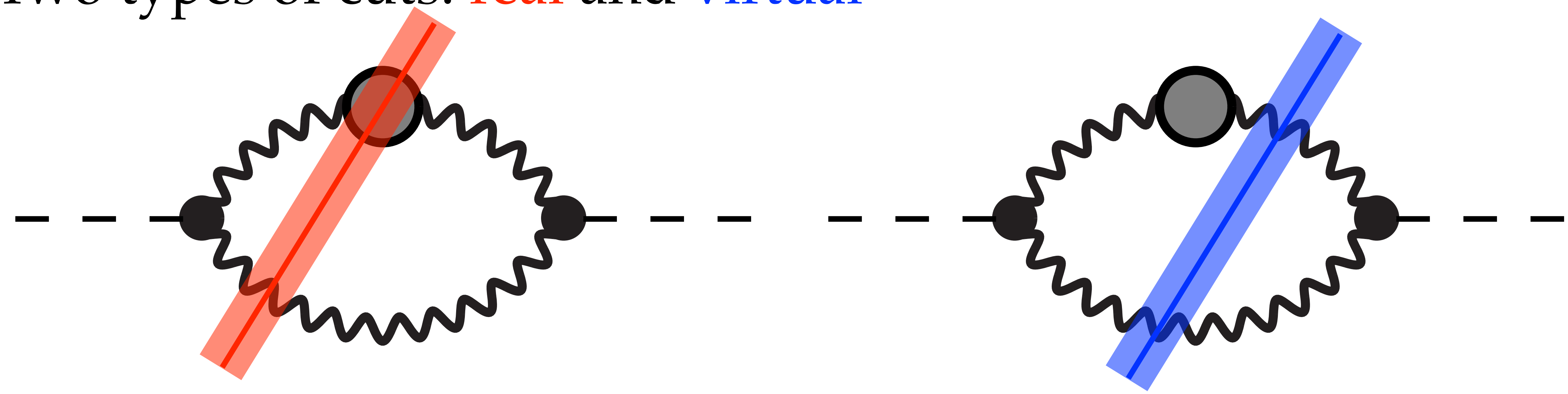
- Two types of cuts: real and virtual



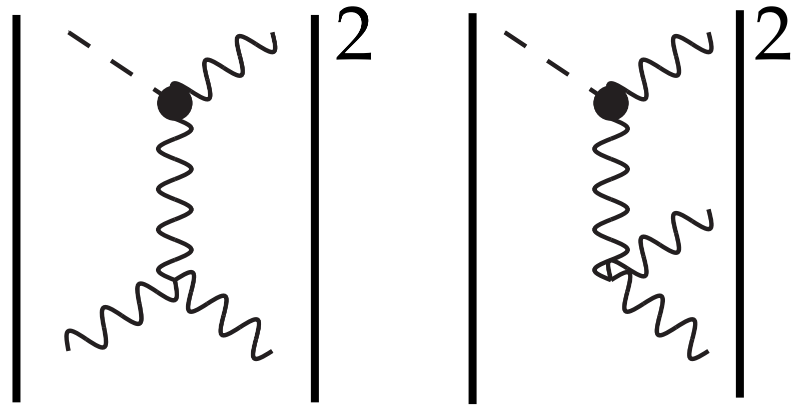
Figures adapted from Laine Vuorinen Zhu (2011)

Thermal corrections to the width

- Two types of cuts: real and virtual



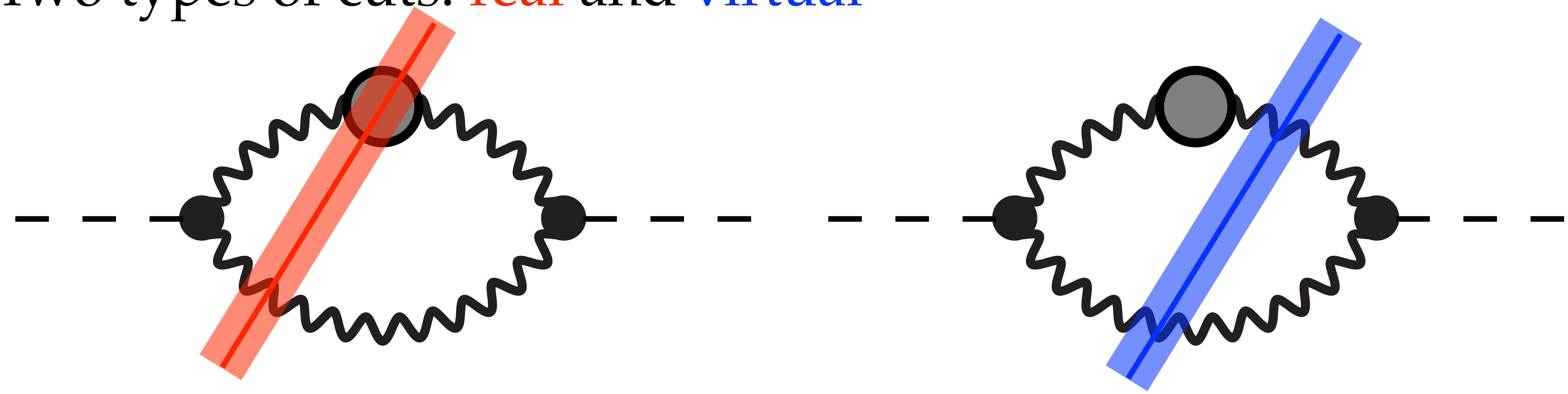
- Real cuts: $Hp_1 \to p_2 p_3$ *parton scattering*, $H \to p_1 p_2 p_3$ *three-body decay*



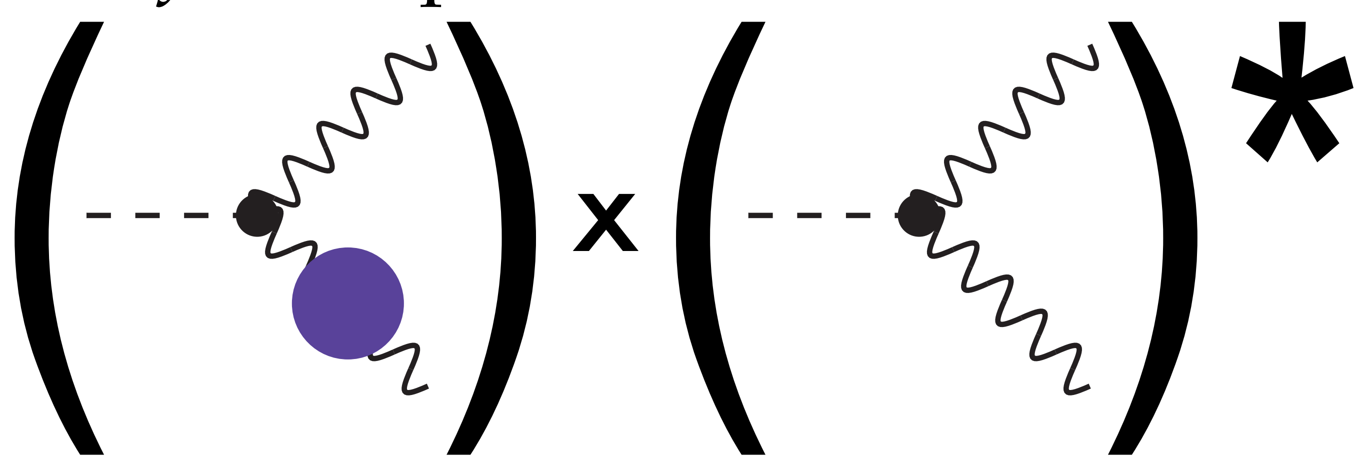
The incoming or one of the three outgoing partons can now be thermal, $p \sim T$

Thermal corrections to the width

- Two types of cuts: real and virtual



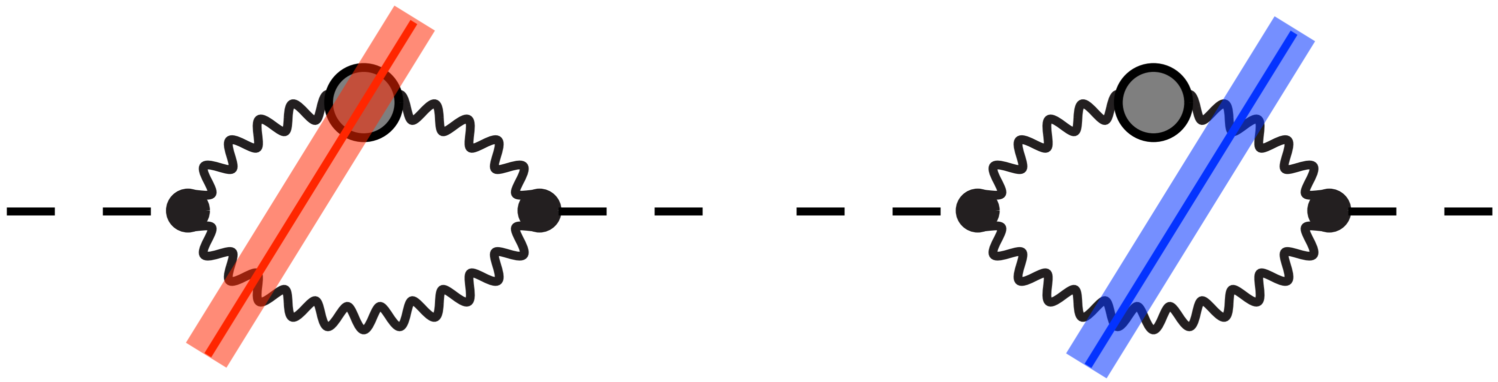
- Virtual cuts: *interference* processes



Quantum interference of the Born process with a one-loop thermal correction (self-energy or vertex)

Thermal corrections to the width

- Two types of cuts: real and virtual



- The parton scattering, three-body decay and interference processes are separately soft/collinear divergent. Only the sum is IR safe and finite

- Two consequences
    - The partial result for any single process is scheme-dependent and unphysical. Only the sum is finite and physical
    - An explicit calculation is going to be challenging

Two-loop thermal spfs

- The good news is that no finite-temperature resummations are needed when $K^2 \gtrsim T^2$. Long history for these types of calculations, dating back to thermal dilepton production

  Baier Pire Schiff (1988), Altherr Aurenche Becherrawy (1989)
  Gabellini Grandou Poizat (1990) (zero $\boldsymbol{k}$)
  Laine (2013) (finite $\boldsymbol{k}$)

- Even better news: so far we have been using $M_H \gg T$ between the lines. What happens if we exploit it heavily? $\Rightarrow$ **OPE for spf asymptotics** at $K^2 \gg T^2$

  Caron-Huot **PRD79** (2009)

OPE at large K^2

- QCD sum rules: in the deep space-like regime (Euclidean) $-K^2 \gg \Lambda^2_{QCD}$ the Euclidean two point function can be written as a series of local gauge-invariant operators (low-energy scale) times Wilson coefficients (high-energy scale): OPE

$$G_E(k_E) = \int d^4x_E e^{-ix_E \cdot k_E} \langle J(x_E) J(0) \rangle \sim \sum_n \langle \mathcal{O}_n \rangle \frac{c_n}{k_E^{d_n}}$$

Novikov Shifman Vainshtein Zakharov (1985)

- Caron-Huot (2009): this approach also works at large positive $K^2 \gg T^2$, thereby obtaining

$$\rho_J(k_E) = 2\,\mathrm{Im} G_E(-i(k^0 + i\epsilon)) \quad \Rightarrow \quad \rho_J(k_0) \sim \sum_n \langle \mathcal{O}_n \rangle \, 2\,\mathrm{Im}\left[\frac{c_n}{(-ik_0)^{d_n}}\right]$$

OPE at large K^2

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$$\rho_J(k_0) \sim \sum_n \langle \mathcal{O}_n \rangle \, 2\,\mathrm{Im}\left[\frac{c_n}{(-ik_0)^{d_n}}\right]$$

- In general knowing asymptotics in one direction (Euclidean) does not translate into knowing it in any other direction. This works because
    - Heuristically, in both cases, the locality of the operators is a consequence of the large scale separation between the short time of the high-energy processes and the long one of the low-energy processes

Sum rules at large K^2

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- In general knowing asymptotics in one direction (Euclidean) does not translate into knowing it in any other direction. This works because
    - Mathematically, because $\rho_J(k^0)$ admits an asymptotic expansion in inverse powers of k^0, just like the Euclidean one

Sum rules at large K^2

- Caron-Huot (2009): this approach also works at large positive $K^2 \gg T^2$, thereby obtaining

$$\rho_J(k_0) \sim \sum_n \langle \mathcal{O}_n \rangle \, 2\,\mathrm{Im}\left[\frac{c_n}{(-ik_0)^{d_n}}\right]$$

- What's left to do: determination of the operators and Wilson coefficients (also in Caron-Huot (2009))

- What to expect: first gauge-invariant local operators in QCD are dimension 4. Wilson coefficients start at $O(\alpha_s)$

$$\rho_J^T(k^0) \sim \alpha_s \frac{T^4}{k_0^4} \sim \alpha_s \frac{T^4}{M_H^4}$$

Thermal Higgs width from sum rules

- What to expect: first gauge-invariant local operators in QCD are dimension 4. Wilson coefficients start at $O(\alpha_s)$

- These operators are the dimension-four quark and gluon condensates, e.g.

$$\rho^T_{\mathcal{O}_g}(K) = \frac{2\alpha_s}{3}\frac{K_\mu K_\nu}{K^2}\left[2C_F T_f^{\mu\nu} - \left(n_f T_F + \frac{3}{2}b_0\right) T_g^{\mu\nu}\right] - \pi T^\mu{}_\mu$$

traceless parts of the quark and gluon contributions to the stress-energy tensor, $O(T^4)$, gluonic trace part, $O(\alpha_s{}^2 T^4)$

- At leading order

$$\langle T_g^{00}\rangle = 3\langle T_g^{ii}\rangle = \frac{\pi^2 T^4}{15} d_A \qquad \langle T_f^{00}\rangle = 3\langle T_f^{ii}\rangle = \frac{7\pi^2 T^4}{60} d_F$$

Thermal Higgs width from sum rules

- The sum rule results are thus, for the Higgs at rest in the medium

$$\delta\Gamma_{H\to gg} = -\Gamma^{\rm vac}_{H\to gg}\,\alpha_{\rm s}\,\frac{T^4}{M_H^4}\,\frac{112\,\pi^3}{45}\left(8-n_f^T\right)$$

$$\delta\Gamma_{H\to b\bar b} = -\Gamma^{\rm vac}_{H\to b\bar b}\,\alpha_{\rm s}\,\frac{T^4}{M_H^4}\,\frac{128\,\pi^3}{135}$$

- At any realistic energy $T/M_H<0.01$. These corrections are extremely tiny

- When the Higgs is not at rest, the expressions above get multiplied by $1+\frac{4}{3}\frac{k^2}{M_H^2}$

JG Wiedemann **PRD99** (2019)

Validity region

- A first obvious condition is $K^2 \gg T^2$. From brute-force computations of similar spfs for all $K^2 \gtrsim T^2$ the OPE works for $K^2 \gtrsim 15T^2$ Laine Vepsäläinen Vuorinen Zhu (2010-13)

- The medium however requires that both k^0+k and k^0-k be much larger than T. As $k^0=(k^2+M^2)^{1/2}$, the first is trivially satisfied since $K^2 \gg T^2$. The latter corresponds to $k \ll M^2/(2T)$

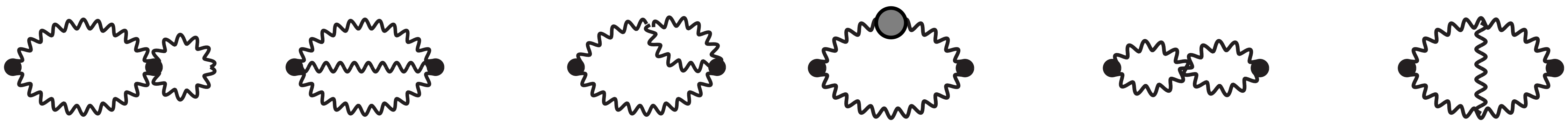
- Three computational regimes
    - Sum rules for $k^0-k \gg T$
    - Unresummed perturbation theory for $k^0-k \gtrsim T$
    - Resummations (HTL, LPM) below that

Unresummed computations

- To learn a couple of things about these computations, and remind ourselves of how it is easy to get large unphysical, scheme-dependent results if virtual processes are neglected, let us look at the explicit computation of the spf of O_g in the quenched limit for all $K^2 \gtrsim T^2$ at $k=0$ in
  Laine Vuorinen Zhu **JHEP1109** (2011)

- The motivation of that calculation was that more knowledge of the stress-energy spectral function (that is what O_g is) is needed to have better control over lattice reconstruction of these spfs, whose IR limits determine the bulk and shear viscosity of the QGP

Unresummed computations



- Evaluate each cut (real or virtual) of the two-loop graphs in a scheme defined by adding an extra mass λ to one of the propagators. Technically very intricate evaluation.

- Upon summing all of the cuts a finite expression is recovered for $\lambda \to 0$. It is a set of one- and two-dimensional integrations to be carried out numerically
Laine Vuorinen Zhu **JHEP1109** (2011)

- For ***illustration***, we undid their last step: we evaluated the real and virtual cuts separately in their scheme in the $k^0 \gg T \gg \lambda$ limit, where integrations can be done analytically

Unresummed computations

- For the real $Hg\to gg$ cut we find

$$
\begin{aligned}
\delta\rho_{\mathcal{O}_g}(k_0,\lambda)\Big|_{Hg\to gg} =& 3\alpha_s\Bigg\{\frac{k_0^4}{4\pi^2}\left[\frac{2\pi T}{\lambda}-\ln^2\left(\frac{\pi T}{\lambda}\right)+2(\ln(4)-\gamma_E)\ln\left(\frac{\lambda}{4\pi T}\right)+2\gamma_1-\frac{\pi^2}{6}-\ln^2(4)\right]\\
&-\frac{k_0^3T}{8\pi^2}\left[\ln^2\left(\frac{2k_0T}{\lambda^2}\right)-10\ln\left(\frac{T}{\lambda}\right)\ln\left(\frac{4T}{\lambda}\right)+18\gamma_1-\frac{4\pi^2}{3}+9\gamma_E^2-10\ln^2(2)\right]\\
&+\frac{k_0^2T^2}{12}\left[-144\ln(A)+4\ln\left(\frac{64\pi^3k_0T^3}{\lambda^4}\right)+11\right]\\
&-\frac{k_0T^3}{\pi^2}\left[\zeta(3)\left(\ln\left(\frac{k_0^4}{16T^4}\right)+4\gamma_E-15\right)-4\zeta'(3)\right]\\
&+\frac{8T^4}{45\pi^2}\left[\pi^4\left(3\ln\left(\frac{k_0T}{\lambda^2}\right)-3\gamma_E-\frac{1}{2}+\ln(8)\right)+270\zeta'(4)\right]+\mathcal{O}\left(\frac{T^5}{k_0}\right)\Bigg\}
\end{aligned}
$$

$k^0 = M_H$

scheme-dependent

- Power-law and log divergences in λ
- Terms at order $(k^0)^n$, $n>0$, all cancel with the contributions of the $H\to ggg$ and virtual cuts

Unresummed computations

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\end{aligned}
$$

$$k^0 = M_H$$

scheme-dependent

- Only one term survives to contribute to the full, scheme-independent result

$$
\delta\rho_{\mathcal{O}_g}(k_0)\Big|_{\text{tot}} = \delta\rho_{\mathcal{O}_g}(k_0,\lambda)\Big|_{\text{virt}} + \delta\rho_{\mathcal{O}_g}(k_0,\lambda)\Big|_{H\to ggg} + \delta\rho_{\mathcal{O}_g}(k_0,\lambda)\Big|_{Hg\to gg} = -\frac{88\pi^2\alpha_s T^4}{15}
$$

- It is $(T/M_H)^4\ \lambda/T$ smaller than the leading, scheme-dependent behaviour from a single cut/process

Conclusions

- Thermal modifications to the partonic width of the Higgs boson at temperatures much smaller than the mass are a **tiny correction**

$$\delta\Gamma \sim \Gamma_{\rm vac} \times \alpha_{\rm s} \times \frac{T^4}{M_H^4} \times \# \times \left(1 + \frac{4k^2}{3M_H^2}\right)$$

for non-relativistic to mildly relativistic Higgs bosons

- Is the signal/background ratio enhanced then?

Berger Gao Jueid Zhang **PRL122** (2018)

Conclusions

- Thermal modifications to the partonic width of the Higgs boson at temperatures much smaller than the mass are a **tiny correction**
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for non-relativistic to mildly relativistic Higgs bosons

- The OPE (or EFTs) are an excellent tool for computations of deeply time-like spectral functions. They predict the leading thermal behaviour from considerations of gauge invariance and locality alone

- Explicit diagrammatic calculations are intricate