Status of net-proton fluctuation measurements with ALICE and long-term perspectives

Mesut Arslandok
Physikalisches Institut Universität Heidelberg

- ✓ Why fluctuations?
- ✓ How to link experiment to theory?
- ✓ Experimental Challenges
- ✓ Results from ALICE
- ✓ Future plans

Rencontre Ions Lourds, Institut de Physique Nucléaire, Orsay, France
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UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

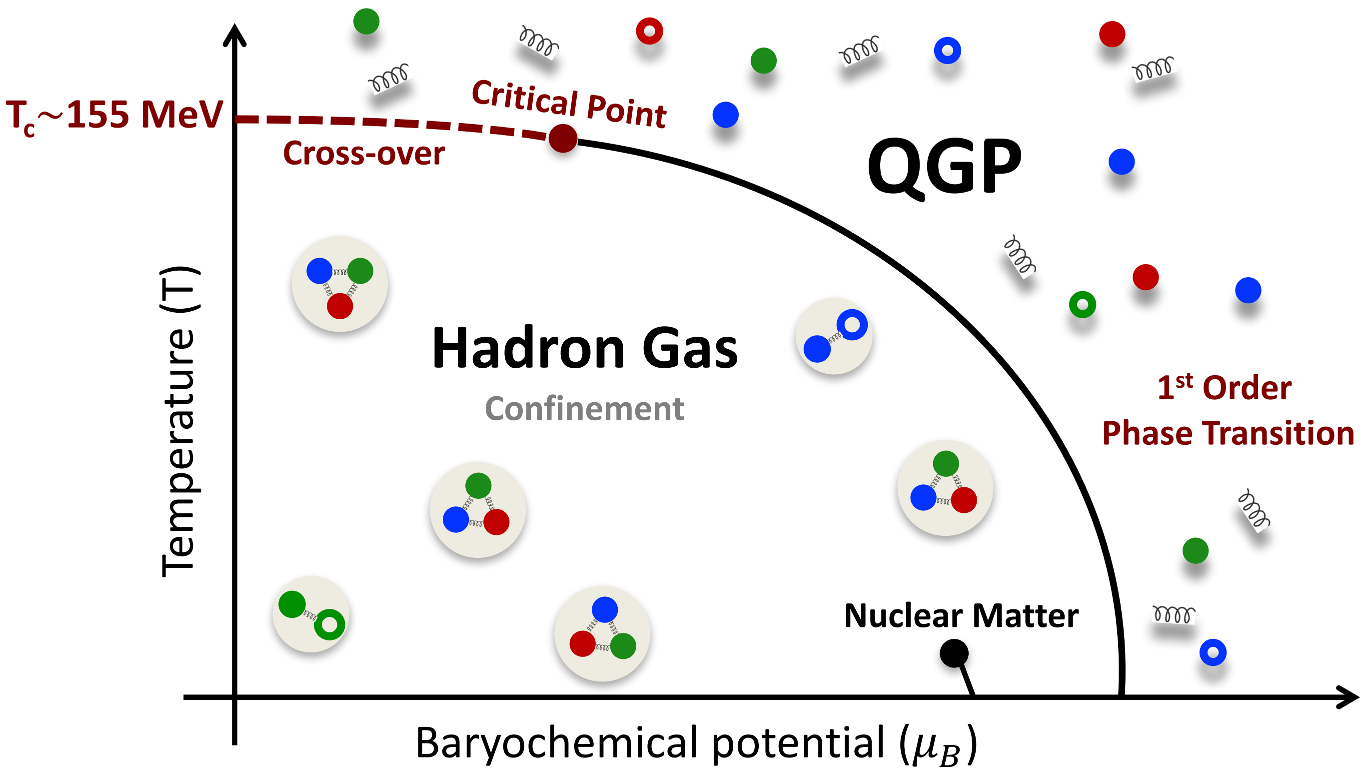
isoQuant SFB1225

BMBF Forschungsschwerpunkt ALICE Experiment 202

ALICE

Motivation #1

QCD phase diagram



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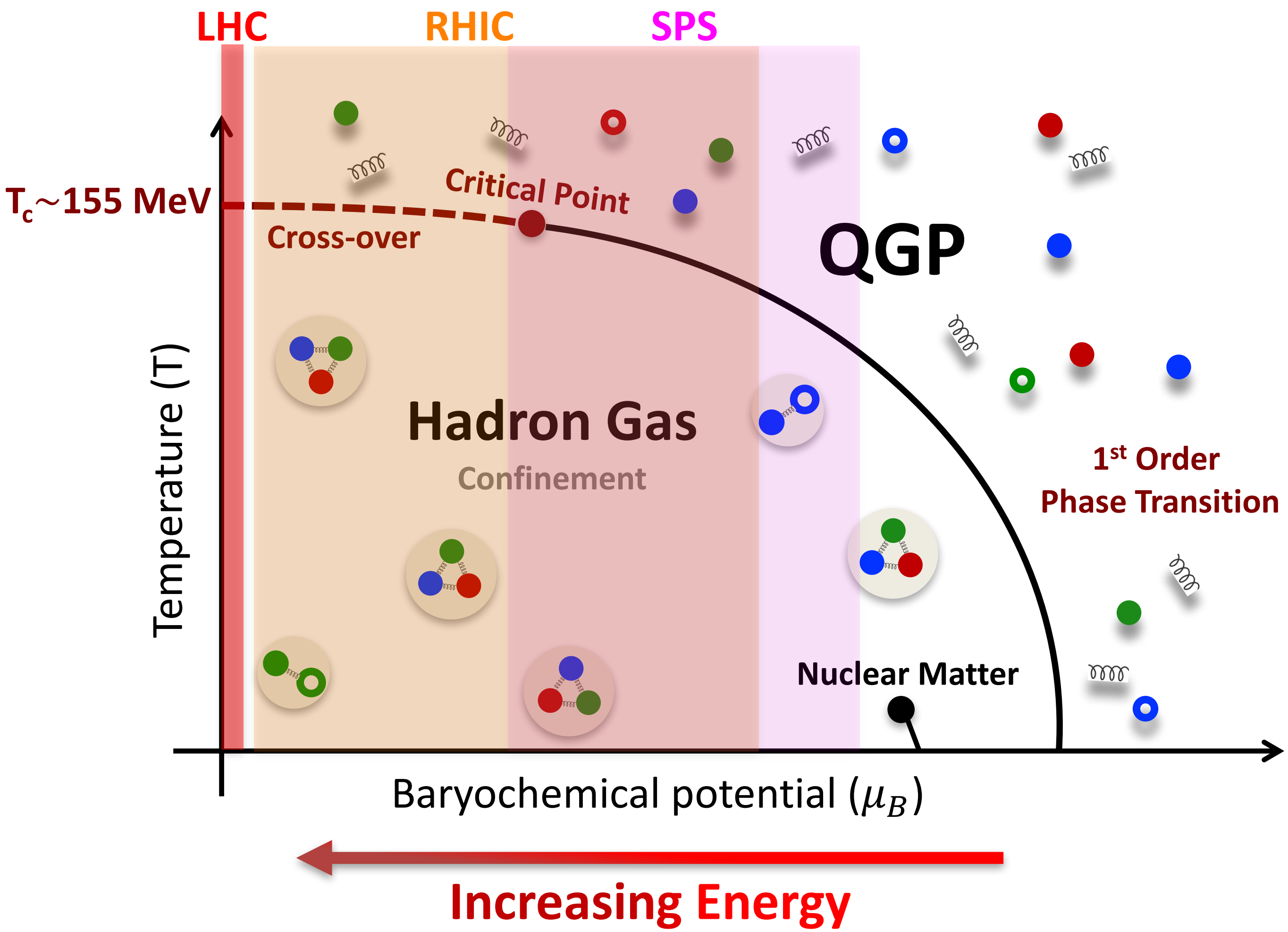
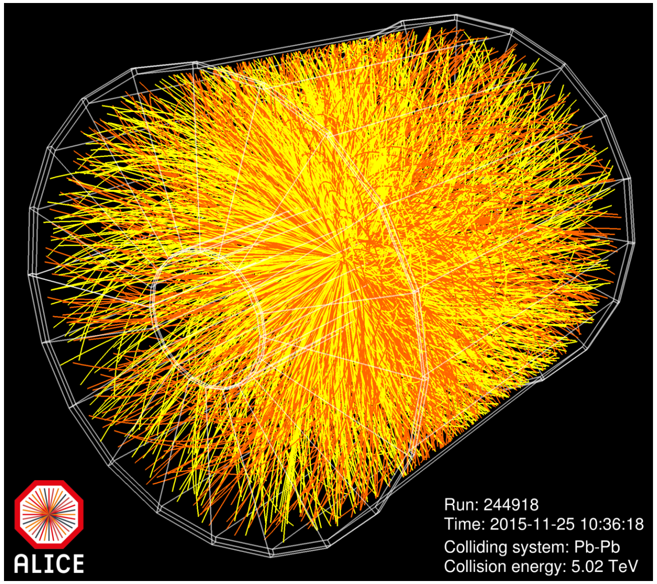


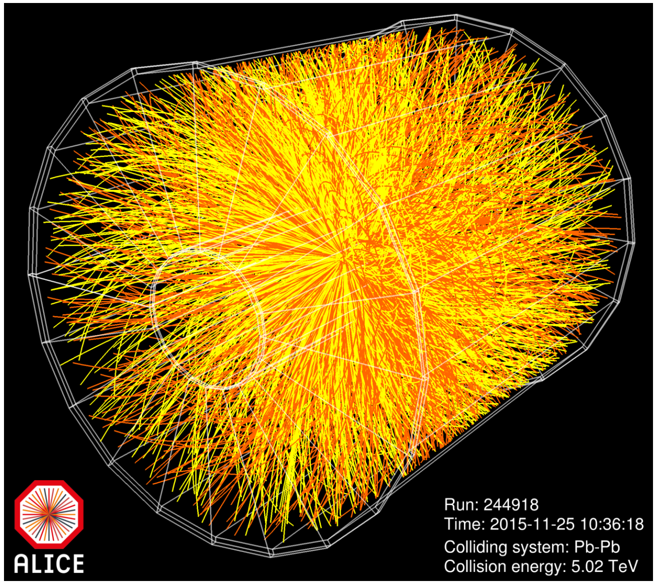
Why fluctuations?

Multiplicity distributions



$\sim$**15000 charged particles** are detected in one central Pb-Pb collision

Multiplicity distributions



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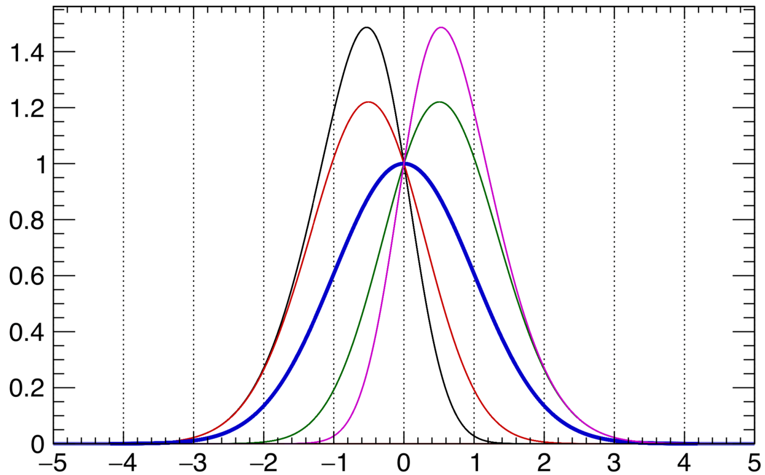
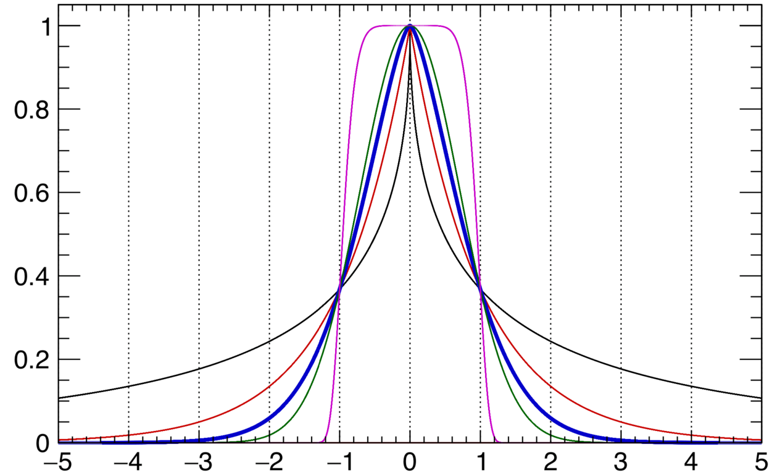
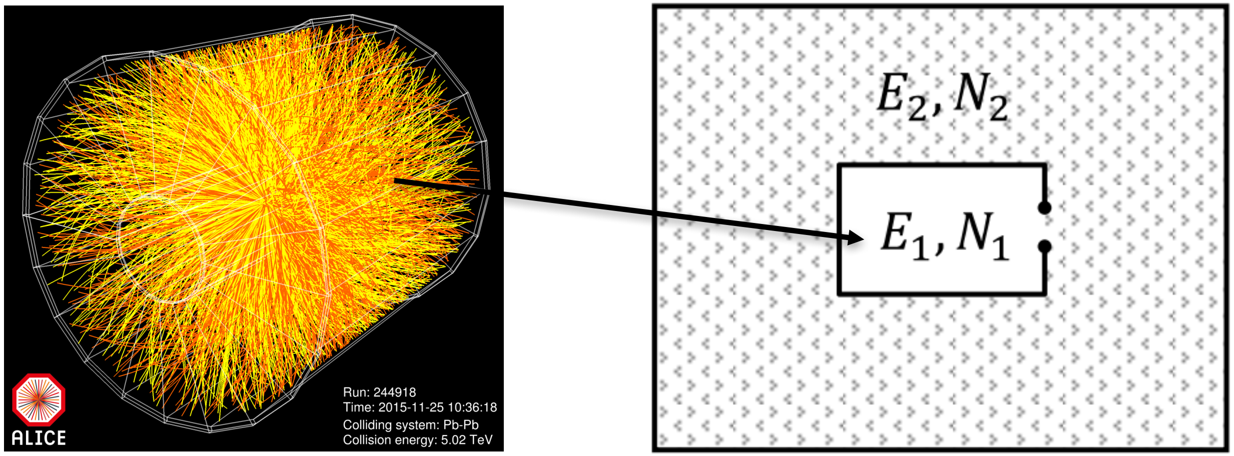
Moments of the multiplicity distributions





What kind of a system we are talking about?



<u>Grand canonical ensemble</u> where particles are in a thermal equilibrium

- Energy (*E*) and number of particles (*N*) are **not conserved** in each microstate

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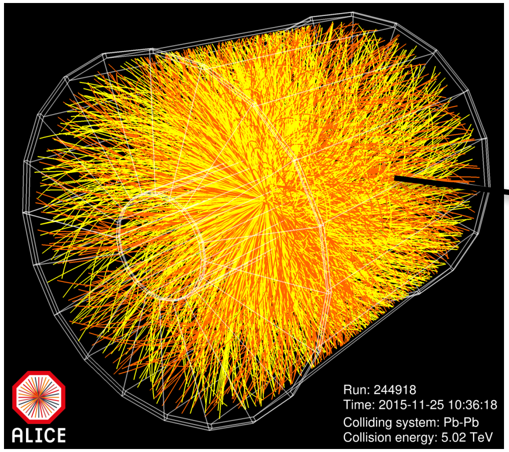

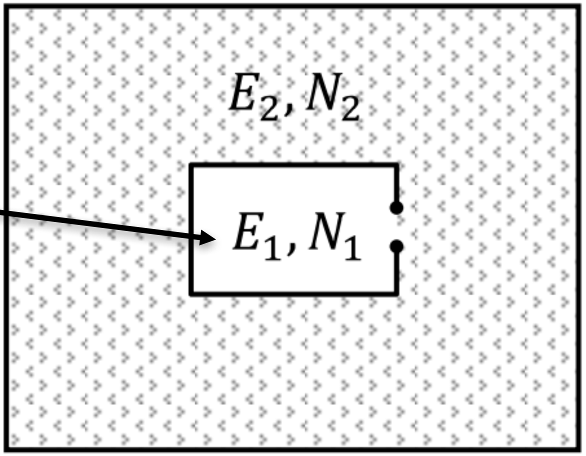

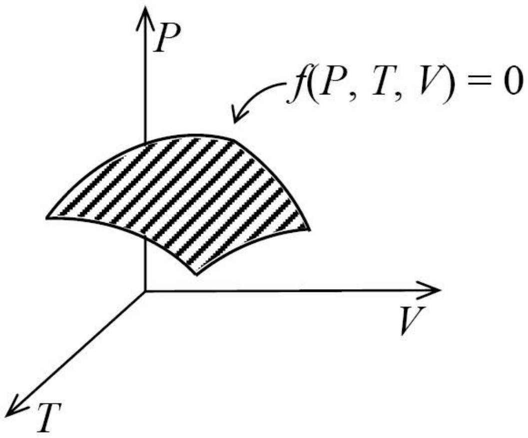

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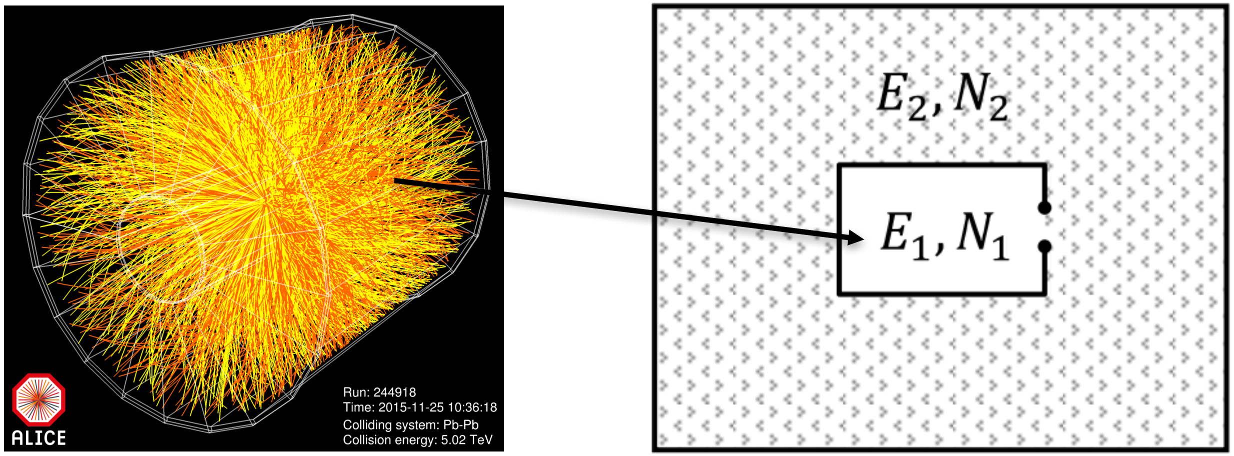
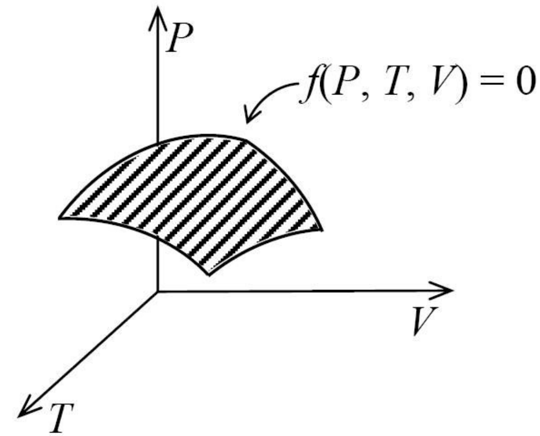




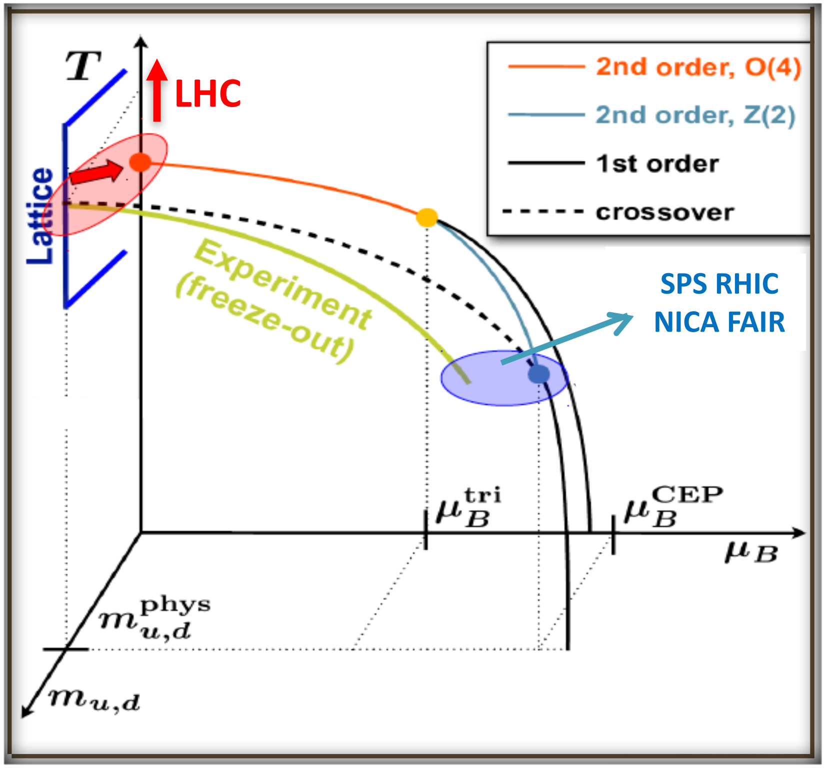
Grand canonical ensemble where particles are in a thermal equilibrium

- Energy (*E*) and number of particles (*N*) are **not conserved** in each microstate
- EOS can be represented **by a surface** in the state space spanned by P, V and T
- Conservation laws are applied **on average**
- Chemical potential **(μ)**, Volume **(V)** and Temperature (T) are constant
- For a given state E_j and N_j grand canonical partition function

$$Z_{GCE}(T,V,\mu) = \sum_j \exp\left[-\frac{E_j - \mu N_j}{T}\right] \Rightarrow \langle N \rangle = \sum_j N_j p_j = T \frac{\partial \ln Z_{GCE}}{\partial \mu}\bigg|_V$$

Motivation #2:
How to link experiment to theory?

Closer look at QCD Phase diagram: **Nature of chiral phase transition**



F. Karsch, Schleching 2016

small u, d quark masses

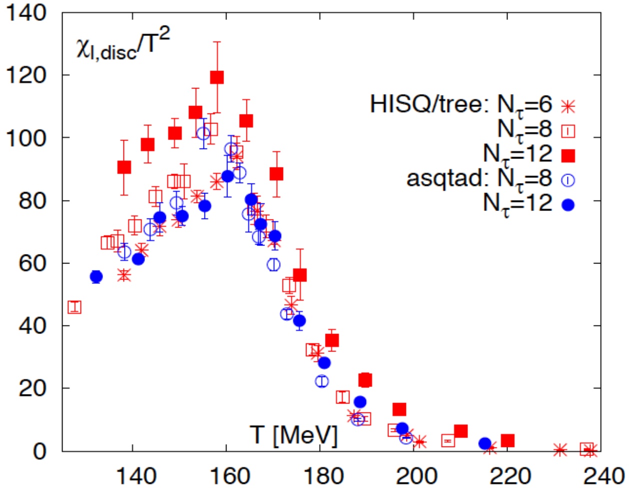
↔

vicinity to 2nd order O(4) criticality

pseudocritical features possible

Criticality & Link to Lattice QCD

A. Andronic, P. Braun-Munzinger, J. Stachel and K. Redlich
Nature 561, 321–330 (2018)



$$T_{pc} = 156.5 \pm 1.5 MeV$$

$$\langle\bar{\psi}\psi\rangle_l^{n_f=2} = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$

$$\chi_{m,l} = \frac{\partial}{\partial m_l}\langle\bar{\psi}\psi\rangle_l^{n_f=2}$$

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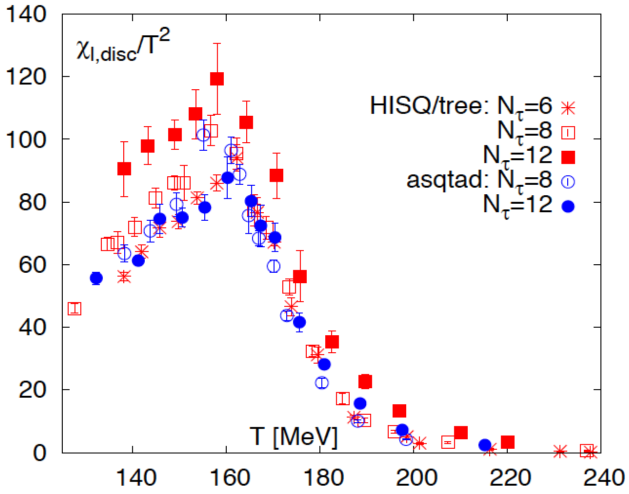

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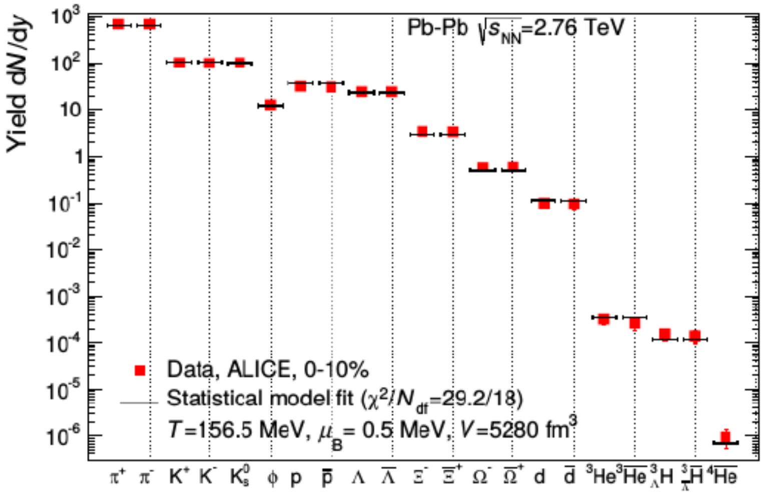
HotQCD Collaboration
Phys.Rev. D85 (2012) 054503, Phys.Lett. B795 (2019) 15



$$T_{fo}^{ALICE} = 156.5 \pm 3 MeV$$

$$\langle N_i \rangle = V \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i\right)/T\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_I I_i$$

$$\chi^2 = \sum_{k=1}^{n} \frac{\left(\langle N_k^{\exp}\rangle - \langle N_k^{HRG}\rangle\right)^2}{\sigma_k^2}$$

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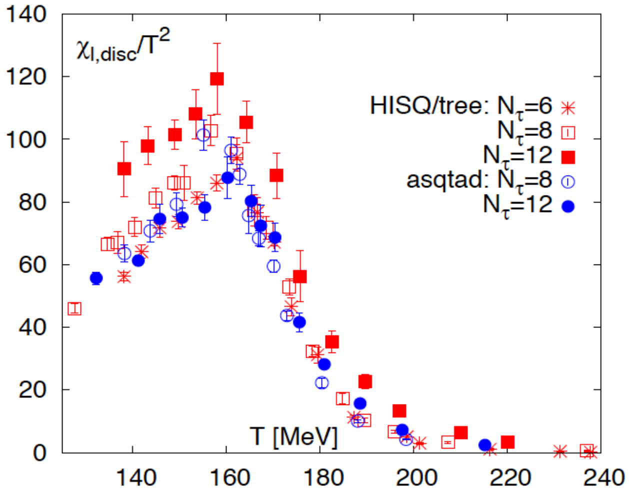

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Chemical freeze-out at the phase boundary!

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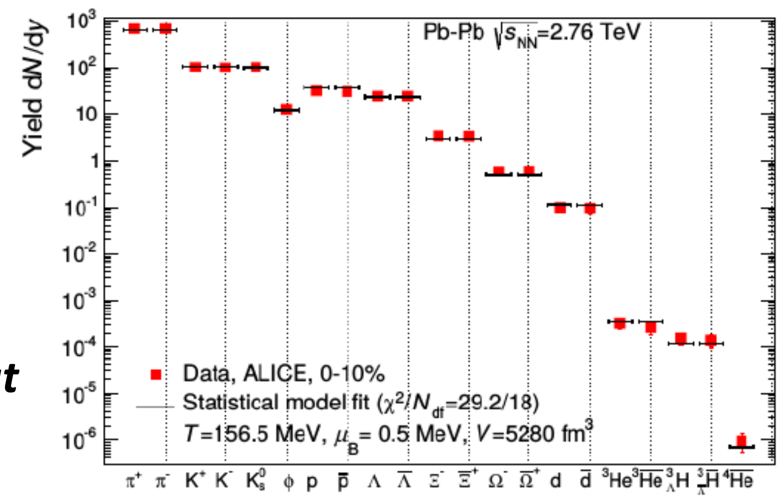

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Chemical freeze-out near T_{pc} → motivation to look for higher order moments

Criticality & Link to Lattice QCD

LQCD

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z\left(V, T, \mu_{B,Q,S}\right) \Rightarrow \hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial\left(\mu_N/T\right)^n}$$

Susceptibilities

Criticality & Link to Lattice QCD

LQCD | Experiment

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Susceptibilities

$$\Rightarrow \hat{\chi}_2^B = \frac{\kappa_2\left(\Delta N_B\right)}{VT^3}$$

Cumulants

$$\Rightarrow \frac{\kappa_4\left(\Delta N_B\right)}{\kappa_2\left(\Delta N_B\right)} = \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

Higher orders

P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

Criticality & Link to Lattice QCD

LQCD | **Experiment**

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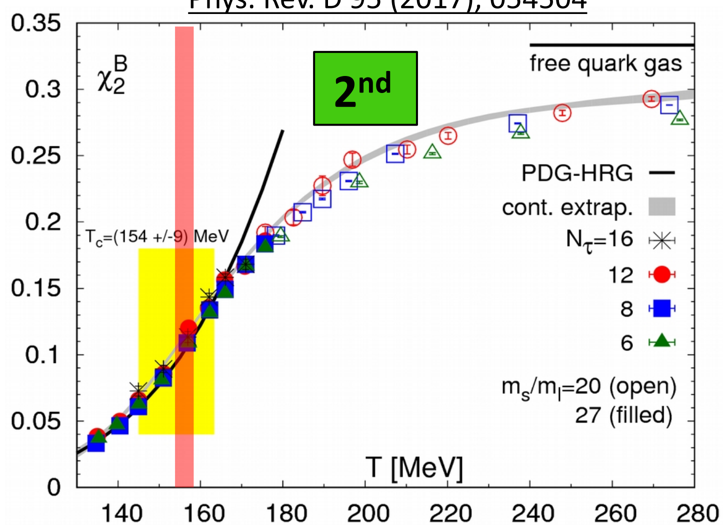

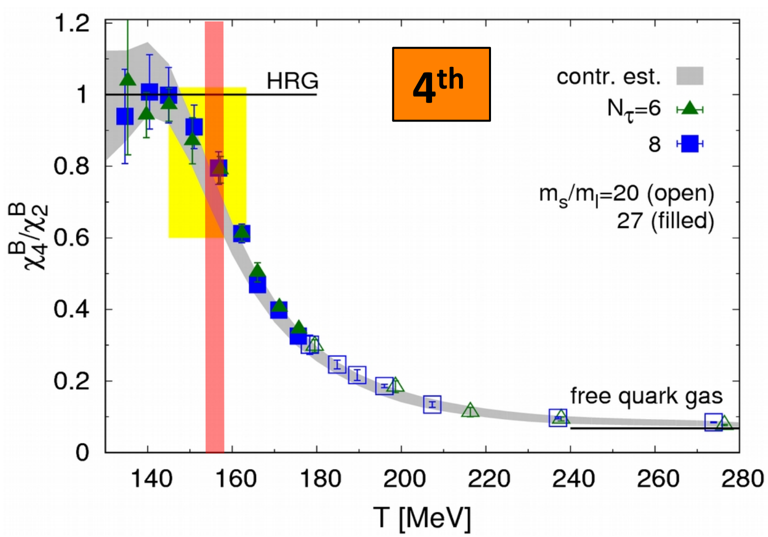

At 4th order LQCD shows a **deviation (~30% from unity)** from Hadron Resonance Gas (HRG)

What is the baseline?

Skellam distribution

$$X = N_B - N_{\overline{B}}$$

- **r^{th} central moment:**

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

- **First four cumulants**

$$\kappa_1 = \langle X \rangle, \quad \kappa_2 = \mu_2,$$

$$\kappa_3 = \mu_3, \quad \kappa_4 = \mu_4 - 3\mu_2^2$$

- **Uncorrelated Poisson limit:**

$$\langle N_B N_{\overline{B}} \rangle = \langle N_B \rangle \langle N_{\overline{B}} \rangle$$

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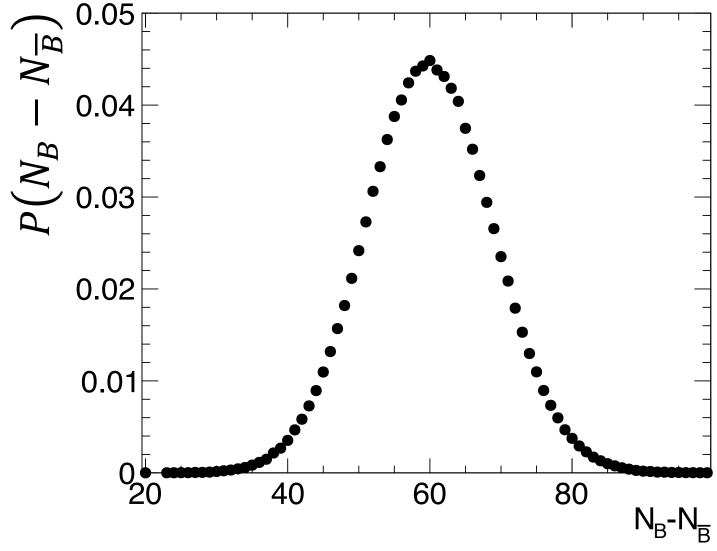


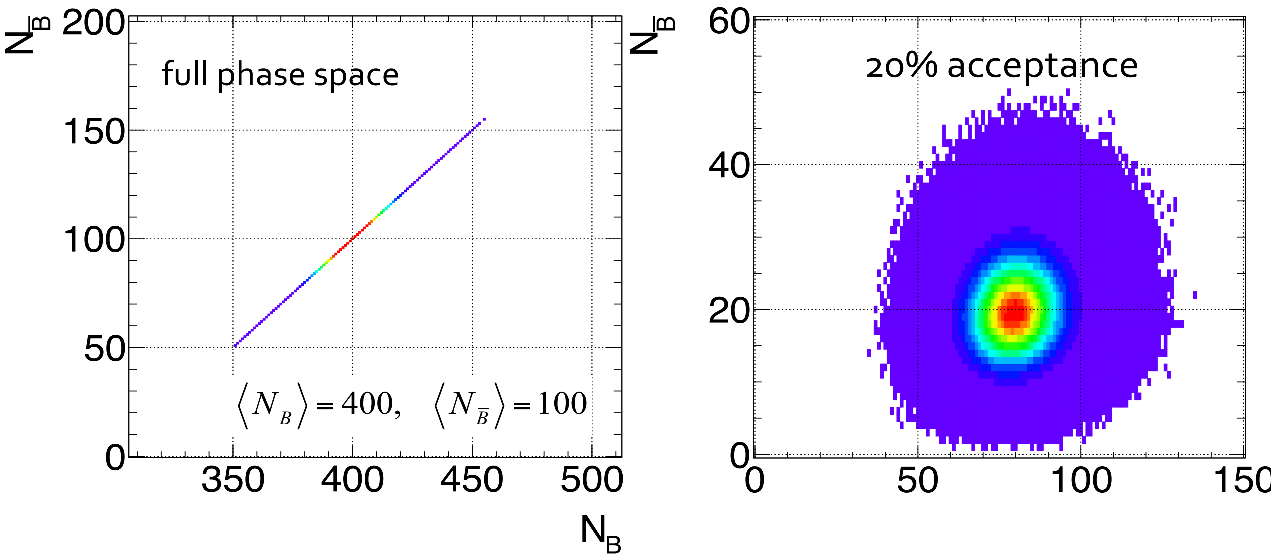
Difference between two independent Poissonian distributions

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\overline{B}} \rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \frac{\langle n_B \rangle - \langle n_{\bar{B}} \rangle}{\langle n_B \rangle + \langle n_{\bar{B}} \rangle}$$

Importance of acceptance

- Fluctuations of net-baryons appear only inside **finite acceptance**



P. Braun-Munzinger, A. Rustamov, J. Stachel, QM18, NPA 982 (2019) 307-310

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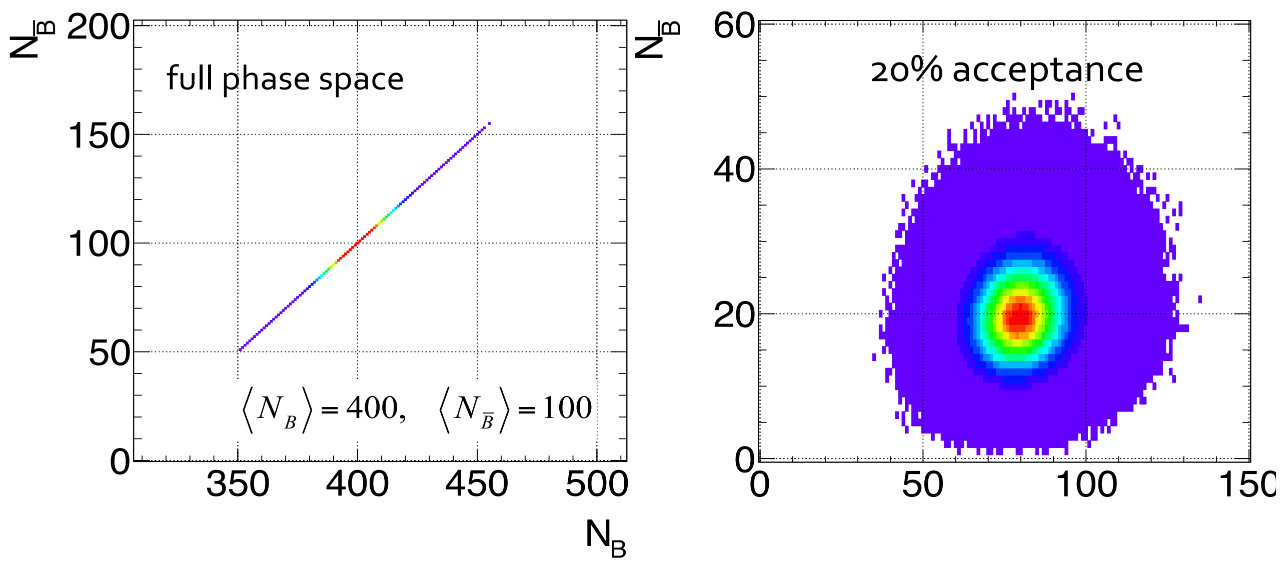

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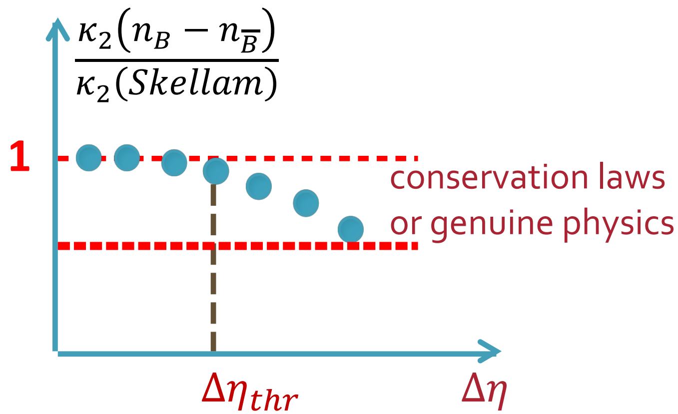


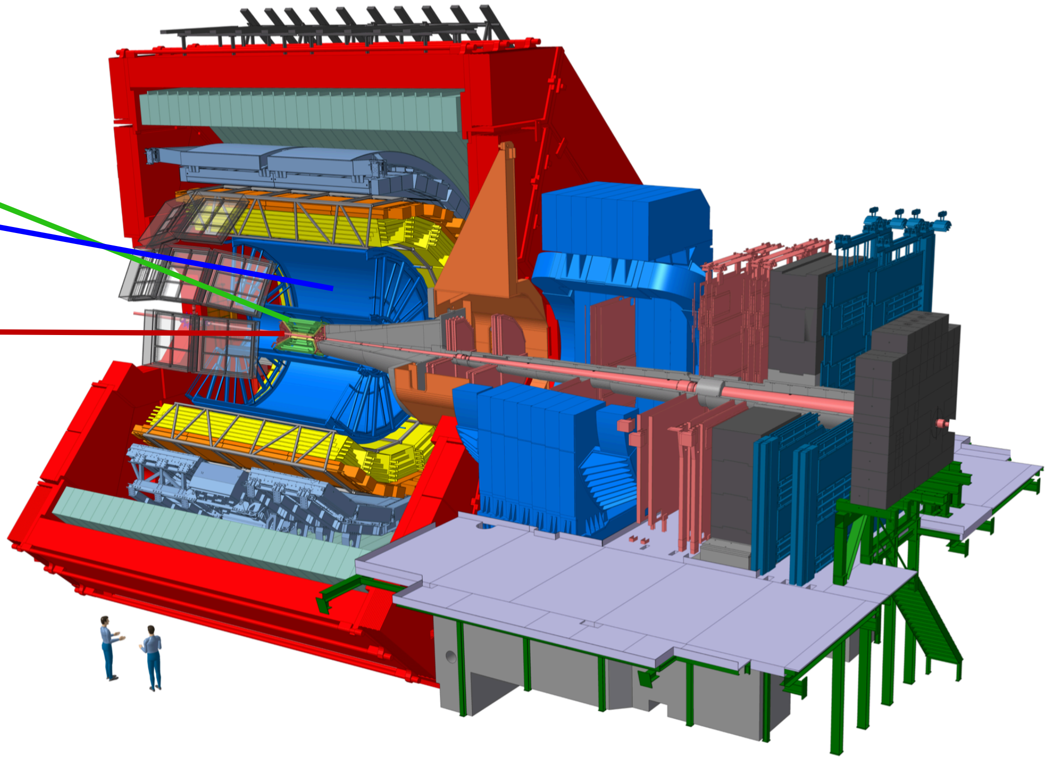
➢ **In the limit of very small acceptance**
→ only Poissonian fluctuations

From data to physics

A Large Ion Collider Experiment

Main detectors used:

- **I**nner **T**racking **S**ystem **(ITS)**
  → Tracking and vertexing
- **T**ime **P**rojection **C**hamber **(TPC)**
  → Tracking and
  Particle Identification (PID)
- **V**ertex **0** **(V0)**
  → Centrality determination



Data Set:

- $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV, ~78 M events
- $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV, ~12 M events

Kinematic acceptance:

- $0.6 < p < [1.5, 2]$ GeV/c
- $|\eta|$<0.2, 0.4, … , 0.8

The Method

“ **The 1st** was never to accept anything for true which I did not clearly know to be such; that is to say, carefully to avoid precipitancy and prejudice, and to comprise nothing more in my judgment than what was presented to my mind so clearly and distinctly as to exclude all ground of doubt.

The 2nd, to divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution.

The 3rd, to conduct my thoughts in such order that, by commencing with objects the simplest and easiest to know, I might ascend by little and little, and, as it were, step by step, to the knowledge of the more complex; assigning in thought a certain order even to those objects which in their own nature do not stand in a relation of antecedence and sequence.

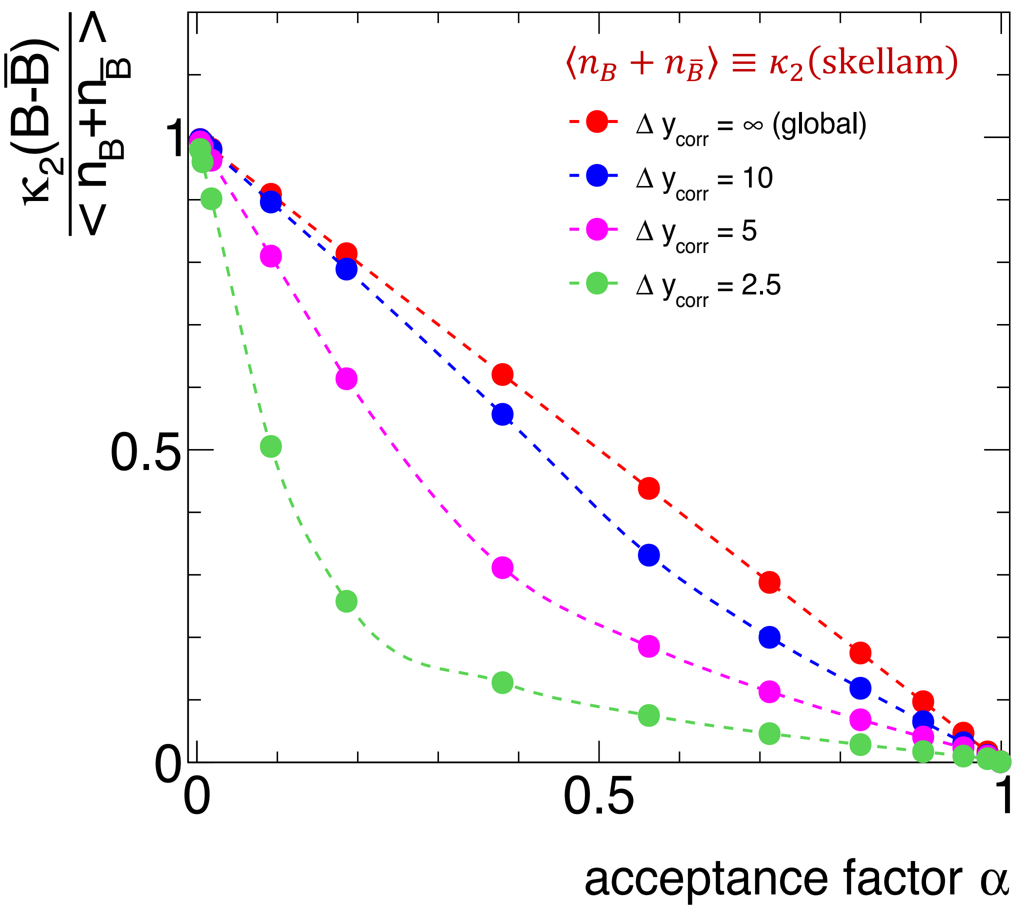
The last, in every case to make enumerations so complete, and reviews so general, that I might be assured that nothing was omitted.”

Experimental Challenges

Baryon number conservation

➢ **Baryon number conservation** imposes subtle correlations

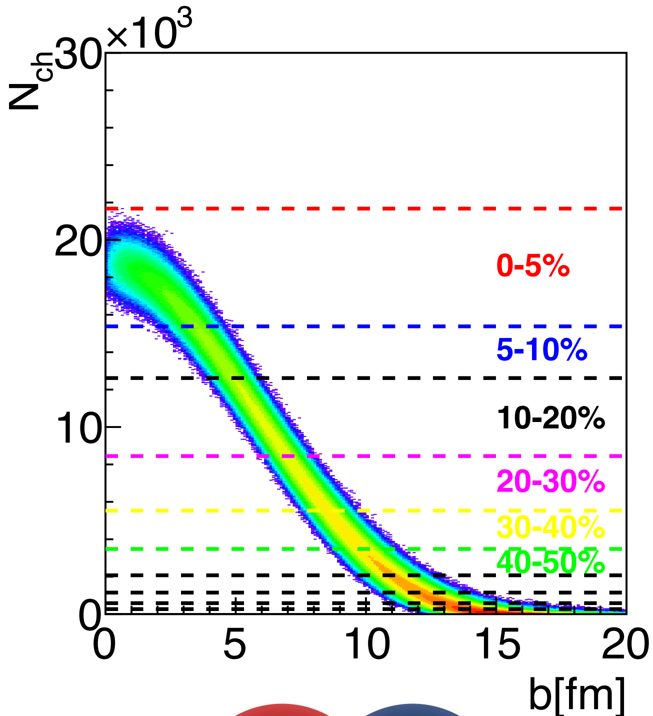
P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032

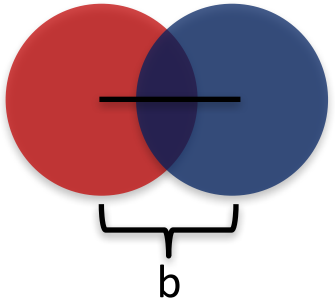


$$\alpha = \frac{\langle N_B^{acc} \rangle}{\langle N_B^{4\pi} \rangle}$$

$$|y_{\bar{B}} - y_B| < \frac{\Delta y_{corr}}{2}$$

Volume Fluctuates





P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A 960 (2017) 114-130

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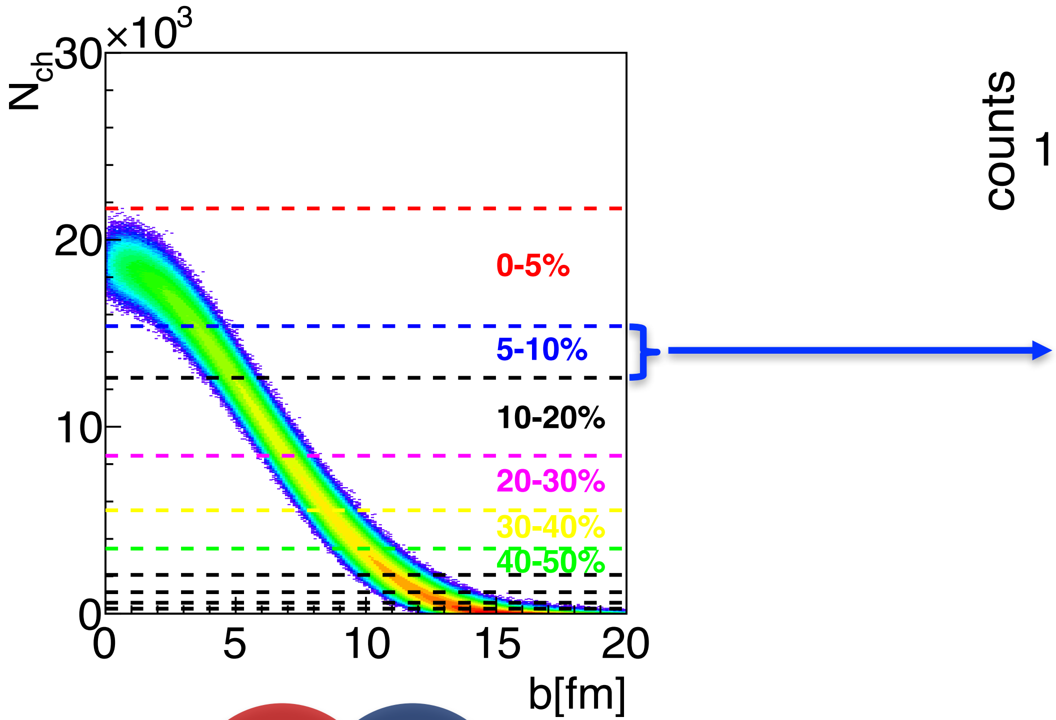
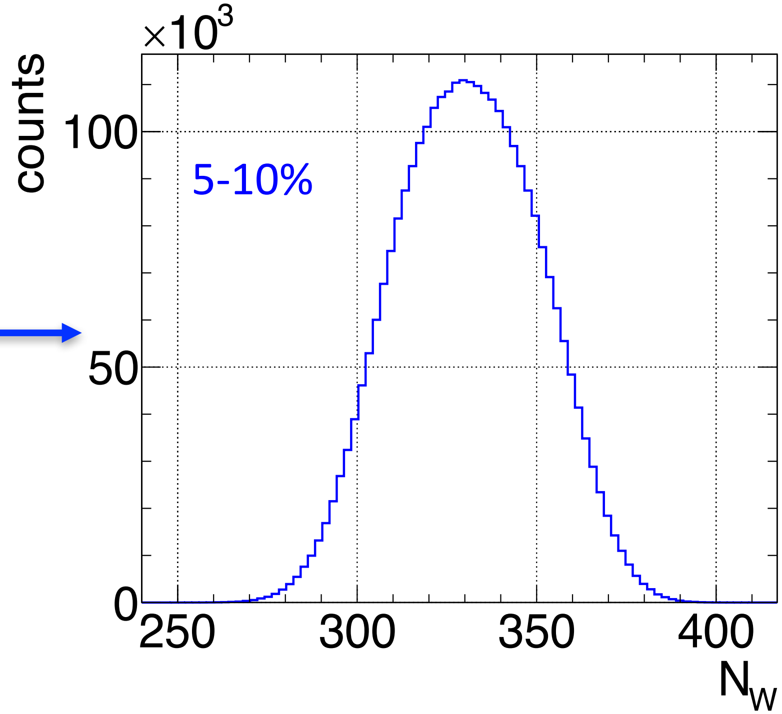
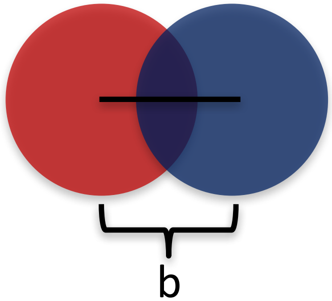


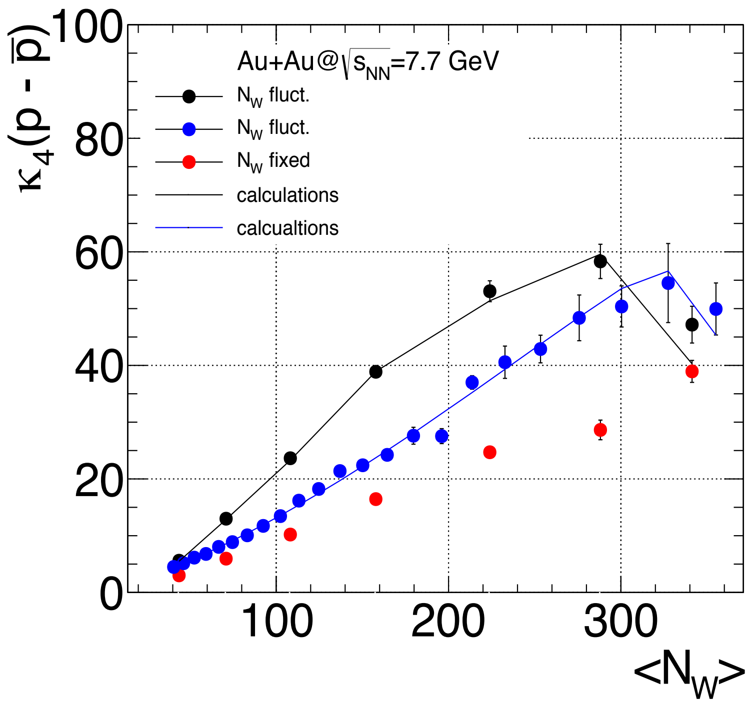


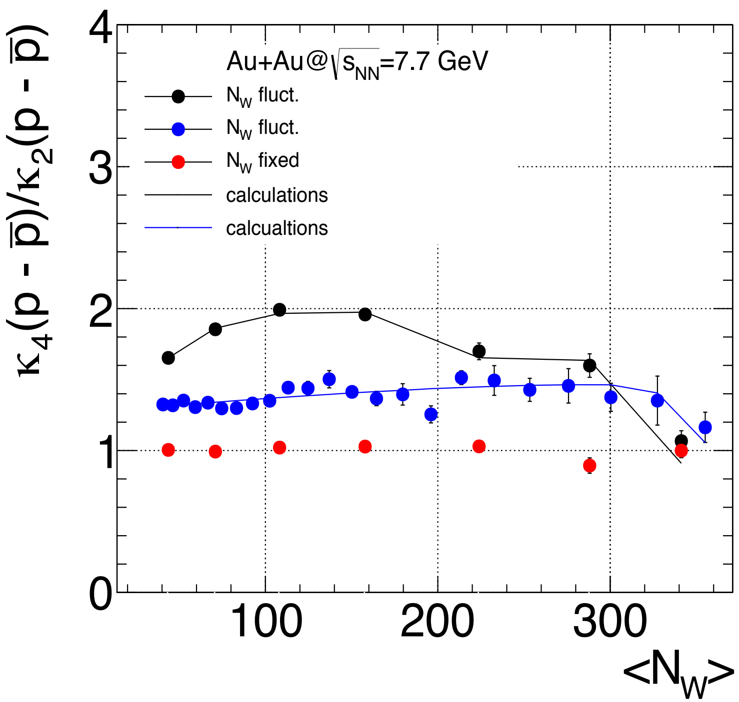


$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} \stackrel{?}{=} \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B}$$

P. Braun-Munzinger, A. Rustamov, J. Stachel, Nucl. Phys. A 960 (2017) 114-130

Volume Fluctuations at RHIC energies

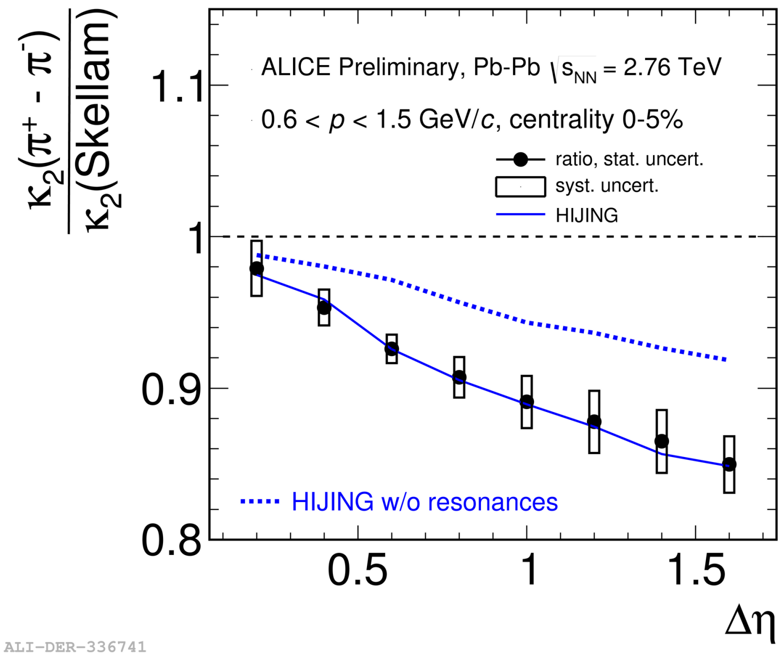




- Participant fluctuations will be present even in the limit of very fine centrality bins
- Incoherent addition of data from intervals with very small centrality bin width will eliminate true dynamical fluctuations.

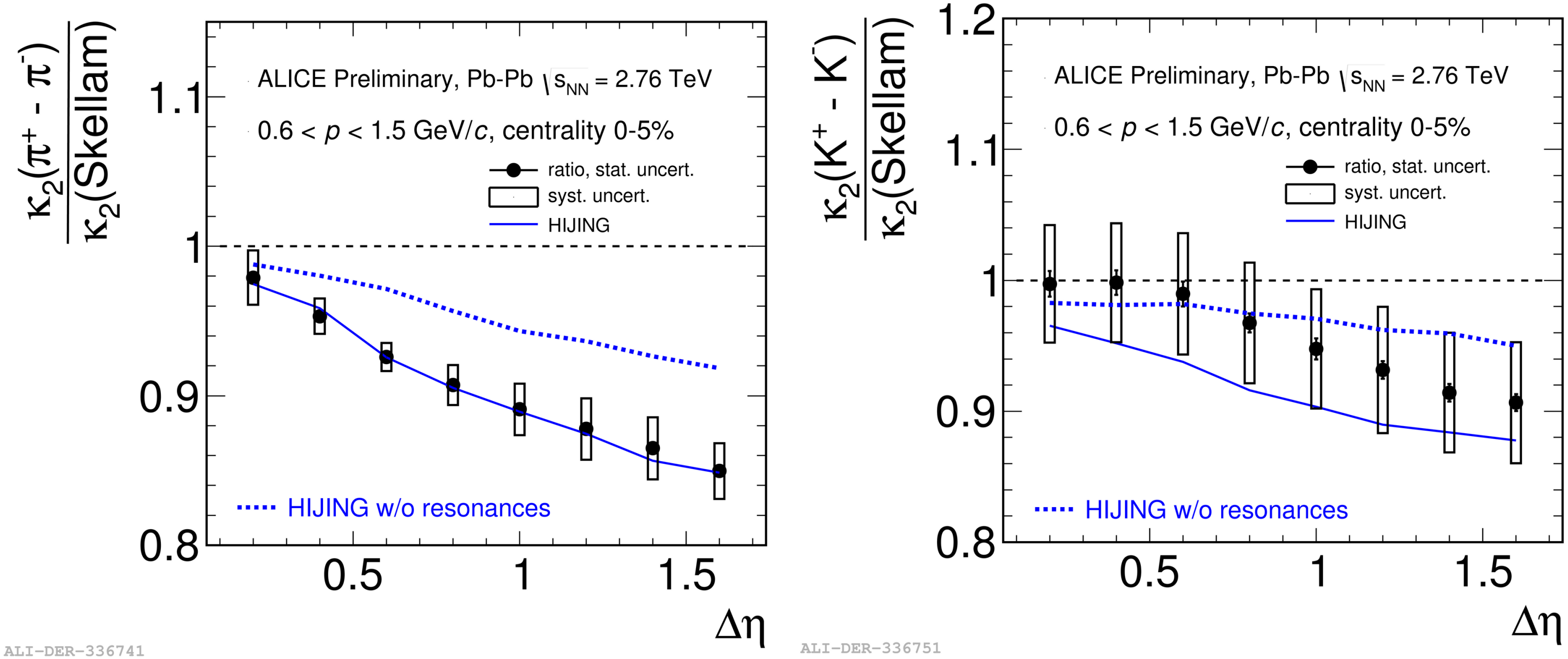
P. Braun-Munzinger, A. Rustamov, J. Stachel, Nuclear Physics A 960 (2017) 114–130

Effect of resonances



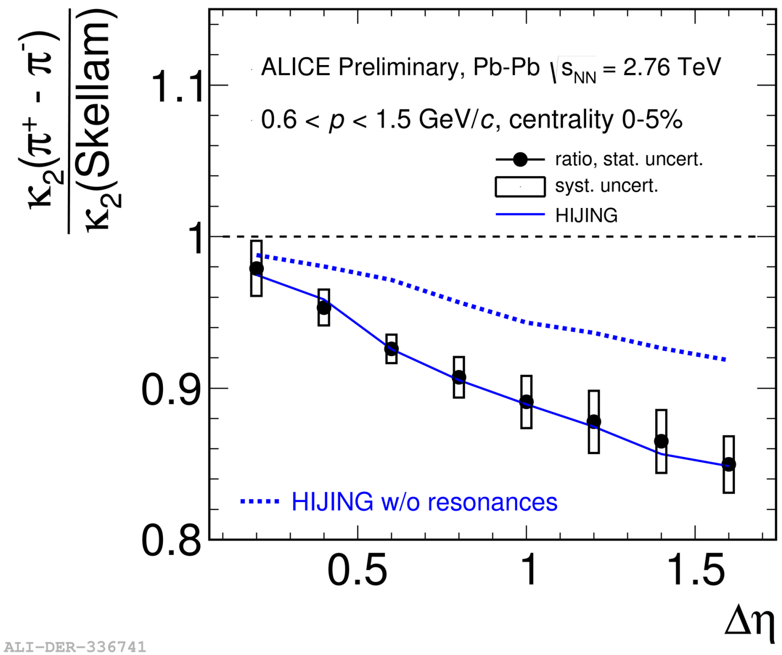
➢ **Net-electric-charge:** → Strongly dominated by **resonance contributions**

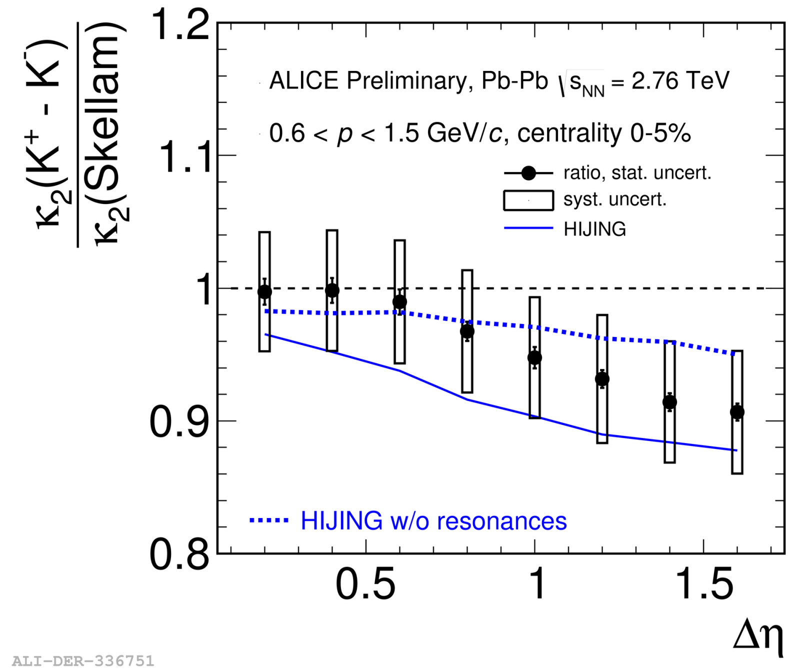
Effect of resonances



- **Net-electric-charge:** → Strongly dominated by **resonance contributions**
- **Net-strangeness:** → Kaons are dominated by **ϕ-decay**

Effect of resonances

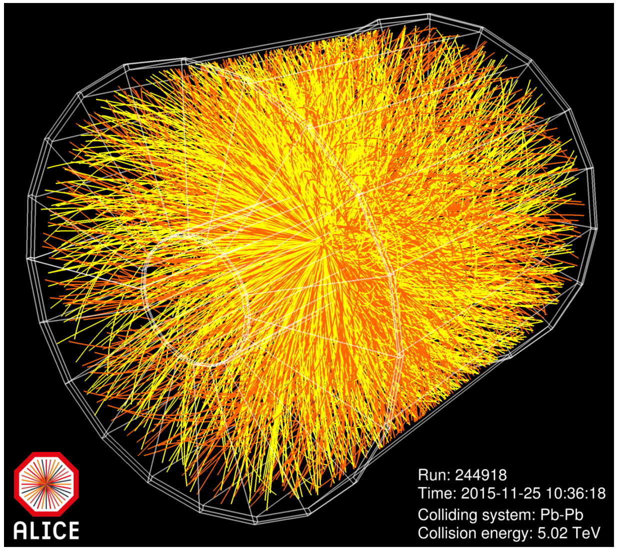


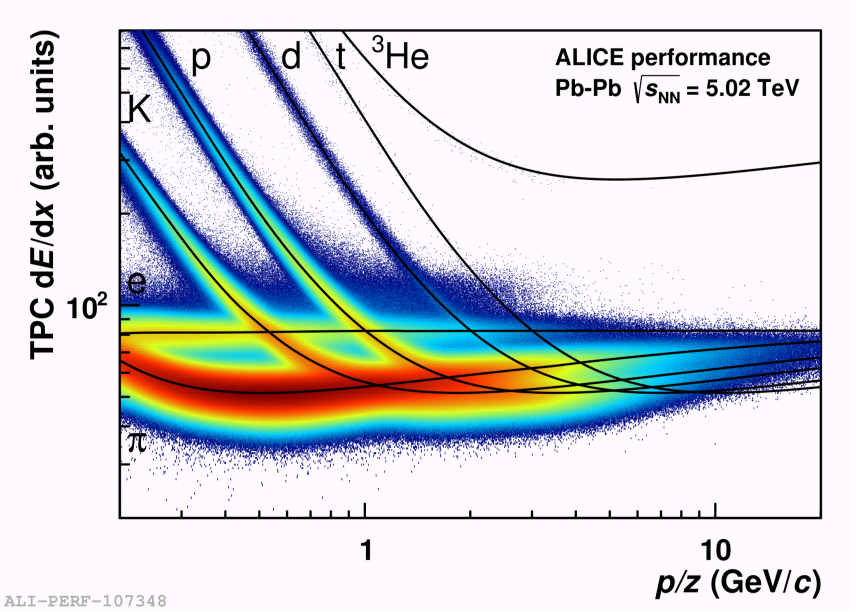


- **Net-electric-charge:** → Strongly dominated by **resonance contributions**
- **Net-strangeness:** → Kaons are dominated by **ϕ-decay**
- **Net-baryon:**
  - → Due to **isospin randomization,** at $\sqrt{s_{\mathrm{NN}}} > 10$ GeV **net-baryon** fluctuations can be obtained from corresponding **net-proton** measurements (M. Kitazawa, and M. Asakawa, Phys. Rev. C 86, 024904 (2012))
  - → No resonance feeding $\mathrm{p} + \bar{\mathrm{p}}$
  - → **Best candidate for measuring charge susceptibilities**

Particle Identification?

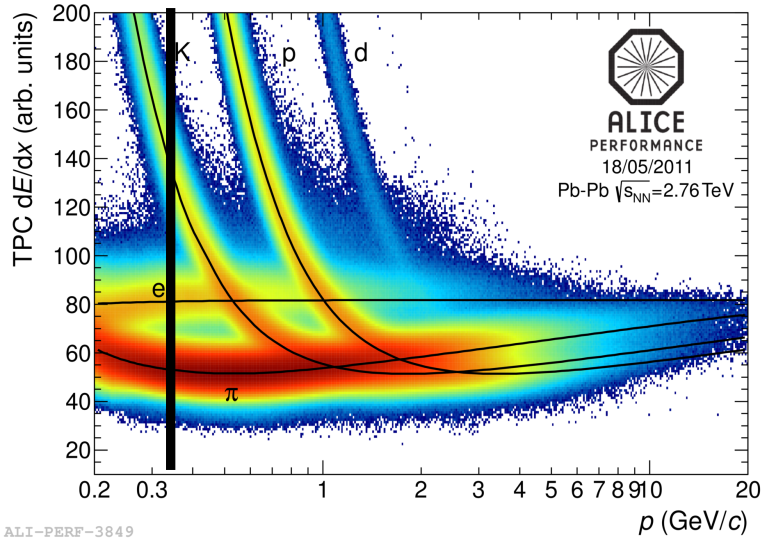
via specific energy loss as function of momentum in the TPC

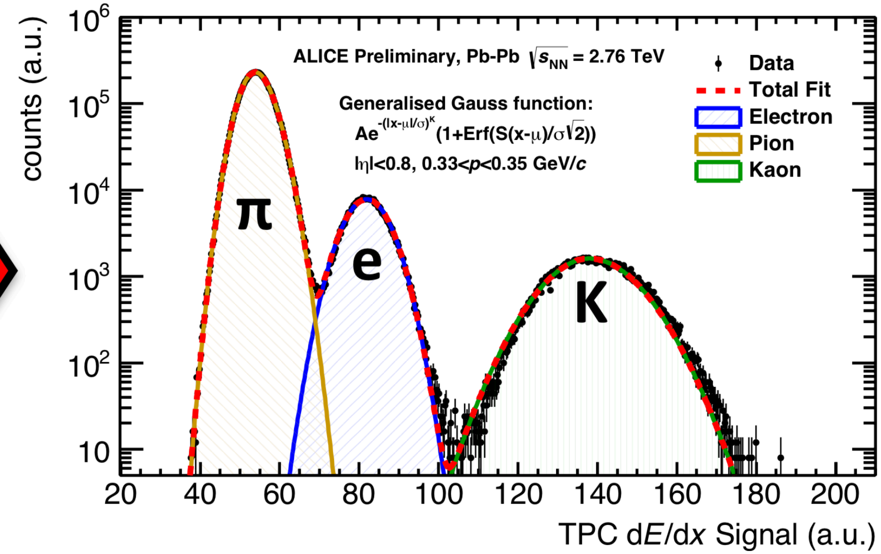




Cut based vs Identity method

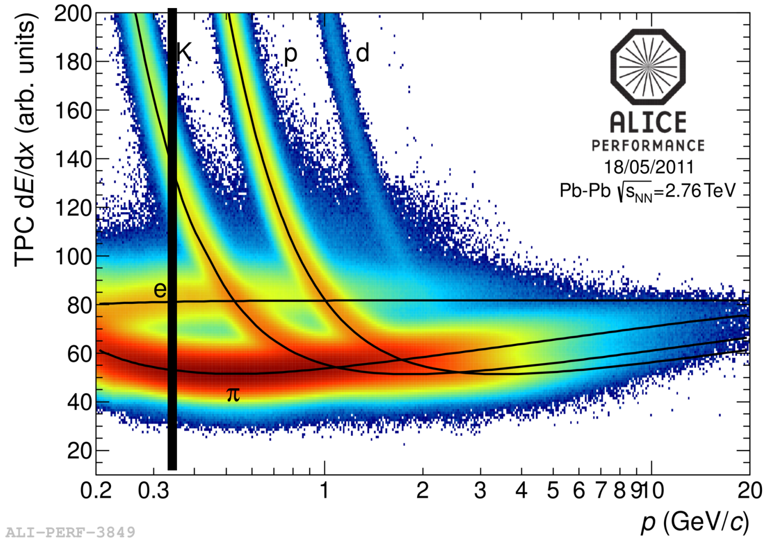
Cut-based approach: count tracks of a given particle type

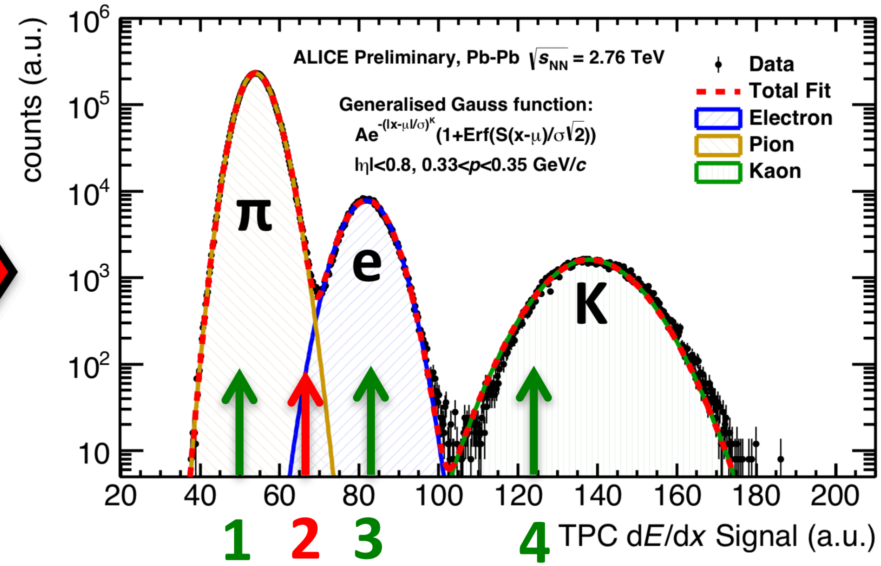




Cut based vs Identity method

Cut-based approach: count tracks of a given particle type
Identity method: count probabilities to be of a given particle type

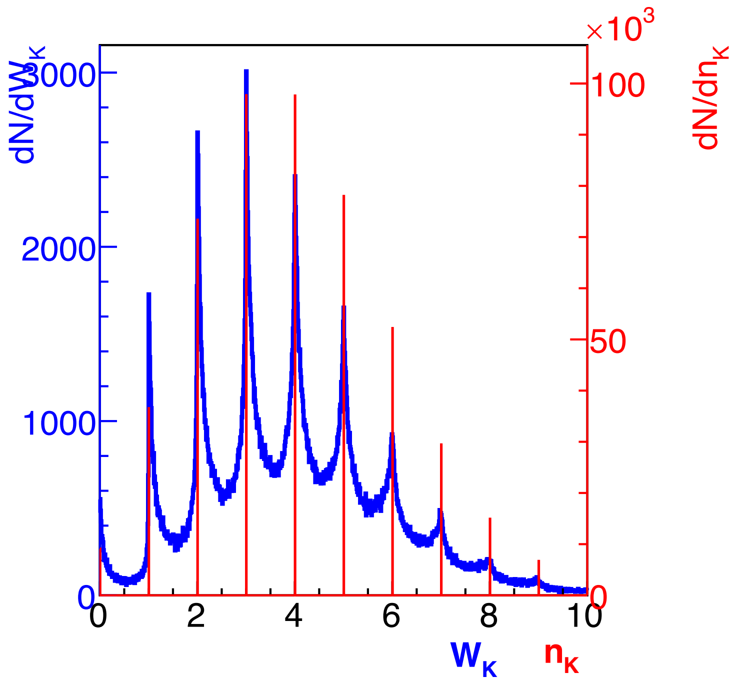




$\omega_\pi^{(1)} = 1$, $\omega_\pi^{(2)} \cong 0.6$, $\omega_\pi^{(3)} = 0$, $\omega_\pi^{(4)} = 0$ $\Rightarrow$ $W_\pi = 1.6 \neq N_\pi$ $\Rightarrow$ $\langle N_j^n \rangle = A^{-1} \langle W_j^n \rangle$

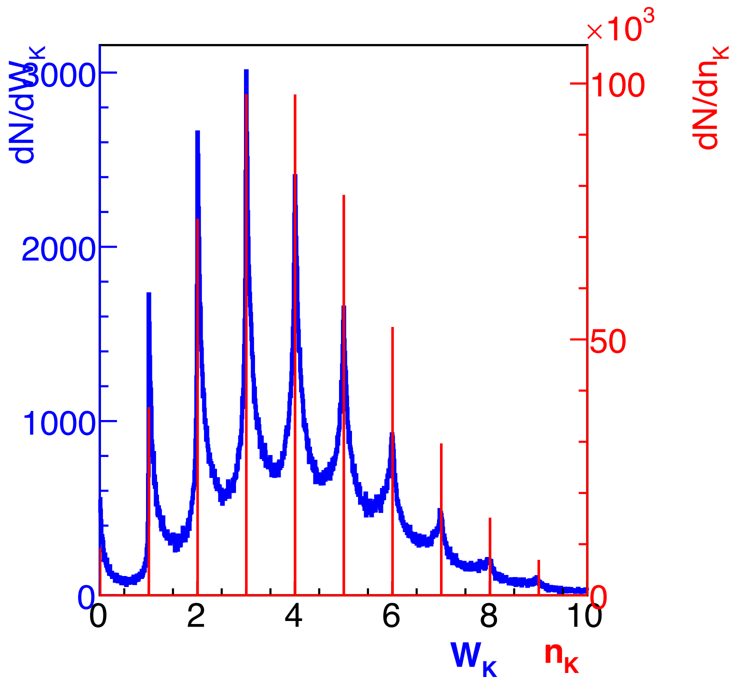
A. Rustamov, M. Gazdzicki, M. I. Gorenstein, PRC 86, 044906 (2012), PRC 84, 024902 (2011)
M. Arslandok, A. Rustamov, NIM A, 946, (2019), 162622

Cut based vs Identity method



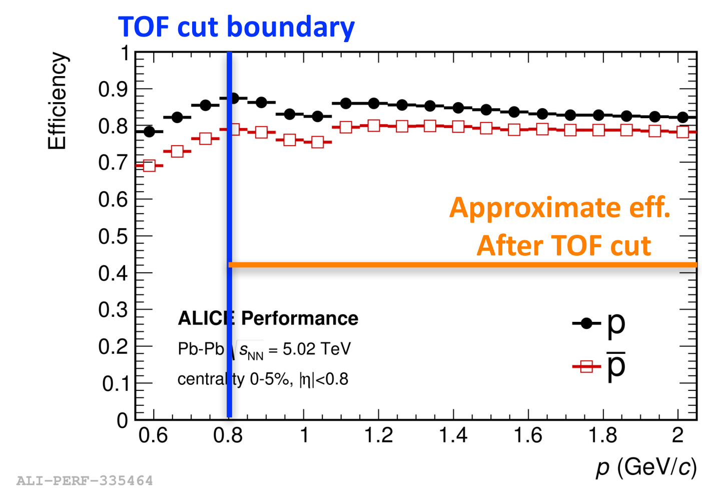
$$\langle N_j^n \rangle = A^{-1} \langle W_j^n \rangle$$

Cut based vs Identity method



$$\langle N_j^n \rangle = A^{-1} \langle W_j^n \rangle$$

- **Cut based approach**
  - Use additional detector information or reject a given phase space bin
  - Challenge: efficiency correction and contamination
- **Identity Method**
  - Gives folded multiplicity distribution
  - Easier to correct inefficiencies
  - Ideal approach for low momentum (p<2 GeV/c)

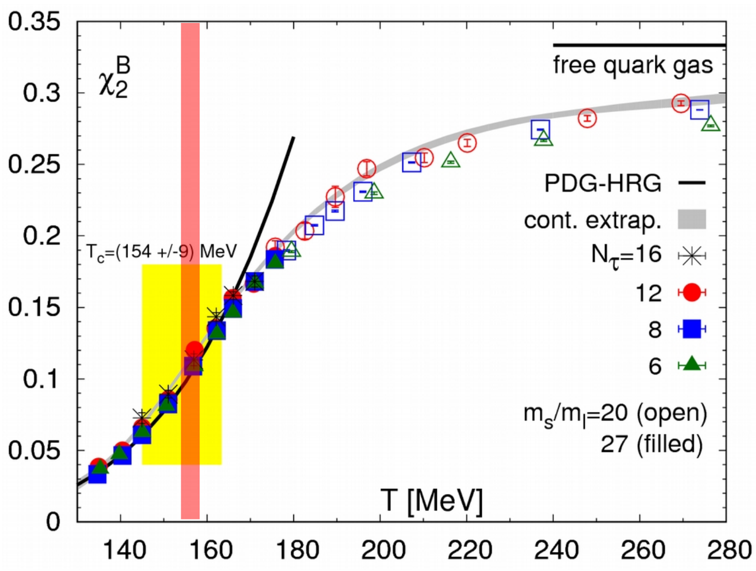


Recent results

1st and 2nd order cumulants

LQCD expectations:

- ✓ 1st moments → $T_{pc} = T_{freeze\text{-}out} = \sim 156$ MeV
- ✓ 2nd moments → No deviation from HRG at T_{pc}



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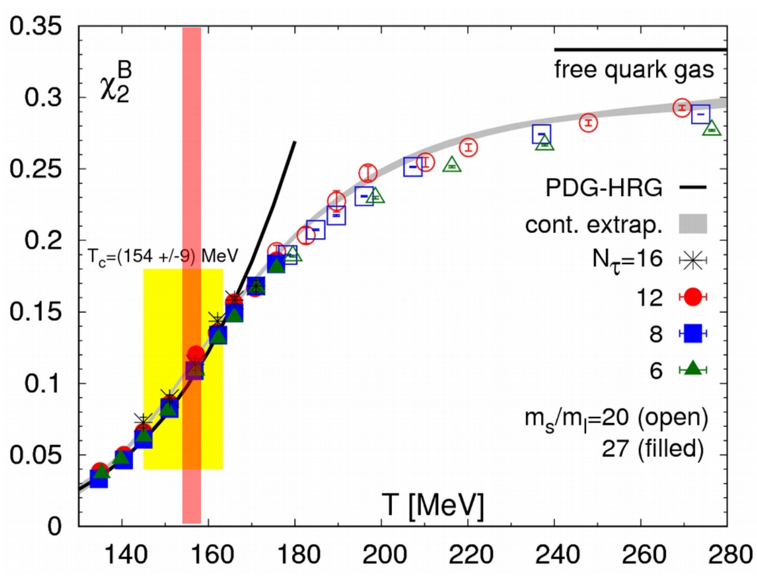
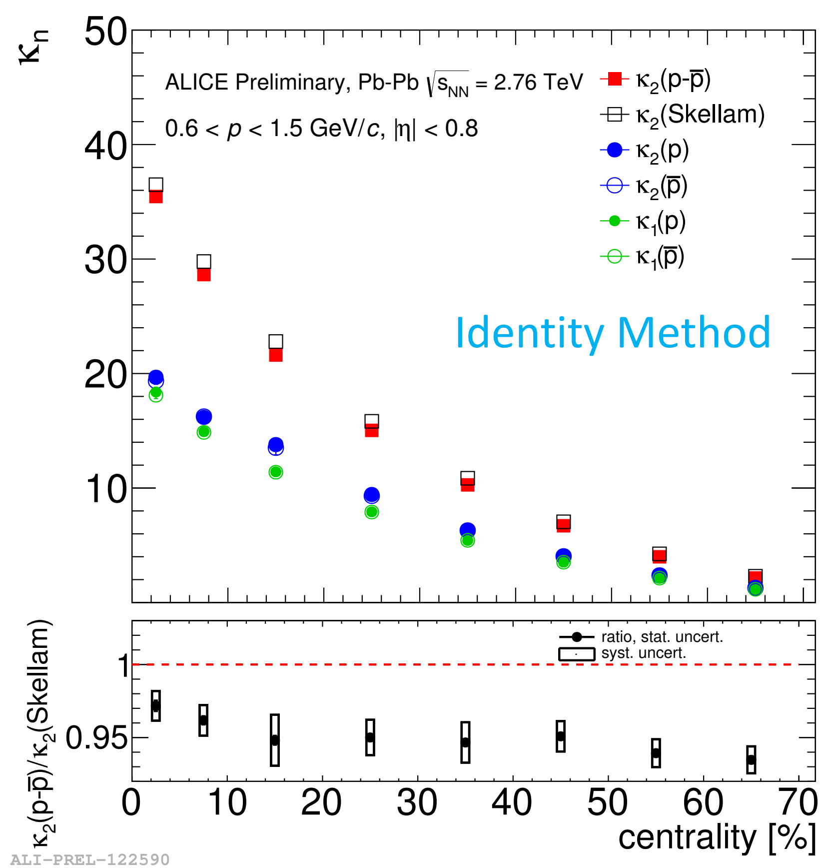
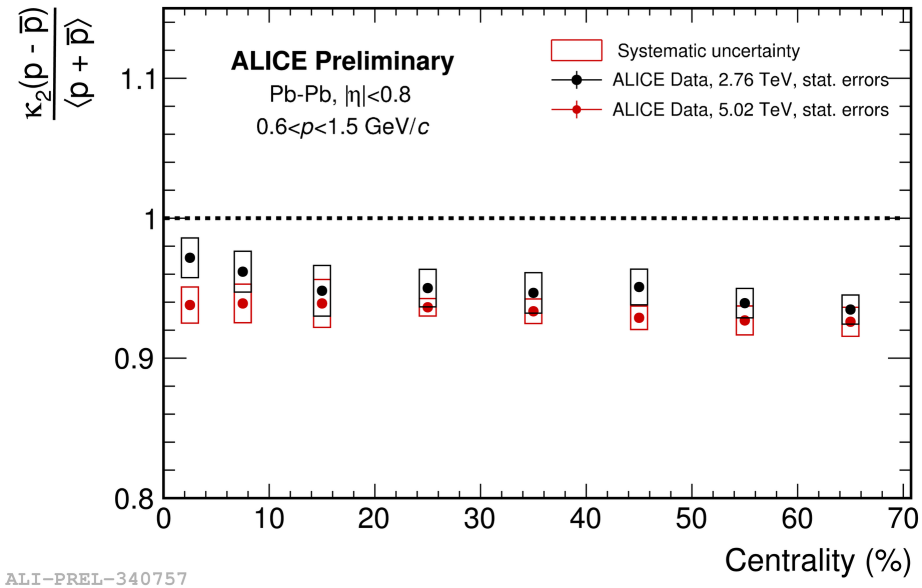


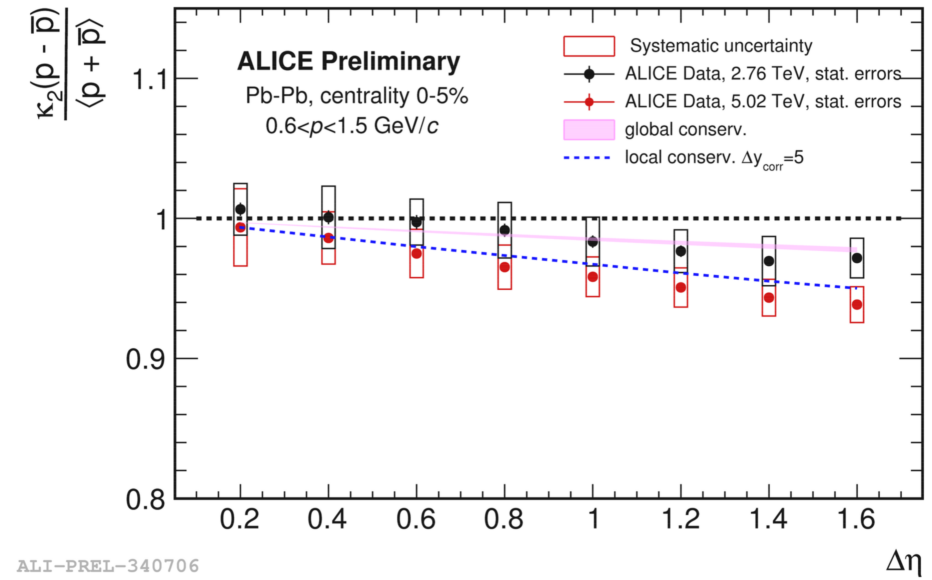
$$\kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p})$$

$$\kappa_2(p-\bar{p}) = \kappa_2(p) + \kappa_2(\bar{p}) - 2\left(\langle p\bar{p}\rangle - \langle p\rangle\langle\bar{p}\rangle\right)$$



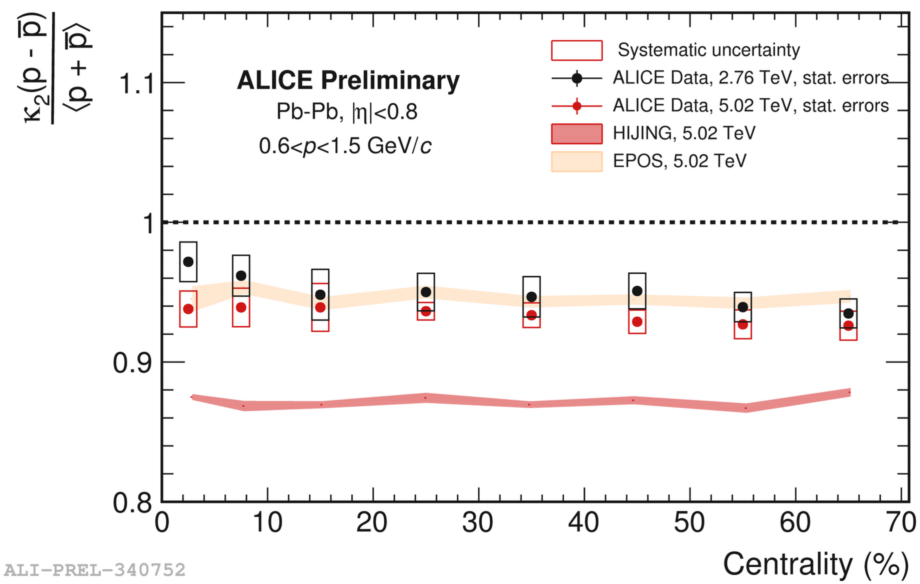
2nd order cumulants

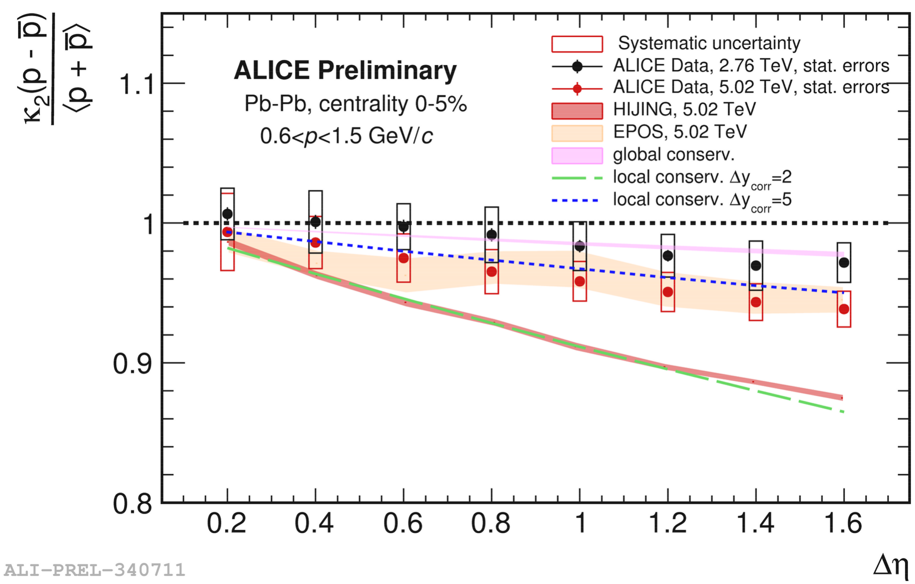




- Deviation from Skellam baseline is due to **baryon number conservation**
- ALICE data suggest **long range correlations,** $\Delta y = \pm 2.5$ unit **or longer**

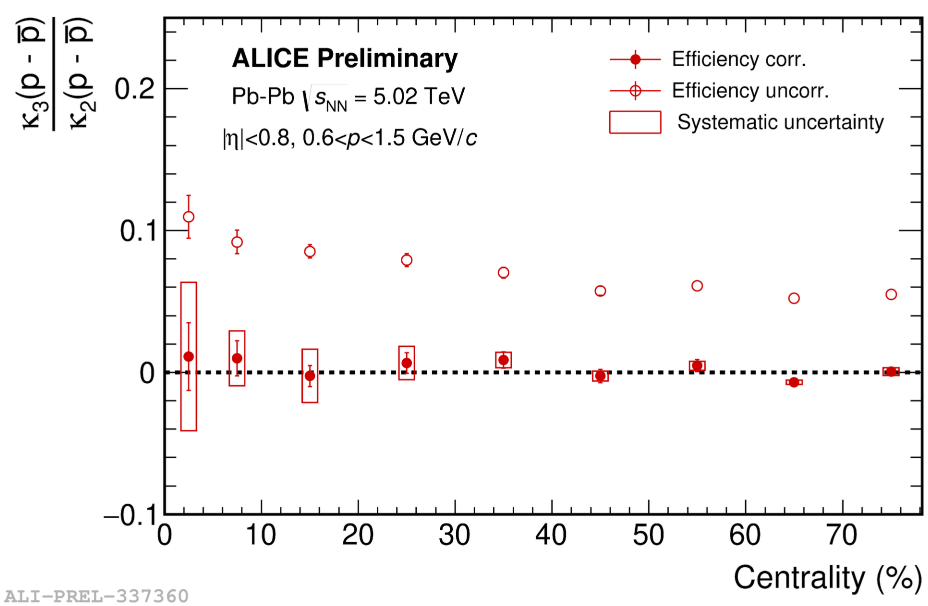
2nd order cumulants

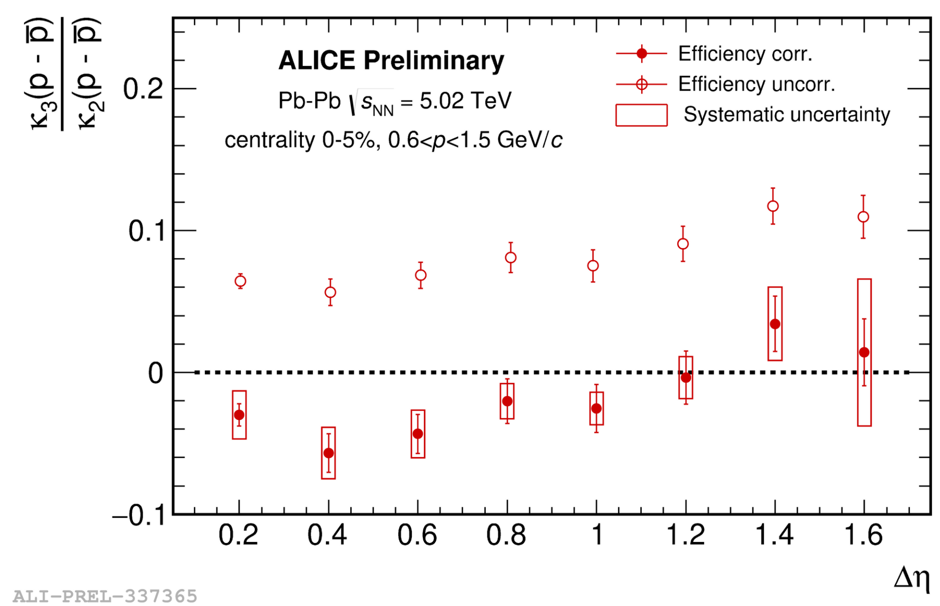




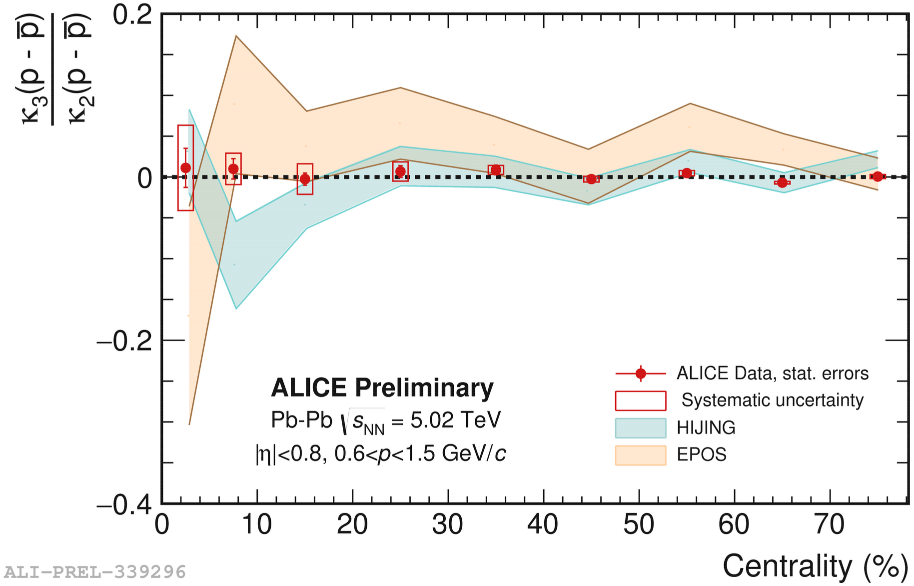
- Deviation from Skellam baseline is due to **baryon number conservation**
- ALICE data suggest **long range correlations,** $\Delta y = \pm 2.5$ unit **or longer**
- EPOS agrees with ALICE data but HIJING deviates significantly
    - Event generators based on <u>string fragmentation</u> (HIJING) conserve baryon number over $\Delta y = \pm 1$ unit

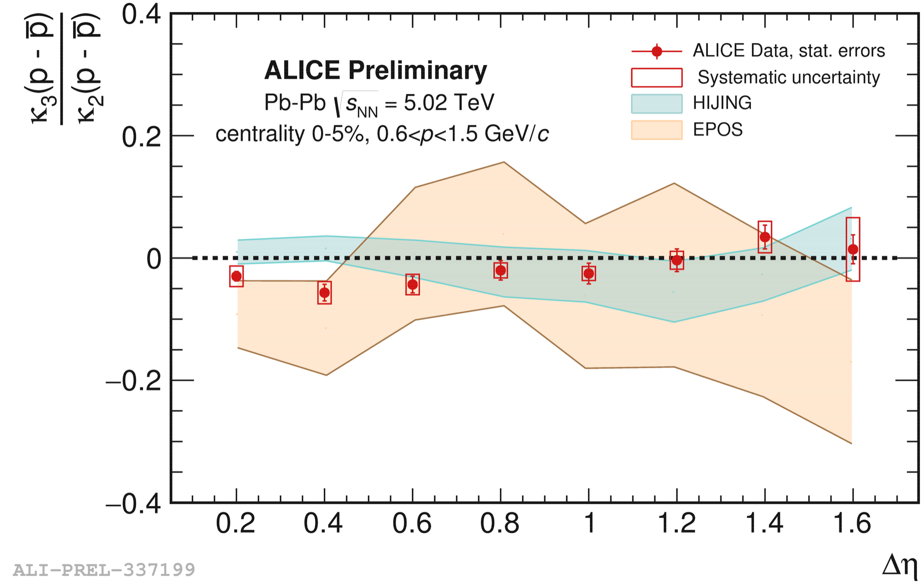
3rd order cumulants





- **Data** agree with Skellam baseline **“0”** as a function of centrality and pseudorapidity
- Achieved precision of **better than 5%**



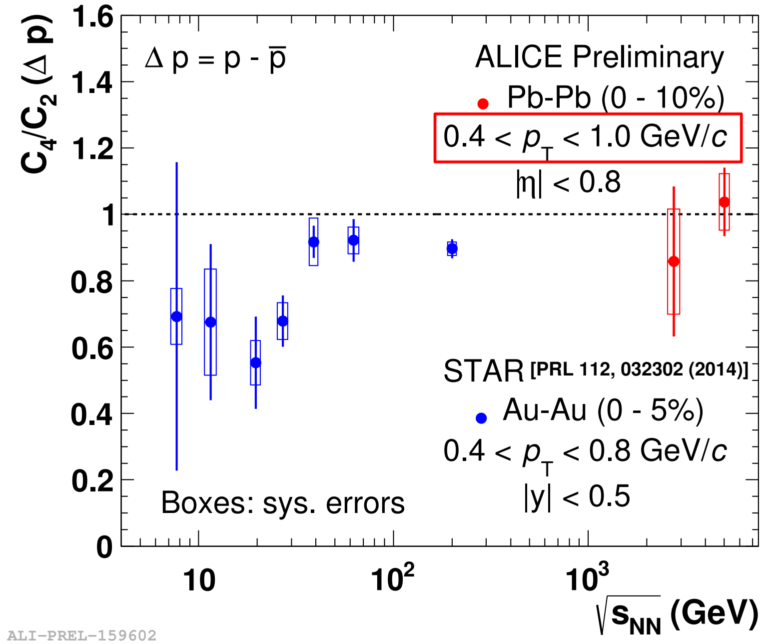


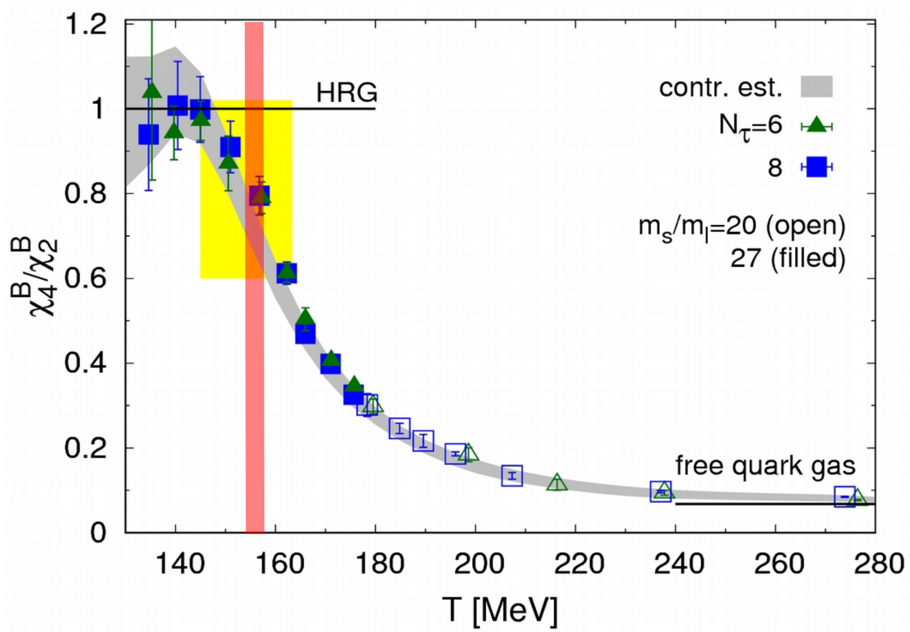
- **Data** agree with Skellam baseline **"0"** as a function of centrality and pseudorapidity
- Achieved precision of **better than 5%**
- **EPOS and HIJING in agreement with data**
  - Both models conserve global charge → net-p within acceptance is ~0

4^{th} order cumulants of net-p

C_3/C_2 and C_4/C_2 agree with Skellam at LHC energies?

- Small acceptance
- Low statistics
- Cut-based approach for PID

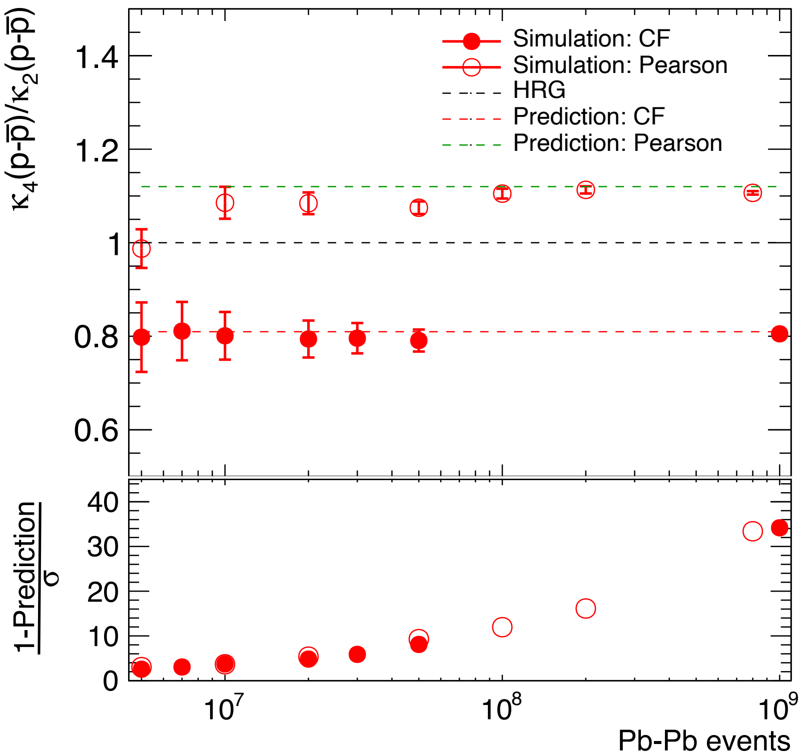


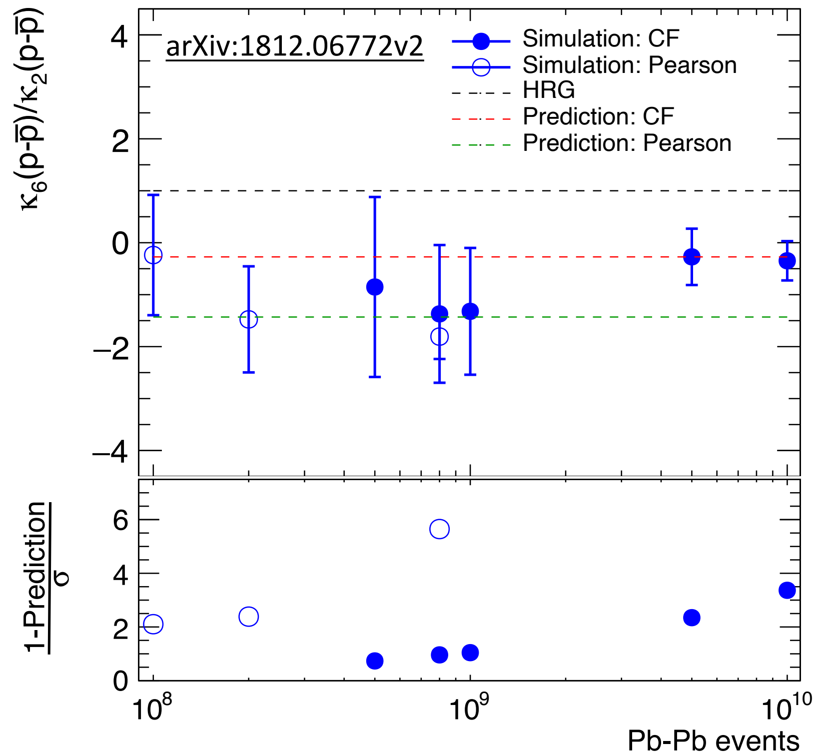


Analysis within a larger kinematic acceptance using Identity Method is in progress

After ALICE upgrade

- **New ITS:** better vertexing
- **Faster TPC:** MWPC → GEMs
- Record minimum-bias Pb-Pb data at 50kHz
    - Order of magnitude more events
- 6^{th} order and may be beyond





Summary

- **Net-electric-charge fluctuations:** Challenge are the dominant **resonance contributions**
- **Net-proton fluctuations:**
  - ✓ **1st order:** $T_{fo}^{ALICE} \sim T_{pc}^{LQCD}$
  - ✓ **2nd order:** Deviation from Skellam baseline is due to baryon number conservation
    - ALICE data suggests **long range correlations**
  - ✓ **3rd order:** Agrees with Skellam baseline **"0"** as a function of centrality and pseudorapidity
    - Achieved precision of **better than 5%** for the κ_3/κ_2 results is promising for the higher order cumulants
- **Up to 3rd order** ALICE data agree with the LQCD expectations

Summary

- **Net-electric-charge fluctuations:** Challenge are the dominant **resonance contributions**
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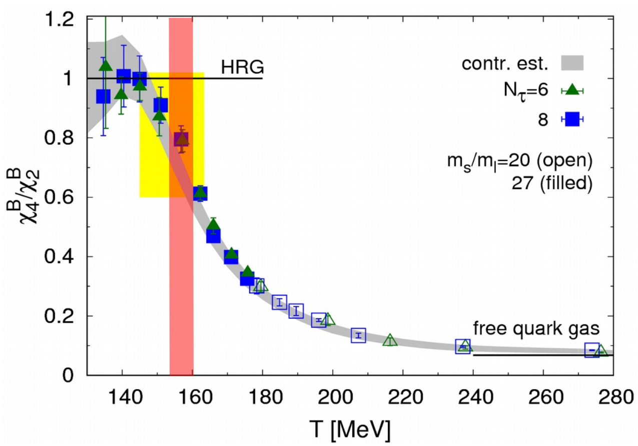
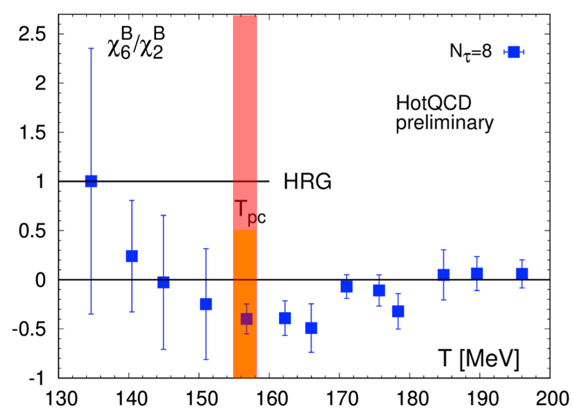
- **Up to 3rd order** ALICE data agree with the LQCD expectations

Holy grail: see critical behavior in 6th and higher order cumulants





RUN1: 2nd order (~13M min. bias events)
RUN2: 4th order (~150M central events)
RUN3: **6th ...** (>1000M central events)

Open Questions

Experiment

- Efficiency correction
  ⟶ realistic detector simulations
- Volume fluctuations
  ⟶ centrality resolution
- Effect of resonances
- Measurement at low energies
- Systematic uncertainties
- …

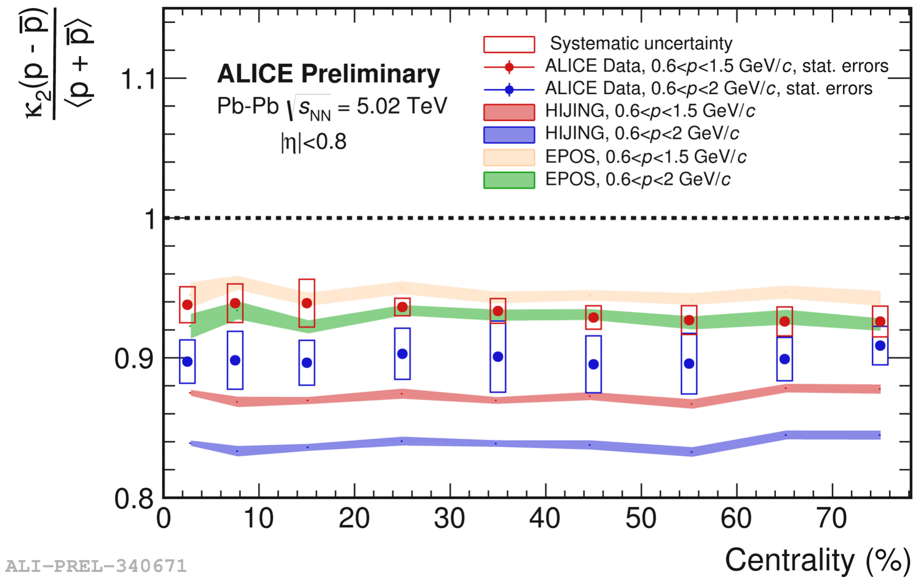
Theory

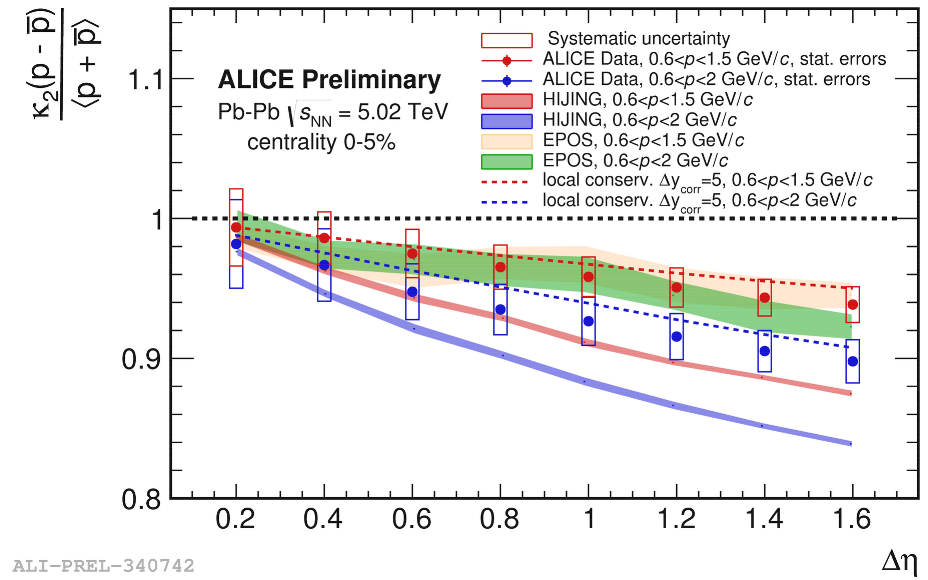
- Efficiency correction
  ⟶ unfolding or …
- Volume fluctuations
- Effect of resonances
- Measurement at low energies
  ⟶ baryon stopping, deuteron formation …
- Effect of hydrodynamic evolution
- …

- Adam Bzdak et. al., arXiv:1906.00936
- Probing the Phase Structure of Strongly Interacting Matter: Theory and Experiment, https://indico.gsi.de/event/7994/overview

BACKUP

2^{nd} order cumulants of net-p: Acceptance dependence

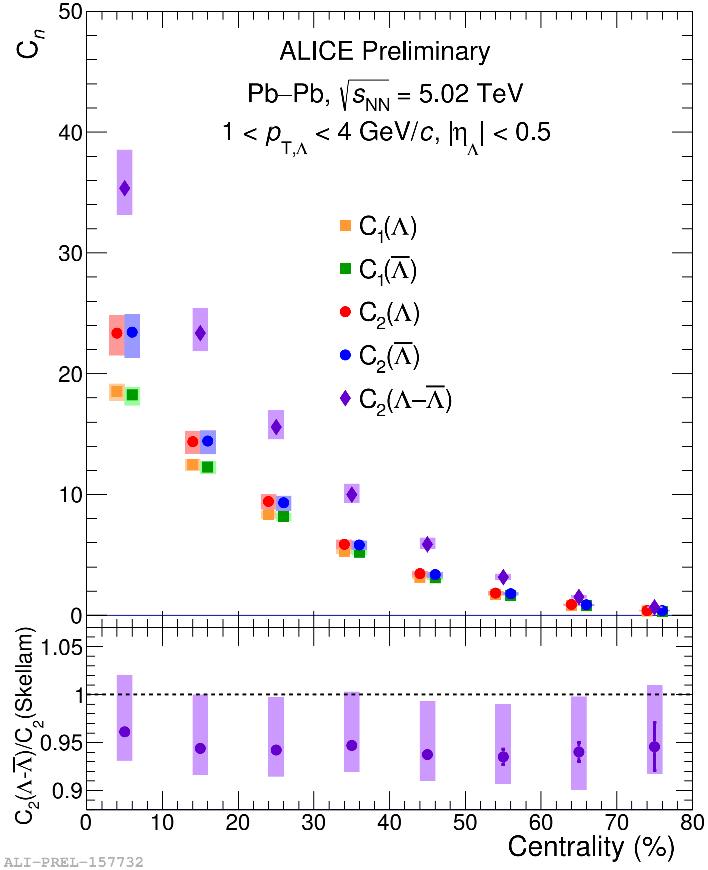




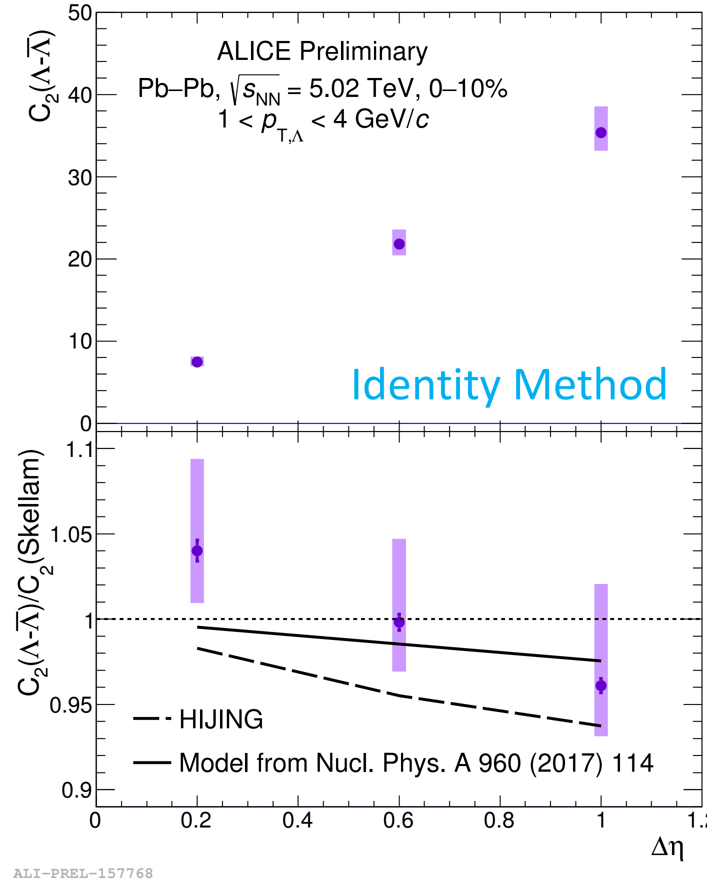
- Consistent with the baryon number conservation picture
  - Increase in fraction of accepted $p, \bar{p}$ -> stronger constraint of fluctuations due to baryon number conservation
- EPOS & HIJING show this drop qualitatively

2^{nd} order cumulants of net-Λ at LHC

$$\kappa_2(n-\overline{n}) \equiv C_2(n-\overline{n})$$



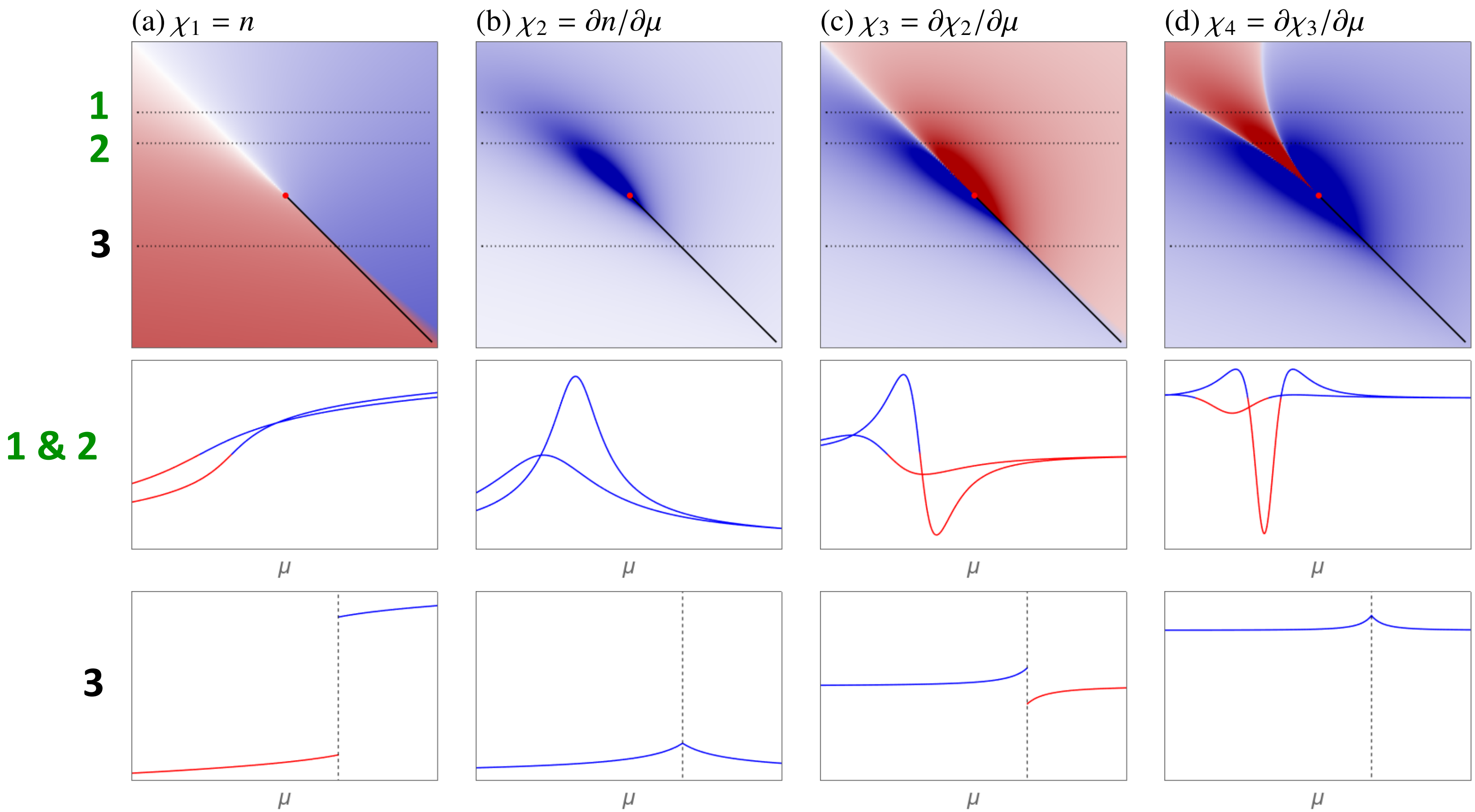
Alice Ohlson, QM2018, NPA 982 (2019) 299

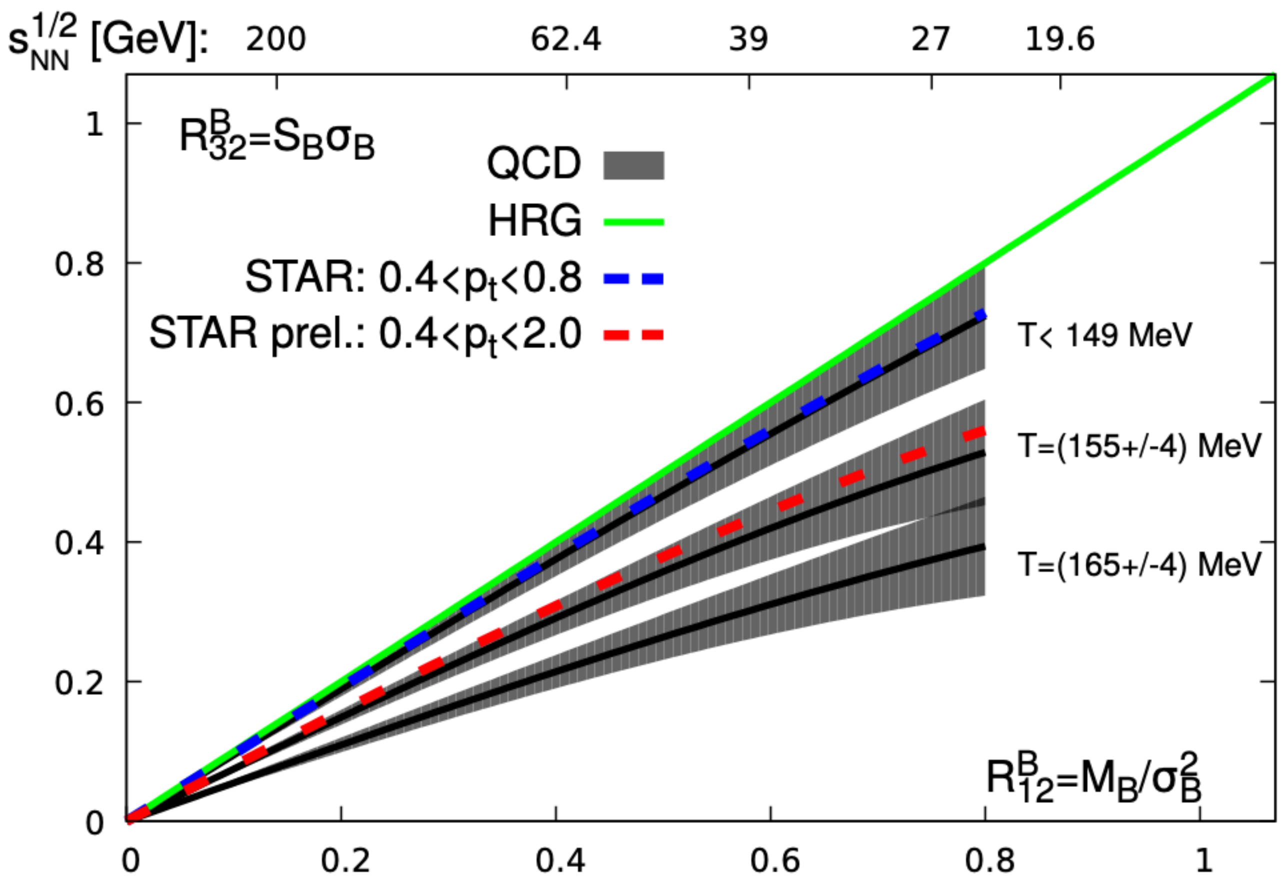


- Similar trend as for **net-p**
- **Better precision** is needed to see the impact of strangeness conservation

Thermodynamic susceptibilities

$$\langle N \rangle = \sum_j N_j p_j = T \frac{\partial \ln Z_{GCE}}{\partial \mu}\bigg|_V \Rightarrow n \equiv \frac{\langle N \rangle}{V} = \left(\frac{\partial P}{\partial \mu}\right)_T \Rightarrow \chi_k = \left(\frac{\partial^k P}{\partial \mu^k}\right)_T = \left(\frac{\partial^{k-1} n}{\partial \mu^{k-1}}\right)_T$$




$s_{NN}^{1/2}$ [GeV]: 200 62.4 39 27 19.6
$R_{32}^B=S_B\sigma_B$
QCD
HRG
STAR: 0.4<p_t<0.8
STAR prel.: 0.4<p_t<2.0
T< 149 MeV
T=(155+/-4) MeV
T=(165+/-4) MeV
$R_{12}^B=M_B/\sigma_B^2$
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Cross Cumulants

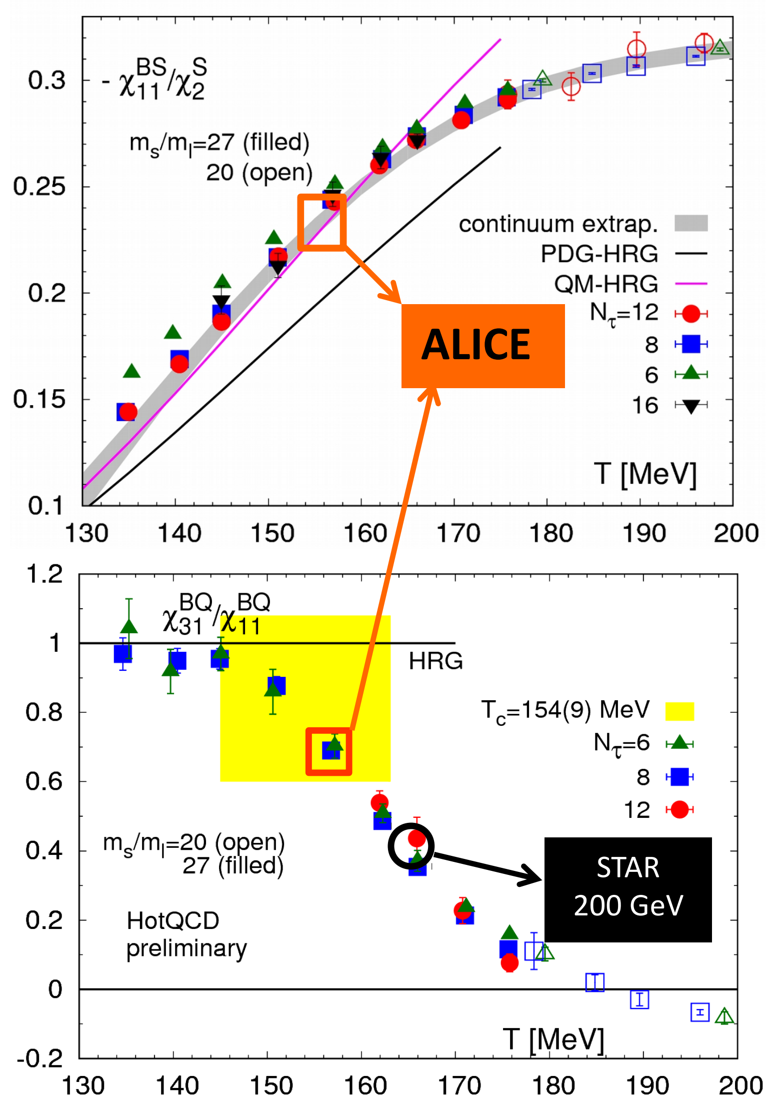
- Taylor expansion of the **QCD** pressure

$$\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Cumulants of **net-charge fluctuations and correlations**

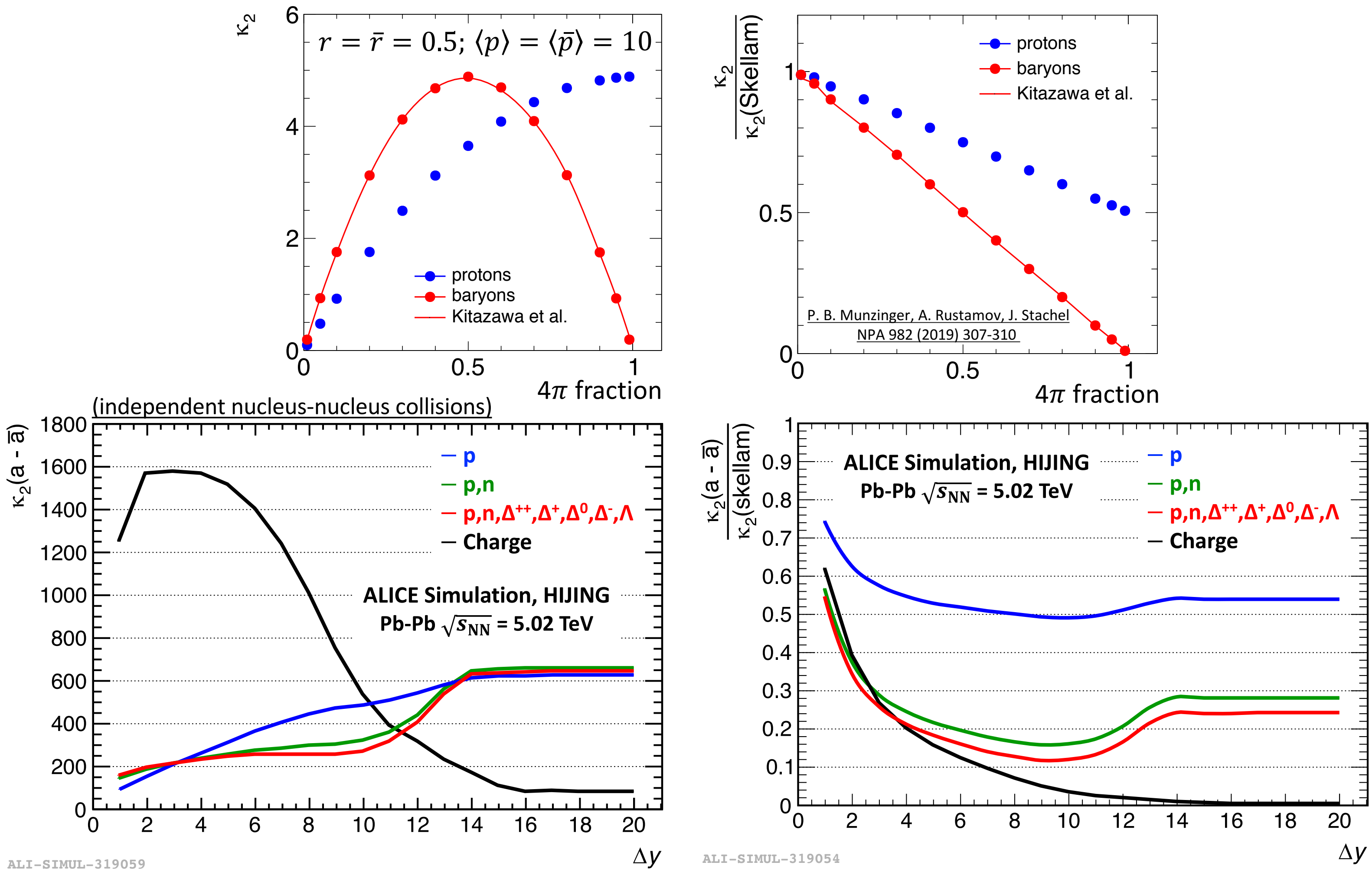
$$\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$



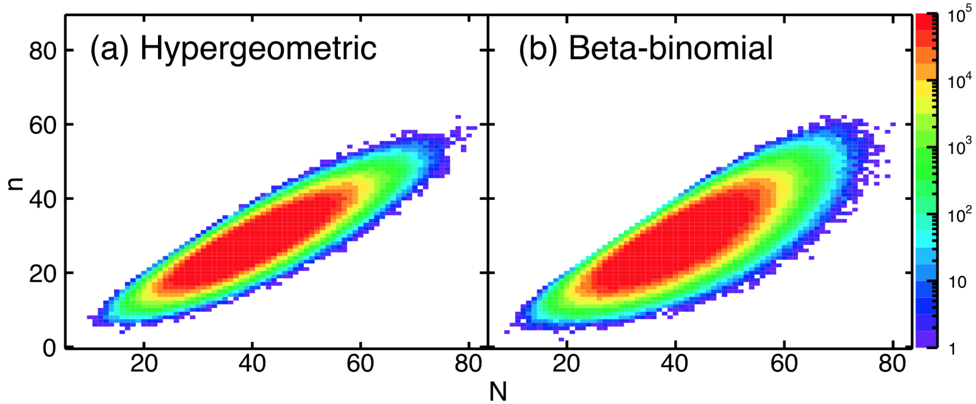
F. Karsch, QM17, arXiv:1706.01620

Which acceptance?

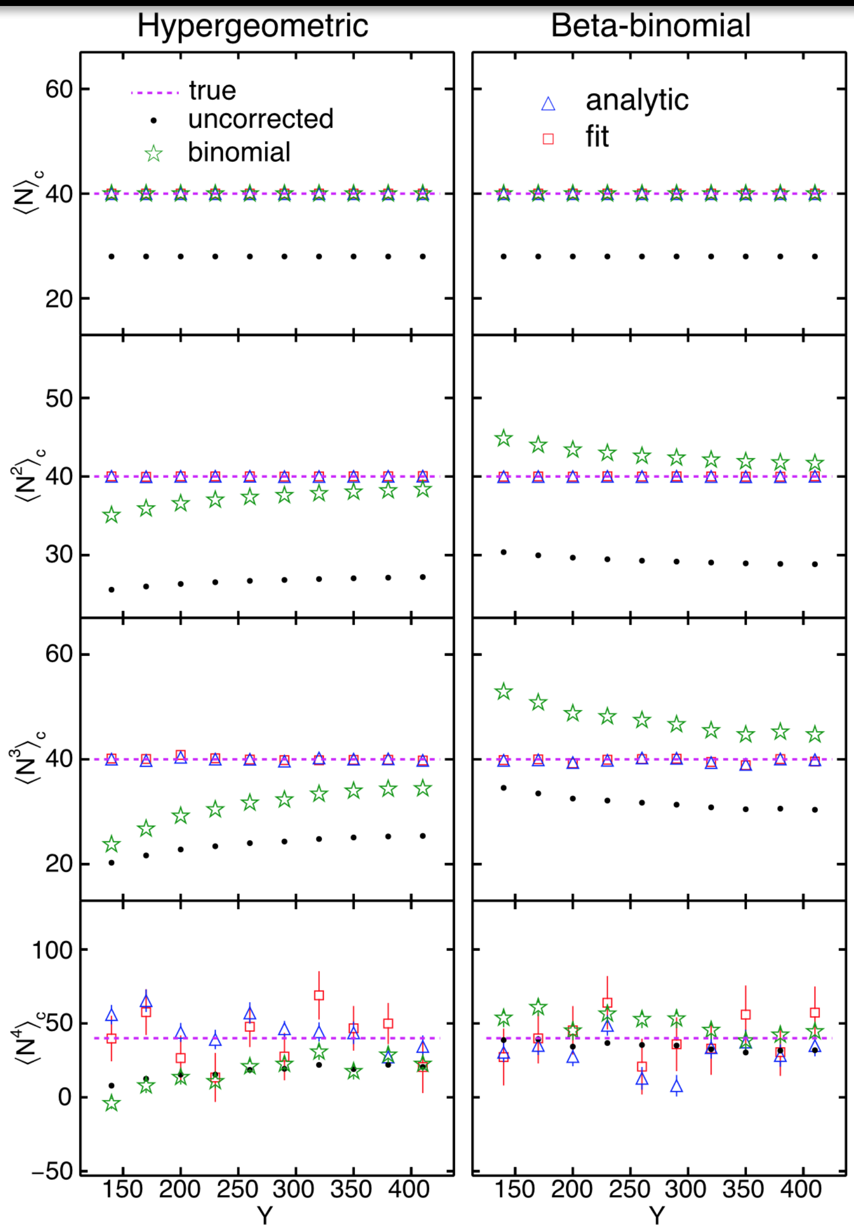
- Due to **isospin randomization,** at $\sqrt{s_{NN}}$ > 10 GeV **net-baryon** fluctuations can be obtained from corresponding **net-proton** measurements (M. Kitazawa, and M. Asakawa, Phys. Rev. C 86, 024904 (2012))



What if efficiency loss is not binomial?



Draw N balls from the urn without returning balls to the urn	In each draw, when one draws a white ball, two white balls are returned to the urn

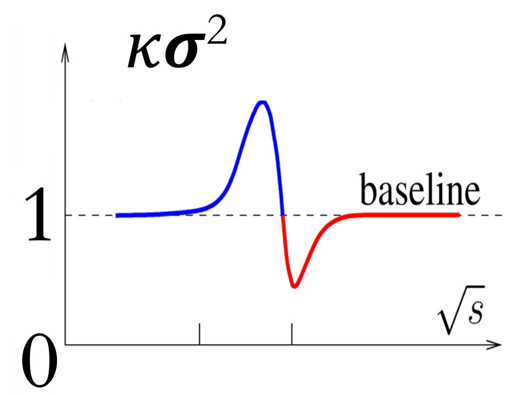


[T. Nonaka, M. Kitazawa, S. Esumi, Nucl.Instrum.Meth. A906 (2018) 10-17]
[T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)]
[Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)]

Expectations for the 3rd and 4th order cumulants

At RHIC:

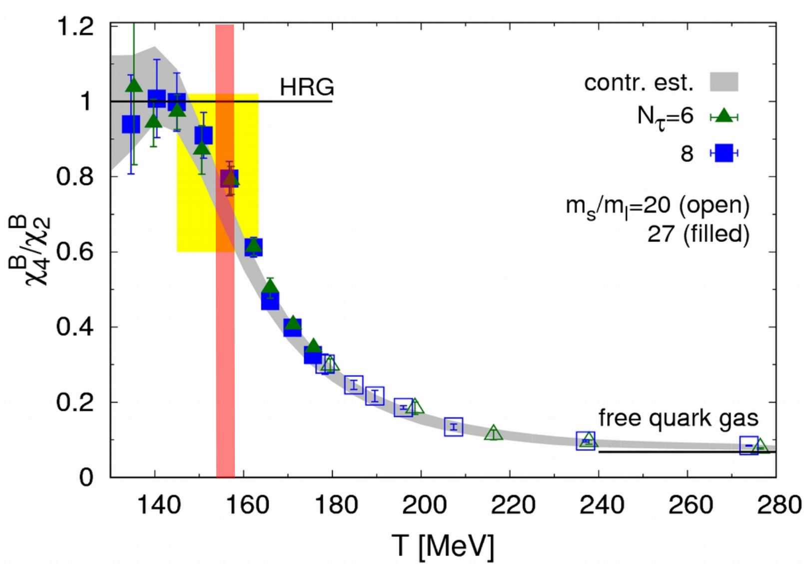
Non-monotonic behavior as a function of energy



M. Stephanov
PRL102, 032301 (2009), PRL107, 052301 (2011)

At LHC:

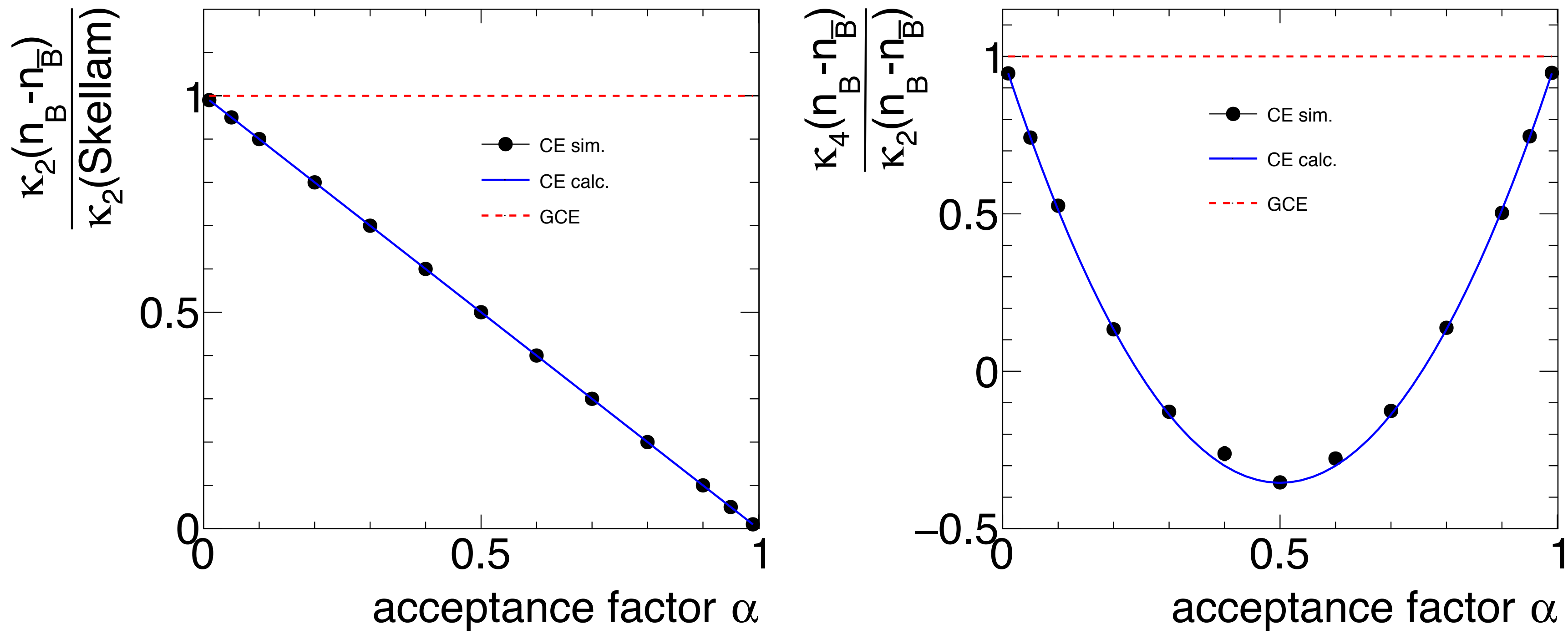
~30% difference between LQCD and HRG at T_{pc}



Effect of baryon number conservation at 4th order?

CE:Canonical ensemble

GCE: Grand canonical ensemble

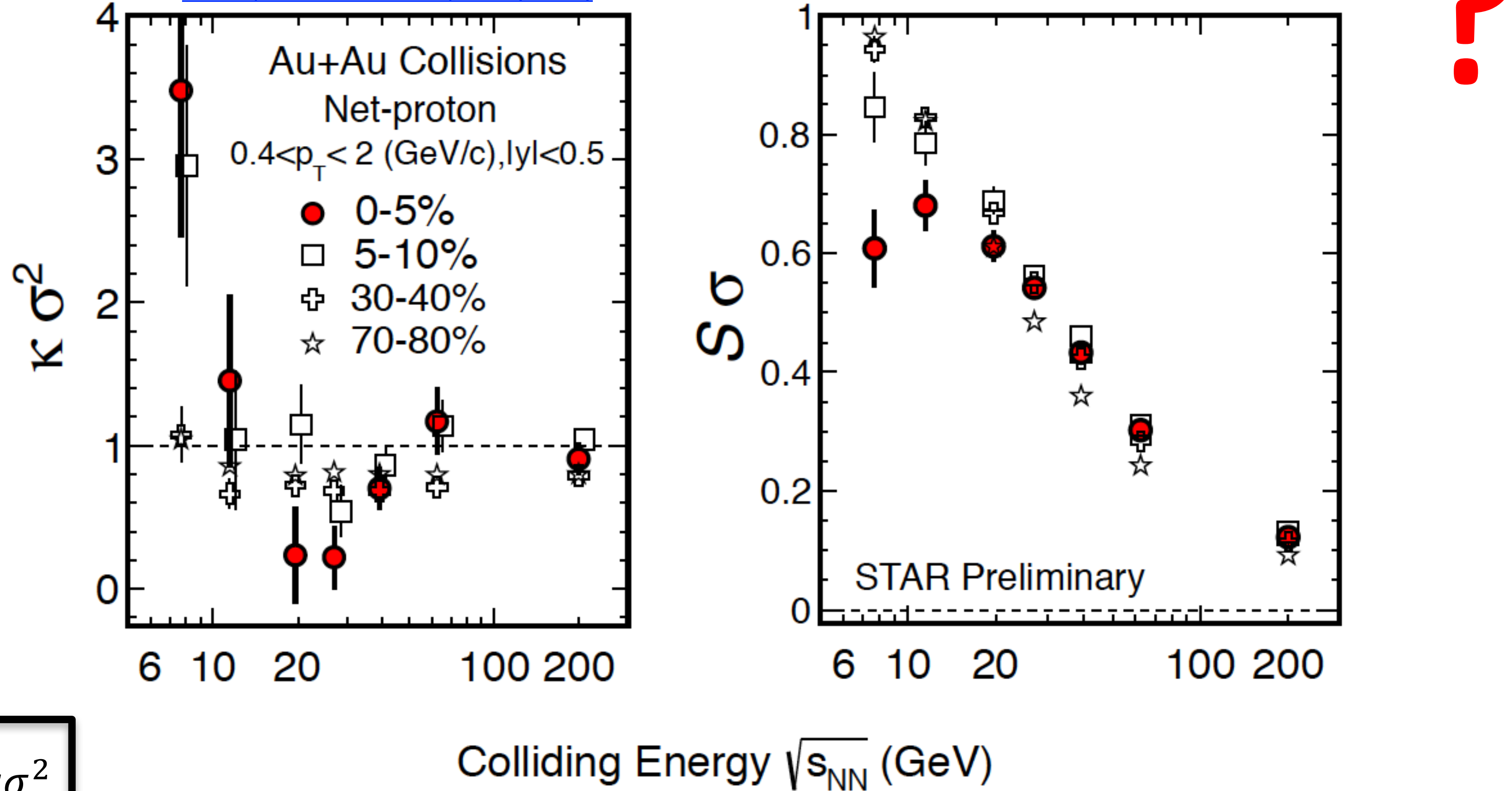


- **Small acceptance** → small multiplicities → approach to Poissonian limit
- Acceptance is more crucial for the **4th cumulant**

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310

3^{rd} and 4^{th} order cumulants of net-p at RHIC

X. Luo, PoS CPOD2014, 019 (2015)

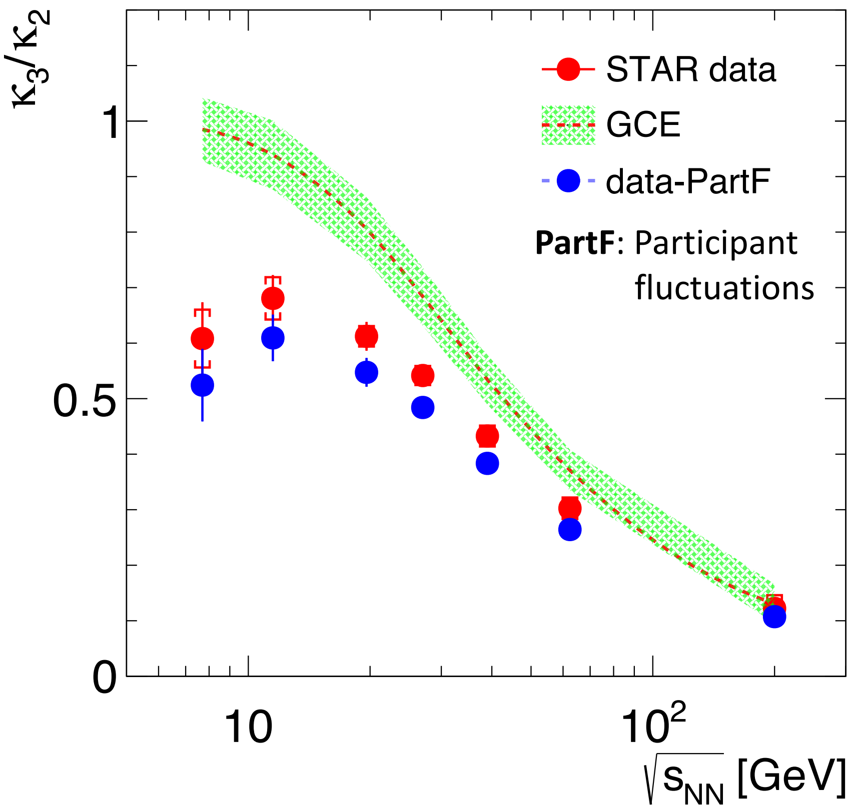


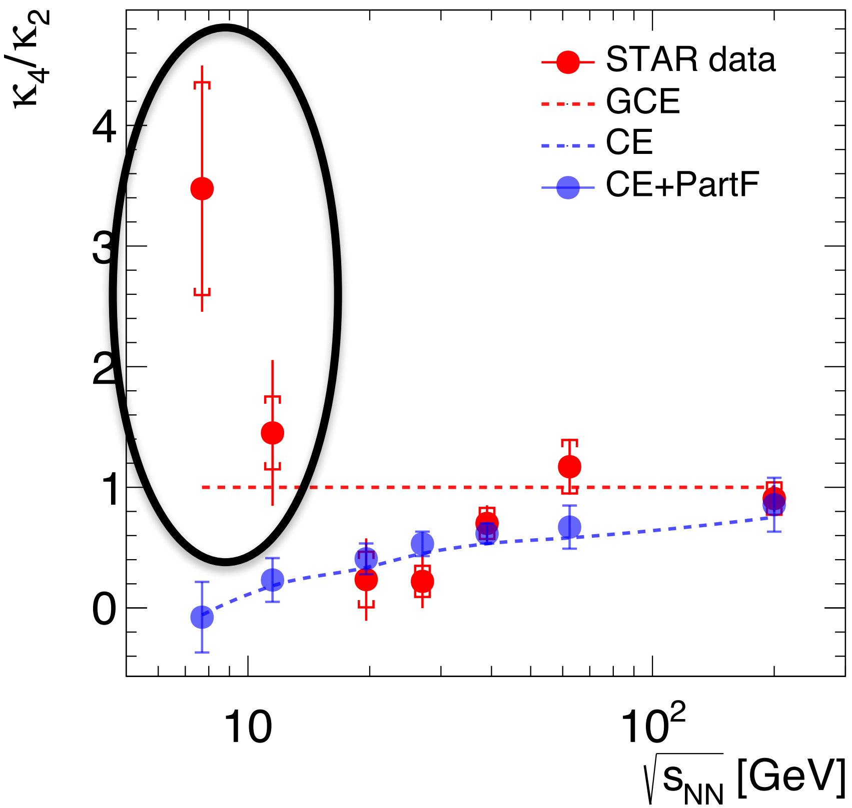
$$\frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$$

$$\frac{\kappa_3}{\kappa_2} = S\sigma$$

Effect of baryon number conservation

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310

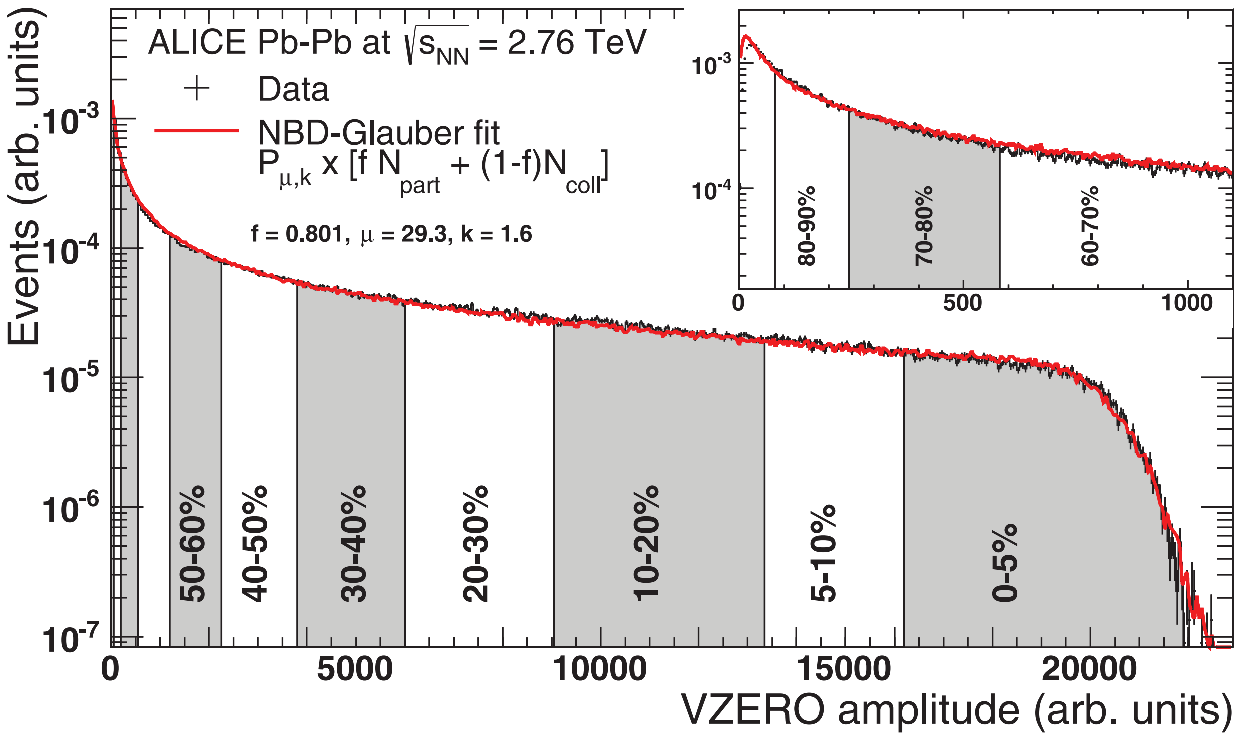


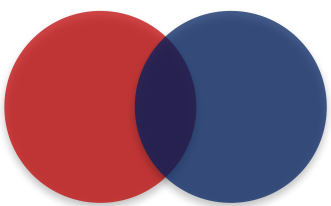


- κ_3/κ_2 and κ_4/κ_2 cannot be simultaneously explained for the lowest two energies
- Possible biases due to efficiency correction procedure and cut based approach

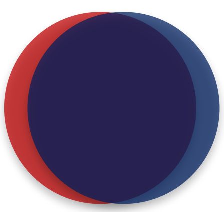
Volume in experiment? ⟶ "Centrality"

ALICE: Phys.Rev. C88 (2013) no.4, 044909



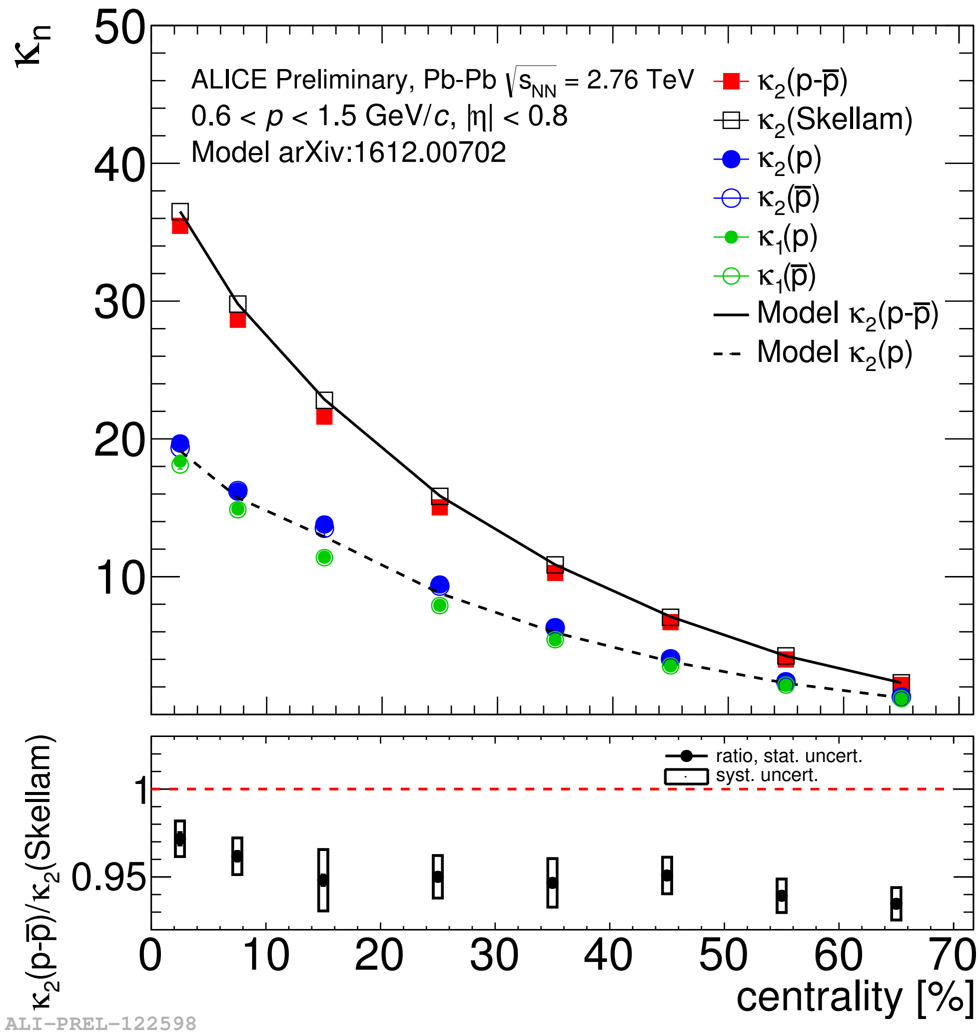


Peripheral collision



Central Collision

“Model” vs ALICE Data



Input to the Model

$\kappa_1(p)$, $\kappa_1(\bar{p})$

centrality selection procedure

Predictions

——— $\kappa_2(p-\bar{p})$

– – – $\kappa_2(p)$

participants

$$\kappa_2(N_B) = \langle N_W \rangle \kappa_2(n_B) + \langle n_B \rangle^2 \kappa_2(N_W)$$

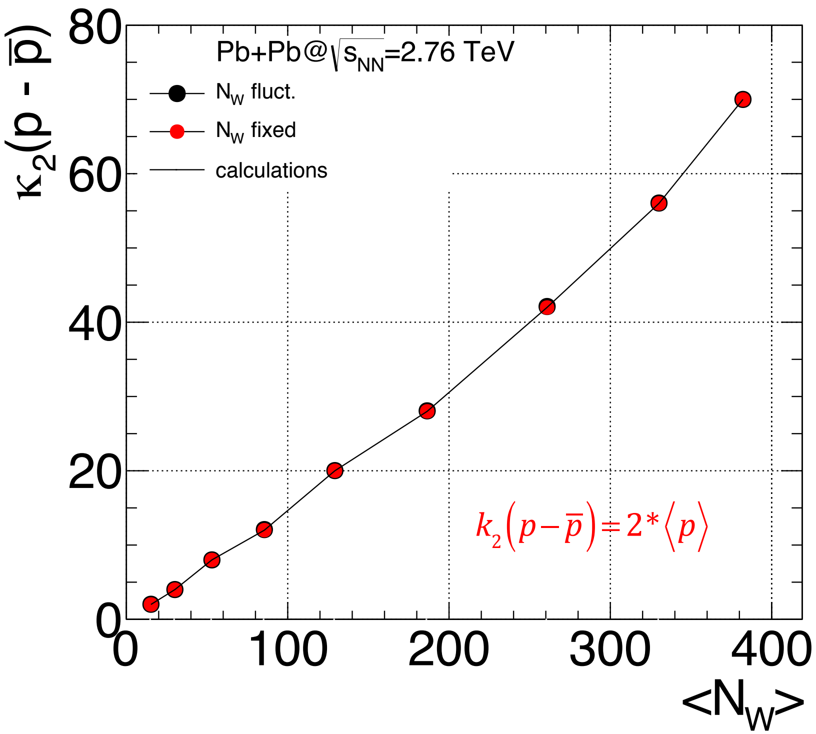
from single participant

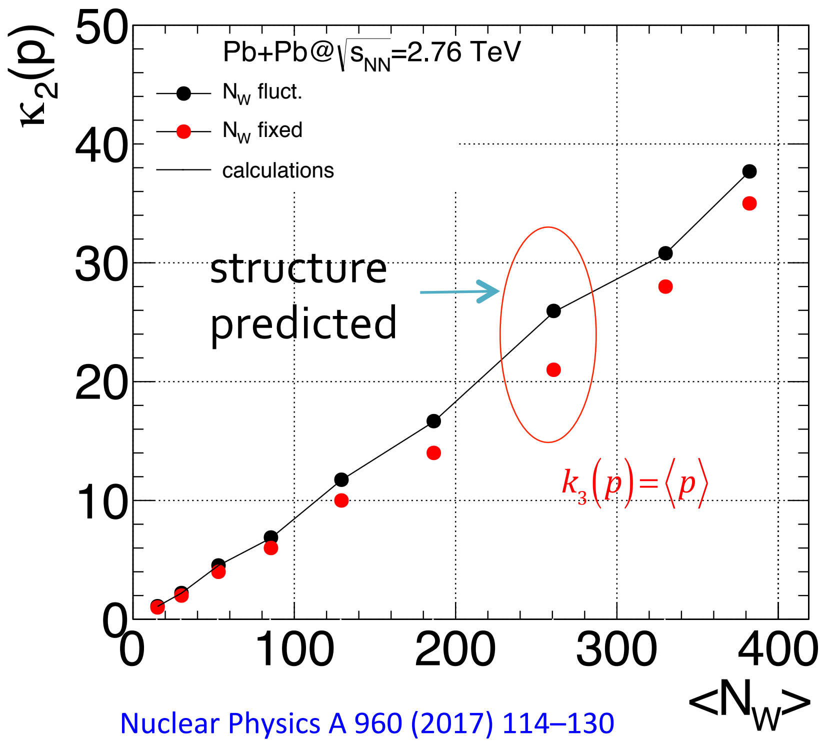
Consistent predictions for net-protons, protons and antiprotons

P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

Volume Fluctuations: 2nd order

150*10^6 Events





Nuclear Physics A 960 (2017) 114–130

$$k_2(p-\bar{p}) = \langle N_w \rangle k_2(n-\bar{n}) + \langle n-\bar{n} \rangle^2 k_2(N_w)$$

vanishes for ALICE

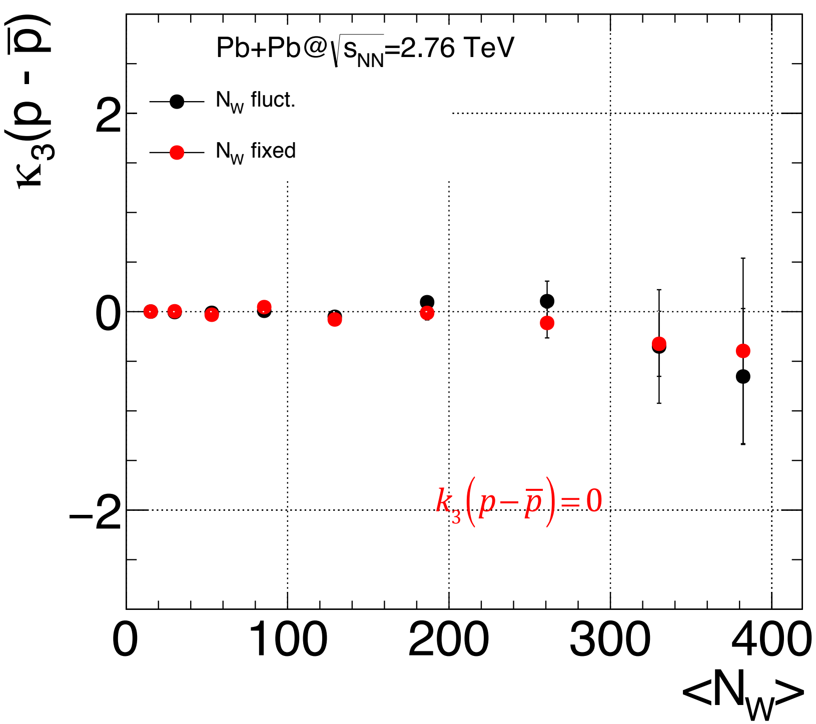
$$k_2(p) = \langle N_w \rangle k_2(n) + \langle n \rangle^2 k_2(N_w)$$

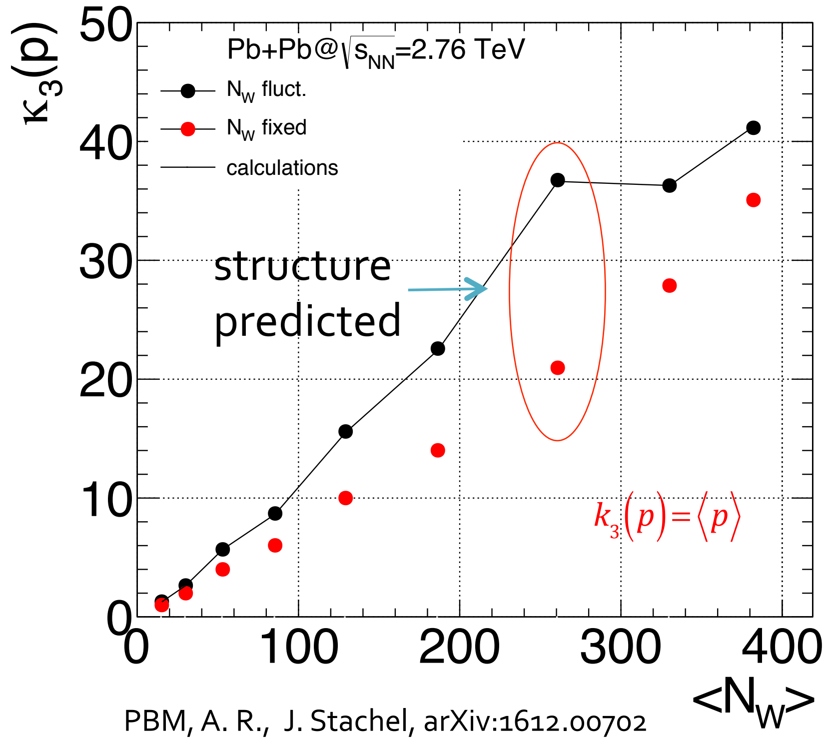
does not vanish

$n, \bar{n}$ from single wounded nucleon

Volume Fluctuations: 3rd order

150*10^6 Events





PBM, A. R., J. Stachel, arXiv:1612.00702

$$k_3(p-\bar{p})=\langle N_w \rangle k_3(n-\bar{n})+\langle n-\bar{n} \rangle (\ldots)$$

vanishes for ALICE

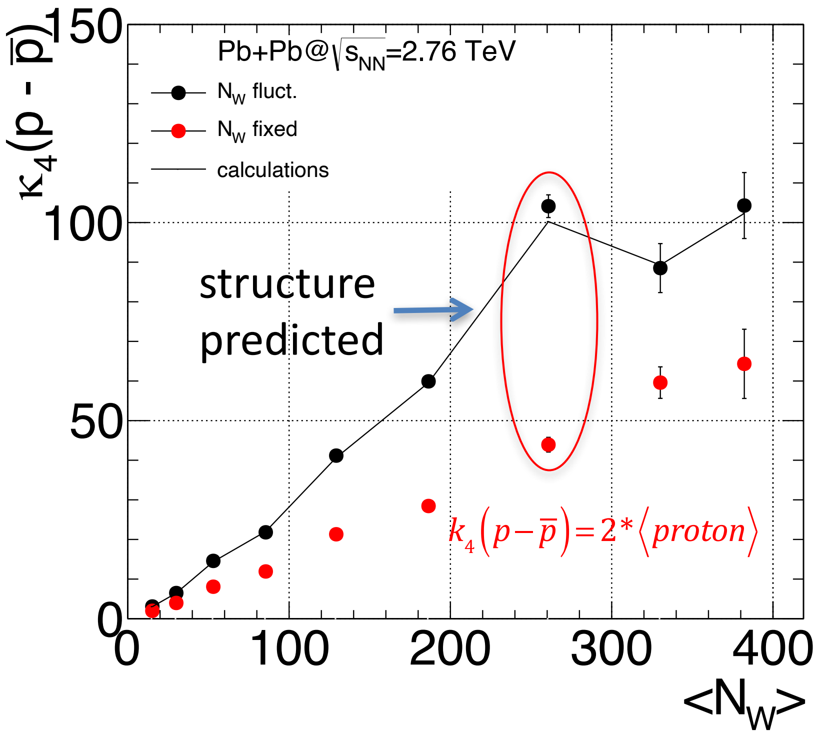
$$k_3(p)=\langle N_w \rangle k_3(n)+\langle n \rangle (\ldots)$$

does not vanish

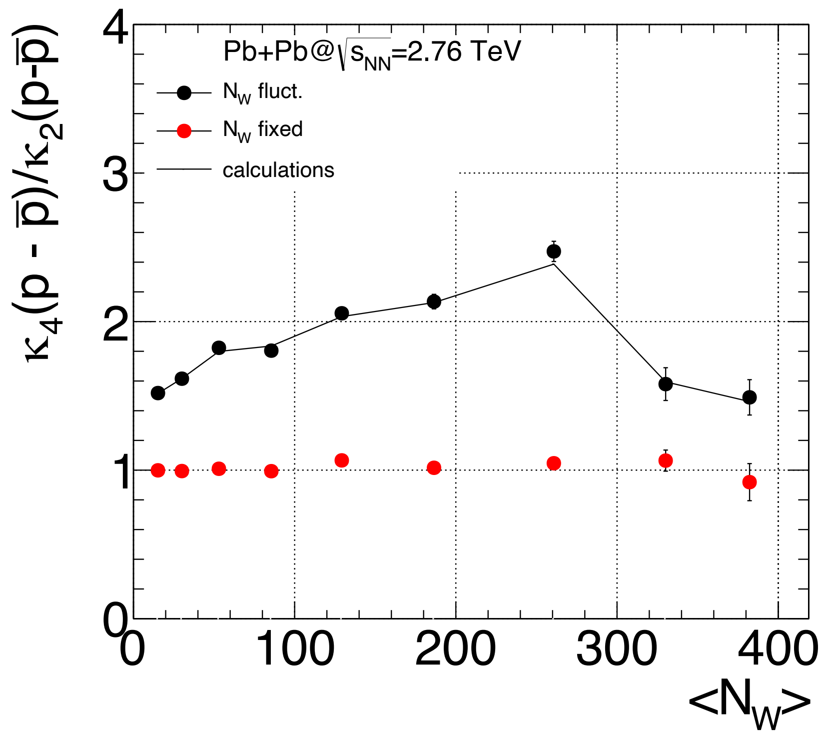
$n,\bar{n}$ from single wounded nucleon

Volume Fluctuations: 4th order

150*10^6 Events



P. Braun-Munzinger, A. Rustamov, J. Stachel
Nuclear Physics A 960 (2017) 114–130

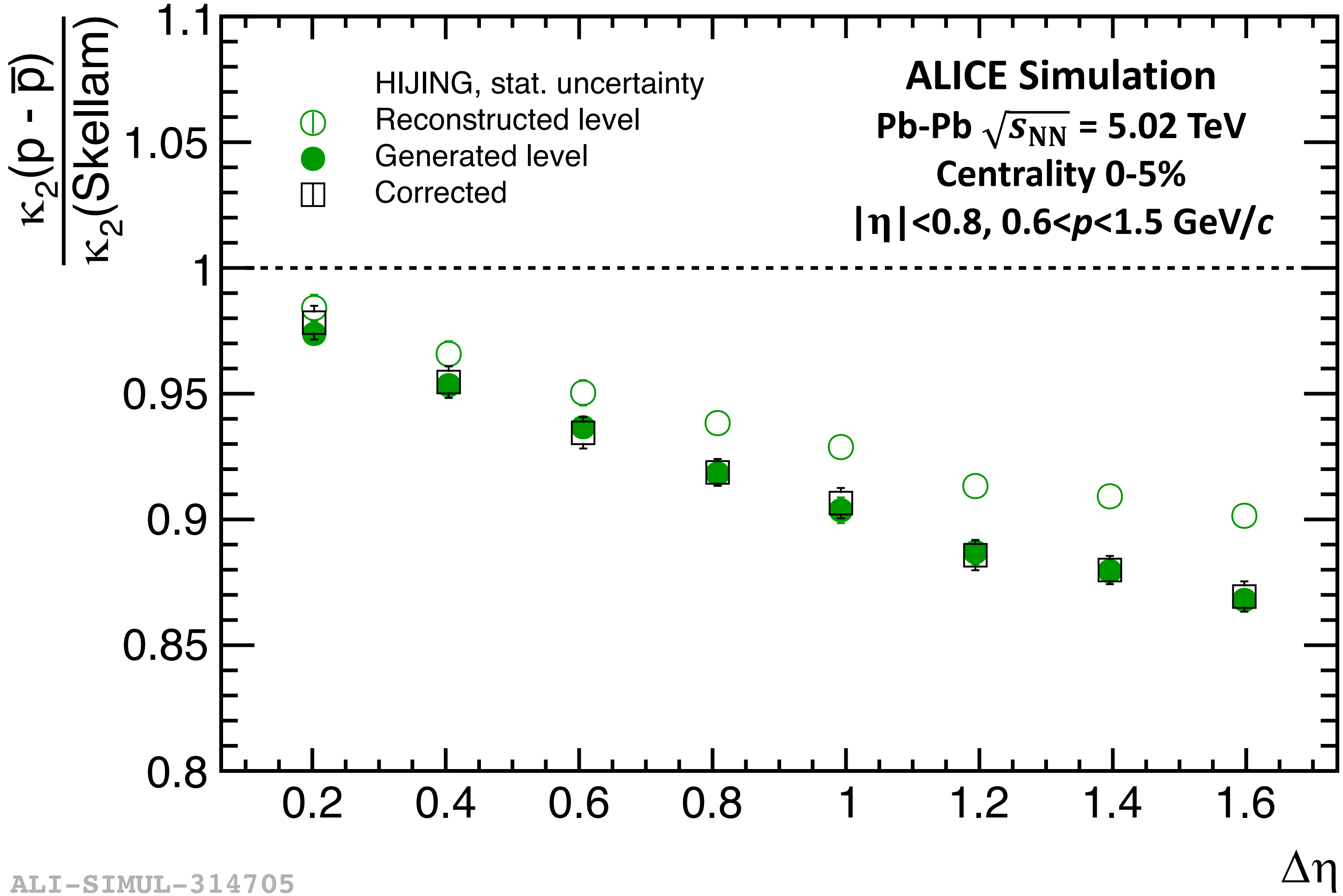


$$k_4(p-\bar{p}) = \langle N_w \rangle k_4(n-\bar{n}) + 3k_2(n-\bar{n})^2 k_2(N_w) + \langle n-\bar{n} \rangle(\ldots)$$

$n, \bar{n}$ → from single wounded nucleon

vanishes for ALICE

Efficiency correction: $\kappa_2(p-\bar{p})/\kappa_2(Skellam)$

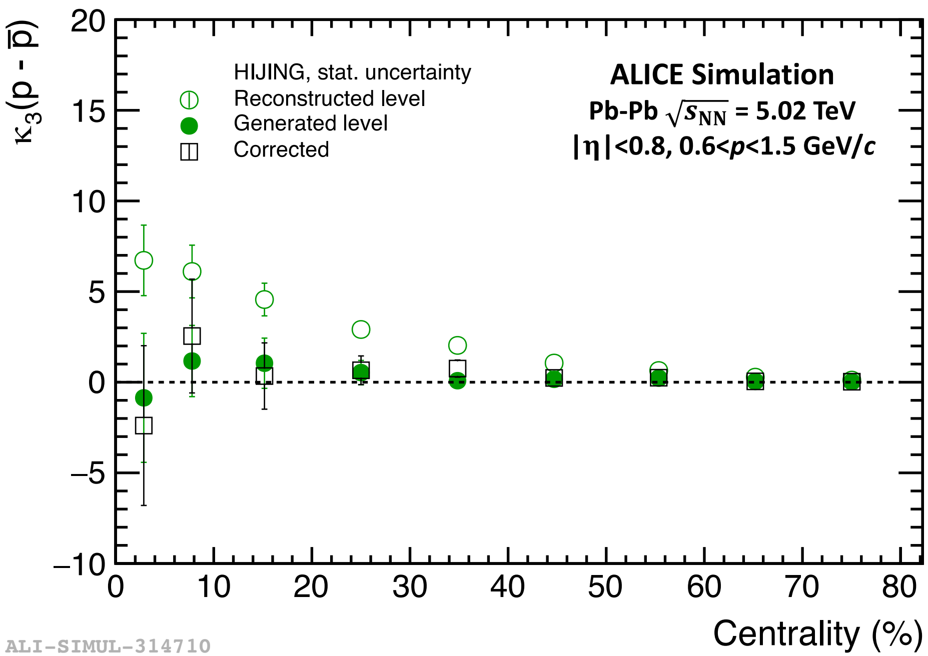


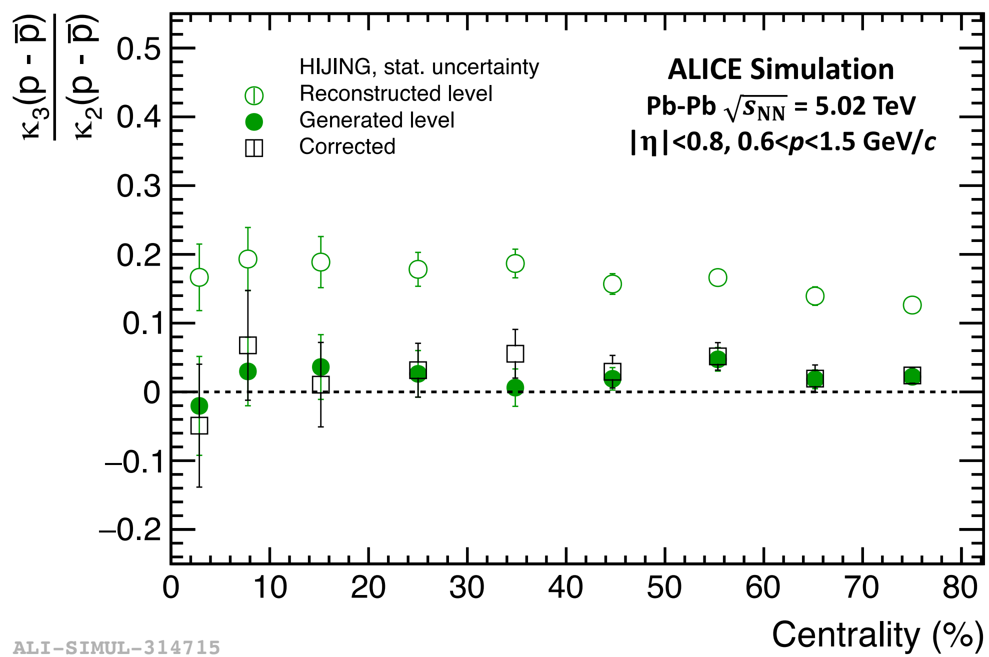
Efficiency correction with binomial assumption:

T. Nonaka, M. Kitazawa, S. Esumi, Phys. Rev. C 95, 064912 (2017)

Adam Bzdak, Volker Koch, Phys. Rev. C86, 044904 (2012)

Efficiency correction: $\kappa_3(p-\bar{p})/\kappa_2(p-\bar{p})$





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- Probability of measuring n_B baryons in the acceptance:

$$B\left(n_B; N_B, \alpha\right) = \frac{N_B!}{n_B!\left(N_B - n_B\right)!}\alpha^{n_B}\left(1-\alpha\right)^{N_B - n_B} \qquad \alpha = \frac{\langle N_B^{acc}\rangle}{\langle N_B^{4\pi}\rangle}$$

- Multiplicity distribution in the acceptance:

$$P(n_B) = \sum_{N_B} B(n_B; N_B, \alpha) P(N_B)$$

- The moments of the measured baryon distributions can be then calculated

$$\langle n_B \rangle = \sum_{n_B=0}^{\infty} n_B P(n_B) = \alpha \langle N_B \rangle\,,$$

$$\langle n_B^2 \rangle = \sum_{n_B=0}^{\infty} n_B^2 P(n_B) = \alpha^2 \langle N_B^2 \rangle + \alpha(1-\alpha)\langle N_B \rangle$$

$$\frac{\kappa_2\left(n_B - n_{\bar{B}}\right)}{\kappa_2\left(Skellam\right)} = \frac{\kappa_2\left(n_B - n_{\bar{B}}\right)}{\alpha\left(\langle N_B\rangle + \langle N_{\bar{B}}\rangle\right)} = \alpha\frac{\overset{0}{\cancel{\kappa_2\left(N_B - N_{\bar{B}}\right)}}}{\langle N_B\rangle + \langle N_{\bar{B}}\rangle} + 1 - \alpha$$

MC implementation of canonical ensemble

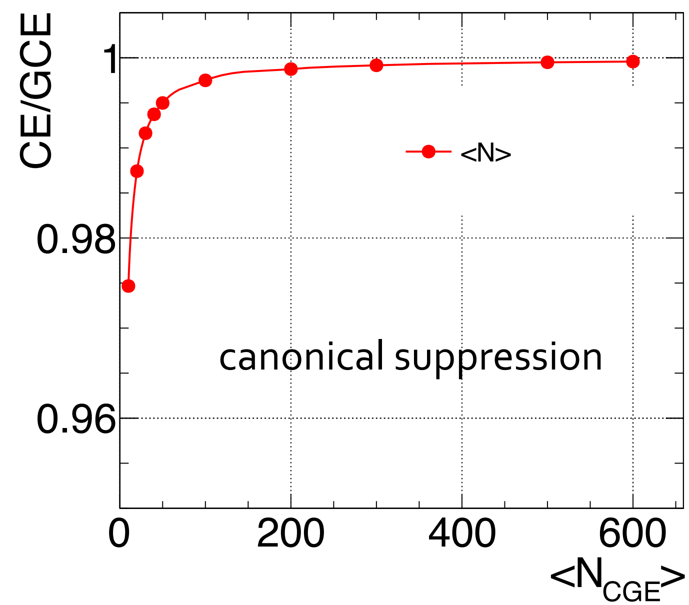
Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V,T,\mu)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}=e^{2z\cosh\left(\mu/T\right)},\quad \lambda_{B,\bar{B}}=e^{\pm\frac{\mu}{T}}$$

$$Z_{CE}(V,T,B)=\sum_{N_B=0}^{\infty}\sum_{N_{\bar{B}}=0}^{\infty}\frac{(\lambda_B z)^{N_B}}{N_B!}\frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta\left(N_B-N_{\bar{B}}-B\right)=I_B(2z)\Big|_{\lambda_B=\lambda_{\bar{B}}=1}$$

$$\left\langle N_{B,\bar{B}}\right\rangle_{GCE}=\lambda_{B,\bar{B}}\frac{\partial\ln Z_{GCE}}{\partial\lambda_{B,\bar{B}}}=e^{\pm\frac{\mu}{T}}z,\quad z=\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}$$

$$\left\langle N_{B,\bar{B}}\right\rangle_{CE}=\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\frac{I_{B\mp1}\left(2\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\right)}{I_B\left(2\sqrt{\left\langle N_B\right\rangle_{GCE}\left\langle N_{\bar{B}}\right\rangle_{GCE}}\right)}$$



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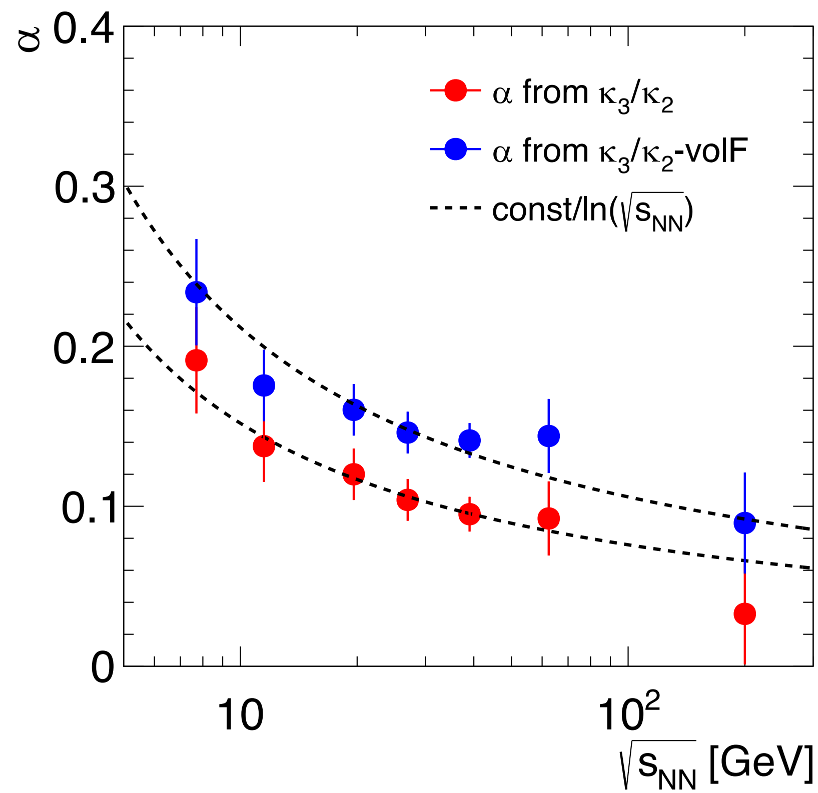
P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA 982 (2019) 307-310

Results from STAR vs Our predictions

Acceptances: $\alpha_{\sqrt{s}=7.7GeV} = 0.19 \pm 0.03, \quad \alpha_{\sqrt{s}=19.6GeV} = 0.12 \pm 0.016$

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1-2\alpha), \quad \frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle n_B + n_{\bar{B}} \rangle_{CE}} \left[\langle n_B \rangle_{GCE} \langle n_{\bar{B}} \rangle_{GCE} - \langle n_B \rangle_{CE} \langle n_{\bar{B}} \rangle_{CE} \right] \right)$$



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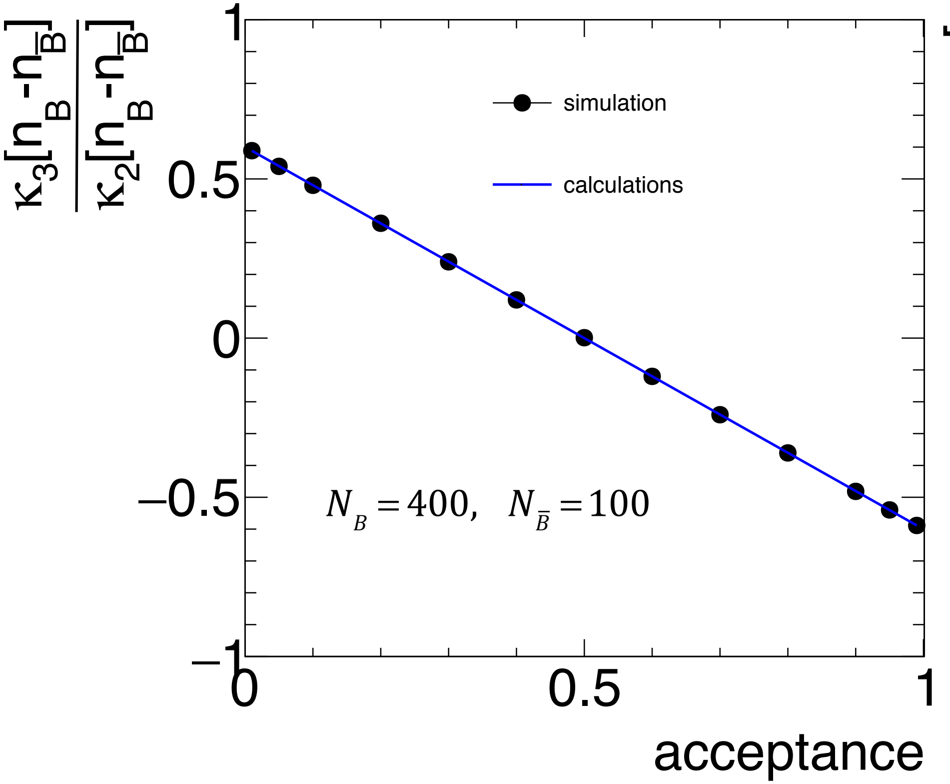
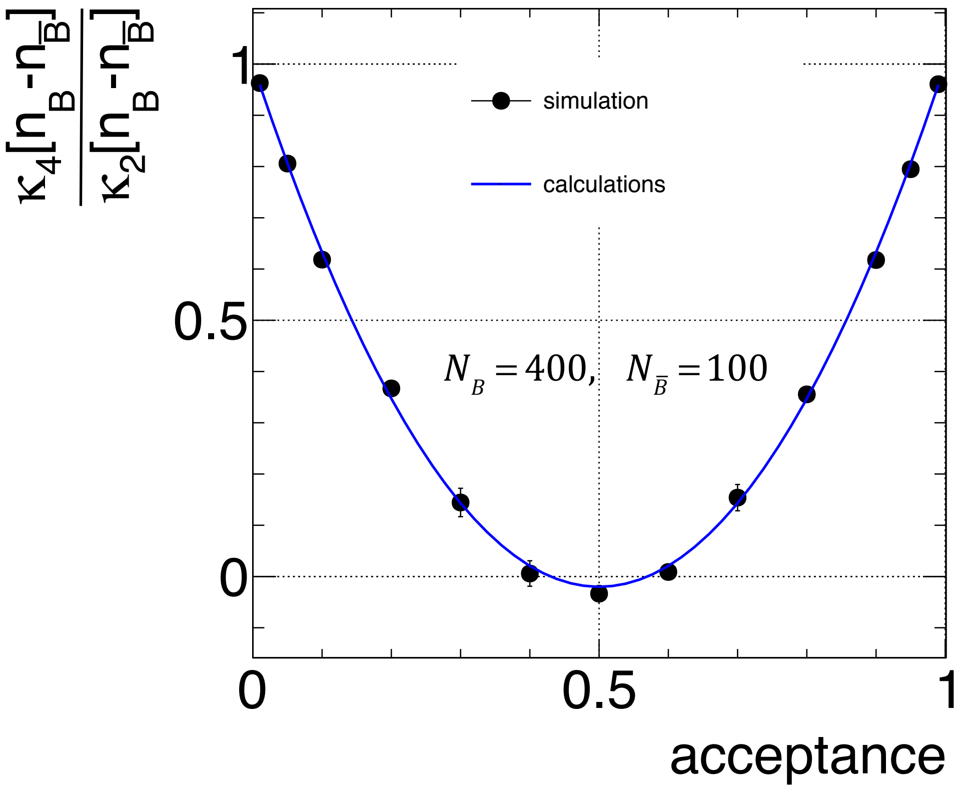
3rd and 4th cumulants

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1-2\alpha) \xrightarrow{\langle n_{\bar{B}} \to 0 \rangle} (1-2\alpha)$$





$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1-\alpha)\left(1 - \frac{2}{\langle n_B + n_{\bar{B}} \rangle_{CE}} \left[\langle n_B \rangle_{GCE} \langle n_{\bar{B}} \rangle_{GCE} - \langle n_B \rangle_{CE} \langle n_{\bar{B}} \rangle_{CE} \right] \right)$$

Free energy density

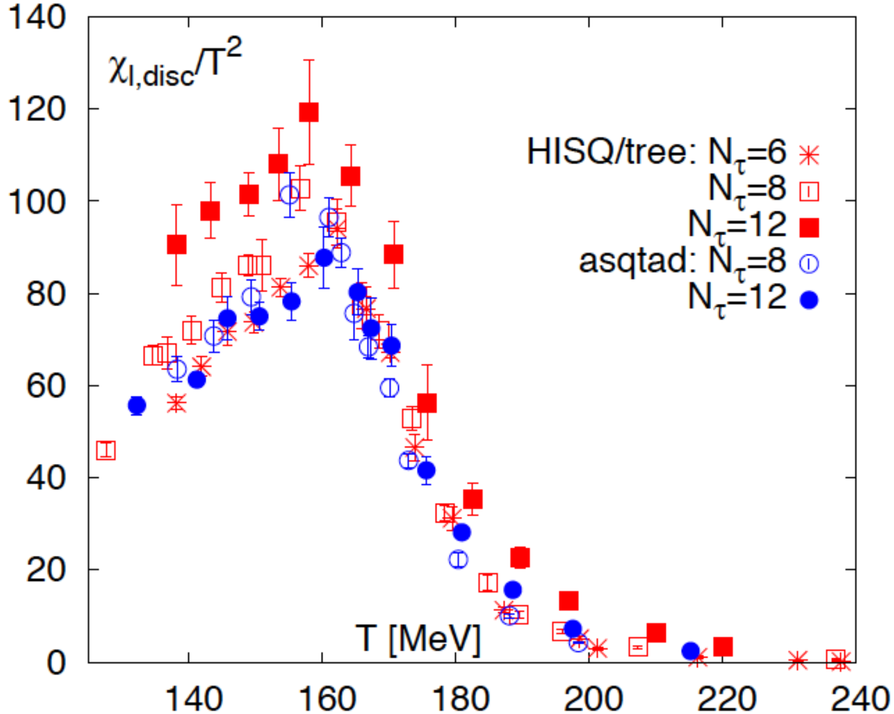
$$f = -\frac{T}{V}\ln Z$$

2-flavor light quark chiral condensate

$$\langle\bar{\psi}\psi\rangle_l^{n_f=2} = \frac{T}{V}\frac{\partial \ln Z}{\partial m_l}$$

Chiral susceptibility (sum of connected and disconnected Feynman diagrams)

$$\chi_{m,l} = \frac{\partial}{\partial m_l}\langle\bar{\psi}\psi\rangle_l^{n_f=2}$$



"The disconnected part of the light quark susceptibility describes the fluctuations in the light quark condensate"