Radiative $p_\perp$-broadening of high energy quarks and gluons in QCD matter

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B. Wu, JHEP **1110***, 029 (2011) [arXiv:1102.0388];*

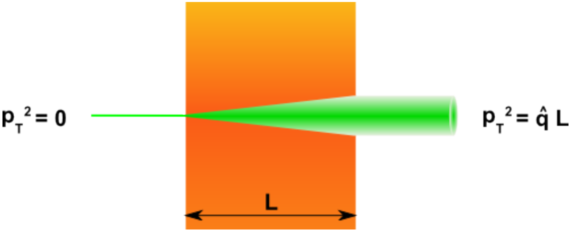
T. Liou, A. H. Mueller and B. Wu, Nucl. Phys. A 916 (2013) 102-125 [arXiv:1304.7677].

Outline

1. **Motivation**
2. **Radiative $p_\perp$-broadening in single scattering**
3. **Radiative $p_\perp$-broadening in multiple scattering**
4. **Conclusions**

Motivation

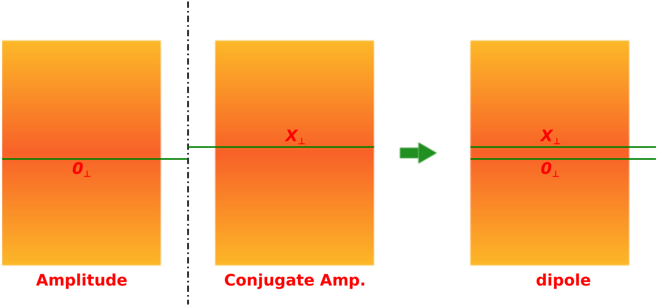
- $p_\perp$**-broadening of high energy partons**
  - $\langle p_\perp^2 \rangle$ **at leading order**



Brownian motion of partons in the transverse plane.

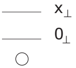
Motivation

- $p_\perp$-broadening of high energy partons
  - Dipole picture



Dipole size = relative coordinates in amplitude to that in its conjugate!.

  - Dipole cross-section



$$= \frac{\alpha_s C_R}{\pi} \int d^2 q_\perp \, |V(q_\perp)|^2 \left(e^{i q_\perp \cdot x_\perp} - 1 \right) .$$

Motivation

- **Basic formula**
  - $p_\perp$**-broadening**

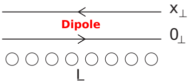
$$\langle p_\perp^2 \rangle \equiv \int d^2 p_\perp p_\perp^2 \frac{dN}{d^2 p_\perp} = -\left. \nabla^2 S(x_\perp) \right|_{x_\perp = 0}$$

with

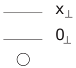
$$S(x_\perp) \equiv \int d^2 p_\perp e^{i p_\perp \cdot x_\perp} \frac{dN}{d^2 p_\perp}.$$

  - **Leading order: multiple scattering** $(L \gg \lambda \gg \frac{1}{\mu})$

$S(x_\perp) =$



$= \exp\Big\{ \rho L \Big($



$\Big)\Big\} \simeq e^{-\frac{1}{4}\hat{q} L x_\perp^2}$

Motivation

- **Transport coefficient $\hat{q}$**
  - **Definition**

$$\hat{q} \equiv \frac{1}{\lambda_R}\int d^2q_\perp \frac{q_\perp^2}{\sigma_R}\frac{d\sigma_R}{d^2q_\perp} \sim \frac{\langle q_\perp^2\rangle}{\lambda}$$

$$\propto g^2 \int d^2q_\perp d^2y_\perp dy^+ e^{-iq^- y^+ + iq_\perp \cdot y_\perp} \langle F_i^{a+}(y^+, y_\perp) F_i^{a+}(0)\rangle,$$

where the differential cross-section in single scattering is

$$\frac{d\sigma_R}{d^2q_\perp} = \frac{\alpha_s C_R}{\pi}\left|A^-(q_\perp)\right|^2.$$

  - $\langle p_\perp^2\rangle$ **at LO**

$$\langle \mathbf{p}_\perp^2\rangle = \hat{\mathbf{q}}\mathbf{L}$$

Motivation

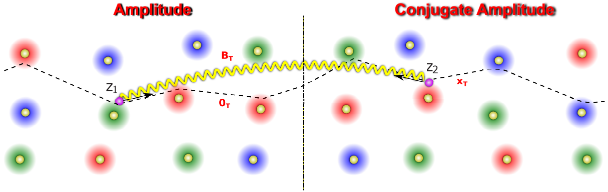
- $p_\perp$-broadening and Radiative energy loss(the LPM effect) at LO
  - Formation time

$$t_{form} \simeq \frac{\omega}{k_\perp^2} \simeq \frac{\omega}{\hat{q}\, t_{form}} \Leftrightarrow t_{form} = \sqrt{\frac{\omega}{\hat{q}}}.$$

  - Energy loss

$$\mathbf{\Delta E \sim \alpha_s N_c \omega_c \simeq \alpha_s N_c \hat{q} L^2 = \alpha_s N_c \langle p_\perp^2 \rangle L}.$$

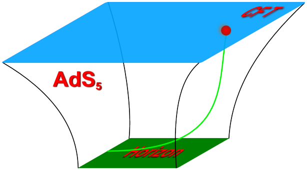
  - **Does it still hold at NLO?**



Motivation

- **Radiative corrections: parametrically BIG!**

- **In AdS/CFT**

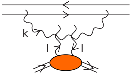


$p_\perp$-broadening is **radiation dominated**!

F. Dominguez, C. Marquet, A. H. Mueller, B. Wu and B. -W. Xiao, Nucl. Phys. A **811**, 197 (2008) [arXiv:0803.3234].

- **Q: $p_\perp$-broadening in pQCD?**
- **A: Double logarithmically enhanced by radiation!**

$$\langle p_\perp^2 \rangle_{rad} \quad =$$



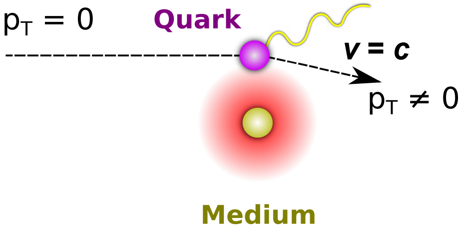
$$= \quad \frac{\alpha_s N_c}{8\pi} \hat{q} L \log^2 \left(\frac{L}{l_0} \right)^2$$

BDMPS: single scattering missing: *B. Wu, JHEP* **1110**, *029 (2011) [arXiv:1102.0388].*

Zakharov Completely right:*T. Liou, A. H. Mueller and B. Wu, Nucl. Phys. A* **811**, *197 (2008) [arXiv:1304.7677].*

Single scattering

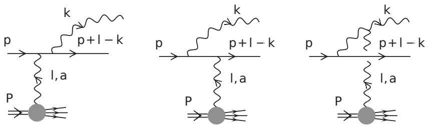
- **Let us start with a dilute medium**



Single scattering

- **Real gluon emission**

$$\frac{dN_r}{d^2 p_{R\perp}} \equiv \left| \quad \cdots \quad \right|^2$$

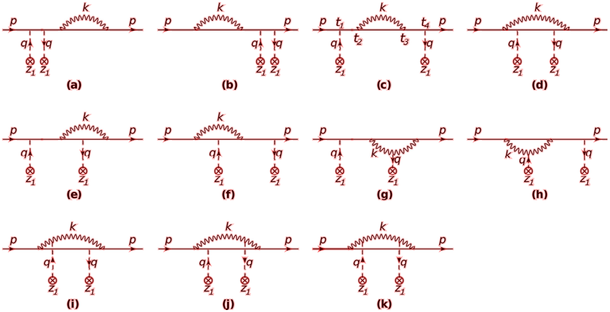


$$= \frac{L}{\lambda_R} \frac{\alpha_s N_c}{\pi^2} \int \frac{d\omega}{\omega} \int d^2 k_\perp \frac{l_\perp^2}{k_\perp^2 p_{R\perp}^2} \frac{1}{\sigma_R} \frac{d\sigma_R}{d^2 l_\perp} \bigg|_{\vec{l}_\perp = \vec{k}_\perp + \vec{p}_{R\perp}}$$

$$\langle p_{R\perp}^2 \rangle_r = \underbrace{\frac{\alpha_s N_c L}{\pi} \int \frac{d\omega}{\omega} \int dk_\perp^2 \frac{1}{k_\perp^2}}_{\text{Double-log}} \underbrace{\frac{1}{\lambda_R} \int d^2 l_\perp \frac{l_\perp^2}{\sigma_R} \frac{d\sigma_R}{d^2 l_\perp}}_{\hat{q}} = \langle k_\perp^2 \rangle .$$

Single scattering

- **Including virtual gluon emission**

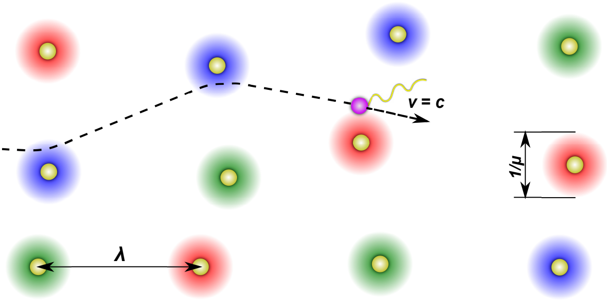


- **The final answer: with a double-log integration!**

$$\langle p_\perp^2 \rangle_{rad} = \underbrace{\frac{\alpha_s N_c L}{\pi} \int \frac{d\omega}{\omega} \int dk_\perp^2 \frac{1}{k_\perp^2}}_{\text{Double-log}} \underbrace{\frac{1}{\lambda_R} \int dl_\perp^2 \, \theta(k_\perp^2 - l_\perp^2) \frac{l_\perp^2}{\sigma_R} \frac{d\sigma_R}{dl_\perp^2}}_{\hat{q}}$$

Multiple scattering

- **Decorrelated multiple scattering:** $\lambda \gg \frac{1}{\mu}$



Short gluon formation time $z \equiv \frac{\omega}{k_\perp^2} \Rightarrow$ result in single scattering!

HOW SHORT ($z \lesssim \lambda$)? Answer: not always.

Multiple scattering

- **Integrating out medium particles** $\Leftrightarrow \hat{q}$
  - **Integrate out the medium particle by the following assumptions**
    - **Uncorrelated multiple scattering**

      $$\langle A^-(x,z)A^-(y,z')\rangle \propto \delta(z-z') \Rightarrow \text{Infrared cutoff } l_0$$

      The same assumption as that in the Langevin equation.
    - **Gaussian distribution of scatterers**

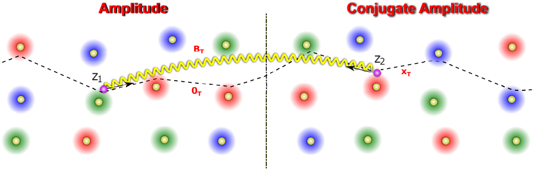
      $$\langle A^-A^-A^-A^- \cdots\rangle = \langle A^-A^-\rangle\langle A^-A^-\rangle \cdots$$

  The medium effects are described (only) by $\hat{q}$!

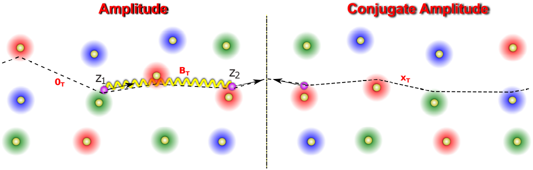
  - **Focus only on the hard probe: high energy quark(partons)**

Multiple scattering

- **Medium-induced gluon emission**

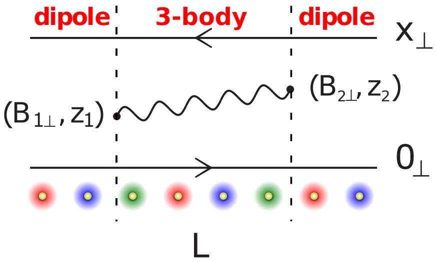


- **Medium-induced "self-energy"**



Multiple scattering

- **Using the dipole-like picture**



- **One needs to solve for 3-body propagator**

$$\left(i\frac{\partial}{\partial z} - H\right) G\left(B_\perp, z; B_{1\perp}, z_1\right) = i\delta(z - z_1)\delta(B_\perp - B_{1\perp})$$

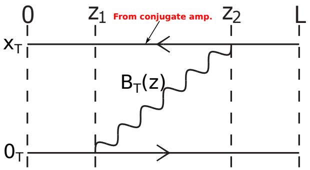
with

$$H \simeq -\frac{\nabla^2_{B_\perp}}{2\omega} - \frac{i}{4}\hat{q}_R\left\{x_\perp^2 + \frac{C_A}{2C_R}\left[B_\perp^2 + (B_\perp - x_\perp)^2 - x_\perp^2\right]\right\}$$

Multiple scattering

- **Real gluon emission**

$S_r(x_\perp) \equiv$



$$= 2\alpha_s C_F \mathrm{Re} \int \frac{d\omega}{\omega} \int_0^L dz(L-z)$$

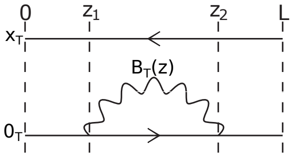
$$\times \nabla_{B_{1\perp}} \cdot \nabla_{B_{2\perp}} \Big[e^{-\frac{1}{4}\hat{q}x_\perp^2 (L-z)} G(B_{2\perp}, z_2; B_{1\perp}, z_1)$$

$$- G_0(B_{2\perp}, z_2; B_{1\perp}, z_1) \Big] \Big|_{B_{1\perp}=0, B_{2\perp}=x_\perp}$$

Multiple scattering

- **Medium-induced "self-energy"**

$$S_v(x_\perp) \equiv$$



$$= -2\alpha_s C_F \mathrm{Re} \int \frac{d\omega}{\omega} \int_0^L dz(L-z)$$

$$\times \nabla_{B_{1\perp}} \cdot \nabla_{B_{2\perp}} \Big[e^{-\frac{1}{4}\hat{q}x_\perp^2 (L-z)} G(B_{2\perp}, z_2; B_{1\perp}, z_1)$$

$$- G_0(B_{2\perp}, z_2; B_{1\perp}, z_1) \Big] \Big|_{B_{1\perp}=0=B_{2\perp}}$$

As a result, we have

$$S_{tot} = e^{-\frac{1}{4}\hat{q}Lx_\perp^2} + S_r(x_\perp) + S_v(x_\perp) + \cdots$$

with

$$S_r(0) + S_v(0) = 0 \Leftarrow \text{Probability conservation!}$$

Multiple scattering

- **Correction from radiation (real + virtual gluons)**

$$\langle p_\perp^2 \rangle_{rad} = -\nabla^2 \left[S_r(x_\perp) + S_v(x_\perp)\right]\Big|_{x_\perp=0}$$
$$= \mathrm{Re}\frac{i\alpha_s N_c}{\pi}\int d\omega \int_{t_0}^{L} dz\, \frac{L-z}{z^3}\left\{\left(\frac{\omega_0 z}{\sin\omega_0 z}\right)^3\left[4-\sin^2\omega_0 z\right]-4\right\}$$

where

$$\omega_0 = \frac{1+i}{\sqrt{2}}\sqrt{\frac{\hat{q}}{\omega}}$$

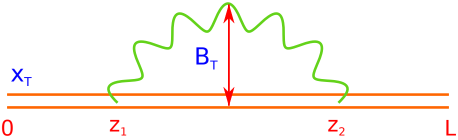
- **Short formation time means:** $z \lesssim \frac{1}{|\omega_0|} \simeq t_{form} = \sqrt{\frac{\omega}{\hat{q}}}$

$$\langle p_\perp^2 \rangle_{rad} \simeq \frac{\alpha_s N_c}{\pi}\hat{q}L\int \frac{d\omega}{\omega}\int \frac{dz}{z}$$

$z \simeq \frac{\omega}{k_\perp^2} \Rightarrow$ **result from single scattering!**

Multiple scattering

- **Various kinetic regions**



$$z = z_2 - z_1 \sim \frac{\omega}{k_\perp^2} \sim \omega B_\perp^2, \quad t_{form} \sim \sqrt{\frac{\omega}{\hat{q}}}, \quad x_\perp^2 \sim \frac{1}{\hat{q}L}.$$

  - **Single scattering**

$$z \lesssim t_{form} \Leftrightarrow B_\perp^2 \lesssim \frac{1}{\hat{q}z} \Leftrightarrow \omega \gtrsim \hat{q}z^2$$

  - **Double log region**

$$B_\perp^2 \gtrsim x_\perp^2 \Leftrightarrow \omega \lesssim \hat{q}Lz$$

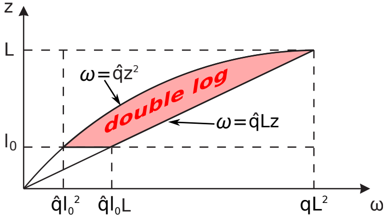
  - **l_0: the scale of the validity of the formalism**

    Cold matter: size of the nucleons; Hot matter: inverse of the temperature.

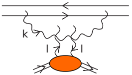
Multiple scattering

- **Setting limits of integration:** **Multiple scattering**

- **Double log region**



- **Double logarithmical result:** $B_\perp^2 \gtrsim 1/\hat{q}L$

$\langle p_\perp^2 \rangle_{rad} =$ 

$$= \frac{\alpha_s N_c}{\pi} \hat{q}L \int_{l_0}^{L} \frac{dz}{z} \int_{\hat{q}z^2}^{\hat{q}Lz} \frac{d\omega}{\omega}$$

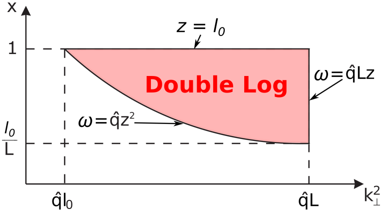
$$= \frac{\alpha_s N_c}{8\pi} \hat{q}L \log^2 \left(\frac{L}{l_0}\right)^2$$

- **Resummation of Double Logs**

- **Transformation**

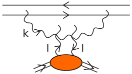
$$\begin{cases} x = \frac{l_0}{z} \\ k_\perp^2 = \frac{\omega}{z} \end{cases}$$

- **Double log region**



- **Double logarithmical result: again**

$$\langle p_\perp^2 \rangle_{rad} =$$

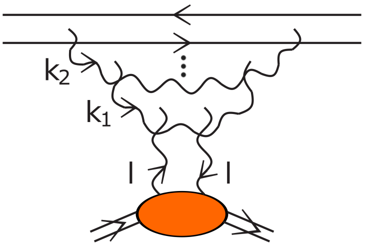


$$= \frac{\alpha_s N_c}{\pi} \hat{q} L \int_{\hat{q} l_0}^{\hat{q} L} \frac{dk_\perp^2}{k_\perp^2} \int_{\frac{\hat{q} l_0}{k_\perp^2}}^{1} \frac{dx}{x}$$

$$= \frac{\alpha_s N_c}{4\pi} \hat{q} L \frac{1}{1!2!} \log^2 \left(\frac{L}{l_0} \right)^2$$

Multiple scattering

- **Resummation of Double Logs**
  - **Many gluon emission**



  - **Leading double log region:**
    - **Formation time:** $x_1 \geq x_2 \geq x_3 \geq \cdots$
    - **Transverse momentum:** $k_{1\perp} \leq k_{2\perp} \leq k_{3\perp} \leq \cdots$
    - **"Single scatter":** $x_1 \geq \frac{\hat{q} l_0}{{k_{1\perp}}^2}, x_2 \geq \frac{\hat{q} l_0}{{k_{2\perp}}^2}, x_3 \geq \frac{\hat{q} l_0}{{k_{3\perp}}^2} \cdots$

Multiple scattering

- **Resummation of Double Logs**

  For n gluon emission

$$\langle p_\perp^2 \rangle_{ng} = \hat{q} L \frac{1}{n!(n+1)!} \left[\frac{\alpha N_c}{4\pi} \log^2 \frac{L^2}{l_0^2} \right]^n$$

  As a result

$$\langle p_\perp^2 \rangle = \hat{q} L \sqrt{\frac{4\pi}{\alpha_s N_c}} \frac{1}{\ln \frac{L^2}{l_0^2}} I_1 \left[\sqrt{\frac{\alpha_s N_c}{\pi}} \ln \frac{L^2}{l_0^2} \right]$$

and

$$S(x_\perp) = 1 - \frac{1}{4} \langle p_\perp^2 \rangle x_\perp^2 + O(x_\perp^4)$$

Conclusions and Perspectives

1. **Radiative $p_\perp$-broadening is double logarithmically enhanced**

$$\langle p_\perp^2 \rangle_{rad} = \frac{\alpha_s N_c}{8\pi} \hat{q} L \log^2\left(\frac{L}{l_0}\right)^2$$

2. **Leading double log can be resummed!**

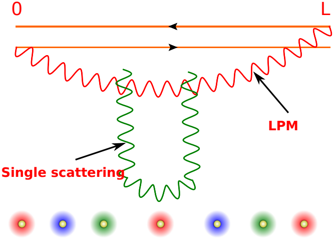
$$\boxed{\langle p_\perp^2 \rangle = \hat{q} L \sqrt{\frac{4\pi}{\alpha_s N_c}} \frac{1}{\ln \frac{L^2}{l_0^2}} I_1\left[\sqrt{\frac{\alpha_s N_c}{\pi}} \ln \frac{L^2}{l_0^2}\right] \simeq \left[1 + \frac{\alpha_s N_c}{8\pi} \log^2\left(\frac{L}{l_0}\right)^2\right] \hat{q} L}$$

3. **Imply a parametrical large radiative energy loss at NLO?**

Positive!

Conclusions and Perspectives

1. **Radiative energy loss at NLO**



$$\Delta E \sim \alpha_s N_c \omega_c \times \alpha_s N_c \ln^2 \frac{L}{l_0} \simeq (\alpha_s N_c)^2 \hat{q} L^2 \ln^2 \frac{L}{l_0}.$$

In progress with Yacine, Jean-Paul, Fabio, Edmond· · ·