# How the heck is it possible that a system emitting only a dozen particles can be described by fluid dynamics?

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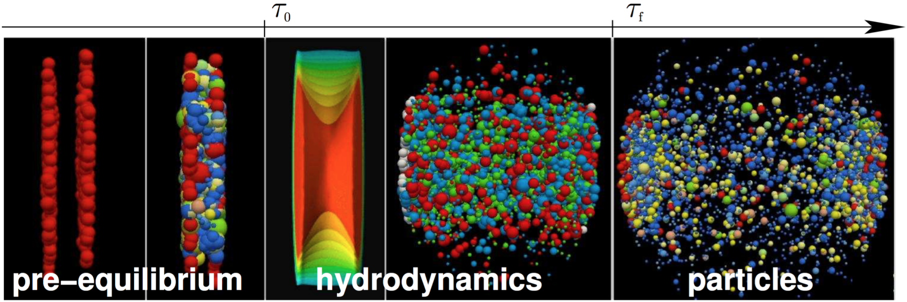
**Saclay/Orsay Heavy Ion Meeting**

**IPN Orsay, June 7, 2018**

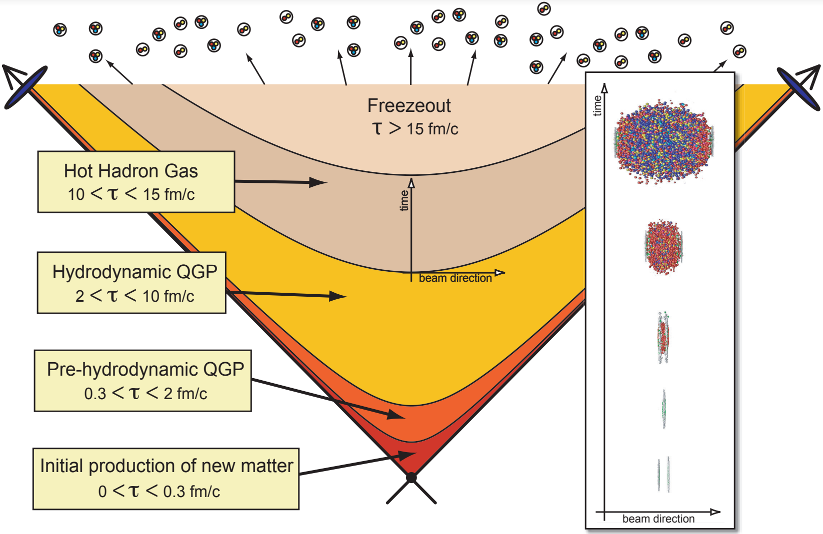
BEST COLLABORATION JETSCAPE

# Overview

# Evolution of a heavy-ion collision



# Space-time diagram of a heavy-ion collision




(After M. Strickland, arXiv:1410.5786)

# Overview

# Hydrodynamic tools for heavy-ion collisions

- Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions

## Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity $\eta$, neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = \big(e(x)+p(x)\big)u^\mu(x)u^\nu(x) - g^{\mu\nu}p(x) + \pi^{\mu\nu}.$$

$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to $2\eta$ times the shear tensor $\nabla^{\langle\mu}u^{\nu\rangle}$ on a microscopic kinetic time scale $\tau_\pi$:

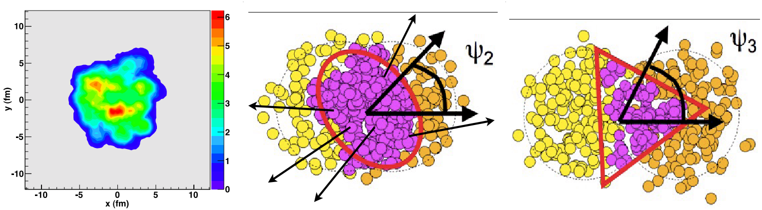
$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) + \dots$$

where $D \equiv u^\mu\partial_\mu$ is the time derivative in the local rest frame.

Kinetic theory relates $\eta$ and $\tau_\pi$, but for a strongly coupled QGP neither $\eta$ nor this relation are known $\Longrightarrow$ treat $\eta$ and $\tau_\pi$ as independent phenomenological parameters.
For consistency: $\tau_\pi\theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

# Event-by-event shape and flow fluctuations rule!

(Alver and Roland, PRC81 (2010) 054905)

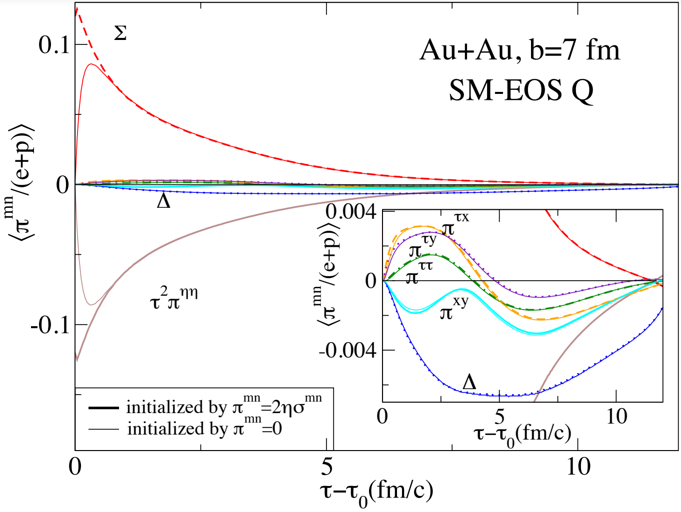



- Each event has a different initial shape and density distribution, characterized by different set of harmonic eccentricity coefficients $\varepsilon_n$
- Each event develops its individual hydrodynamic flow, characterized by a set of harmonic flow coefficients $v_n$ and flow angles $\psi_n$
- At small impact parameters fluctuations ("hot spots") dominate over geometric overlap effects (Alver & Roland, PRC81 (2010) 054905; Qin, Petersen, Bass, Müller, PRC82 (2010) 064903)

Definition of flow coefficients:

$$\frac{dN^{(i)}}{dy\,p_T dp_T\,d\phi_p}(b) = \frac{dN^{(i)}}{dy\,p_T dp_T}(b)\left(1 + 2\sum_{n=1}^{\infty} \boldsymbol{v_n^{(i)}(y, p_T; b)}\cos(\phi_p - \Psi_n^{(i)})\right).$$

# Large shear stress throughout the QGP phase!

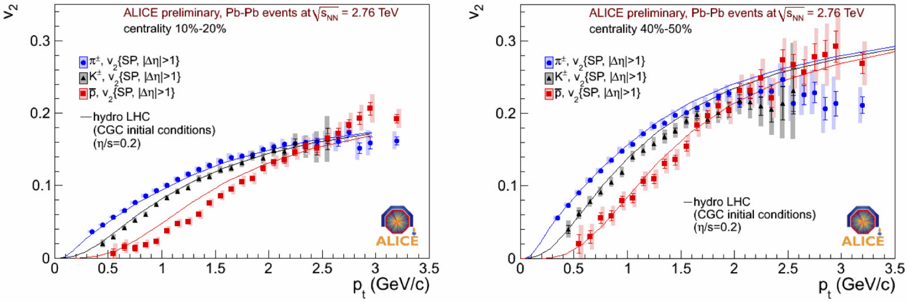



VISH2+1 (from H. Song's PhD thesis (2009))

# Hydrodynamics – a theory with predictive power

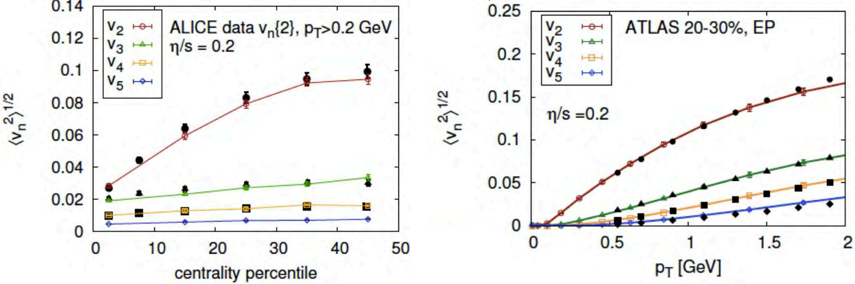
After tuning initial conditions and viscosity at RHIC to obtain a good description of all soft hadron data simultaneously (Song et al. 2010) the first LHC spectra and elliptic flow measurements were successfully **pre**dicted:

ALICE, Quark Matter 2011 (VISH2+1 prediction: Shen et al., PRC84 (2011) 044903)



# Towards a Standard Model of the Little Bang

B. Schenke: QM2012




Schenke, Tribedy, Venugopalan, Phys.Rev.Lett. 108:25231 (2012)

With inclusion of sub-nucleonic quantum fluctuations and pre-equilbrium dynamics of gluon fields:

→ outstanding agreement between data and model

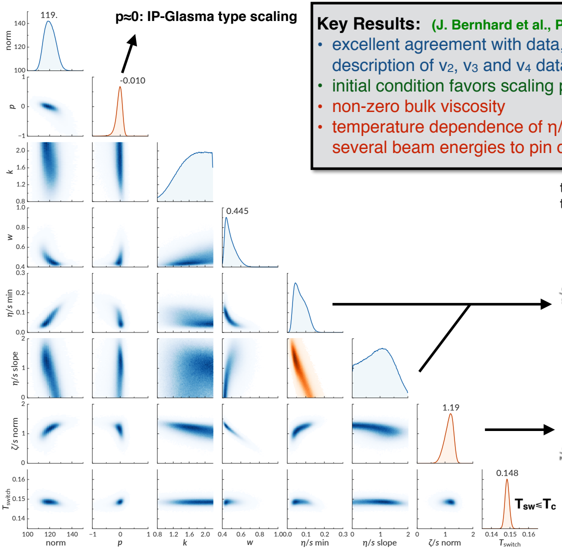
**Rapid convergence on a standard model of the Little Bang!**

Perfect liquidity reveals in the final state initial-state gluon field correlations of size $1/Q_s$ (sub-hadronic)!

# Hydrodynamic tools for heavy-ion collisions

- Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions
- It has been successfully used in a Bayesian analysis of LHC Pb+Pb collision data for putting meaningful constraints on the initial conditions and medium properties of QGP created in heavy-ion collisions:

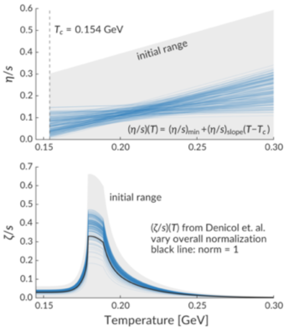
# Calibrated Posterior Distribution




**Key Results:** (J. Bernhard et al., PRC94 (2016) 024907)

- excellent agreement with data, simultaneous description of $v_2$, $v_3$ and $v_4$ data
- initial condition favors scaling properties of IP-Glasma
- non-zero bulk viscosity
- temperature dependence of η/s requires data at several beam energies to pin down

**temperature-dependent viscosities from the calibrated posterior:**



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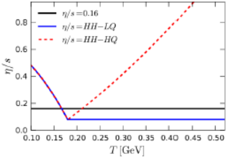
- It has been successfully used in a Bayesian analysis of LHC Pb+Pb collision data for putting meaningful constraints on the initial conditions and medium properties of QGP created in heavy-ion collisions:
- It works even in "small" collision systems:

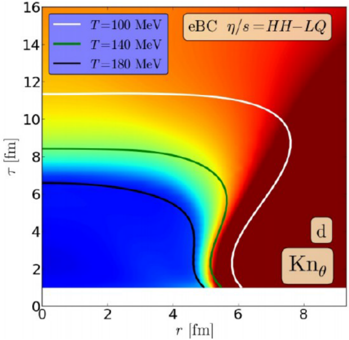
# "Small" systems aren't really small:

Niemi & Denicol, arXiv:1404.7327

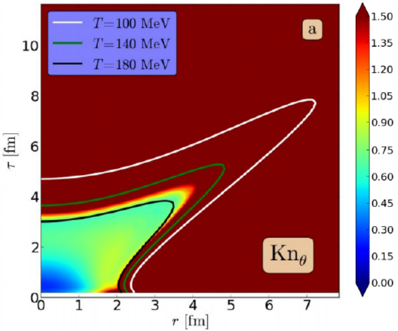
$$\mathrm{Kn} = \tau_{\mathrm{micro}}\,\theta = \tau_{\mathrm{micro}} / \tau_{\mathrm{macro}}$$




**Pb+Pb**



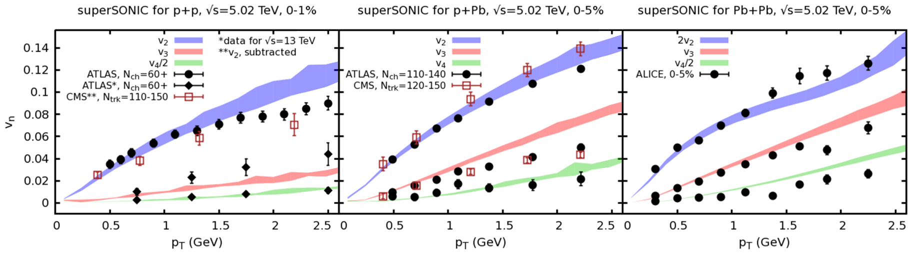

**p+Pb**




**At freeze-out, collisions with similar charged multiplicity $dN_{\mathrm{ch}}/d\eta$ have similar freeze-out volumes!**

# Flow in Pb+Pb, p+Pb and even p+p at the LHC!

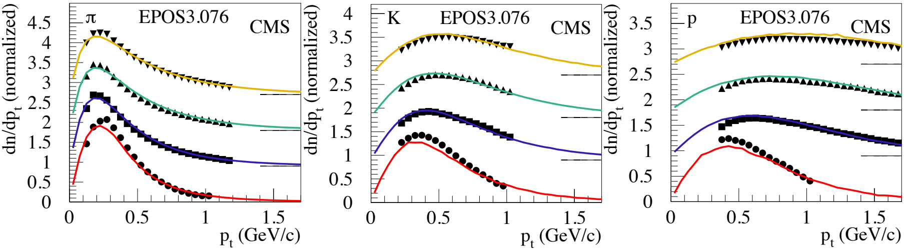
R.D. Weller, P. Romatschke, Phys. Lett. B 774 (2017) 351




Requires fluctuating proton substructure (gluon clouds clustered around valence quarks (K. Welsh et al. PRC94 (2016) 024919))

# Radial flow in pp collisions at the LHC

Werner, Guiot, Karpenko, Pierog (EPOS3), PRC 89 (2014) 064903;
Data: CMS Collaboration (8, 84, 160, 235 charged tracks)



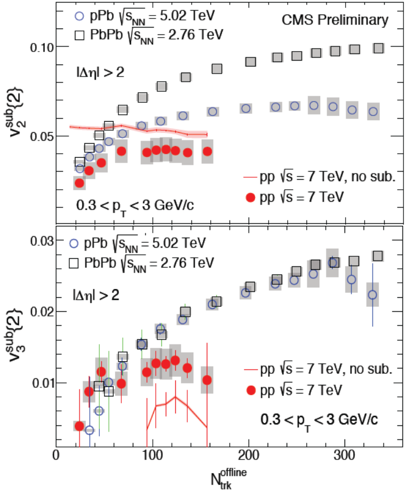

Elliptic flow (double ridge) discovered in high-multiplicity pp by CMS at 7 TeV (and confirmed by ATLAS at 13 TeV) also reproduced by EPOS.

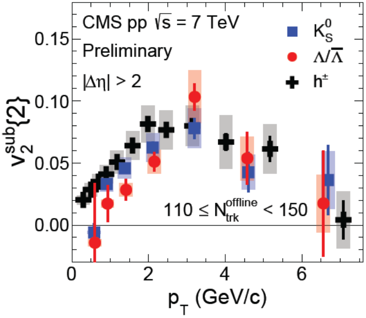
# Everything flows? Why? How?

CMS Collaboration, Quark Matter 2015

Z. Chen    CMS-HIN-15-009

Flow parameter analysis




- $v_2(\mathrm{pp}) < v_2(\mathrm{pPb}) < v_2(\mathrm{PbPb})$
- $v_3(\mathrm{pp}) \approx v_3(\mathrm{pPb}) \approx v_3(\mathrm{PbPb})$, but $v_3(\mathrm{pp})$ deviates for $N_{trk}^{offline} \gtrsim 90$
- Mass ordering for $v_2^{sub}\{2\}$ at low $p_T$

# Questions about the hydrodynamic picture

- Why does it work?
- How does it work?
- Where does it stop working?
- What about lower energies? Will it work without creation of a QGP?
- What about smaller collision systems? What is the smallest droplet of strongly interacting matter at a given collision energy that behaves hydrodynamically?
- Can we modify the theory to make it work even better?

Innumerable studies of relativistic viscous fluid dynamics have been made in the last decade; reviewing them and the conclusions they yield would take an entire semester course. Let me pick out a small subset that address the "unreasonable effectiveness" of the hydrodynamic framework that we have witnessed.

# Overview

# Heavy-ion collisions provide a particular challenge:

- Relativistic viscous hydrodynamics has become the workhorse of dynamical modeling of ultra-relativistic heavy-ion collisions
- However, the kinematics of ultra-relativistic heavy-ion collisions introduces a complication that severely limits the applicability of standard viscous relativistic fluid dynamics:
  **large viscous stresses caused by large initial anisotropies between the longitudinal and transverse expansion rates and by critical dynamics near the quark-hadron phase transition**

# Hydrodynamics from kinetic theory

Both simultaneously valid if weakly coupled and small pressure gradients.
Form of hydro equations remains unchanged for strongly coupled systems.

**Boltzmann Equation in Relaxation Time Approximation (RTA):**

$$p^\mu \partial_\mu f(x,p) = C(x,p) = \frac{p\cdot u(x)}{\tau_{\rm rel}(x)}\Big(f_{\rm eq}(x,p) - f(x,p)\Big)$$

For conformal systems $\tau_{\rm rel}(x) = c/T(x) = 5\eta/(\mathcal{S}\,T) \equiv 5\bar\eta/T(x)$.

**Macroscopic currents:**

$$j^\mu(x) = \int_p p^\mu\, f(x,p) \equiv \langle p^\mu \rangle; \qquad T^{\mu\nu}(x) = \int_p p^\mu\, p^\nu\, f(x,p) \equiv \langle p^\mu p^\nu \rangle$$

where $$\int_p \cdots \equiv \frac{g}{(2\pi)^3}\int \frac{d^3p}{E_p} \cdots \equiv \langle \dots \rangle$$

# Hydrodynamics for strongly anisotropic expansion:

**Account for large viscous flows by including their effect already at leading order in the Chapman-Enskog expansion:**

Expand the solution $f(x,p)$ of the Boltzmann equation as

$$f(x,p) = f_0(x,p) + \delta f(x,p) \qquad \Big(\big|\delta f/f_0\big| \ll 1\Big),$$

$$f_0(x,p) = f_0\left(\frac{\sqrt{p_\mu \Omega^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\tilde{T}(x)}\right),$$

where $p_\mu \Omega^{\mu\nu}(x) p_\nu = m^2 + (1{+}\xi_\perp(x))p^2_{\perp,\mathrm{LRF}} + (1{+}\xi_L(x))p^2_{z,\mathrm{LRF}}$

- $\tilde{T}(x)$, $\tilde{\mu}(x)$ are the effective temperature and chemical potential in the LRF, Landau matched to energy and particle density, $e$ and $n$.
- $\xi_{\perp,L}$ parametrize the momentum anisotropy in the LRF, Landau matched to the transverse and longitudinal pressures, $P_\perp$ and $P_L$. (McNelis, Bazow, UH, arXiv:1803.01810)
- $P_\perp$ and $P_L$ encode the bulk viscous pressure, $\Pi = (2P_\perp{+}P_L)/3 - P_{\mathrm{eq}}$, and the largest shear stress component, $P_L{-}P_\perp$.

# A variety of hydrodynamic approximations:

Different hydrodynamic approaches can be characterized by the different assumptions they make about the dissipative corrections and/or the different approximations they use to derive their dynamics from the underlying Boltzmann equation:

- **Ideal hydro:** local momentum isotropy ($\xi_{\perp,L} = 0$), $\Pi^{\mu\nu} = V^\mu = 0$.
- **Navier-Stokes (NS) theory:** local momentum isotropy ($\xi_{\perp,L} = 0$), ignores microscopic relaxation time by postulating instantaneous constituent relations for $\Pi^{\mu\nu}$, $V^\mu$.
- **Israel-Stewart (IS) theory:** local momentum isotropy ($\xi_{\perp,L} = 0$), evolves $\Pi^{\mu\nu}$, $V^\mu$ dynamically, keeping only terms linear in $\mathrm{Kn} = \lambda_{\mathrm{mfp}}/\lambda_{\mathrm{macro}}$
- **Denicol-Niemi-Molnar-Rischke (DNMR) theory:** improved **IS theory** that keeps nonlinear terms up to order $\mathrm{Kn}^2$, $\mathrm{Kn} \cdot \mathrm{Re}^{-1}$ when evolving $\Pi^{\mu\nu}$, $V^\mu$.
- **Third-order Chapman-Enskog expansion (Jaiswal 2013):** local momentum isotropy ($\xi_{\perp,L} = 0$), keeping terms up to third order when evolving $\Pi^{\mu\nu}$, $V^\mu$.
- **Anisotropic hydrodynamics (aHydro):** allows for leading-order local momentum anisotropy ($\xi_{\perp,L} \neq 0$), evolved according to equations obtained from low-order moments of BE, but ignores residual dissipative flows: $\Pi^{\mu\nu} = V^\mu = 0$.
- **Viscous anisotropic hydrodynamics (vaHydro):** improved **aHydro** that additionally evolves residual dissipative flows $\Pi^{\mu\nu}$, $V^\mu$ with **IS** or **DNMR theory**.

# Testing the various hydrodynamic approximations against exact solutions of the underlying microscopic dynamics

# BE for systems with highly symmetric flows: I. Bjorken flow

- Longitudinal boost invariance, transverse homogeneity ("physics on the light cone", no transverse flow) $\Longrightarrow$ $\boldsymbol{u^\mu = (1,0,0,0)}$ in Milne coordinates $(\tau, r, \phi, \eta)$ where $\tau = (t^2{-}z^2)^{1/2}$ and $\eta = \frac{1}{2}\ln[(t{-}z)/(t{+}z)]$ $\Longrightarrow$ $\boldsymbol{v_z = z/t}$
- Metric: $ds^2 = d\tau^2{-}dr^2 - r^2 d\phi^2 - \tau^2 d\eta^2$, $\qquad g_{\mu\nu} = \mathrm{diag}(1, -1, -r^2, -\tau^2)$
- Symmetry restricts possible dependence of distribution function $f(x,p)$ (Baym '84, Florkowski et al. '13, '14):

$$f(x,p) = f(\tau; p_\perp, w) \qquad \text{where} \qquad w = tp_z - zE = \tau m_\perp \sinh(y{-}\eta).$$

- RTA BE simplifies to ordinary differential equation

$$\partial_\tau f(\tau; p_\perp, w) = -\frac{f(\tau; p_\perp, w) - f_{\mathrm{eq}}(\tau; p_\perp, w)}{\tau_{\mathrm{rel}}(\tau)}.$$

- **Solution:**

$$f(\tau; p_\perp, w) = D(\tau,\tau_0) f_0(p_\perp, w) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\mathrm{rel}}(\tau')} D(\tau,\tau')\, f_{\mathrm{eq}}(\tau'; p_\perp, w)$$

where $\qquad D(\tau_2,\tau_1) = \exp\left(-\int_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\mathrm{rel}}(\tau'')}\right).$

# BE for systems with highly symmetric flows: II. Gubser flow

- Longitudinal boost invariance, azimuthally symmetric radial dependence ("physics on the light cone" with azimuthally symmetric transverse flow)
  (Gubser '10, Gubser & Yarom '11)
  $\Longrightarrow \boldsymbol{u^\mu = (1,0,0,0)}$ in de Sitter coordinates $(\rho, \theta, \phi, \eta)$ where
  $\rho(\tau, r) = -\sinh^{-1}\left(\frac{1-q^2\tau^2+q^2r^2}{2q\tau}\right)$ and $\theta(\tau, r) = \tan^{-1}\left(\frac{2qr}{1+q^2\tau^2-q^2r^2}\right)$.
- $\Longrightarrow \boldsymbol{v_z = z/t}$ and $\boldsymbol{v_r = \frac{2q^2\tau r}{1+q^2\tau^2+q^2r^2}}$ where $q$ is an arbitrary scale parameter.
- Metric: $d\hat{s}^2 = ds^2/\tau^2 = d\rho^2 - \cosh^2\rho\,(d\theta^2 + \sin^2\theta\, d\phi^2) - d\eta^2$,
  $g_{\mu\nu} = \mathrm{diag}(1,\ -\cosh^2\rho,\ -\cosh^2\rho\,\sin^2\theta,\ -1)$
- Symmetry restricts possible dependence of distribution function $f(x,p)$

$$f(x,p) = f(\rho; \hat{p}_\Omega^2, \hat{p}_\eta) \quad \text{where} \quad \hat{p}_\Omega^2 = \hat{p}_\theta^2 + \frac{\hat{p}_\phi^2}{\sin^2\theta} \quad \text{and} \quad \hat{p}_\eta = w.$$

- With $T(\tau, r) = \hat{T}(\rho(\tau, r))/\tau$ RTA BE simplifies to the ODE

$$\frac{\partial}{\partial\rho} f(\rho; \hat{p}_\Omega^2, \hat{p}_\varsigma) = -\frac{\hat{T}(\rho)}{c}\left[f\left(\rho; \hat{p}_\Omega^2, \hat{p}_\varsigma\right) - f_{\rm eq}\left(\hat{p}^\rho/\hat{T}(\rho)\right)\right].$$

- **Exact solution (formally similar to an analogous solution for Bjorken flow):**
  $f(\rho; \hat{p}_\Omega^2, w) = D(\rho, \rho_0) f_0(\hat{p}_\Omega^2, w) + \frac{1}{c}\int_{\rho_0}^{\rho} d\rho'\, \hat{T}(\rho')\, D(\rho, \rho')\, f_{\rm eq}(\rho'; \hat{p}_\Omega^2, w)$

# Hydrodynamic equations for systems with Gubser flow:

- The exact solution for $f$ can be worked out for any "initial" condition $f_0(\hat{p}^2_\Omega, w) \equiv f(\rho_0; \hat{p}^2_\Omega, w)$. We here use equilibrium initial conditions, $f_0 = f_{\mathrm{eq}}$.

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- By taking hydrodynamic moments, the exact $f$ yields the exact evolution of all components of $T^{\mu\nu}$. Here, $\Pi^{\mu\nu}$ has only one independent component, $\pi^{\eta\eta}$.

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- By taking hydrodynamic moments, the exact $f$ yields the exact evolution of all components of $T^{\mu\nu}$. Here, $\Pi^{\mu\nu}$ has only one independent component, $\pi^{\eta\eta}$.
- This exact solution of the BE can be compared to solutions of the various hydrodynamic equations in de Sitter coordinates, using identical initial conditions.
  - **Ideal:** $\hat{T}_{\rm ideal}(\rho) = \frac{\hat{T}_0}{\cosh^{2/3}(\rho)}$
  - **NS:** $\frac{1}{\hat{T}}\frac{d\hat{T}}{d\rho} + \frac{2}{3}\tanh\rho = \frac{1}{3}\bar{\pi}(\rho)\tanh\rho$ (viscous $T$-evolution) with $\bar{\pi} \equiv \hat{\pi}^\eta_\eta/(\hat{T}\hat{\mathcal{S}})$ and $\hat{\pi}_{NS} = \frac{4}{3}\hat{\eta}\tanh\rho = \frac{4}{15}\hat{\tau}_{\rm rel}\tanh\rho$
  - **IS:** $\frac{d\bar{\pi}}{d\rho} + \frac{\bar{\pi}}{\hat{\tau}_{\rm rel}} = \frac{4}{15}\tanh\rho - \frac{4}{3}\bar{\pi}^2\tanh\rho$
  - **DNMR:** $\frac{d\bar{\pi}}{d\rho} + \frac{\bar{\pi}}{\hat{\tau}_{\rm rel}} = \frac{4}{15}\tanh\rho + \frac{10}{21}\bar{\pi}\tanh\rho - \frac{4}{3}\bar{\pi}^2\tanh\rho$
  - **3rd-order CE:** $\frac{d\bar{\pi}}{d\rho} + \frac{\bar{\pi}}{\hat{\tau}_{\rm rel}} = \frac{4}{15}\tanh\rho + \frac{10}{21}\bar{\pi}\tanh\rho - \frac{412}{147}\bar{\pi}^2\tanh\rho$
  - **aHydro:** see M. Nopoush et al., PRD 91 (2015) 045007
  - **vaHydro:** $\frac{d\bar{\pi}}{d\rho} + \frac{\bar{\pi}}{\hat{\tau}_{\rm rel}} = \frac{5}{12}\tanh\rho + \frac{4}{3}\bar{\pi}\tanh\rho - \frac{4}{3}\bar{\pi}^2\tanh\rho - \frac{4}{3}\mathcal{F}(\bar{\pi})$

(M. Martinez et al., PRC 95 (2017) 054907)

# Exact BE vs. hydrodynamic approximations

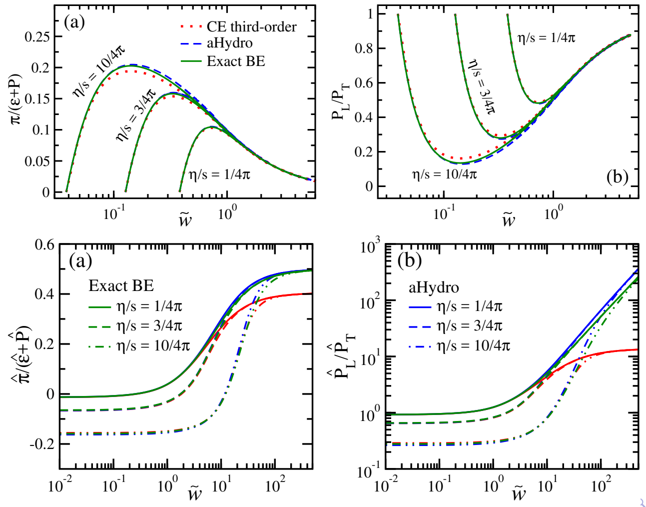
Chattopadhyay, UH, Pal, Vujanovic, arXiv:1801.07755

**Bjorken:**

$$\tilde{w} = \frac{\tau\, T(\tau)}{4\pi\eta/s}$$

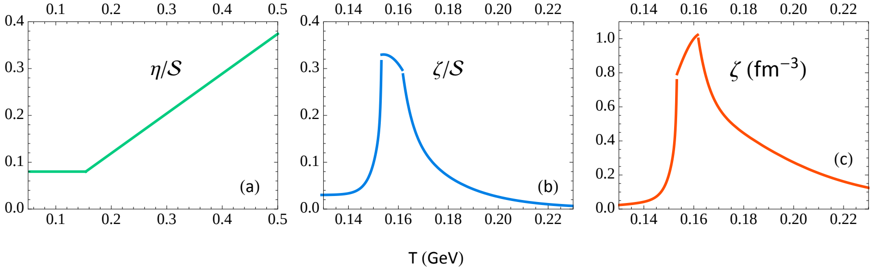
**Gubser:**

$$\tilde{w} = \frac{2\tanh\rho}{\hat{T}(\rho)}\,\frac{4\pi\eta}{s}$$



# vHydro vs. aHydro for (0+1)-d Bjorken flow with bulk visc.

McNelis, Bazow, UH, arXiv:1803.01810



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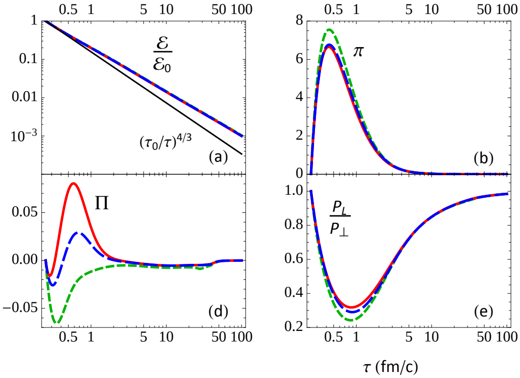
McNelis, Bazow, UH, arXiv:1803.01810

**Equilibrium IC:**

red solid: aHydro

blue long-dashed: vHydro

green short-dashed: vHydro with different transport coefficients




Using transport coefficients from the same microscopic theory, standard viscous and anisotropic hydrodynamic evolutions are very similar in (0+1)-d, even for large viscous stresses.

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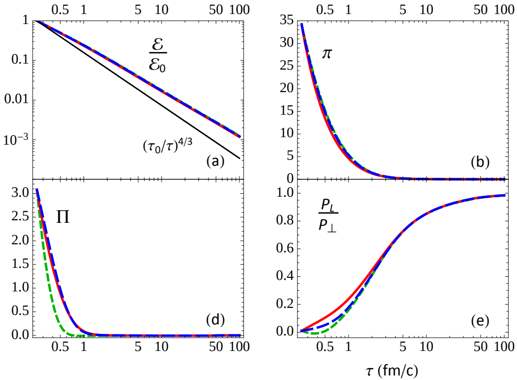


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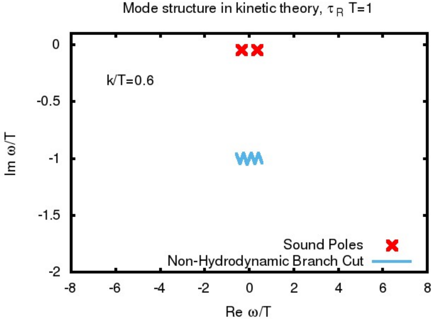
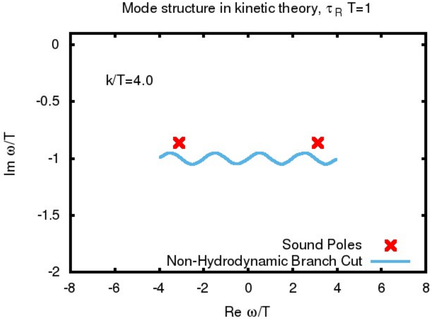
**(3+1)-d with fluctuating initial conditions? Stay tuned!**

# Breakdown of hydrodynamics at short length scales

Romatschke, EPJC 76 (2016) 352; Romatschke[2], arXiv:1712.05815




- Hydrodynamic (sound) poles disappear for $k/T > 4.5$
- Assuming $T_i \sim 600$ MeV at LHC energies (could be higher in high-multiplicity *pp*)
  $\Longrightarrow$ **short wavelength structures with $\lambda < 0.5$ fm are no longer accurately evolved with hydrodynamics**

# Overview

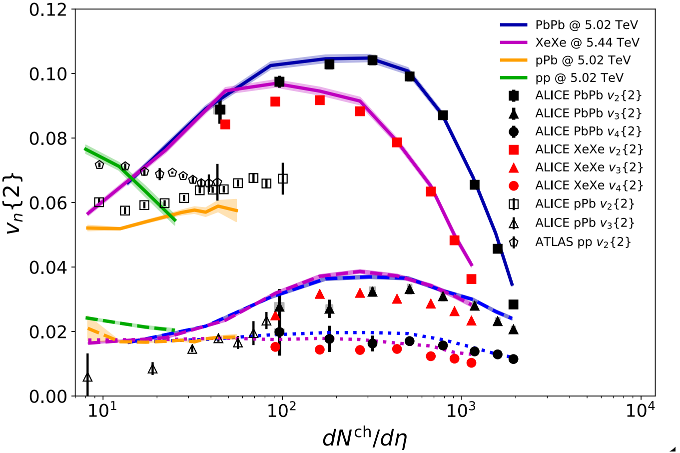
# Summary

- Viscous relativistic hydrodynamics provides a **robust, reliable, efficient and accurate** description of QGP evolution in heavy-ion collisions.
- It is valid even when the expansion is fast and highly anisotropic, causing large local momentum anisotropies $\Longrightarrow$ **local momentum isotropy and thermalization not strictly required**.
- While first-order viscous corrections are large in nuclear collisions, especially in small systems, they can be handled efficiently in an **optimized anisotropic hydrodynamic approach** that accounts for local momentum anisotropies at leading order; residual dissipative flows remain small.
- **New exact solutions of the Boltzmann equation** enable powerful tests of the efficiency and accuracy of various hydrodynamic expansion schemes, providing strong support for the **validity and robustness** of second-order viscous hydrodynamics (especially their anisotropic variants).
- Hydrodynamics appears to break down only at length scales significantly smaller than the proton size, $\boldsymbol{\lambda < \frac{4}{3}\tau_{\rm rel} \sim 0.5\,\mathrm{fm}}$ (assuming $\frac{\eta}{s} = \frac{2}{4\pi}$).

# Thank you!

# "One fluid that rules them all" (Weller & Romatschke 2017)

Schenke, Quark Matter 2018 (Schenke, Shen, Tribedy, in preparation)




**Except for pp, hydro describes all collision systems at all "centralities".**