Gluon TMDs in UPCs at low-x

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ÉCOLE POLYTECHNIQUE

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Reminder on transverse momentum dependent parton distribution functions (TMDs)

Primordial example: semi-inclusive DIS (SIDIS)

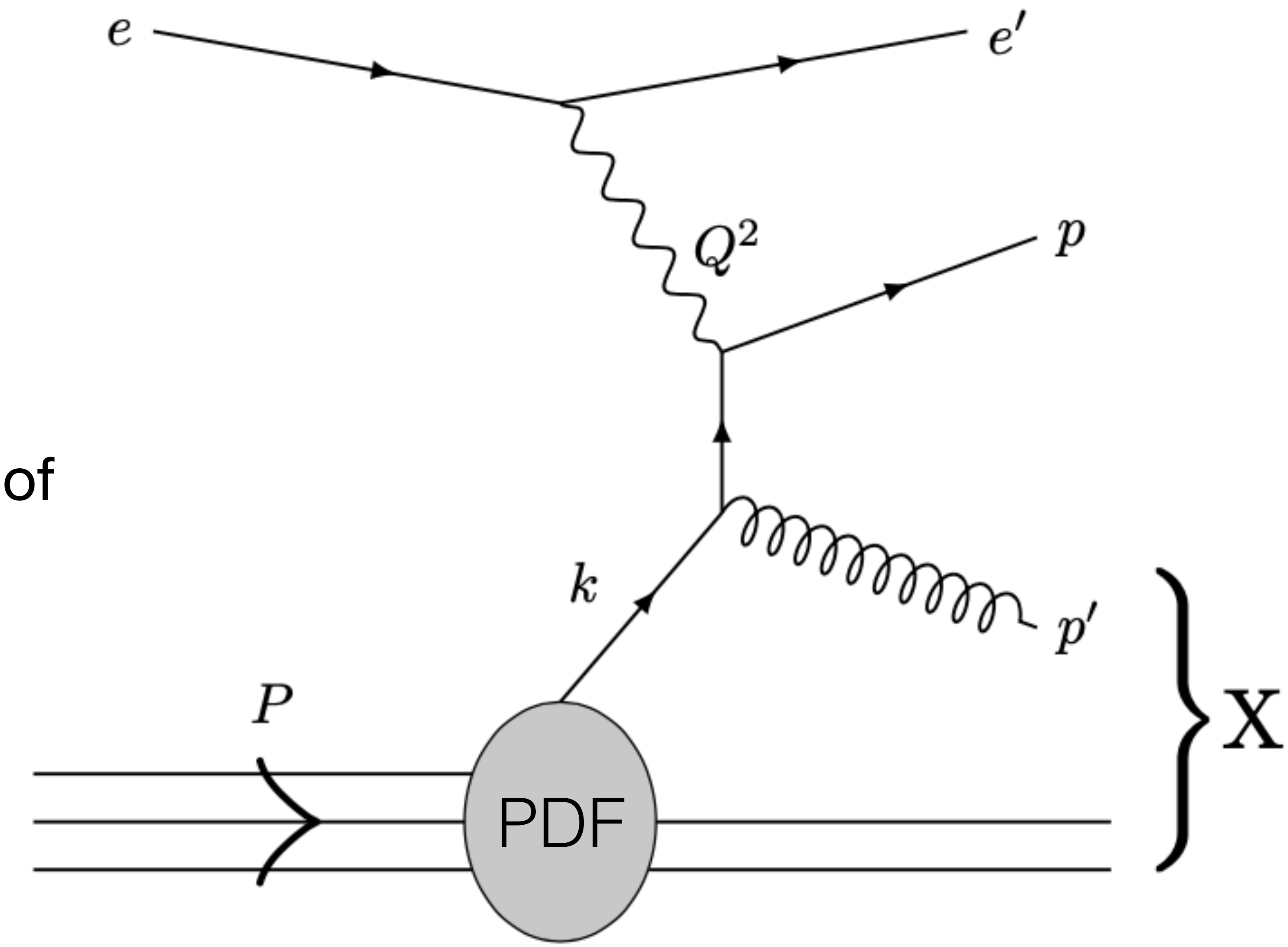
$Q^2 = -(e - e')^2$

is the hard scale which allows for a perturbative description ($\alpha_s(Q^2) \ll 1$)

Transverse momentum $p_\perp$ of the outgoing hadron (generated by recoil off unobserved parton) sets another scale

If $Q \sim p_\perp$, **collinear factorization** applies:

$$x = \frac{Q^2}{2P \cdot q} \simeq \frac{k^+}{P^+}$$



$$\sigma^{\gamma^* p \to hX} = \hat{\sigma}(Q^2) \otimes \sum_i f_i(x, Q^2) \otimes \sum_j D_j(z, Q^2)$$

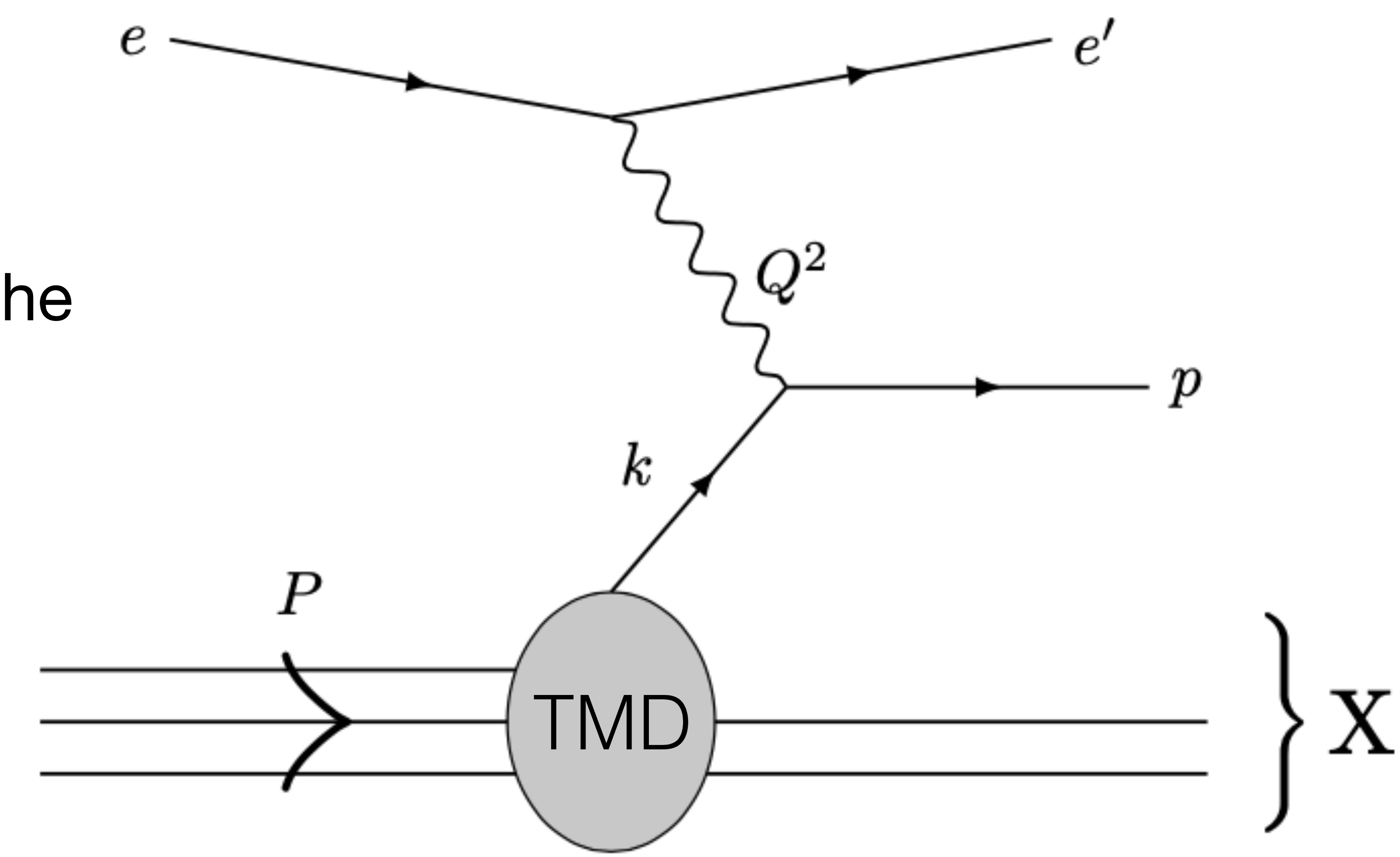
hard part **PDFs** **FFs**

DGLAP evolution

Primordial example: semi-inclusive DIS (SIDIS)

What if $p_\perp \ll Q$? If transverse momentum is generated by soft emissions or stems from the primordial motion of the parton inside the hadron?

Need to resum large logs and introduce transverse momentum dependent PDFs (TMDs)



TMD factorization:

$$\sigma^{\gamma^* p \to hX} = \hat{\sigma}(Q^2) \otimes \int d^2\mathbf{k}_\perp \sum_i f_i(x, k_\perp, Q^2) \otimes \sum_j D_j(z, p_\perp - k_\perp, Q^2)$$

hard part **TMD PDFs** **TMD FFs**

Collins-Soper-Sterman (CSS) evolution

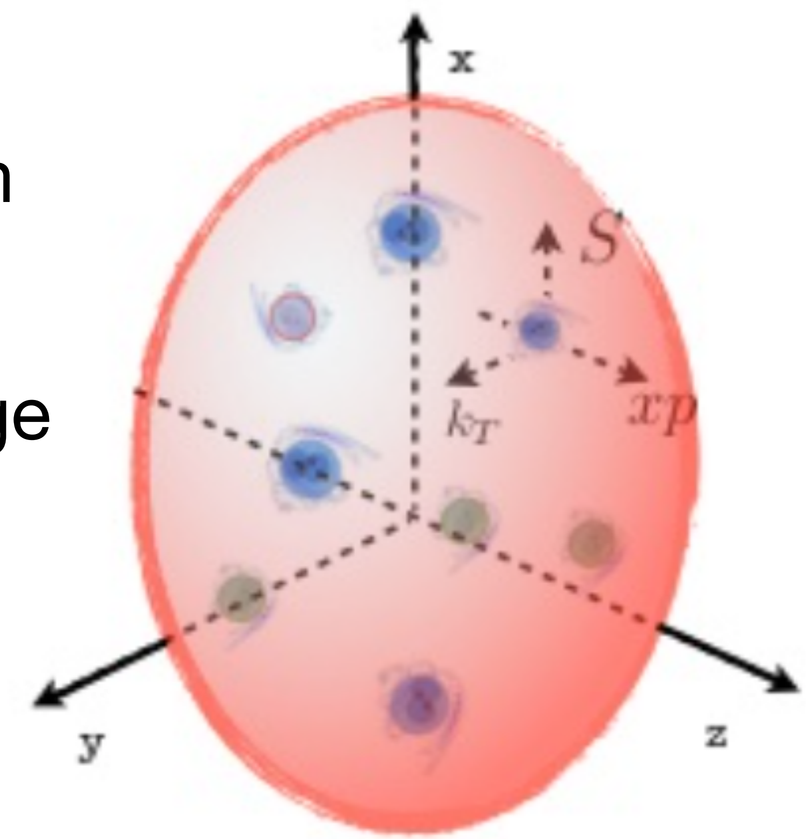
Transverse momentum dependent PDFs (TMDs)

PDFs parameterize longitudinal structure of hadron

TMDs parameterize 3D momentum structure + spin correlations

Many intricacies: process dependence due to gauge invariance, complicated evolution, etc.

Gluon TMDs are experimentally (almost) completely unknown! Play in important role in particular at low-*x* and in quarkonium production



GLUONS	*unpolarized*	*circular*	*linear*
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	h_{1T}^g, $h_{1T}^{\perp g}$

Mulders & Rodrigues (2001)
Collins (2011)
Angelez-Martinez et al. (2015)

Heavy-quark pair photoproduction at small x

QCD at high energies or small x

The *energy* is the largest scale in the process

$$s \gg Q^2$$

Need to resum large logarithms

$$\ln \frac{s}{Q^2} \simeq \ln \frac{1}{x} \gg 1$$

Low-x evolution equation (BFKL) predicts very fast growth of the gluon distribution:

$$x\mathcal{G}(x, Q^2) \simeq \frac{1}{x^{2.77\frac{\alpha_s N_c}{\pi}}}$$

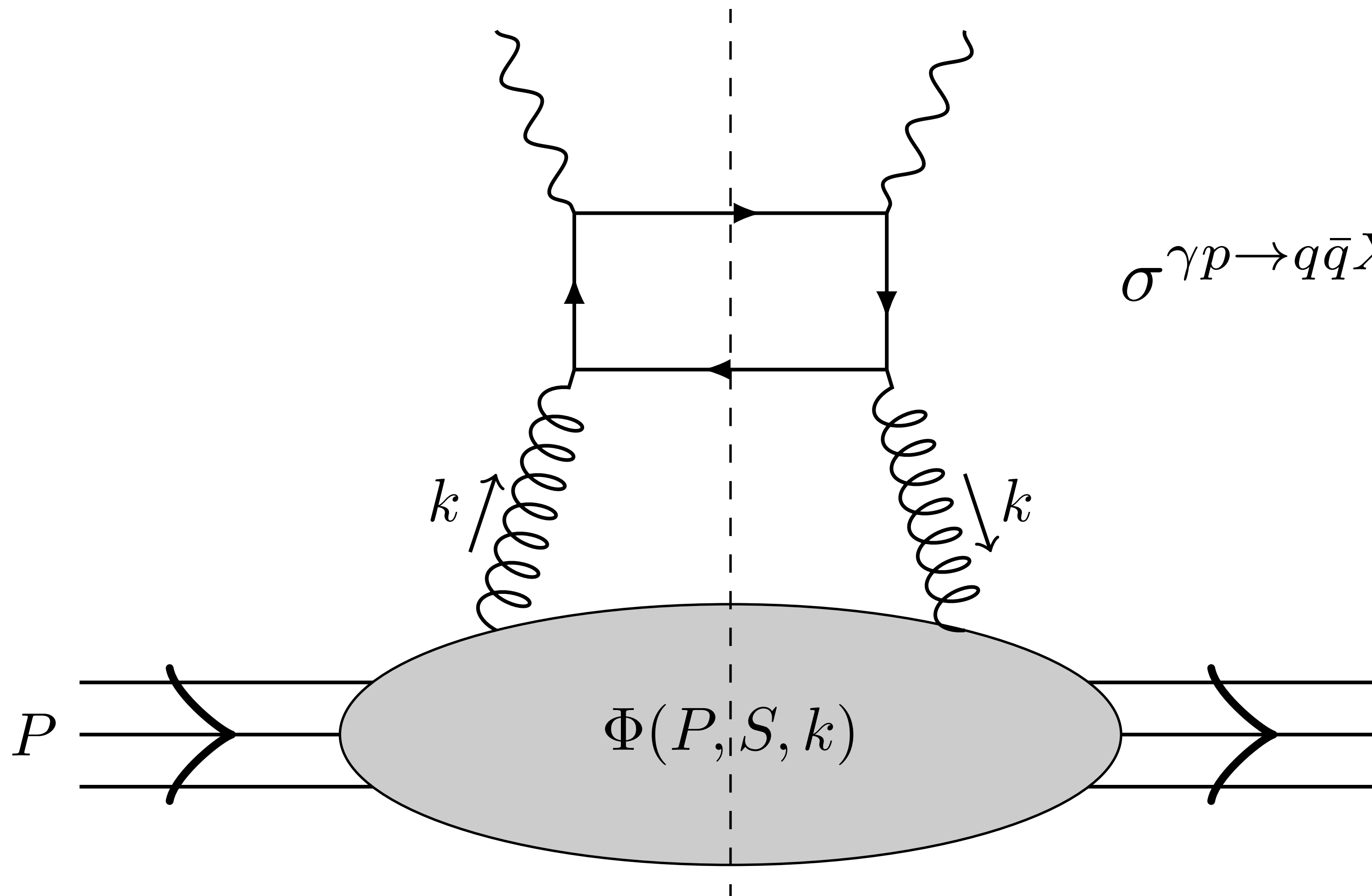
At high enough density, semiclassical regime is reached where

$$g_s A \sim 1$$

This regime is characterized by dynamically generated hard saturation scale Q_s, low-x evolution becomes nonlinear: BK-JIMWLK equations

McLerran, Venugopalan, Balitsky, Kovchegov, Iancu, Jalilian-Marian, Kovner, Leonidov, Weigert (1994-2002)

$\gamma p \to Q\bar{Q}X$ in TMD factorization



$$\sigma^{\gamma p \to q\bar{q}X} \propto |\mathcal{M}|^2 \otimes \Phi$$

$$\Phi^{\mu\nu}(P, k, S) = \sum_X \int \mathrm{d}^4\xi \, e^{i(k-P+P_X)\xi} \langle P, S \big| A^\mu(0) \big| X \rangle \langle X \big| A^\nu(0) \big| P, S \rangle$$

$$\propto -g_T^{\mu\nu} \mathcal{F}(x, \mathbf{k}) + \left(2 \frac{k^\mu k^\nu}{\mathbf{k}^2} + g_T^{\mu\nu} \right) \mathcal{H}(x, \mathbf{k})$$

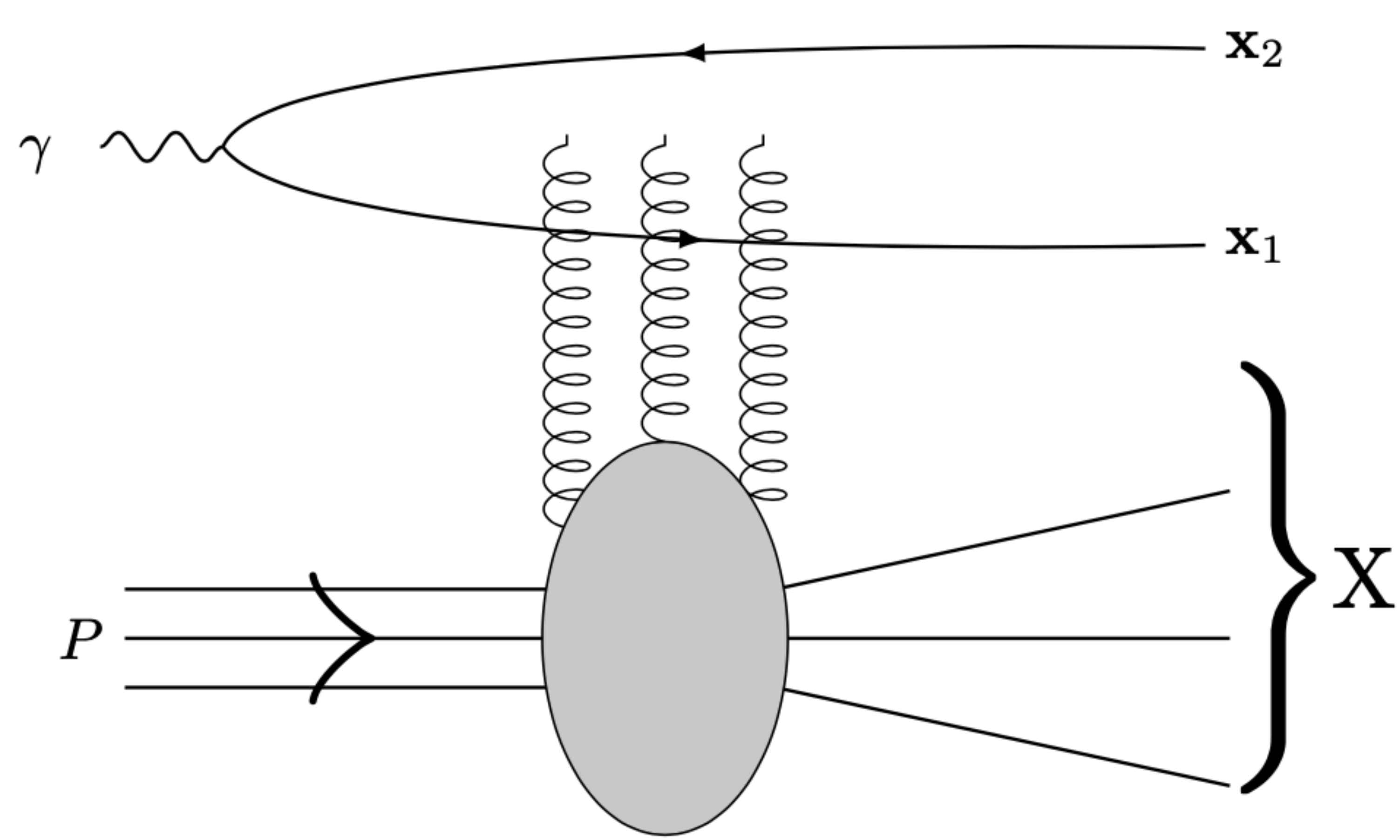
Gluon correlator for unpolarized proton sensitive to both unpolarized and linearly polarized gluon TMD!

$$f_1^g(x, \mathbf{k}) = \frac{1}{2x} \mathcal{F}(x, \mathbf{k})$$

$$h_1^{\perp g}(x, \mathbf{k}) = \frac{M_p^2}{2x\mathbf{k}^2} \mathcal{H}(x, \mathbf{k})$$

Mulders & Rodrigues (2001)

Photoproduction of a heavy-quark pair in the CGC



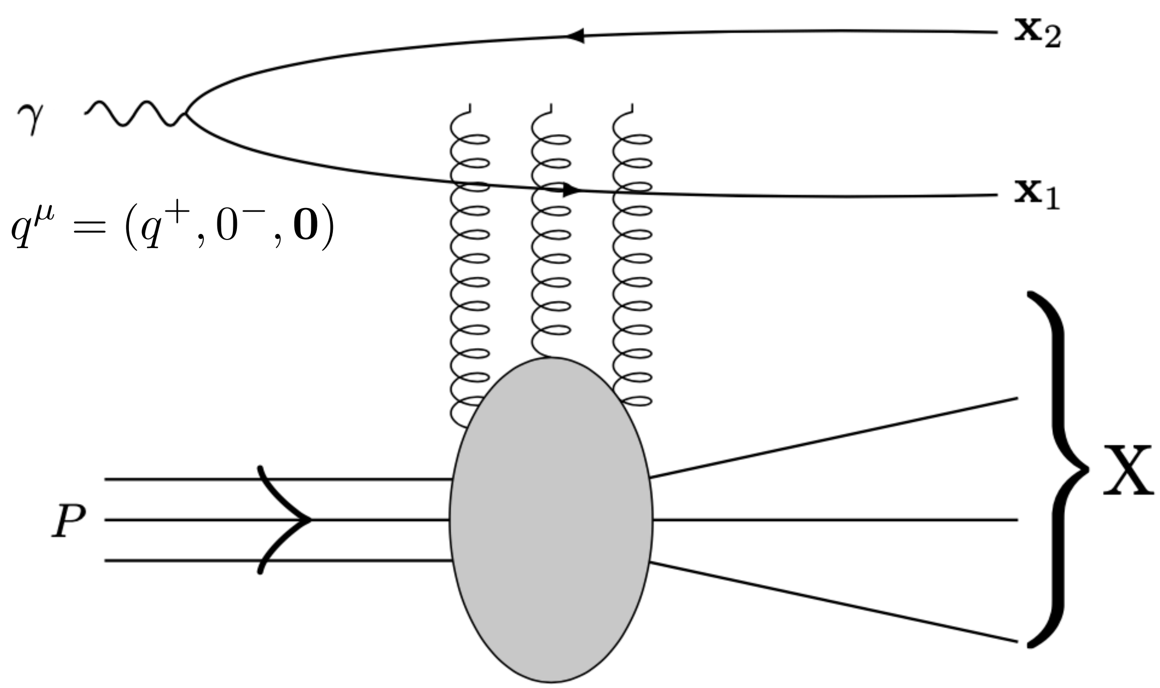
In Mueller's *dipole frame*, the photon dissociates in a quark-antiquark pair long before the scattering off a highly boosted proton

Semiclassical approximation: frozen quark-antiquark dipole interacts with dense classical gluon fields through Wilson lines

Color Glass Condensate (CGC): nonperturbative model of initial state + perturbative nonlinear evolution (BK-JIMWLK)

McLerran, Venugopalan, Balitsky, Kovchegov, Iancu, Jalilian-Marian, Kovner, Leonidov, Weigert (1994-2002)

Photoproduction of a heavy-quark pair in the CGC



$s \gg k_{1_T} \sim k_{2_T} \gg Q_s^2$

a priori no restrictions on the scales formed by the transverse momenta

perturbative $\gamma \to Q\bar{Q}$ splittings

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2\mathbf{k}_1\mathrm{d}^2\mathbf{k}_2} \propto \int_{\mathbf{x}_1'\mathbf{x}_2'\mathbf{x}_1\mathbf{x}_2} e^{-i\mathbf{k}_1(\mathbf{x}_1-\mathbf{x}_1')} e^{-i\mathbf{k}_2(\mathbf{x}_2-\mathbf{x}_2')}$$

$$z = \frac{k_1^+}{q^+}$$

$$\times \sum \varphi(\mathbf{x}_1,\mathbf{x}_2)\varphi^\dagger(\mathbf{x}_1',\mathbf{x}_2')(Q_{\mathbf{x}_2\mathbf{x}_1\mathbf{x}_1'\mathbf{x}_2'} - s_{\mathbf{x}_2\mathbf{x}_1} - s_{\mathbf{x}_1'\mathbf{x}_2'} + 1)$$

eikonal interaction of dipole with semiclassical gluon fields through Wilson lines

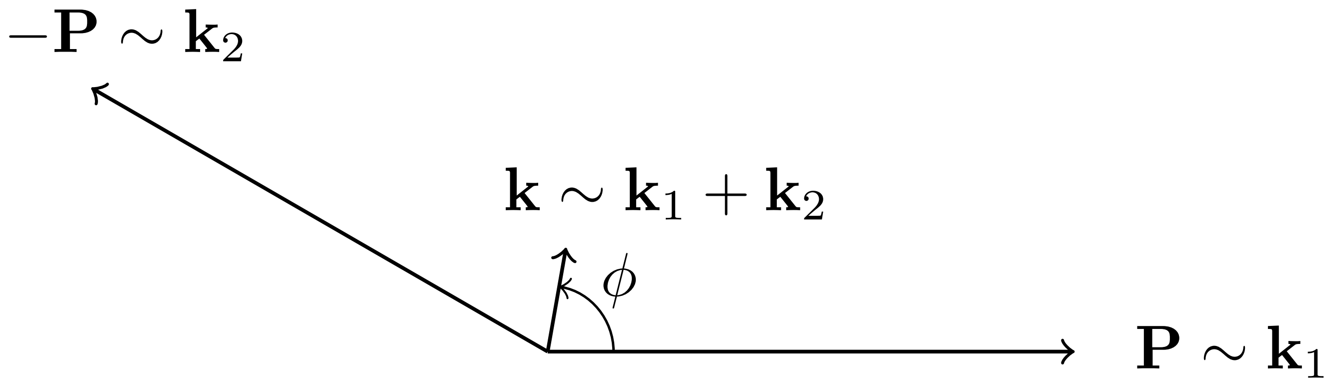
$$S_{(F,A)}(\mathbf{x}) = \mathscr{P} e^{ig_s\int \mathrm{d}x^+ A^-(x^+,\mathbf{x})}$$

$\langle \ldots \rangle_x$ = average over classical gluon fields of the target: nonperturbative info

$$s_{\mathbf{x}_2\mathbf{x}_1} = \frac{1}{N_c}\langle \mathrm{Tr}\, S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{x}_2)\rangle_x$$

$$Q_{\mathbf{x}_2\mathbf{x}_1\mathbf{x}_1'\mathbf{x}_2'} = \frac{1}{N_c}\langle \mathrm{Tr}\, S_F(\mathbf{x}_2)S_F^\dagger(\mathbf{x}_1)S_F(\mathbf{x}_1')S_F^\dagger(\mathbf{x}_2')\rangle_x$$

Correlation limit



In the transverse back-to-back regime, one recovers an additional *small* scale given by the vector sum of the outgoing transverse momenta: $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2 \rightarrow$ obtained from gluons!

Scales: $s \gg \mathbf{k}_1^2 \sim \mathbf{k}_2^2 \sim \mathbf{P}^2 \gg \mathbf{k}^2 \sim Q_s^2$
similar hierarchy as in TMD factorization!

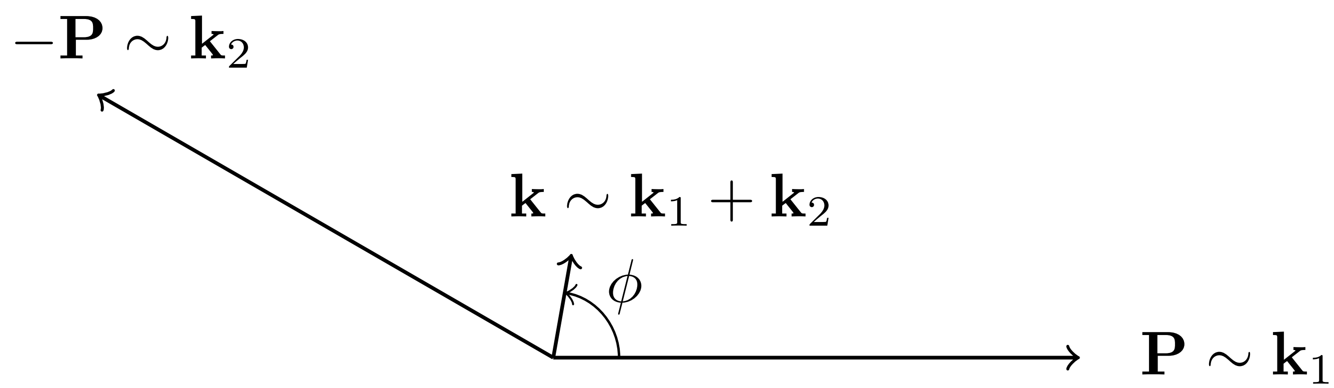
Perform a twist expansion in k/P = “correlation limit”

Wilson line structures involving derivatives:

$$\mathrm{Tr}\left\langle S_F^\dagger(\mathbf{u})\left[\partial_i S_F(\mathbf{u})\right] S_F^\dagger(\mathbf{v})\left[\partial_j S_F(\mathbf{v})\right]\right\rangle_x$$

Dominguez, Marquet, Xiao, Yuan (2011)

Correlation limit



TMD-factorized expression is recovered!

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2\mathbf{k}_1\mathrm{d}^2\mathbf{k}_2} = \frac{\alpha_{\mathrm{em}}\alpha_s}{(\mathbf{P}^2+m^2)^4}\Big[\big((\mathbf{P}^4+m^4)(z^2+\bar{z}^2)+2m^2\mathbf{P}^2\big)\mathcal{F}_{\mathrm{WW}}(x,\mathbf{k}) + 4m^2 z\bar{z}\mathbf{P}^2\cos(2\varphi)\mathcal{H}_{\mathrm{WW}}(x,\mathbf{k})\Big]$$

We would have obtained *exactly* the same result working in TMD factorization and taking the low-*x* limit

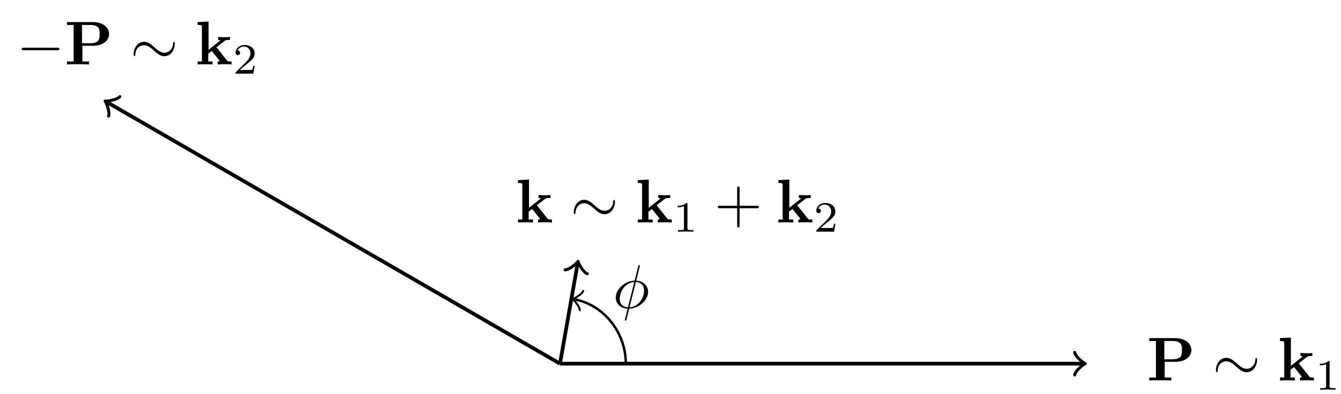
Turns out to work for all $2 \to 2$ processes!

At small-*x,* two approaches to describe our process:

CGC: no factorization, valid for all kinematics $\longleftrightarrow$ TMD: factorization, valid in correlation limit

Dominguez, Marquet, Xiao, Yuan (2011)

Beyond the correlation limit



Can we work at low-*x*, remove the restriction $k \ll P$, but still have a description in terms of gluon TMDs?

Yes! Low-*x improved TMD* framework: re-introduce the kinematical power corrections k/P, resum them on top of the CGC result

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2\mathbf{k}_1\mathrm{d}^2\mathbf{k}_2} = \alpha_{\mathrm{em}}\alpha_s\Big[H^f(z,\mathbf{k}_1,\mathbf{k}_2)\mathcal{F}_{\mathrm{WW}}(x,\mathbf{k}) + H^h(z,\mathbf{k}_1,\mathbf{k}_2)\mathcal{H}_{\mathrm{WW}}(x,\mathbf{k})\Big]$$

Kotko, Kutak, Marquet, Petreska, Sapeta (2015)
Altinoluk, Boussarie, Kotko (2019)
Boussarie, Mehtar-Tani (2020)
Altinoluk, Marquet, PT (in preparation)

Some phenomenology

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2\mathbf{k}_1\mathrm{d}^2\mathbf{k}_2} = \alpha_{\mathrm{em}}\alpha_s\Big[H^f(z,\mathbf{k}_1,\mathbf{k}_2)\mathcal{F}_{\mathrm{WW}}(x,\mathbf{k}) + H^h(z,\mathbf{k}_1,\mathbf{k}_2)\mathcal{H}_{\mathrm{WW}}(x,\mathbf{k})\Big]$$

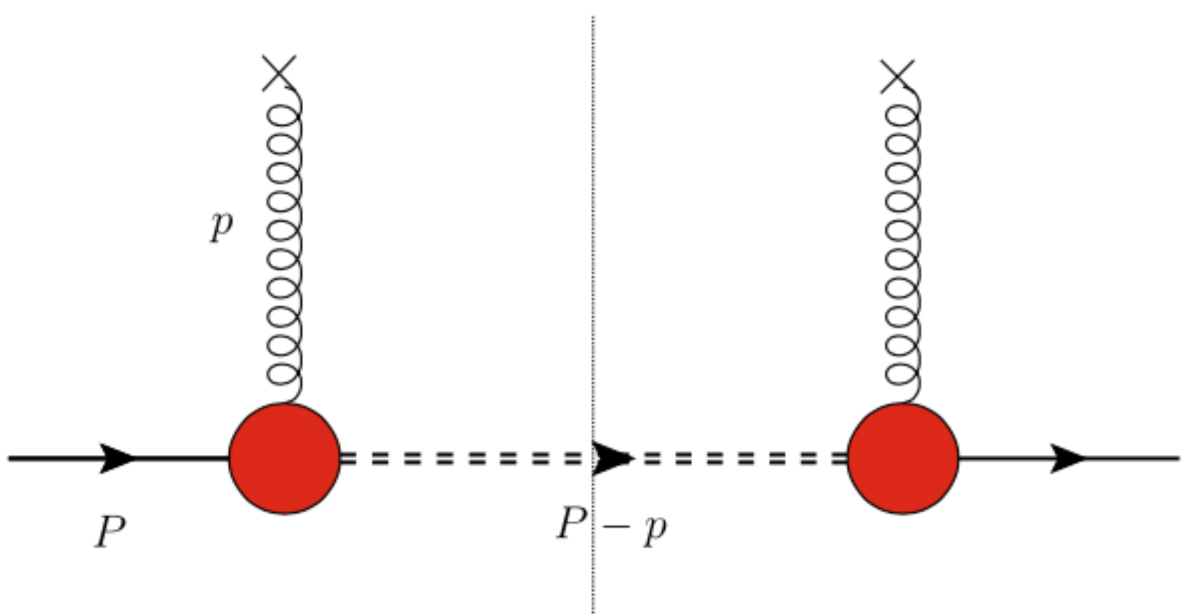
TMDs (as all parton distributions) are *nonperturbative*

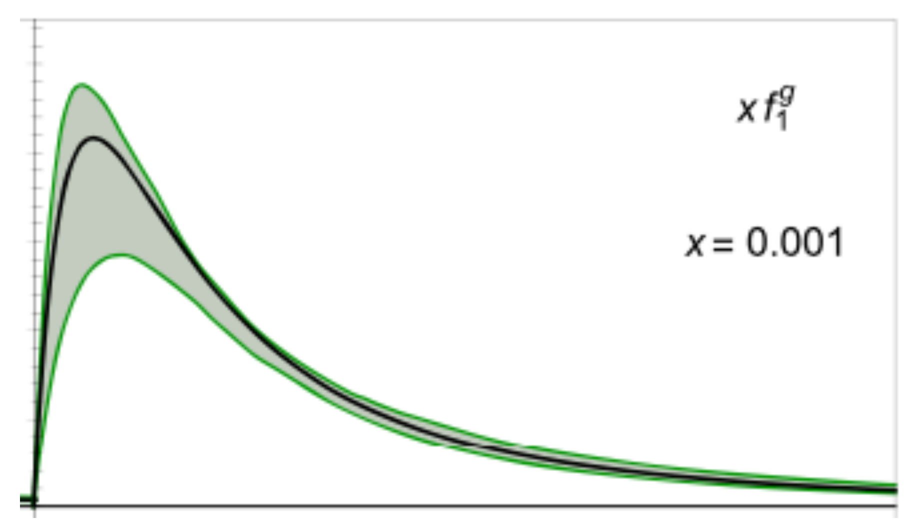
→need to be extracted

Example: exploit angular correlation in TMD-limit,
compare with massless case

$$A^{\cos 2\varphi} = 2\frac{\int \mathrm{d}\varphi \cos 2\varphi\, \mathrm{d}\sigma}{\int \mathrm{d}\varphi\, \mathrm{d}\sigma} = \frac{4m^2 z\bar{z}\mathbf{P}^2}{(\mathbf{P}^4+m^4)(z^2+\bar{z}^2)+2m^2\mathbf{P}^2}\frac{\mathcal{H}_{\mathrm{WW}}(x,\mathbf{k})}{\mathcal{F}_{\mathrm{WW}}(x,\mathbf{k})}$$

We could also *model* the gluon distributions:
in a spectator model:

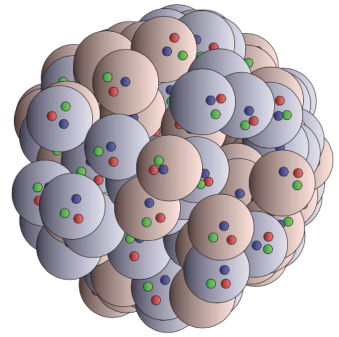
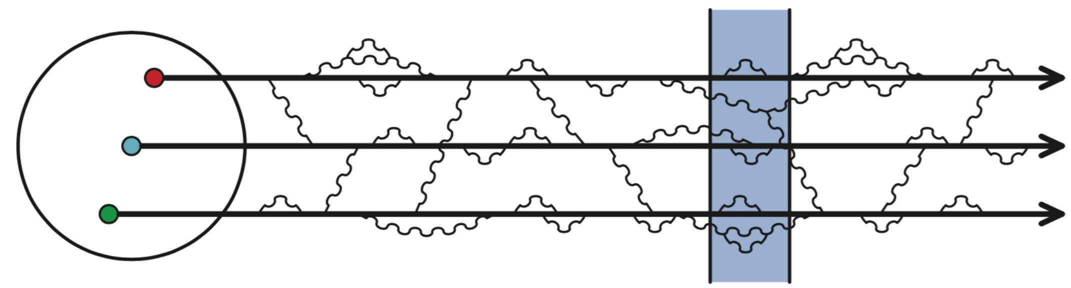




Bacchetta, Celiberto, Radici, PT (2020)

Some phenomenology: MV model

… or, since we are interested in low-*x,* in the MV model:

Simple analytical expressions:

$$\mathcal{F}_{\mathrm{WW}}(x,\mathbf{k}) = \frac{S_\perp C_F}{\alpha_S \pi^3} \int \mathrm{d}r \, \frac{J_0(kr)}{r} \left(1 - e^{-\frac{r^2}{4} Q_{sg}^2 \ln \frac{1}{r^2\Lambda^2}}\right)$$

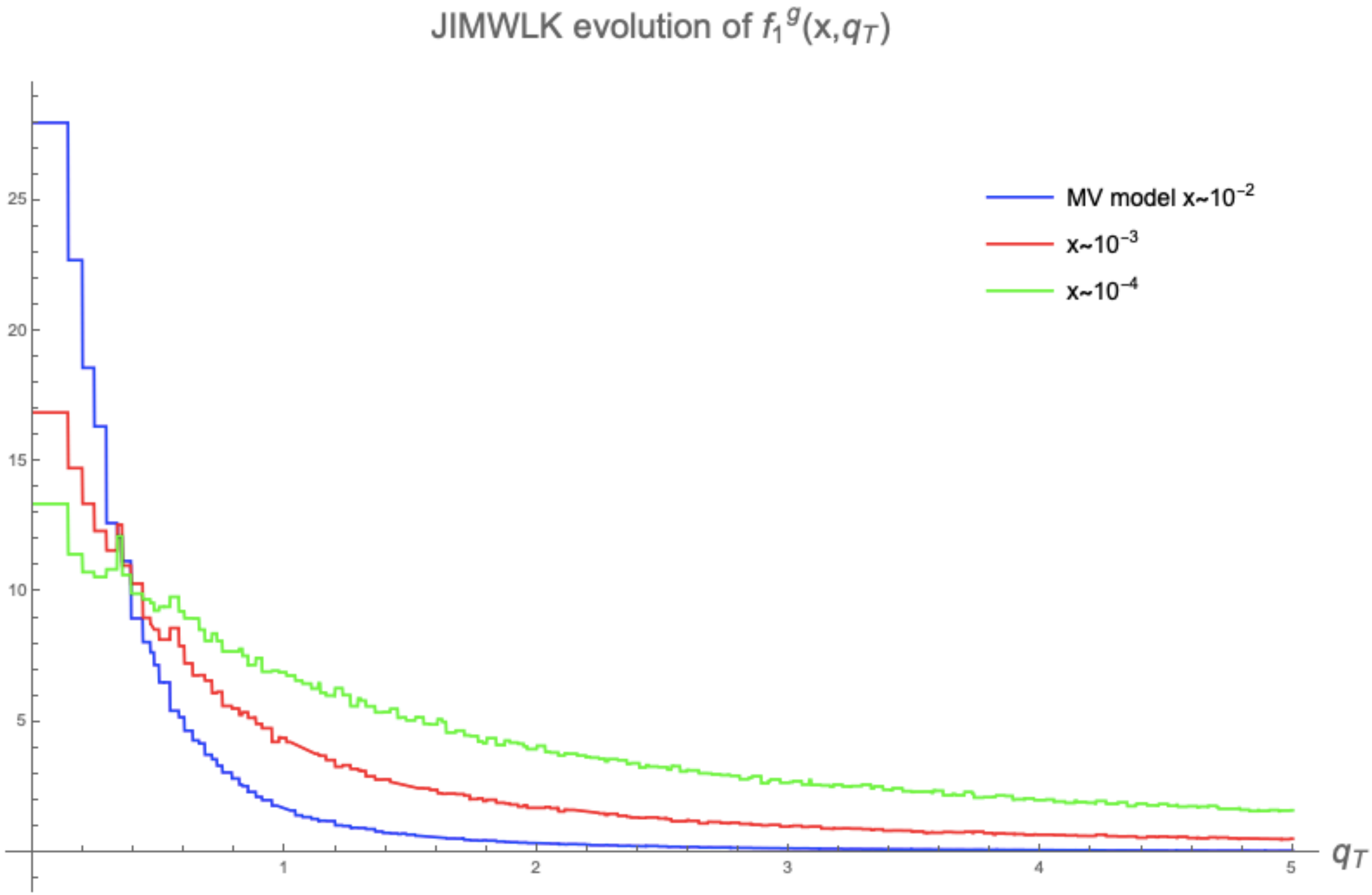
$$\mathcal{H}_{\mathrm{WW}}(x,\mathbf{k}) = \frac{S_\perp C_F}{\alpha_S \pi^3} \int \mathrm{d}r \, \frac{J_2(kr)}{r \ln \frac{1}{r^2\Lambda^2}} \left(1 - e^{-\frac{r^2}{4} Q_{sg}^2 \ln \frac{1}{r^2\Lambda^2}}\right)$$

Free parameters: $S_\perp$ the proton's or nucleus' transverse surface, Q_{sg} the saturation scale, and Λ an IR cutoff

Their nonlinear evolution with *x* can be calculated on the lattice.

McLerran, Venugopalan (1994-'95)
picture from Gelis (2012)

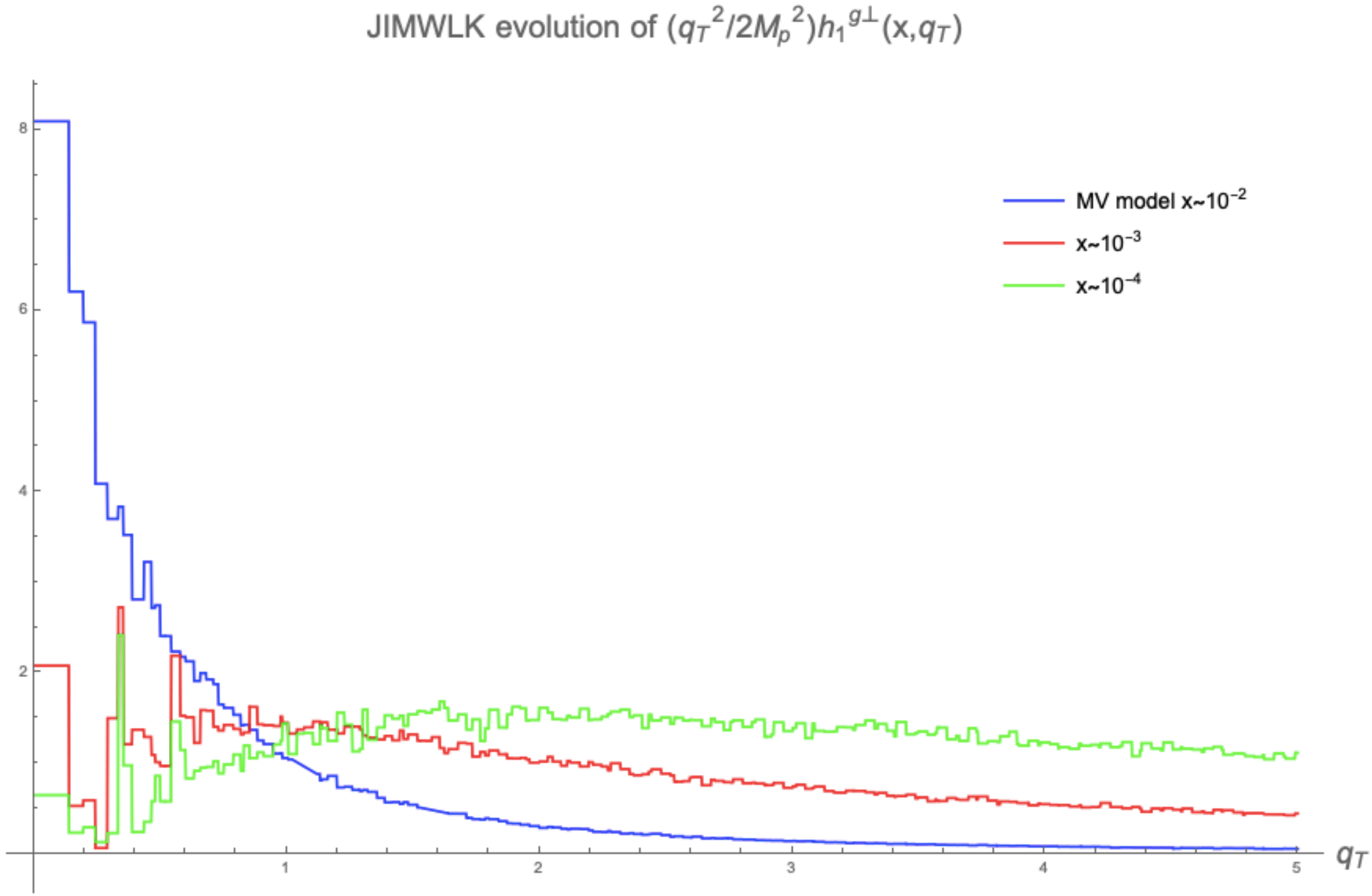
Some phenomenology: gluon TMDs with JIMWLK



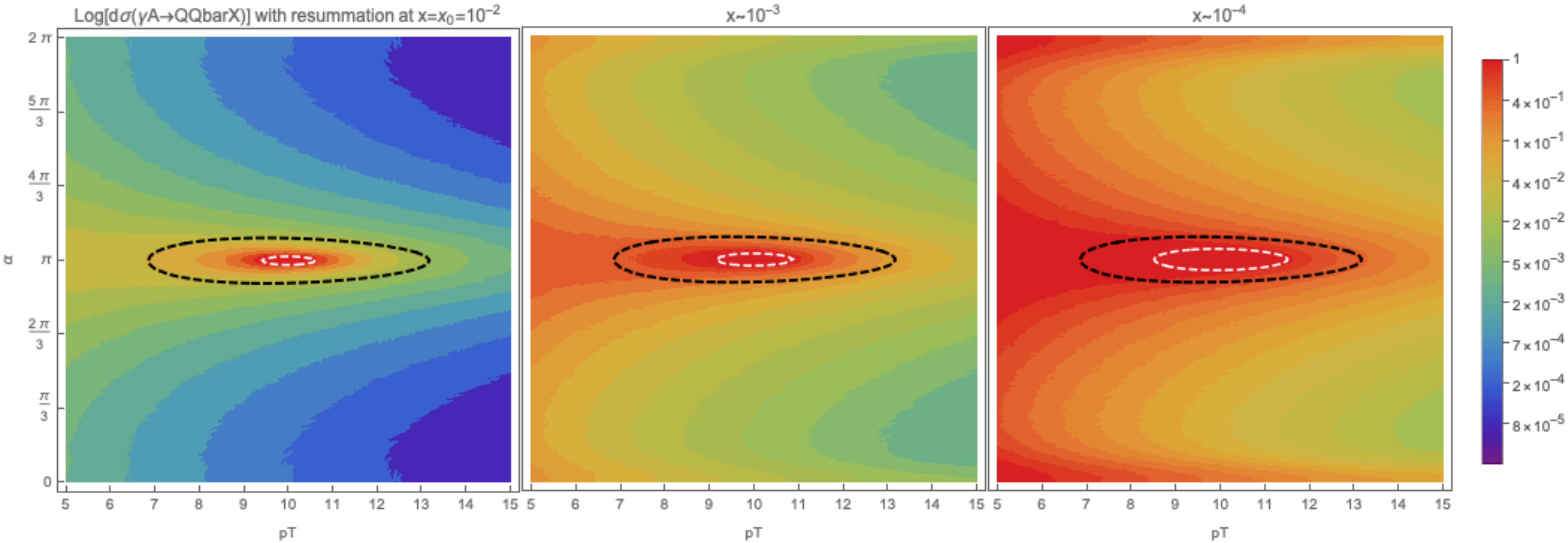
Langevin formulation of JIMWLK: Weigert (2002), Rummukainen; Weigert (2004); Lappi (2008)

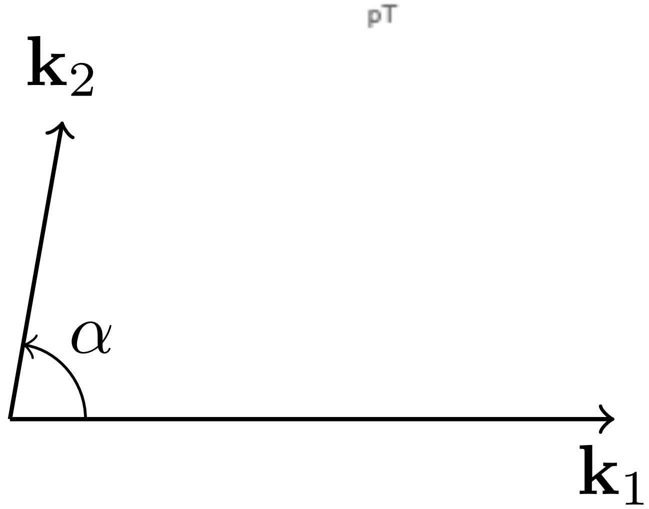
Marquet, Petreska, Roiesnel (2016); Marquet, Roiesnel, PT (2018)

Some phenomenology: gluon TMDs with JIMWLK



Some phenomenology: $\gamma A \to Q\bar{Q}X$ in ITMD





Fix $|\mathbf{k}_1| = 10\,\mathrm{GeV}$, z=1/2, m=1.3 GeV
white dashed lines are saturation scale,
black dashed lines are boundary $|\mathbf{k}| \leq \sqrt{10}\,\mathrm{GeV}$

Altinoluk, Marquet, PT (in preparation)

Towards CGC-TMD at NLO: photoproduction of three jets

Towards NLO: three-jet photoproduction

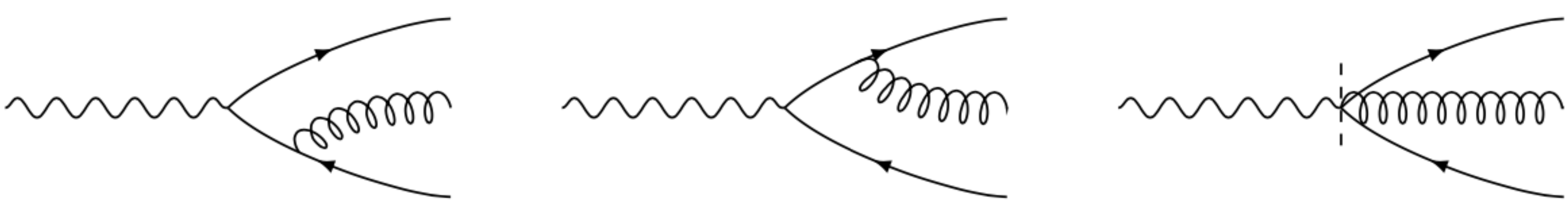
Cross section is given by:

$$2k_1^+ 2k_2^+ 2k_3^+ (2\pi)^9 2\pi\delta\Big(p^+ - \sum_{j=1}^{3} k_j^+\Big) 2p^+ \frac{\mathrm{d}\sigma^{\gamma A \to q g \bar{q} + X}}{\mathrm{d}^3\vec{k}_1\, \mathrm{d}^3\vec{k}_2\, \mathrm{d}^3\vec{k}_3}$$

$$= \frac{1}{2}\, {}_{\mathrm{out}}\Big\langle (\boldsymbol{\gamma})\,[\vec{p}\,]_\lambda \Big| N_q(\vec{k}_1) N_g(\vec{k}_2) N_{\bar{q}}(\vec{k}_3) \Big| (\boldsymbol{\gamma})\,[\vec{p}\,]_\lambda \Big\rangle_{\mathrm{out}}$$

Expansion of the photon Fock state

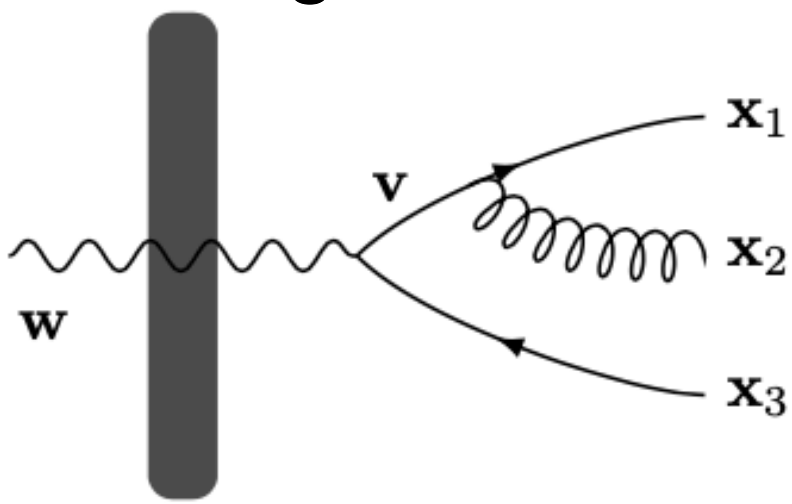
$$|\boldsymbol{\gamma}\rangle_{\mathrm{dressed}} = Z_0 |\boldsymbol{\gamma}\rangle_0 + g_e Z_1 |\mathbf{q\bar{q}}\rangle_0 + g_e g_s Z_2 |\mathbf{q\bar{q}g}\rangle_0 + \mathcal{O}$$



Altinoluk, Boussarie, Marquet, PT (2020)

Eikonal approximation

In our frame, dressed photon hits Lorentz contracted proton or nucleus target = ‘shockwave’



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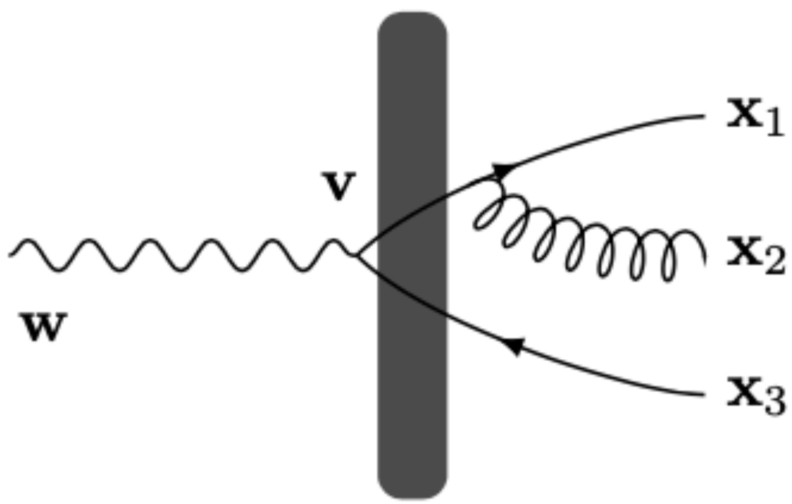
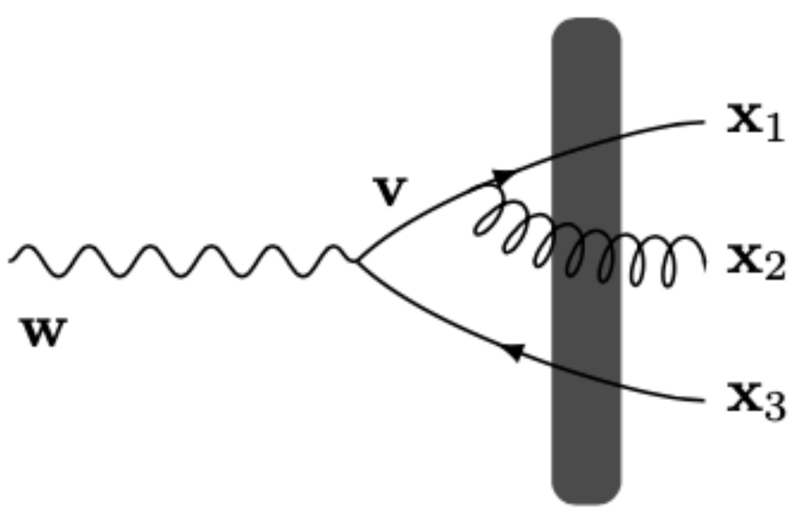
Wilson line:

$$S_{F,A}(\mathbf{x}) = \mathcal{P} e^{ig_s \int \mathrm{d}x^+ A_c^-(\mathbf{x}^+, \mathbf{x}) t^c_{F,A}}$$



$t^c S_F(\mathbf{v}) S_F^\dagger(\mathbf{x}_3)$



$S_F(\mathbf{x}_1) t^d S_A^{dc}(\mathbf{x}_2) S_F^\dagger(\mathbf{x}_3)$

Outgoing photon state

$$
\begin{aligned}
|(\boldsymbol{\gamma})[\vec{p}]_\lambda\rangle_{\mathrm{out}} = g_e g_s \int \frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}k_2^+}{2\pi} \int_{\mathbf{w}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} & |(\mathbf{q})[k_1^+,\mathbf{x}_1]^i_s;(\mathbf{g})[k_2^+,\mathbf{x}_2]^\eta_c;(\bar{\mathbf{q}})[k_3^+,\mathbf{x}_3]^j_{s'}\rangle_D \\
\times \Bigg\{ & \Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} S_A(\mathbf{x}_2)^{dc} - t^c_{ij}\Big) \\
& \times \left(F_q^{(2)} + F_{\bar{q}}^{(2)} + F_C^{(2)}\right)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^{\eta\lambda}_{s's} \\
- \int_{\mathbf{v}} & \Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij} - t^c_{ij}\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^\lambda_{\bar{s}s'} \\
& \times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]^\eta_{s\bar{s}} \\
- \int_{\mathbf{v}} & \Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij} - t^c_{ij}\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\Big]^\lambda_{s\tilde{s}} \\
& \times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]^\eta_{s'\tilde{s}} \Bigg\}
\end{aligned}
$$

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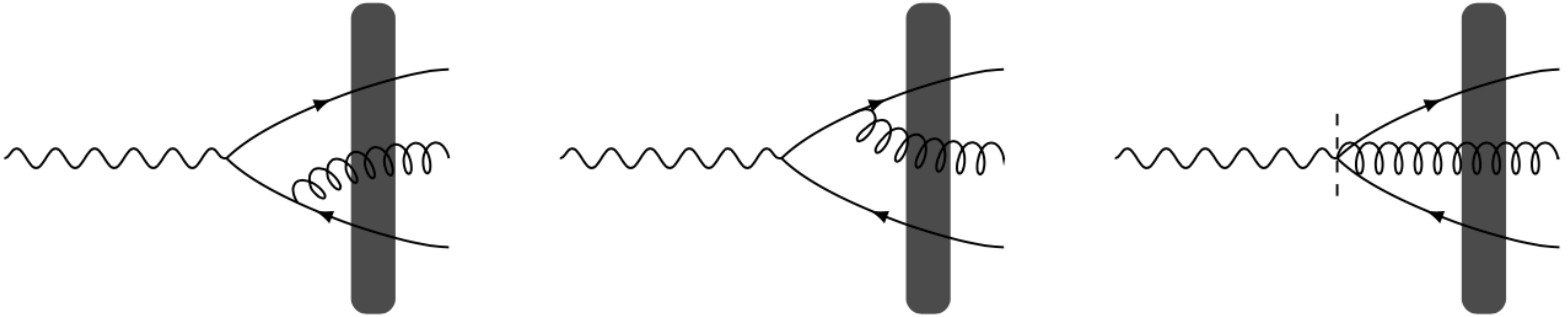
\times \Bigg\{ & \Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} S_A(\mathbf{x}_2)^{dc} - t_{ij}^c\Big) \\
& \times \left(F_q^{(2)} + F_{\bar{q}}^{(2)} + F_C^{(2)}\right)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{s's}^{\eta\lambda} \\
& - \int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij} - t_{ij}^c\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{\bar{s}s'}^{\lambda} \\
& \quad \times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]_{s\bar{s}}^{\eta} \\
& - \int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij} - t_{ij}^c\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\Big]_{s\tilde{s}}^{\lambda} \\
& \quad \times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]_{s'\tilde{s}}^{\eta}\Bigg\}
\end{aligned}
$$



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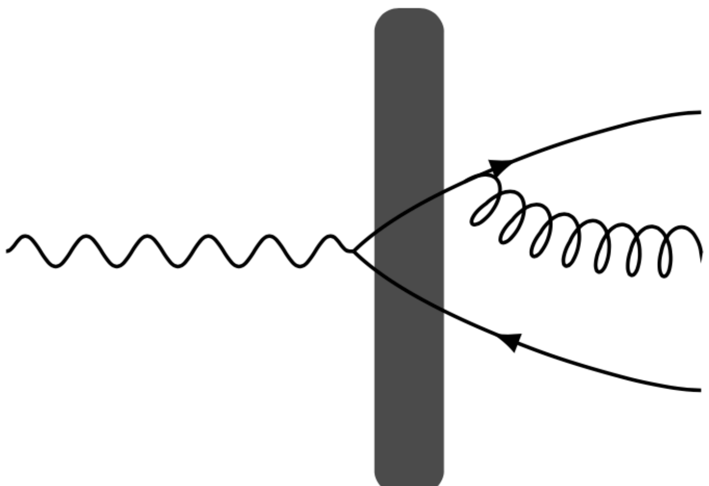
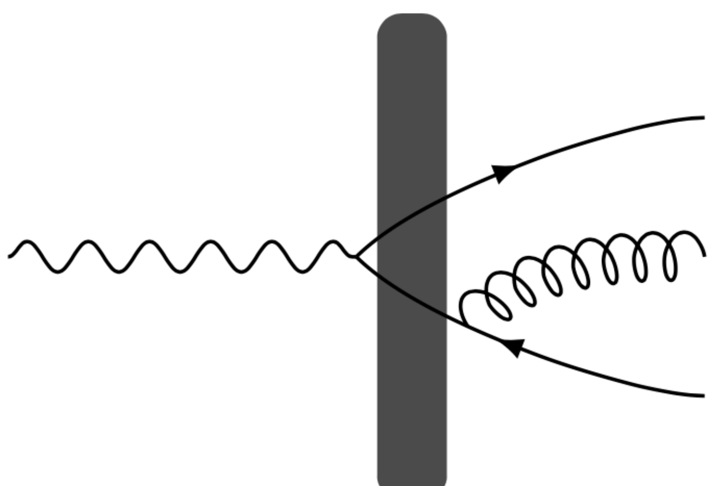
|(\boldsymbol{\gamma})[\vec{p}]_\lambda\rangle_{\text{out}} = g_e g_s \int \frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}k_2^+}{2\pi}\int_{\mathbf{w}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} &|(\mathbf{q})[k_1^+,\mathbf{x}_1]_s^i;(\mathbf{g})[k_2^+,\mathbf{x}_2]_c^\eta;(\bar{\mathbf{q}})[k_3^+,\mathbf{x}_3]_{s'}^j\rangle_D \\
\times \Bigg\{ &\Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} S_A(\mathbf{x}_2)^{dc} - t^c_{ij}\Big) \\
&\times \big(F_q^{(2)} + F_{\bar{q}}^{(2)} + F_C^{(2)}\big)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^{\eta\lambda}_{s's} \\
&- \int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij} - t^c_{ij}\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]^\lambda_{\tilde{s}s'} \\
&\quad\times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]^\eta_{s\tilde{s}} \\
&- \int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij} - t^c_{ij}\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\Big]^\lambda_{s\tilde{s}} \\
&\quad\times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]^\eta_{s'\tilde{s}}\Bigg\}
\end{aligned}
$$



Outgoing photon state

$$
\begin{aligned}
|(\boldsymbol{\gamma})[\vec{p}]_\lambda\rangle_{\text{out}} = g_e g_s \int \frac{\mathrm{d}k_1^+}{2\pi}\frac{\mathrm{d}k_2^+}{2\pi}\int_{\mathbf{w}\mathbf{x}_1\mathbf{x}_2\mathbf{x}_3} & |(\mathbf{q})[k_1^+,\mathbf{x}_1]_s^i;(\mathbf{g})[k_2^+,\mathbf{x}_2]_c^\eta;(\bar{\mathbf{q}})[k_3^+,\mathbf{x}_3]_{s'}^j\rangle_D \\
\times \Bigg\{ & \Big(\big[S_F(\mathbf{x}_1)t^d S_F^\dagger(\mathbf{x}_3)\big]_{ij} S_A(\mathbf{x}_2)^{dc} - t_{ij}^c\Big) \\
& \times \left(F_q^{(2)} + F_{\bar{q}}^{(2)} + F_C^{(2)}\right)\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{w}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{s's}^{\eta\lambda} \\
& - \int_{\mathbf{v}}\Big(\big[t^c S_F(\mathbf{v})S_F^\dagger(\mathbf{x}_3)\big]_{ij} - t_{ij}^c\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+ + k_2^+,\mathbf{w}-\mathbf{v}];(\bar{\mathbf{q}})[k_3^+,\mathbf{w}-\mathbf{x}_3]\Big]_{\bar{s}s'}^{\lambda} \\
& \times F_q^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{v}-\mathbf{x}_1];(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2]\Big]_{s\bar{s}}^{\eta} \\
& - \int_{\mathbf{v}}\Big(\big[S_F(\mathbf{x}_1)S_F^\dagger(\mathbf{v})t^c\big]_{ij} - t_{ij}^c\Big) F_\gamma^{(1)}\Big[(\mathbf{q})[k_1^+,\mathbf{w}-\mathbf{x}_1];(\bar{\mathbf{q}})[k_2^+ + k_3^+,\mathbf{w}-\mathbf{v}]\Big]_{s\tilde{s}}^{\lambda} \\
& \times F_{\bar{q}}^{(1)}\Big[(\mathbf{g})[k_2^+,\mathbf{v}-\mathbf{x}_2];(\bar{\mathbf{q}})[k_3^+,\mathbf{v}-\mathbf{x}_3]\Big]_{s'\tilde{s}}^{\eta}\Bigg\}
\end{aligned}
$$



Differential $\gamma A \to q\bar{q}g + X$ cross section

$$(2\pi)^9 \frac{\mathrm{d}\sigma^{\gamma A \to q\bar{q}g+X}}{\mathrm{d}^3\vec{k}_1 \mathrm{d}^3\vec{k}_2 \mathrm{d}^3\vec{k}_3} = g_e^2 g_s^2 \frac{1}{k_2^+ p^+} 2\pi\delta\Big(p^+ - \sum_{i=1}^{3} k_i^+\Big) \times \Big\langle I_{qq} + I_{\bar{q}\bar{q}} + I_{CC} + 2I_{q\bar{q}} + 2I_{Cq} + 2I_{C\bar{q}} \Big\rangle_{x_A}$$

Differential $\gamma A \to q\bar{q}g + X$ cross section

$$\langle I_{qq} \rangle_{x_A} = \frac{N_c^2}{2} \mathcal{M}_{qq}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\left(\bar{\xi}_3, \frac{\xi_2}{\bar{\xi}_3}\right) \int_{\mathbf{v}\mathbf{v}'} \prod_{i=1}^{3} \int_{\mathbf{x}_i \mathbf{x}_i'} e^{i\mathbf{k}_i \cdot (\mathbf{x}_i' - \mathbf{x}_i)}$$

$$\times A^{\bar{\eta}}(\mathbf{x}_1 - \mathbf{x}_2) A^{\bar{\eta}'}(\mathbf{x}_1' - \mathbf{x}_2')\, \delta^{(2)}\left(\mathbf{v} - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1 - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2\right) \delta^{(2)}\left(\mathbf{v}' - \frac{\xi_1}{\bar{\xi}_3}\mathbf{x}_1' - \frac{\xi_2}{\bar{\xi}_3}\mathbf{x}_2'\right)$$

$$\times \left\{ \left[W_1(\mathbf{x}_1 . \mathbf{x}_2; \mathbf{x}_2', \mathbf{x}_1' | \mathbf{x}_2, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_2') - \frac{1}{N_c^2} W_3(\mathbf{x}_1, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_1') \right] \right.$$

$$\times \mathcal{A}^{\bar{\lambda}}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1 - \mathbf{x}_2\right) \mathcal{A}^{\bar{\lambda}'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1' - \mathbf{x}_2'\right)$$

$$- \left[W_2(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{x}_2, \mathbf{x}_3; \mathbf{x}_3', \mathbf{v}') - \frac{1}{N_c^2} W_3(\mathbf{x}_1, \mathbf{x}_3; \mathbf{x}_3', \mathbf{v}') \right]$$

$$\times \mathcal{A}^{\bar{\lambda}}\left(\xi_3, \mathbf{x}_3 - \mathbf{v}; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1 - \mathbf{x}_2\right) A^{\bar{\lambda}'}(\mathbf{x}_3' - \mathbf{v}')$$

$$- \left[W_2(\mathbf{x}_2', \mathbf{x}_1' | \mathbf{v}, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_2') - \frac{1}{N_c^2} W_3(\mathbf{v}, \mathbf{x}_3; \mathbf{x}_3', \mathbf{x}_1') \right]$$

$$\times A^{\bar{\lambda}}(\mathbf{x}_3 - \mathbf{v}) \mathcal{A}^{\bar{\lambda}'}\left(\xi_3, \mathbf{x}_3' - \mathbf{v}'; \frac{\xi_1}{\bar{\xi}_3}, \mathbf{x}_1' - \mathbf{x}_2'\right)$$

$$\left. - \left(1 - \frac{1}{N_c^2}\right) W_3(\mathbf{v}, \mathbf{x}_3; \mathbf{x}_3', \mathbf{v}')\, A^{\bar{\lambda}}(\mathbf{x}_3 - \mathbf{v}) A^{\bar{\lambda}'}(\mathbf{x}_3' - \mathbf{v}') \right\}$$

Differential $\gamma A \to q\bar{q}g + X$ cross section

$$\langle I_{qq} \rangle_{x_A} = \frac{N_c^2}{2}\mathcal{M}_{qq}^{\bar\eta\bar\eta';\bar\lambda\bar\lambda'}\Big(\bar\xi_3, \frac{\xi_2}{\bar\xi_3}\Big)\int_{\mathbf{v}\mathbf{v}'}\prod_{i=1}^{3}\int_{\mathbf{x}_i\mathbf{x}_i'} e^{i\mathbf{k}_i\cdot(\mathbf{x}_i'-\mathbf{x}_i)}$$
$$\times A^{\bar\eta}(\mathbf{x}_1-\mathbf{x}_2)A^{\bar\eta'}(\mathbf{x}_1'-\mathbf{x}_2')\,\delta^{(2)}\Big(\mathbf{v}-\frac{\xi_1}{\bar\xi_3}\mathbf{x}_1-\frac{\xi_2}{\bar\xi_3}\mathbf{x}_2\Big)\delta^{(2)}\Big(\mathbf{v}'-\frac{\xi_1}{\bar\xi_3}\mathbf{x}_1'-\frac{\xi_2}{\bar\xi_3}\mathbf{x}_2'\Big)$$
$$\times\Big\{\Big[W_1(\mathbf{x}_1.\mathbf{x}_2;\mathbf{x}_2',\mathbf{x}_1'|\mathbf{x}_2,\mathbf{x}_3;\mathbf{x}_3',\mathbf{x}_2')-\frac{1}{N_c^2}W_3(\mathbf{x}_1,\mathbf{x}_3;\mathbf{x}_3',\mathbf{x}_1')\Big]$$
$$\times\mathcal{A}^{\bar\lambda}\Big(\xi_3,\mathbf{x}_3-\mathbf{v};\frac{\xi_1}{\bar\xi_3},\mathbf{x}_1-\mathbf{x}_2\Big)\mathcal{A}^{\bar\lambda'}\Big(\xi_3,\mathbf{x}_3'-\mathbf{v}';\frac{\xi_1}{\bar\xi_3},\mathbf{x}_1'-\mathbf{x}_2'\Big)$$
$$+\ldots$$

W's are combinations of dipoles and quadrupoles:

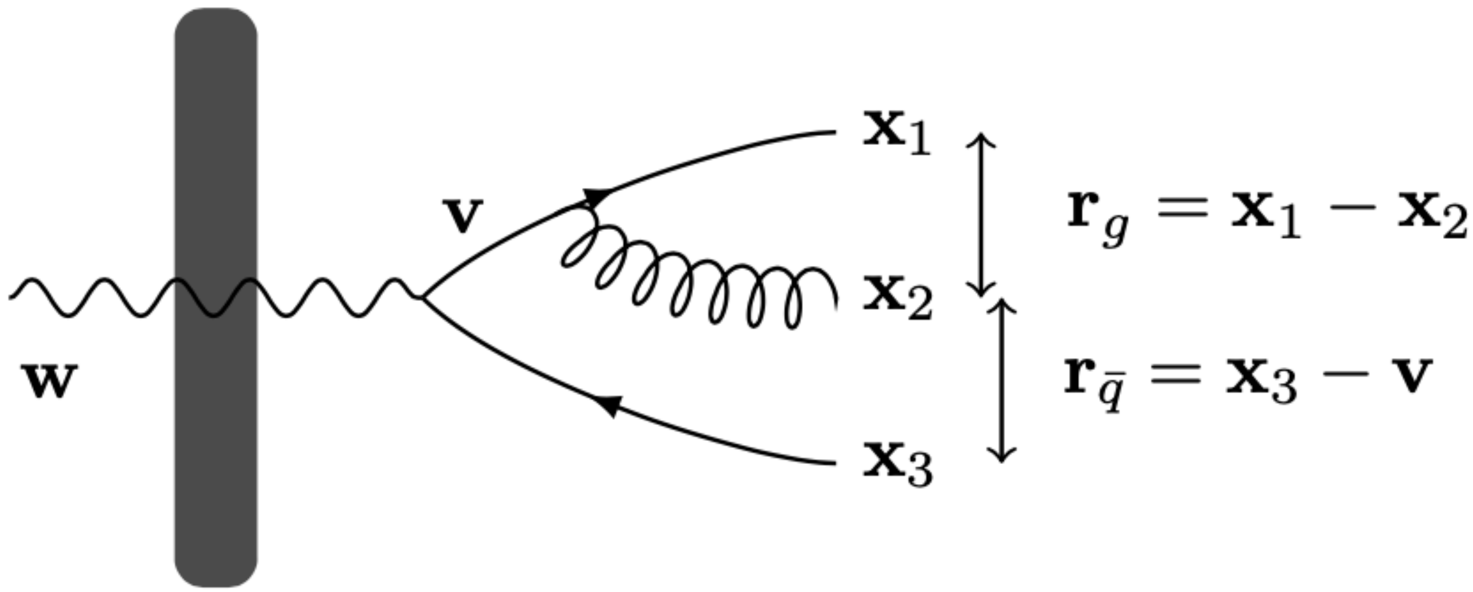
$$s(\mathbf{x},\mathbf{y}) = \Big\langle\frac{1}{N_c}\mathrm{Tr}\big[S_F(\mathbf{x})S_F^\dagger(\mathbf{y})\big]\Big\rangle_{x_A},$$
$$Q(\mathbf{x},\mathbf{y},\mathbf{u},\mathbf{v}) = \Big\langle\frac{1}{N_c}\mathrm{Tr}\big[S_F(\mathbf{x})S_F^\dagger(\mathbf{y})S_F(\mathbf{u})S_F^\dagger(\mathbf{v})\big]\Big\rangle_{x_A}$$

$\langle \ldots \rangle_{x_A}$ is the average over the semiclassical gluon fields of the target → contains the nonperturbative information on the hadron structure

Correlation limit

Defining the total transverse momentum: $\mathbf{q}_T \equiv \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$,
study the limit $|\mathbf{q}_T| \ll |\mathbf{k}_1| \sim |\mathbf{k}_2| \sim |\mathbf{k}_3|$

$$|\mathbf{k}_i| \gg 1 \leftrightarrow |\mathbf{r}_i| \ll 1$$



Perform Taylor expansion around small dipole sizes $\mathbf{r}$
→ derivatives enter Wilson line structures:

$$\mathrm{Tr}\langle S_F^\dagger(\mathbf{x})[\partial_i S_F(\mathbf{x})]S_F^\dagger(\mathbf{y})[\partial_j S_F(\mathbf{y})]\rangle_x$$

Final result

$$(2\pi)^9 \frac{\mathrm{d}\sigma^{\gamma A \to q\bar{q}g+X}}{\mathrm{d}^3\vec{k}_1 \mathrm{d}^3\vec{k}_2 \mathrm{d}^3\vec{k}_3}\bigg|_{\text{corr. limit}} = 2\pi\delta\Big(p^+ - \sum_{i=1}^{3} k_i^+\Big) \, [\mathrm{H}]_{ij}^{\text{total}} \times \left[\frac{1}{2}\delta^{ij}\mathcal{F}_{gg}^{(3)}(x_A, \mathbf{q}_T) + \frac{1}{2}\Big(2\frac{\mathbf{q}_T^i \mathbf{q}_T^j}{\mathbf{q}_T^2} - \delta^{ij}\Big)\mathcal{H}_{gg}^{(3)}(x_A, \mathbf{q}_T)\right]$$

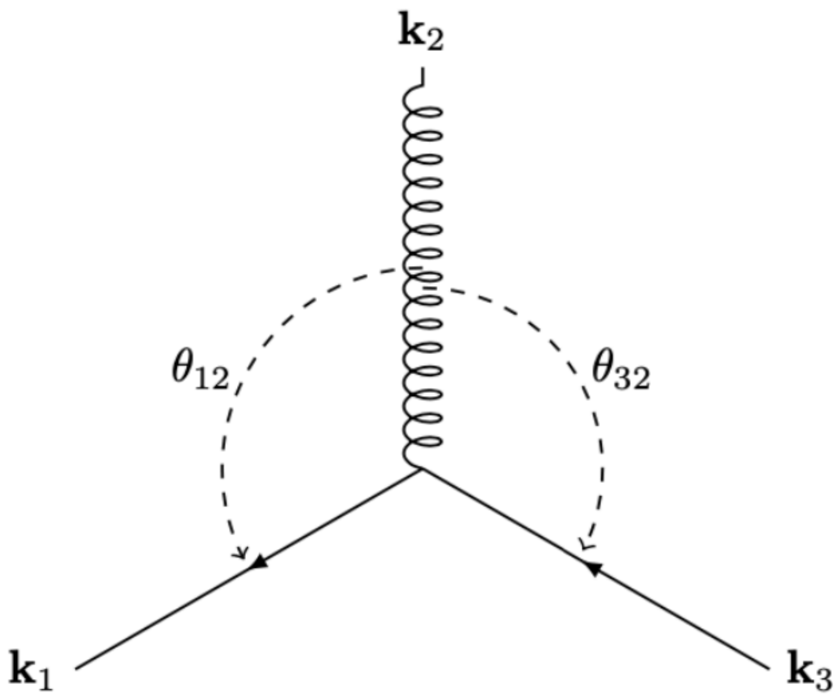
$$[\mathrm{H}]_{ij}^{\text{total}} = N_c\, g_e^2\, g_s^4\, \pi^3\, \frac{1}{k_2^+ p^+}\Big\{\mathcal{M}_{qq}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\Big(\bar{\xi}_3, \frac{\xi_2}{\bar{\xi}_3}\Big)[\mathrm{H}_{qq}]_{ij}^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'} + \mathcal{M}_{\bar{q}\bar{q}}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}\Big(\xi_1, \frac{\xi_2}{\bar{\xi}_1}\Big)[\mathrm{H}_{\bar{q}\bar{q}}]_{ij}^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'}$$
$$+2\,\mathcal{M}_{q\bar{q}}^{\bar{\eta}\bar{\eta}';\bar{\lambda}\bar{\lambda}'}(\xi_1, \xi_2)\,[\mathrm{H}_{q\bar{q}}]_{ij}^{\bar{\lambda}\bar{\lambda}';\bar{\eta}\bar{\eta}'} + \mathcal{M}_{CC}(\xi_1, \xi_2)\frac{1}{\xi_3^2\bar{\xi}_3^2}[\mathrm{H}_{CC}]_{ij}$$
$$+2\,\mathcal{M}_{Cq}(\xi_1, \xi_2)\,\frac{2}{\xi_3\bar{\xi}_3}\,[\mathrm{H}_{Cq}]_{ij}^{\bar{\lambda}\bar{\eta}} + 2\,\mathcal{M}_{C\bar{q}}(\xi_1, \xi_2)\,\frac{2}{\xi_3\bar{\xi}_3}\,[\mathrm{H}_{C\bar{q}}]_{ij}^{\bar{\lambda}\bar{\eta}}\Big\}$$

Again the result is written in the form of a LO TMD factorization cross section.

Let us apply some CGC results to this cross section to do phenomenology.

Final result in the MV model

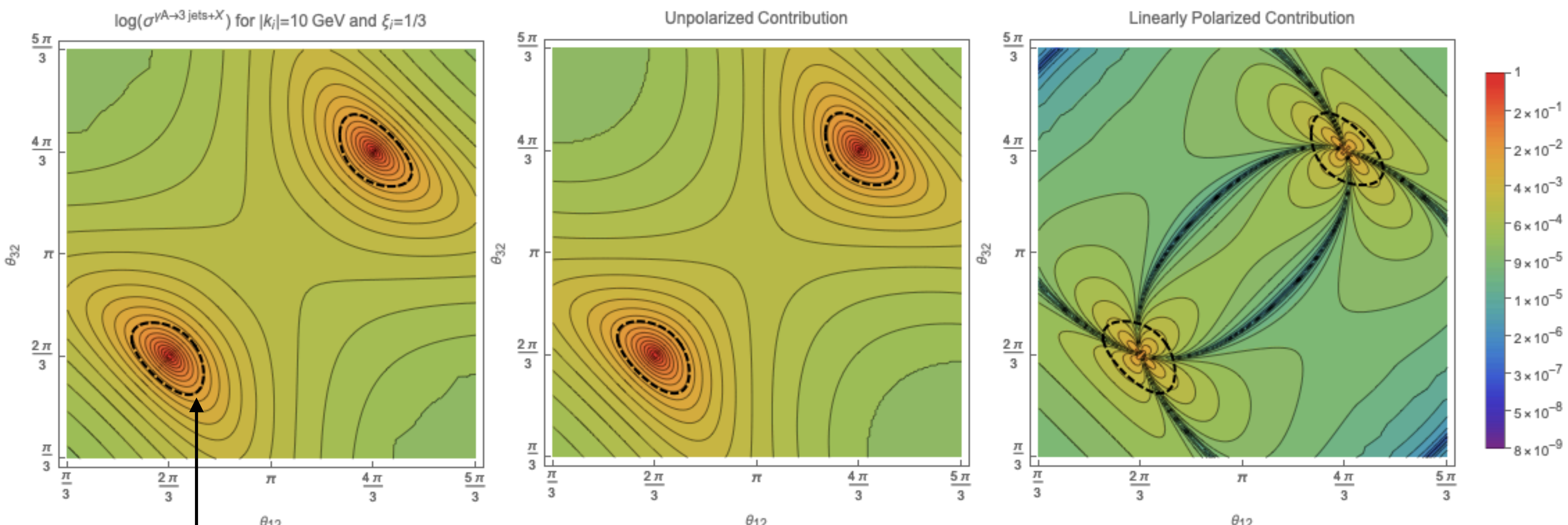
McLerran, Venugopalan (1994)



$$\mathcal{F}_{gg}^{(3)}(x,\mathbf{q}_T^2) = \frac{S_\perp C_F}{\alpha_s \pi^3}\int \mathrm{d}r \frac{J_0(|q_T r|)}{r}\left(1 - e^{-\frac{r^2}{4}Q_{sg}^2 \ln(1/r^2\Lambda^2)}\right)$$

$$\mathcal{H}_{gg}^{(3)}(x,\mathbf{q}_T^2) = \frac{S_\perp C_F}{\alpha_s \pi^3}\int \mathrm{d}r \frac{J_2(|q_T r|)}{r \ln\frac{1}{r^2\Lambda^2}}\left(1 - e^{-\frac{r^2}{4}Q_{sg}^2 \ln(1/r^2\Lambda^2)}\right)$$

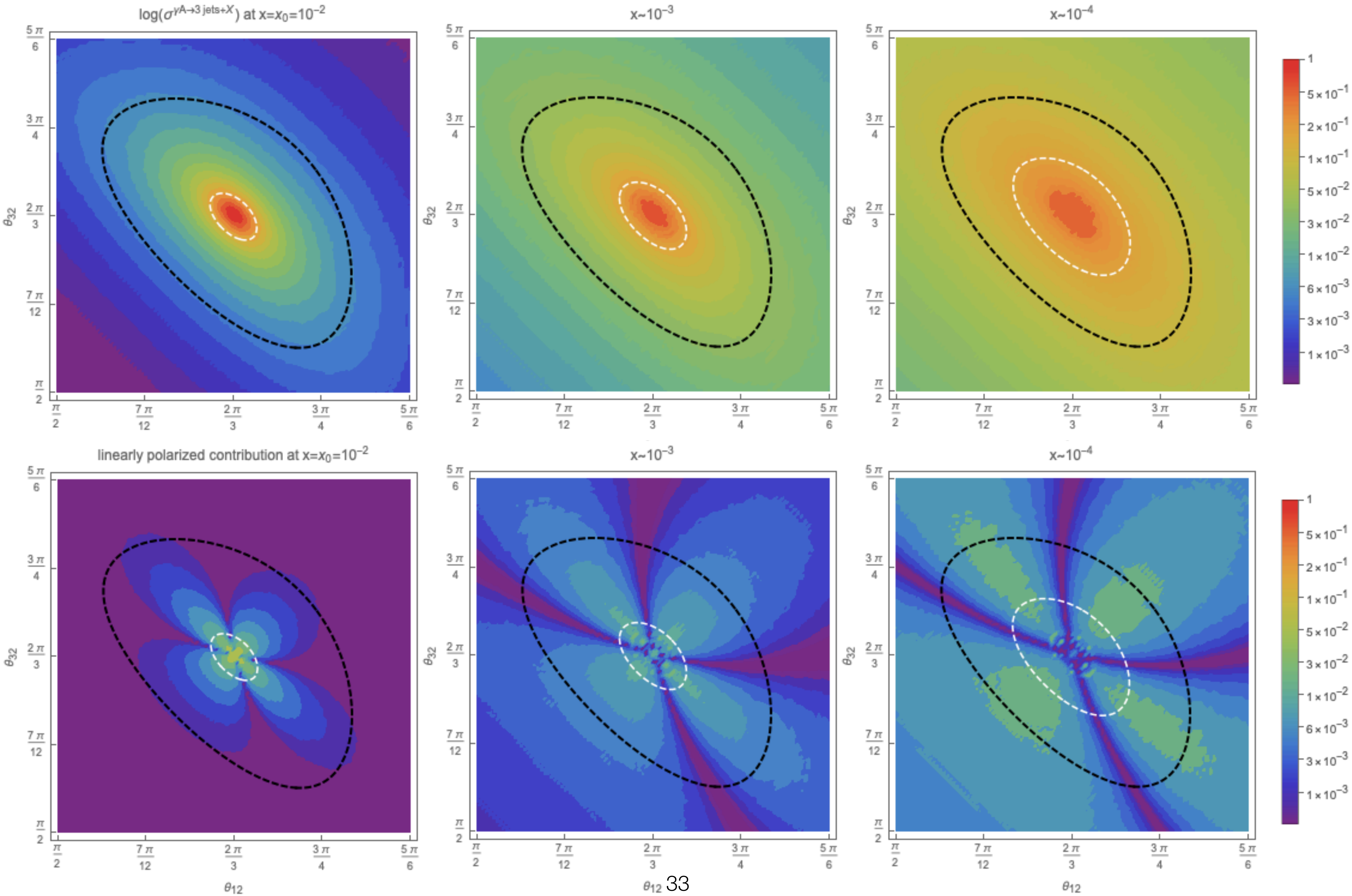
Dominguez, Xiao, Yuan (2011)
Metz, Zhou (2011)



$q_T = \sqrt{10}\,\mathrm{GeV}$

$\xi_i = 1/3$ and $|\mathbf{k}_i| = 10\,\mathrm{GeV}$

JIMWLK evolution of final result



Conclusions and outlook

Conclusions & outlook

Gluon TMDs provide valuable insight in three-dimensional proton structure, unfortunately for the moment no fits available !

Important role in quarkonium production and at low-*x.*

UPCs could be a great probe, in particular at the LHC where one can reach small values of *x*. *See the work by the Krakow group.*

For a serious extraction, precision needs to increase, CSS implemented (see Shu-Yi's work)

Low-*x* improved TMD framework is powerful tool for phenomenology

SIDIS in the CGC at NLO in progress.

Thanks for your attention !