

The Quark-Gluon-Plasma/Black-Hole Holographic Duality

Robi Peschanski

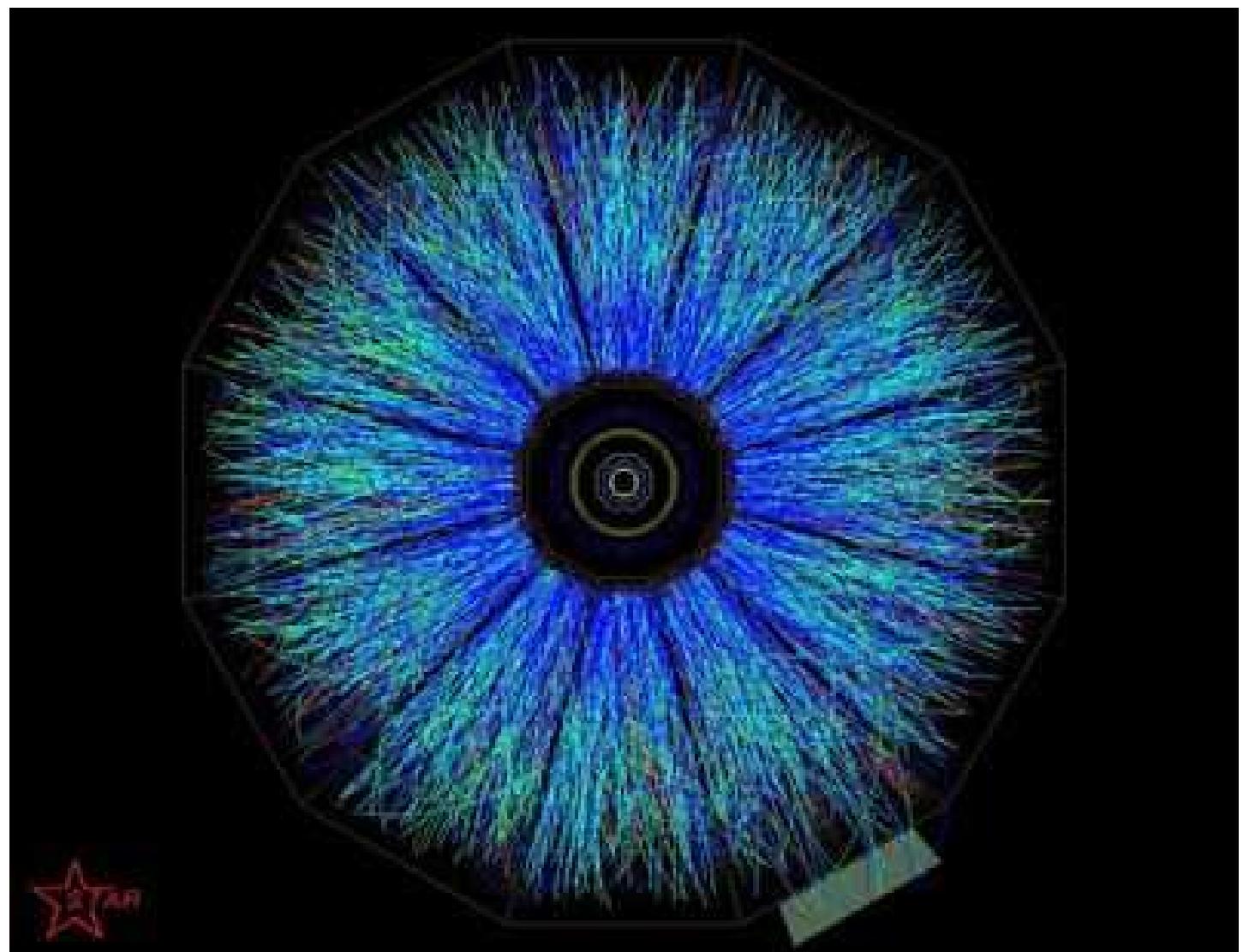
(IPhT, Saclay)

Séminaire général de l'IPhT

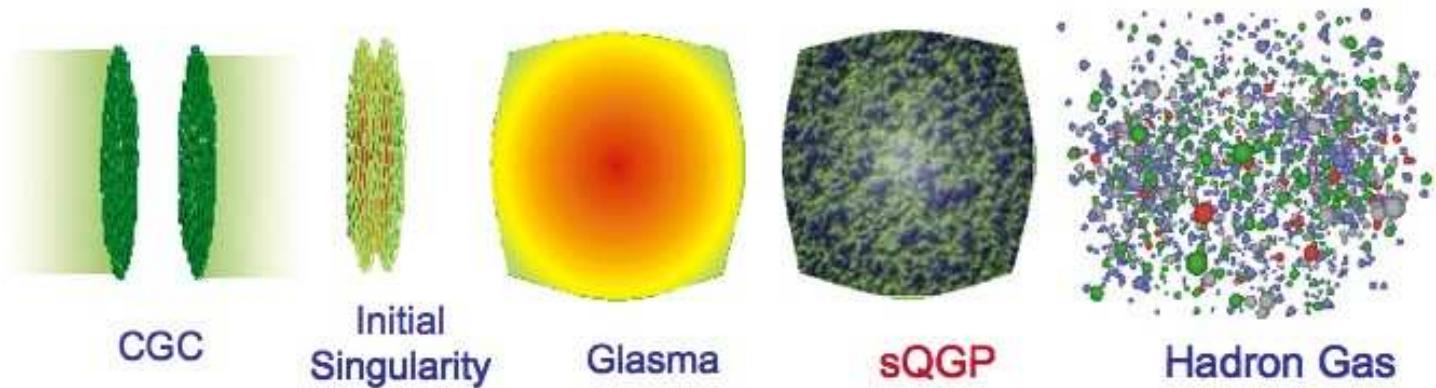
17/11/2009

- Heavy-Ion Reactions and the QGP
The strongly-coupled QGP
- Gauge/Gravity Duality and AdS/CFT correspondence
An Introduction
- QGP/BH Duality: Late time flow
Hydrodynamics
- QGP/BH Duality: Early time flow
Thermalization
- Conclusions and Prospects

Heavy-Ion Collision

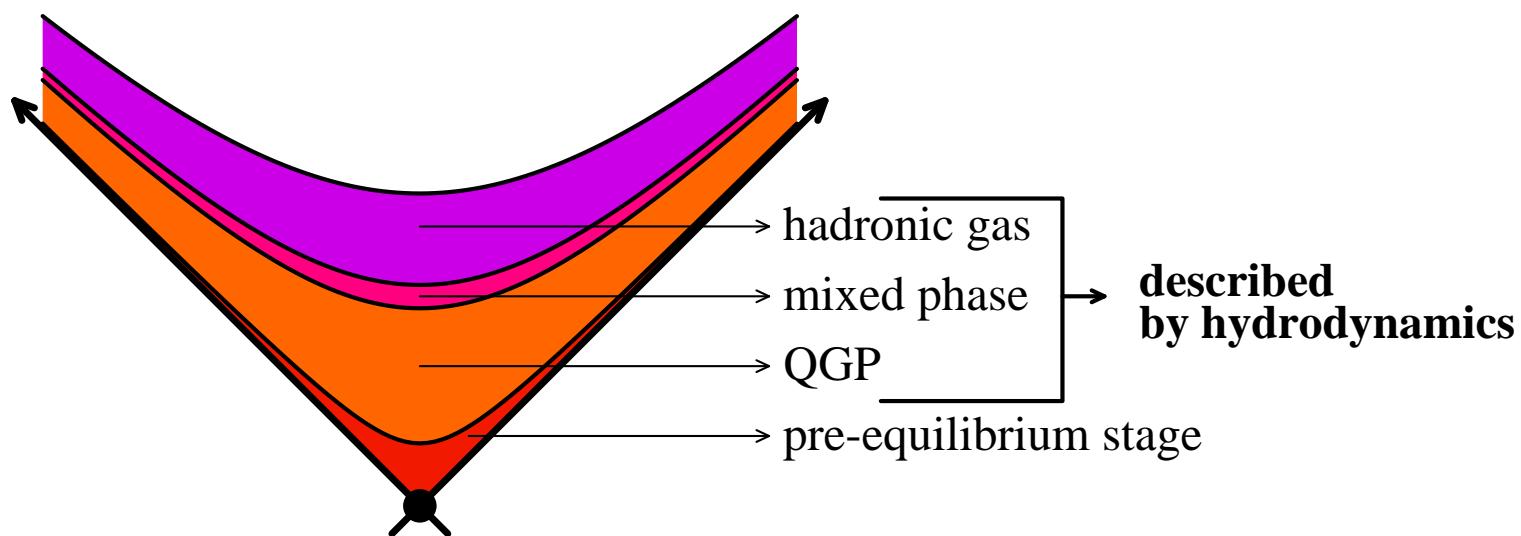


I. QGP Formation in Heavy-Ion Collisions



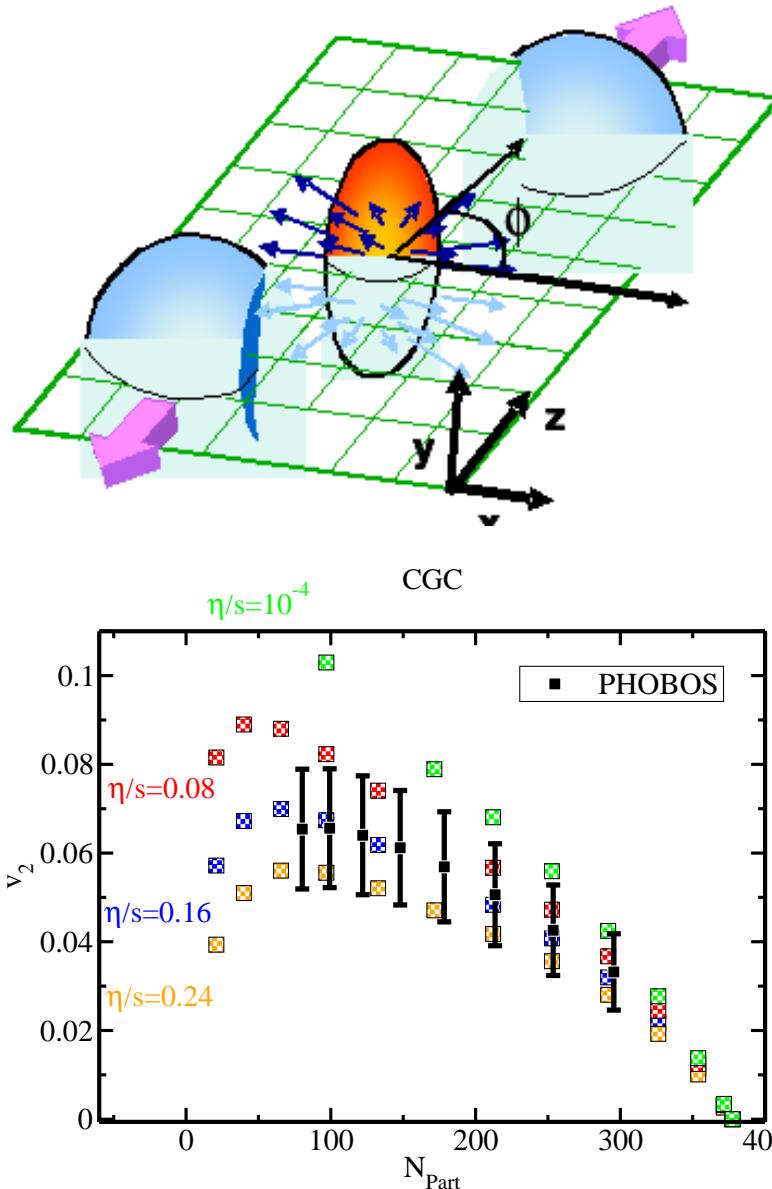
Boost-Invariant Description

Bjorken (1982)



Evidence for Hydrodynamics: Elliptic Flow

Ollitrault (1992)



$$\frac{\partial N}{\partial \Phi} \propto 1 + 2 v_2 \cos 2\Phi$$

Luzum, Romatschke (2008)

(1+3)-d Hydrodynamics

- Energy-Momentum Tensor (perfect fluid)

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - p\eta^{\mu\nu}$$

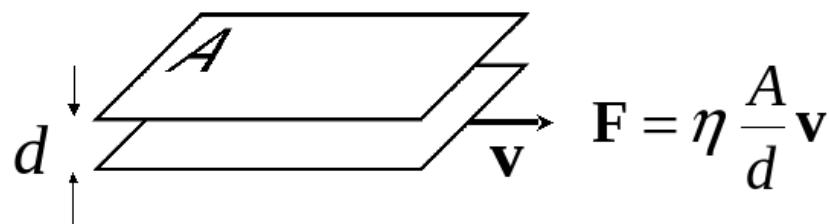
- Relativistic Hydrodynamic Equations (conformal)

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{EM Conservation condition}$$

$$T^{\mu\mu} = 0 : \quad \text{Traceless condition}$$

$$c_{sound}^2 \equiv \partial\epsilon/\partial p = 1/3 : \quad \text{Equation of State}$$

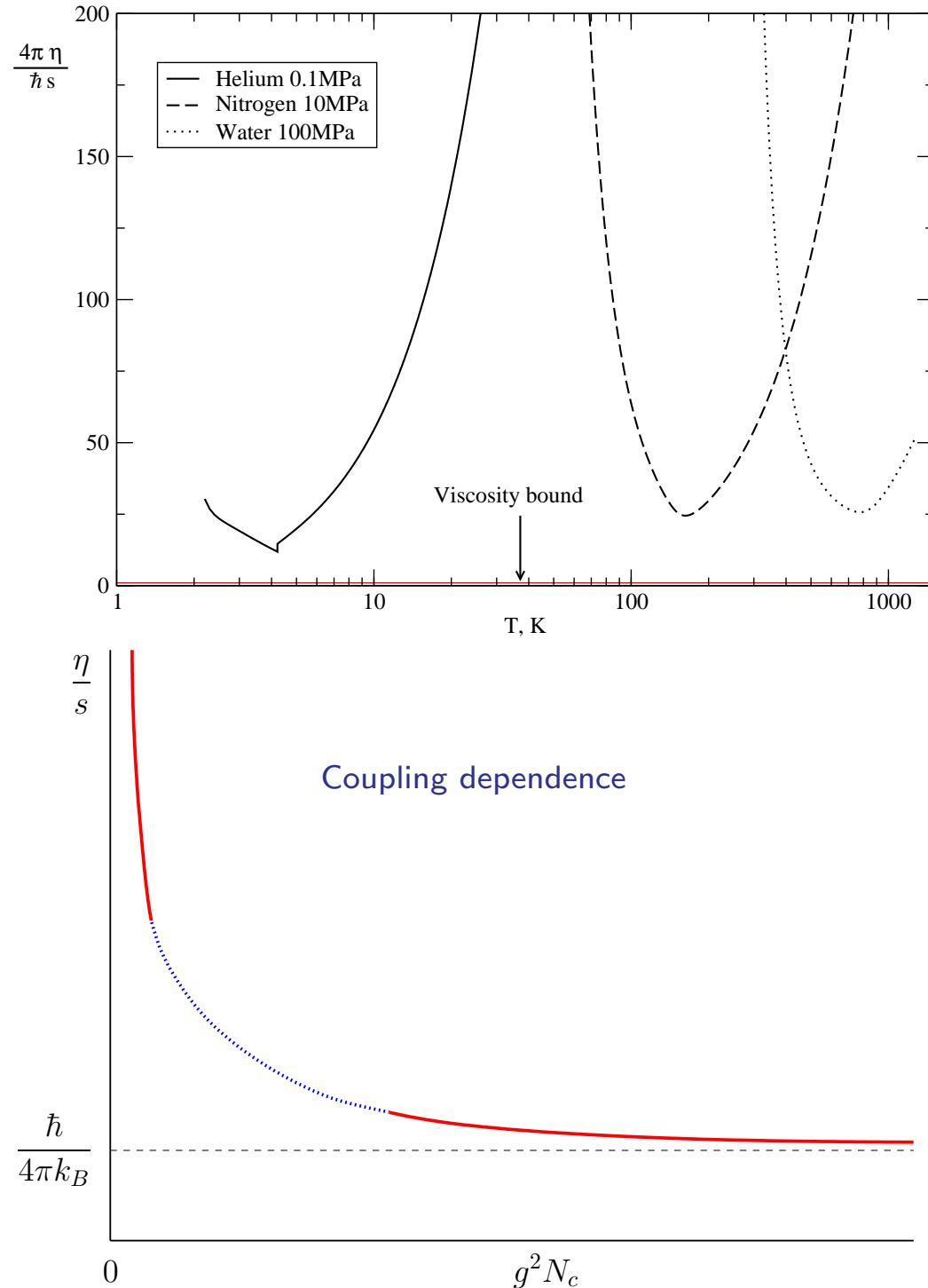
- Viscosity



$$\frac{\eta}{S} = \frac{\rho_{part} \times \lambda_{free} \times \bar{p}_{av}}{\rho_{part} \times k_B} \geq \frac{1}{4\pi} \frac{\hbar}{k_B} = \frac{\eta}{S} \Big|_{AdS/CFT}$$

Viscosity Bound

Kovtun, Son, Starinets (2004)



From Experiments to Theory

Abstracted from RHIC Data :

- Evidence for an Hydrodynamic Flow
- QGP: (Almost) Perfect fluid behaviour \Rightarrow small viscosity
- QGP Formation \Rightarrow fast thermalisation/isotropization

Interest of AdS/CFT :

- QGP: Strongly Coupled Deconfined medium at $T > T_c$
- AdS/CFT as a “gedanken laboratory” for QCD
- Gauge/Gravity: a wider concept

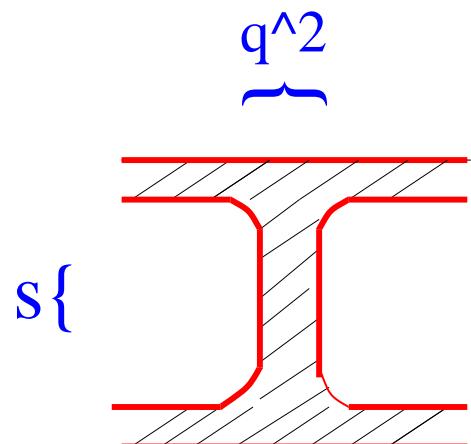
QGP at Strong Coupling : What Gauge/Gravity can tell us?

Strong Interactions and Strings: A Turbulent History

Strings \Leftrightarrow QCD \Leftrightarrow Strings

1968 \Rightarrow 1974 \Rightarrow 1998 \Rightarrow 2008 ...

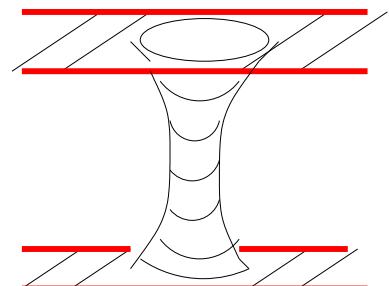
Open String



Veneziano Amplitude

$$A_R(s, q^2)$$

Closed String



Shapiro-Virasoro Amplitude

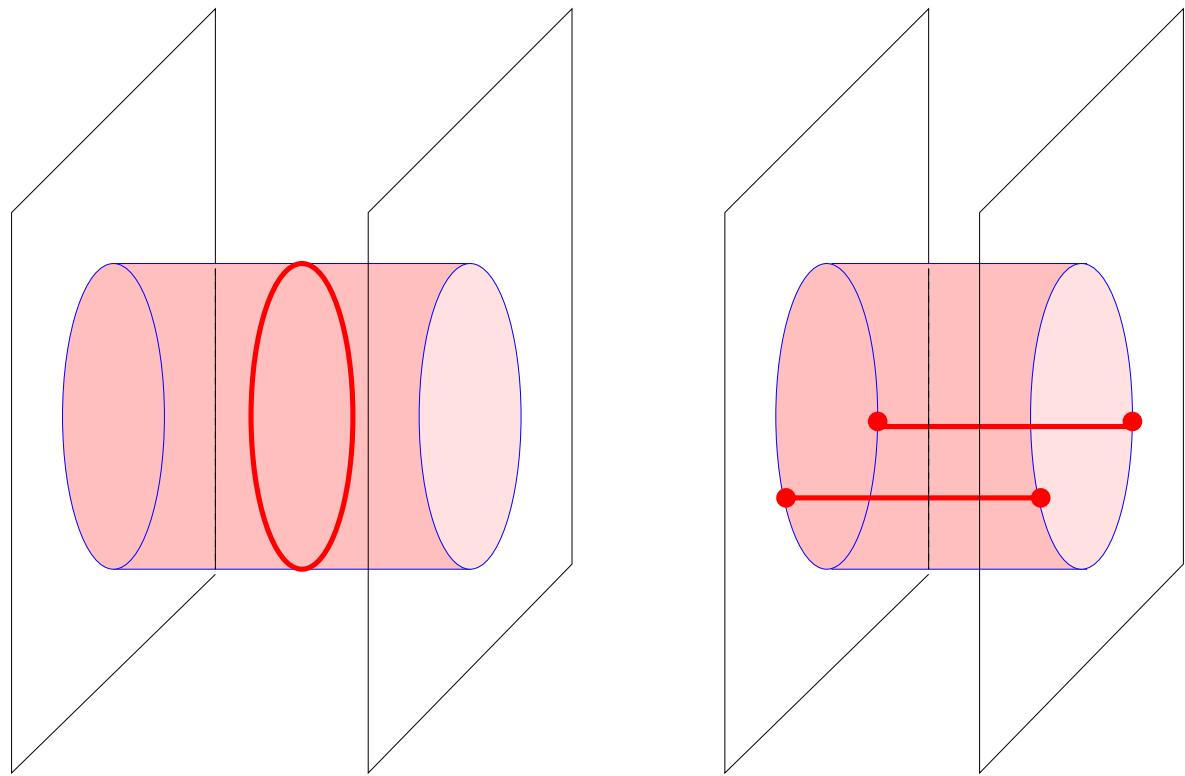
$$A_P(s, q^2)$$

Gauge

Gravity

II. The Gauge-Gravity Duality

Open String \Leftrightarrow Closed String

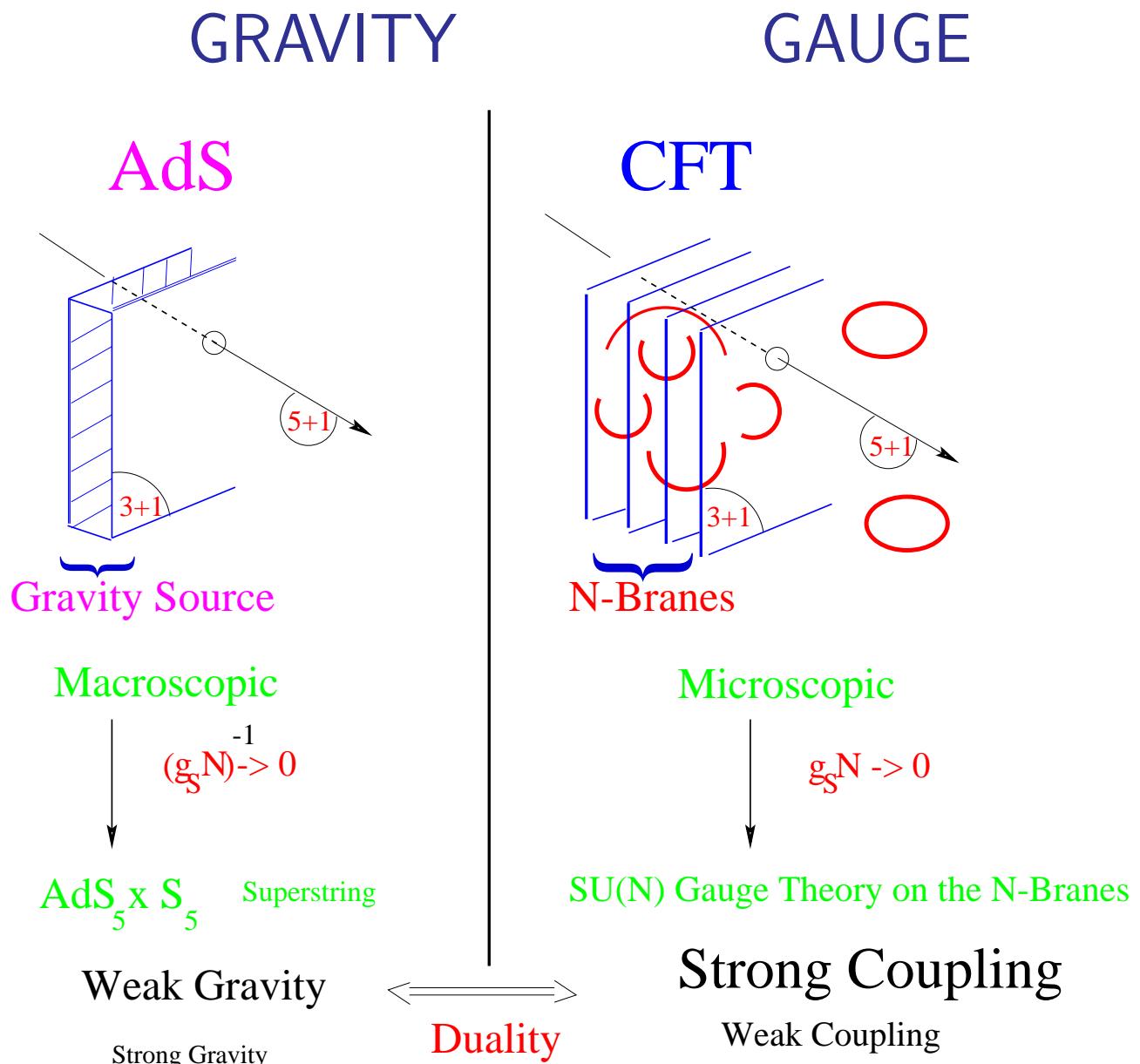


Schomerus, 2006

Closed String \Leftrightarrow *1-loop Open String*
D-Brane “Universe” \Rightarrow *Open String Ending*
Gravity \Leftrightarrow *Gauge*
Large/Small Distance \Rightarrow *Gravity/Gauge Correspondence*

AdS/CFT Correspondence

Maldacena (1997)



WHY $\text{AdS}_5 \otimes S_5$?

- Solution of Gravity for D_3 Branes: Horowitz, Strominger, 1991

$$ds^2 = f^{-1/2}(-dt^2 + \sum_1^3 dx_i^2) + f^{1/2}(dr^2 + r^2 d\Omega_5)$$

“Physical” Branes (d=1+3) + Extra-Dimensions (d=6)

$$f = 1 + \frac{R^4}{r^4}; \quad R^4 = 4\pi\alpha'^2 g_{YM}^2 N_c$$

- “Maldacena breakthrough”: Maldacena, 1997

$$\frac{\alpha'(\rightarrow 0)}{r(\rightarrow 0)} = z \text{ and } R \text{ fixed} \Rightarrow g_{YM}^2 N_c \rightarrow \infty$$

Strong coupling limit

$$ds^2 = \frac{1}{R^2 z^2}(-dt^2 + \sum_{1-3} dx_i^2 + dz^2) + R^2 d\Omega_5$$

Background Structure: $\text{AdS}_5 \otimes S_5$ (same R^2)

HOLOGRAPHY

- Brane/Bulk correspondence: Holographic Matter

Rey and Yee; Maldacena (1998)

Example # 1: Wilson Loops \Leftrightarrow Minimal Surfaces

$$\langle e^{iP \int_C \vec{A} \cdot d\vec{l}} \rangle = \int_{\Sigma} e^{-\frac{\text{Area}(\Sigma)}{\alpha'}} \approx e^{-\frac{\text{Min. Area}}{\alpha'}} \times \text{Fluctuations}$$

- Brane to Bulk: Holographic Renormalization

K.Skenderis (2002)

Example # 2: Adding Matter/Energy on the Brane

$$ds^2 = \frac{g_{\mu\nu}(z) dx^\mu dx^\nu + dz^2}{z^2}$$

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} (= \eta_{\mu\nu}) + z^2 g_{\mu\nu}^{(2)} (= 0) + z^4 \langle T_{\mu\nu} \rangle + z^6 \dots +$$

+ $z^6 \dots +$: from Einstein Eqs.

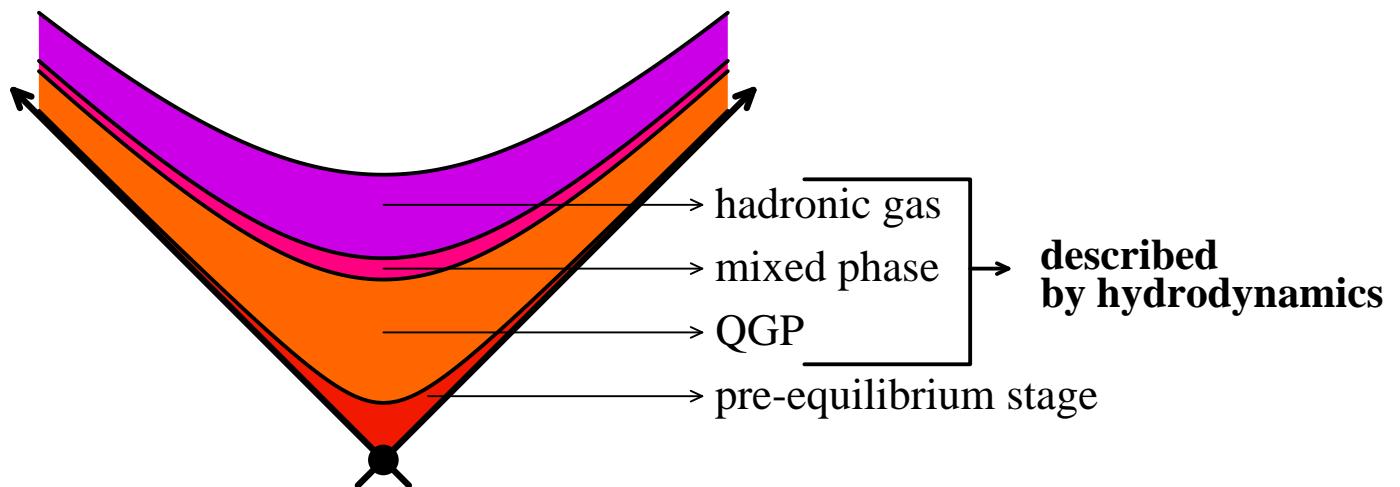
Gauge/Gravity Duality and QGP Dynamics

Janik, R.P.

Janik, Heller, Benincasa, Buchel...

Kovchegov, Taliotis, Albacete,...

Nakamura, Sin, Kinoshita, Mukoyama, Nakamura, Oda, Natsuume, Okamura,...
Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...
.....



$$\tau = \sqrt{x_0^2 - x_1^2} ; y = \frac{1}{2} \log \frac{x_0 + x_1}{x_0 - x_1} ; x_T = x_2, x_3$$

Questions

- What is the Gravity Dual of a Boost-Invariant Flow?
- QGP: (almost) Perfect fluid behaviour, why?
- Universal $\frac{\eta}{S}$, Transport coefficients, Navier-Stokes,...
- Fast Pre-equilibrium stage, why?

III. QGP/BH Duality: The Late Time flow

- Boost-invariant $T_{\mu\nu}$

$$T_{\mu\nu} = \begin{pmatrix} f(\tau) & 0 & 0 & 0 \\ 0 & -\tau^3 \frac{d}{d\tau} f(\tau) - \tau^2 f(\tau) & 0 & 0 \\ 0 & 0 & f(\tau) + \frac{1}{2} \tau \frac{d}{d\tau} f(\tau) & 0 \\ 0 & 0 & 0 & \dots \end{pmatrix}$$

- Proper-time evolution

$f(\tau) \propto \tau^{-S}$: FamilyIndex $T_{\mu\nu} t^\mu t^\nu \geq 0 \Rightarrow 0 < s < 4$

$f(\tau) \propto \tau^{-\frac{4}{3}}$: Perfect Fluid

$f(\tau) \propto \tau^{-1}$: Free streaming

$f(\tau) \propto \tau^{-0}$: Full Anisotropy $\epsilon = p_\perp = -p_L$

- Holographic renormalization:

\Rightarrow Holographic Scaling Variable v at large τ

$$v = \frac{z}{\tau^{S/3}}$$

Calculation of the Gravity Duals

- Boost-Invariant 5-d F-G metric:

$$ds^2 = \frac{-e^{a(\tau,z)}d\tau^2 + \tau^2 e^{b(\tau,z)}dy^2 + e^{c(\tau,z)}dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- Scaling : $v = \frac{z}{\tau^{S/4}}$

$$[a(\tau, z), b(\tau, z), c(\tau, z)] = [a(v), b(v), c(v)] + \mathcal{O}\left(\frac{1}{\tau^\#}\right)$$

$$v(2a'(v)c'(v)+a'(v)b'(v)+2b'(v)c'(v))-6a'(v)-6b'(v)-12c'(v)+vc'(v)^2=0$$

$$3vc'(v)^2+vb'(v)^2+2vb''(v)+4vc''(v)-6b'(v)-12c'(v)+2vb'(v)c'(v)=0$$

$$\begin{aligned} & 2vsb''(v) + 2sb'(v) + 8a'(v) - vsa'(v)b'(v) - 8b'(v) + vsb'(v)^2 + \\ & 4vsc''(v) + 4sc'(v) - 2vs a'(v)c'(v) + 2vsc'(v)^2 = 0 \end{aligned}$$

- Asymptotic Solution

$$a(v) = A(v) - 2m(v)$$

$$b(v) = A(v) + (2s - 2)m(v)$$

$$c(v) = A(v) + (2 - s)m(v)$$

$$A(v) = \frac{1}{2} \left(\log(1 + \Delta(s) v^4) + \log(1 - \Delta(s) v^4) \right) \quad m(v) = \frac{1}{4\Delta(s)} \left(\log(1 + \Delta(s) v^4) - \log(1 - \Delta(s) v^4) \right) \quad \Delta(s) = \sqrt{3s^2 - 8s + 8/24}$$

AdS/CFT: Selection of the Perfect Fluid

- Kreschmann Scalar:

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$$

$$\begin{aligned} \mathfrak{R}^2 = & \frac{4}{(1 - \Delta(s)^2 v^8)^4} \cdot \left[10 \Delta(s)^8 v^{32} - 88 \Delta(s)^6 v^{24} + 42 v^{24} s^2 \Delta(s)^4 + \right. \\ & + 112 v^{24} \Delta(s)^4 - 112 v^{24} \Delta(s)^4 s + 36 v^{20} s^3 \Delta(s)^2 - 72 v^{20} s^2 \Delta(s)^2 + \\ & + 828 \Delta(s)^4 v^{16} + 288 v^{16} \Delta(s)^2 s - 288 v^{16} \Delta(s)^2 - 108 v^{16} s^2 \Delta(s)^2 + \\ & - 136 v^{16} s^3 + 27 v^{16} s^4 - 320 v^{16} s + 160 v^{16} + 296 v^{16} s^2 + 36 v^{12} s^3 + \\ & \left. - 72 v^{12} s^2 - 88 \Delta(s)^2 v^8 + 42 v^8 s^2 + 112 v^8 - 112 v^8 s + 10 \right] + \mathcal{O}\left(\frac{1}{\tau^\#}\right) \end{aligned}$$

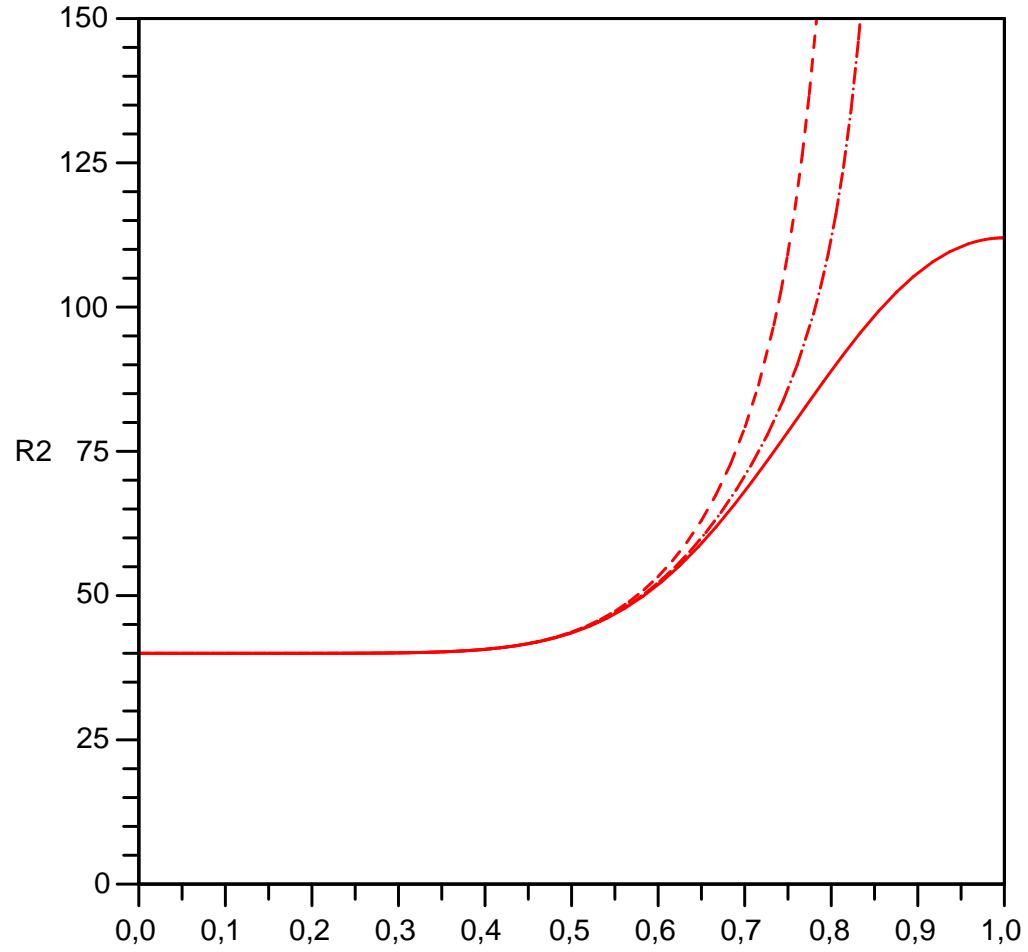
- \mathfrak{R}^2 for $s = \frac{4}{3}$:

$$\mathfrak{R}^2_{\text{perfect fluid}} = \frac{8(5w^{16} + 20w^{12} + 174w^8 + 20w^4 + 5)}{(1 + w^4)^4}$$

where $w = v/\Delta(\frac{4}{3})^{\frac{1}{4}} \equiv \sqrt[4]{3}v$.

AdS/CFT \Rightarrow Perfect Fluid at large τ

Kreschmann Scalar: $\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta}$



$$s = \frac{4}{3} \pm .1$$

“MicroCosmic” Censorship

A nonsingular background selects a moving Black Hole geometry dual to the perfect fluid at large proper-times

The Cooling Plasma/Moving Black Hole Duality

$$v = \frac{z}{\tau^{1/3}}$$

- Asymptotic metric

$$ds^2 = \frac{1}{z^2} \left[-\frac{\left(1 - \frac{e_0}{3} \frac{z^4}{\tau^{4/3}}\right)^2}{1 + \frac{e_0}{3} \frac{z^4}{\tau^{4/3}}} d\tau^2 + \left(1 + \frac{e_0}{3} \frac{z^4}{\tau^{4/3}}\right) (\tau^2 dy^2 + dx_\perp^2) \right] + \frac{dz^2}{z^2}$$

- BH off in the 5th dimension \Leftrightarrow Hwa-Bjorken flow

$$\text{Horizon : } z_h = \left(\frac{3}{e_0}\right)^{\frac{1}{4}} \cdot \tau^{\frac{1}{3}} .$$

$$\text{Temperature : } T(\tau) \sim \frac{1}{z_h} \sim \tau^{-\frac{1}{3}}$$

$$\text{Entropy : } S(\tau) \sim \text{Area} \sim \tau \cdot \frac{1}{z_h^3} \sim const$$

QGP/BH Duality and Viscosity

Static Case

Policastro, Son, Starinets (2001)

Viscosity on the light of duality

Consider a graviton that falls on this stack of N D3-branes

Will be absorbed by the D3 branes.

The process of absorption can be looked at from two different perspectives:



Absorption by D3 branes (\sim viscosity) = absorption by black hole

$$\sigma_{abs}(\omega) \propto \int d^4x \frac{e^{i\omega t}}{\omega} \langle [T_{x_2 x_3}(x), T_{x_2 x_3}(0)] \rangle \Rightarrow \frac{\eta}{s} \equiv \frac{\sigma_{abs}(0)/(16\pi G)}{A/(4G)} = \frac{1}{4\pi}$$

QGP/BH Duality beyond Perfect fluid

Dynamic Case

- Going beyond perfect fluid

In-flow Viscosity, Relaxation time, Transport Coeff., etc...

Janik, Heller, Bak, Benincasa, Buchel, Nakamura, Sin,.....
Kinoshita, Mukoyama, Nakamura, Oda, Natsuume, Okamura,...

$$\boxed{\partial_\tau \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau} + \frac{\eta}{\tau^2} + \dots \Rightarrow \frac{\eta}{s} = \frac{1}{4\pi}}$$

- Going beyond boost-invariance

Fluid/Gravity Duality

Bhattacharyya, Hubeny, Minwalla, Ranganami, Loganayagam,...

$$T_{rescaled}^{\mu\nu} = \underbrace{(\pi T)^4 (\eta^{\mu\nu} + 4u^\mu u^\nu)}_{\text{perfect fluid}} - \underbrace{2(\pi T)^3 \sigma^{\mu\nu}}_{\text{viscosity}} + \\ + \underbrace{(\pi T^2) \left(\log 2 T_{2a}^{\mu\nu} + 2T_{2b}^{\mu\nu} + (2 - \log 2) \left(\frac{1}{3} T_{2c}^{\mu\nu} + T_{2d}^{\mu\nu} + T_{2e}^{\mu\nu} \right) \right)}_{\text{second order hydrodynamics}}$$

- Going Beyond hydrodynamics? Beyond Equilibrium?

IV. QGP/BH Duality: Early-time Flow

- General Boost-Invariant Fefferman-Graham metric:

$$ds^2 = \frac{-e^{a(\tau,z)} d\tau^2 + \tau^2 e^{b(\tau,z)} dy^2 + e^{c(\tau,z)} dx_\perp^2}{z^2} + \frac{dz^2}{z^2}$$

- Einstein Equation:

$$R_{AB} + 4G_{AB} = 0$$

- To be solved: ($\dot{a} = \partial_\tau a; a' = \partial_z a; \dots$)

$$(\tau\tau) : \ddot{b} + 2\ddot{c} - \frac{\dot{a}}{2}(\dot{b} + 2\dot{c}) + \frac{1}{2}(\dot{b}^2 + 2\dot{c}^2) - \frac{1}{\tau}(\dot{a} - 2\dot{b}) = e^a \left\{ a'' - \frac{3a'}{z} + \left(\frac{a'}{2} - \frac{1}{z} \right) (a' + b' + 2c') \right\}$$

$$(yy) : \ddot{b} - \dot{a}\dot{b} + \frac{1}{\tau}(\dot{b} - 2\dot{a}) + \frac{1}{2}(\dot{a} + \dot{b} + 2\dot{c}) \left(\dot{b} + \frac{2}{\tau} \right) = e^a \left\{ b'' - \frac{3b'}{z} + \left(\frac{b'}{2} - \frac{1}{z} \right) (a' + b' + 2c') \right\}$$

$$(\perp\perp) : \ddot{c} - \dot{a}\dot{c} + \frac{\dot{c}}{2} \left(\dot{a} + \dot{b} + 2\dot{c} + \frac{2}{\tau} \right) = e^a \left\{ c'' - \frac{3c'}{z} + \left(\frac{c'}{2} - \frac{1}{z} \right) (a' + b' + 2c') \right\}$$

$$(\tau z) : 2\dot{b}' + 4\dot{c}' + b' \left(\dot{b} + \frac{2}{\tau} \right) + 2\dot{c}c' - a' \left(\dot{b} - 2\dot{c} + \frac{2}{\tau} \right) = 0$$

$$(zz) : a'' + b'' + 2c''' - \frac{1}{z}(a' + b' + 2c') + \frac{1}{2}(a'^2 + b'^2 + 2c'^2) = 0$$

Early-Time; General Features

G.Beuf, M.Heller, R.Janik, R.P., 2009

- No scaling \Rightarrow Dependence on Initial Conditions

$$a(\tau, z) = \dots + z^8 \left\{ -1/16 \tau^{-2} s^2 - 1/6 \tau^{-2} s + 1/6 \tau^{-2} s s \right\} + \frac{z^4}{\tau^s} \left\{ \frac{1}{96} \frac{z^4}{\tau^4} s^2 - \frac{1}{384} \frac{z^4}{\tau^4} s^4 \right\} + \dots$$

scaling part $\{\dots\}$ not dominant when $s=0$

- The metric is singular at all times (including $\tau = 0$!):

$$\text{Set : } u(z^2) = \frac{1}{4z} a'_0(z) \quad v(z^2) = \frac{1}{4z} b'_0(z) \quad w(z^2) = \frac{1}{4z} c'_0(z)$$

$$0 = [u + v + 2w]_0^\infty \equiv \int_0^\infty (u' + v' + 2w') dz^2 \neq -2 \int_0^\infty (u^2 + v^2 + 2w^2) dz^2$$

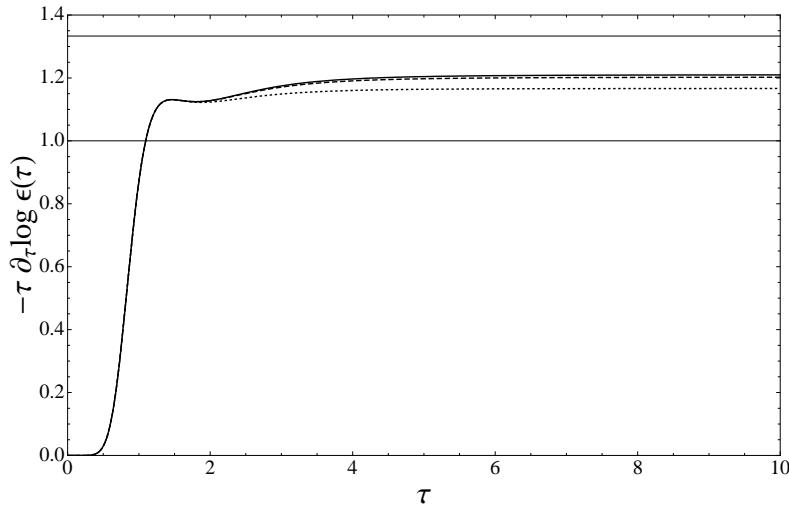
- “MicroCosmic Censorship” \Rightarrow Nonequilibrium Constraints

$$ds^2(z \sim z_{sing}) \sim \frac{1}{z^2} \left(1 - \frac{z}{z_{sing}} \right)^2 d\tau^2 + \dots + \frac{1}{z^2} dz^2$$

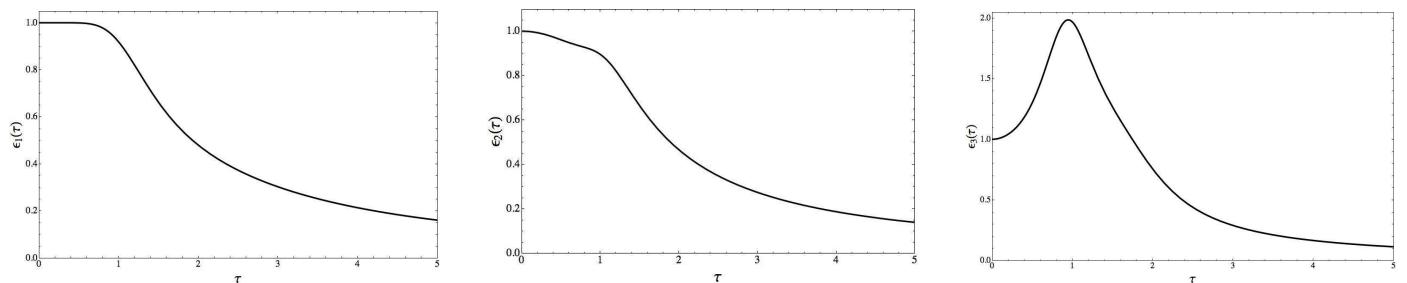
- Looking for solutions with Constraints

Investigations on Thermalization

- “Family Index”: $s = -\tau \partial_\tau \epsilon(\tau)$



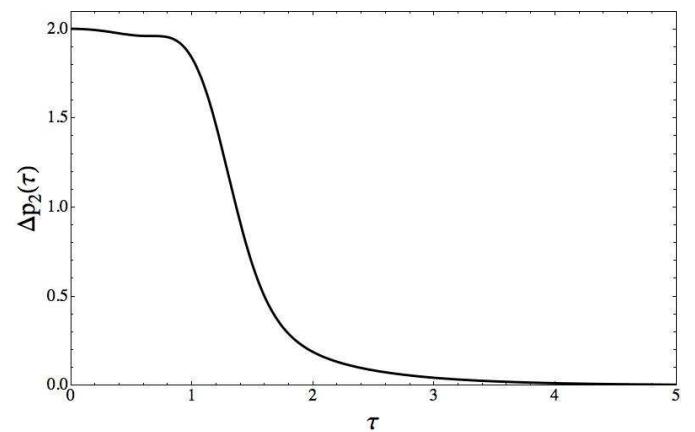
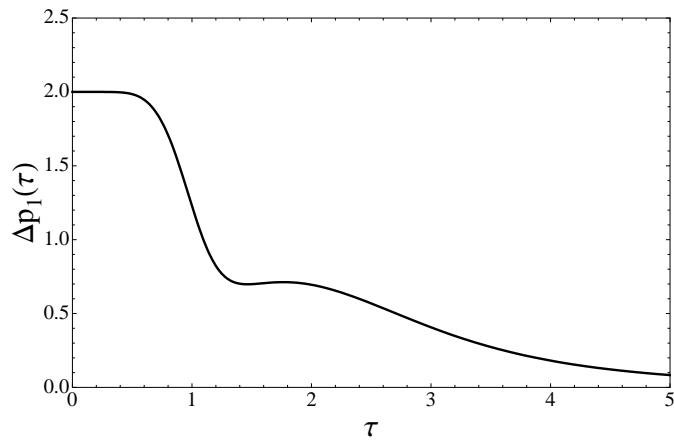
- Dependence on Initial Conditions



$$[v+w- >] A) : \tanh(z^2) - \tan(z^2) \quad B) : \tanh(z^2 + z^8/6) - \tan(z^2) \quad C) : 2/3z^6(1+z^2/2)/(z^2-1)$$

C): Temporary violation of Positivity: $T_{\mu\nu}t^\mu t^\nu \geq 0 \Rightarrow \frac{4\epsilon(\tau)}{\tau} \leq \epsilon'(\tau) \leq 0$

Isotropization of Pressure Density



$$[v+w- >] A) : \tanh(z^2) - \tan(z^2)$$

$$B) : \tanh(z^2 + z^8/6) - \tan(z^2)$$

$$\boxed{\Delta p(\tau) = 1 - \frac{p_{\parallel}(\tau)}{p_{\perp}(\tau)}}$$

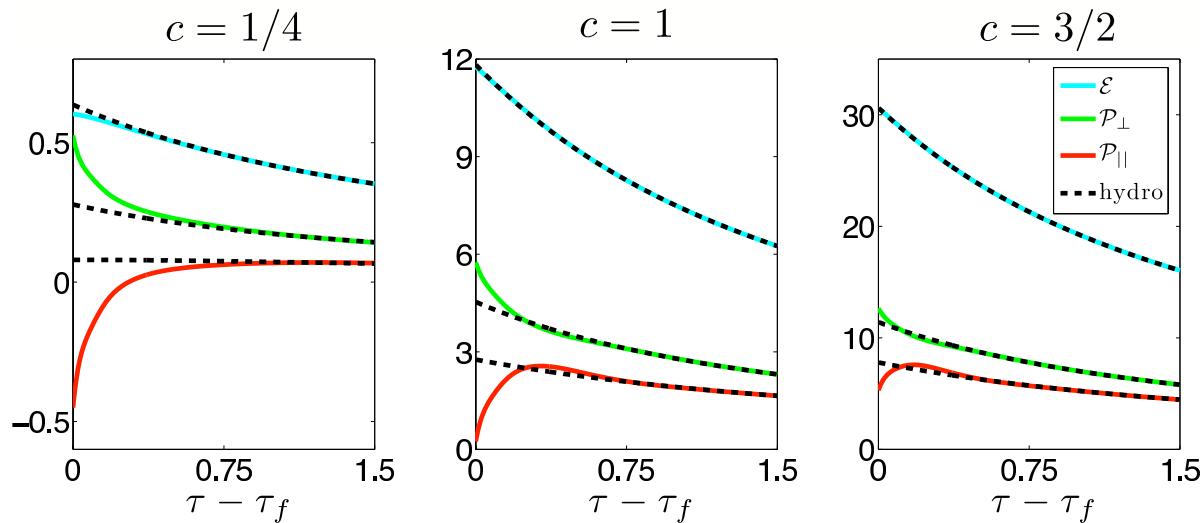
- “Fast” Road towards Isotropization
- Isotropization may stay “some time” Incomplete

Far-from-equilibrium Dynamics: Black hole formation

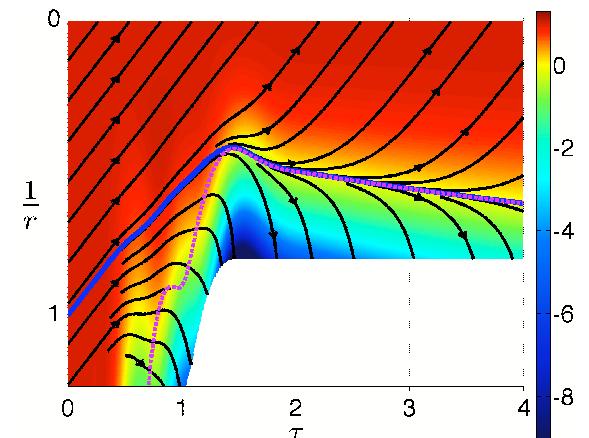
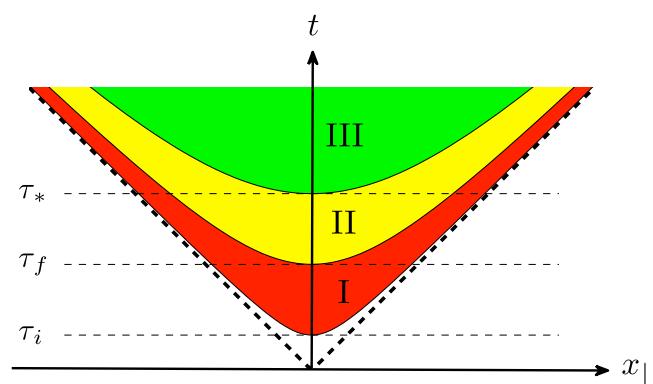
P.M.Chesler, L.G.Yaffe, 2009

- 4d Time-dependent Shear

$$ds^2 = -d\tau^2 + \tau^2 e^{-2c\gamma(\tau)} dy^2 + e^{c\gamma(\tau)} dx_\perp^2$$



- 5d Black hole Formation



I : 4d Deformation II : Anisotropic Relaxation III : Hydro Regime

Conclusions and Prospects

Conclusions:

- **Gauge-Gravity Correspondence**
A promising way to study Boost-Invariant Dynamics
- **Late-time (Hydro)Dynamics**
Scaling, “almost-perfect” fluid, Einstein *vs.* Navier-Stokes
- **Early-time Dynamics**
No scaling, singularity at all times, thermalization studies

Prospects

- **More Numerical Work**
Classifying the thermalization solutions
- **More “Translation” Work**
Relation with initial conditions
- **More Theoretical Work**
Going beyond boost-invariance
- **From S^4 QCD to S^0 QCD ?**
Approaching the “Gravity Dual” of QCD

Why Einstein Eqs. may govern the QGP?

EXTRA SLIDES

Boost-Invariant Viscosity and Relaxation time

R.Janik, R.Janik and M.Heller;

- Shear Viscosity equation (first order)

$$\partial_\tau \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau} + \frac{\eta}{\tau^2}$$

- Asymptotic Expansion of the Black Hole Solution

$$a(\tau, z), b(\tau, z), c(\tau, z) \Rightarrow \sum_n \lambda_n^{a,b,c}(v) \tau^{-2n/3}$$

$$\mathfrak{R}^2 = R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} \Rightarrow \sum_n \mathfrak{R}_n^2 \tau^{-2n/3}$$

- Results

$$\frac{\eta}{S} = \frac{1}{4\pi}$$

Universal viscosity (needs $n \rightarrow 2$)

$$\tau_{Rel} = (1 - \log 2)/2\pi T$$

Relaxation Time (needs $n \rightarrow 3$)

Starinets,

EMERGENCE of the 5d BLACK HOLE

Balasubramanian,de Boer,Minic (2002)

- 4d Perfect Fluid “on the brane”

$$\langle T_{\mu\nu} \rangle \propto g_{\mu\nu}^{(4)} = \begin{pmatrix} 3/z_0^4 = \epsilon & 0 & 0 & 0 \\ 0 & 1/z_0^4 = p_1 & 0 & 0 \\ 0 & 0 & 1/z_0^4 = p_2 & 0 \\ 0 & 0 & 0 & 1/z_0^4 = p_3 \end{pmatrix}$$

- Holographic Renormalisation (Resummed)

Janik,R.P. (2005)

$$ds^2 = -\frac{(1-z^4/z_0^4)^2}{(1+z^4/z_0^4)z^2} dt^2 + (1+z^4/z_0^4) \frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

- \Rightarrow 5d Black Brane with horizon at $z_0 \sim T_0^{-3}$

$$ds^2 = -\frac{1-\tilde{z}^4/\tilde{z}_0^4}{\tilde{z}^2} dt^2 + \frac{dx^2}{\tilde{z}^2} + \frac{1}{1-\tilde{z}^4/\tilde{z}_0^4} \frac{d\tilde{z}^2}{\tilde{z}^2}$$

$$z \rightarrow \tilde{z} = z / \sqrt{1+\frac{z^4}{z_0^4}}$$

Preliminaries (1): Quasi-Normal Modes

R.Janik,R.P., 2006

- Scalar Excitation of a Moving Black Hole

$$\Delta\phi \equiv \frac{1}{\sqrt{-g}} \partial_n (\sqrt{-g} g^{ij} \partial_j \phi) = 0$$

$g_{\mu\nu}$

- Scalar “Quasi-Normal Modes”

$$\phi(\tau, v \equiv z/\tau^{1/3}) = f(\tau) \times \phi(v)$$

$$f(\tau) = \sqrt{\tau} J_{\pm \frac{3}{4}} \left(\frac{3}{2} \omega \tau^{\frac{2}{3}} \right) \sim \tau^{\frac{1}{6}} e^{\frac{3}{2} i \omega \tau^{2/3}}$$

- Short Excitation Decay

$$\frac{\omega_c}{\pi T} \sim 3.1194 - 2.74667 i \Rightarrow \tau \sim \frac{1}{8.3 T}$$

- e-folding conjecture

JJ Friess, SS Gubser, G. Michalogiorgakis, SS Pufu, 2007

$$\tau_{therm} \sim 4\tau_{e-fold} = 4 \times \frac{1}{8.3 T_{peak}} \sim 4 \times .1 fermi$$

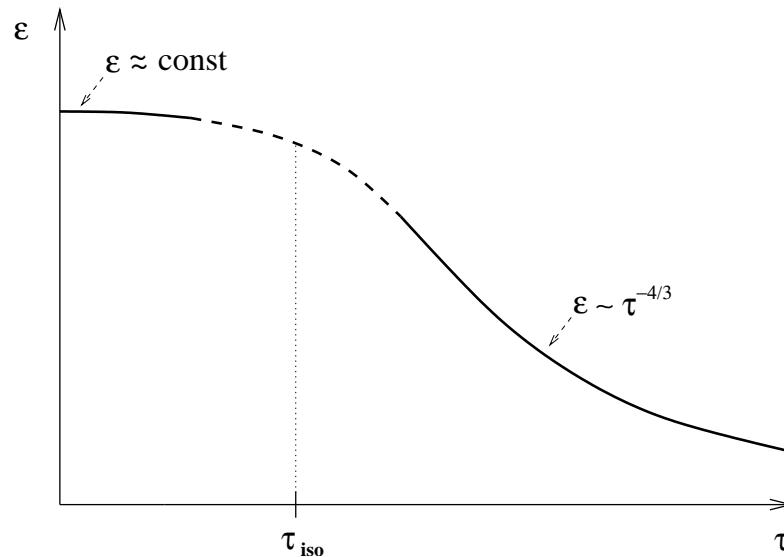
The Black Hole as an “Attractor”

Preliminaries (2): Scaling Solution

Kovchegov, Taliotis, 2007

- Evolution at small ($S = 0$) vs. large ($S = 4/3$) proper-time

Assuming Monodromy \in Regularity



- Evaluation of The Isotropization/Thermalization time

$$\text{Matching : } z_h^{\text{late}}(\tau) = (3/e_0)^{1/4} \equiv z_h^{\text{early}}(\tau) = \tau$$

$$\text{Isotropization : } \tau_{iso} = (3N_c^2/2\pi^2 e_0)^{3/8}$$

$$\text{Typical Scale : } \epsilon(\tau) = e_0 \tau^{4/3}|_{\tau=.6} \sim 15 \text{ GeV fermi}^{-3}$$

$$\Rightarrow \boxed{\tau_{iso} \sim .3 \text{ fermi}}$$