Probabilistic description of in-medium jet evolution

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In collaboration with
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(In preparation)

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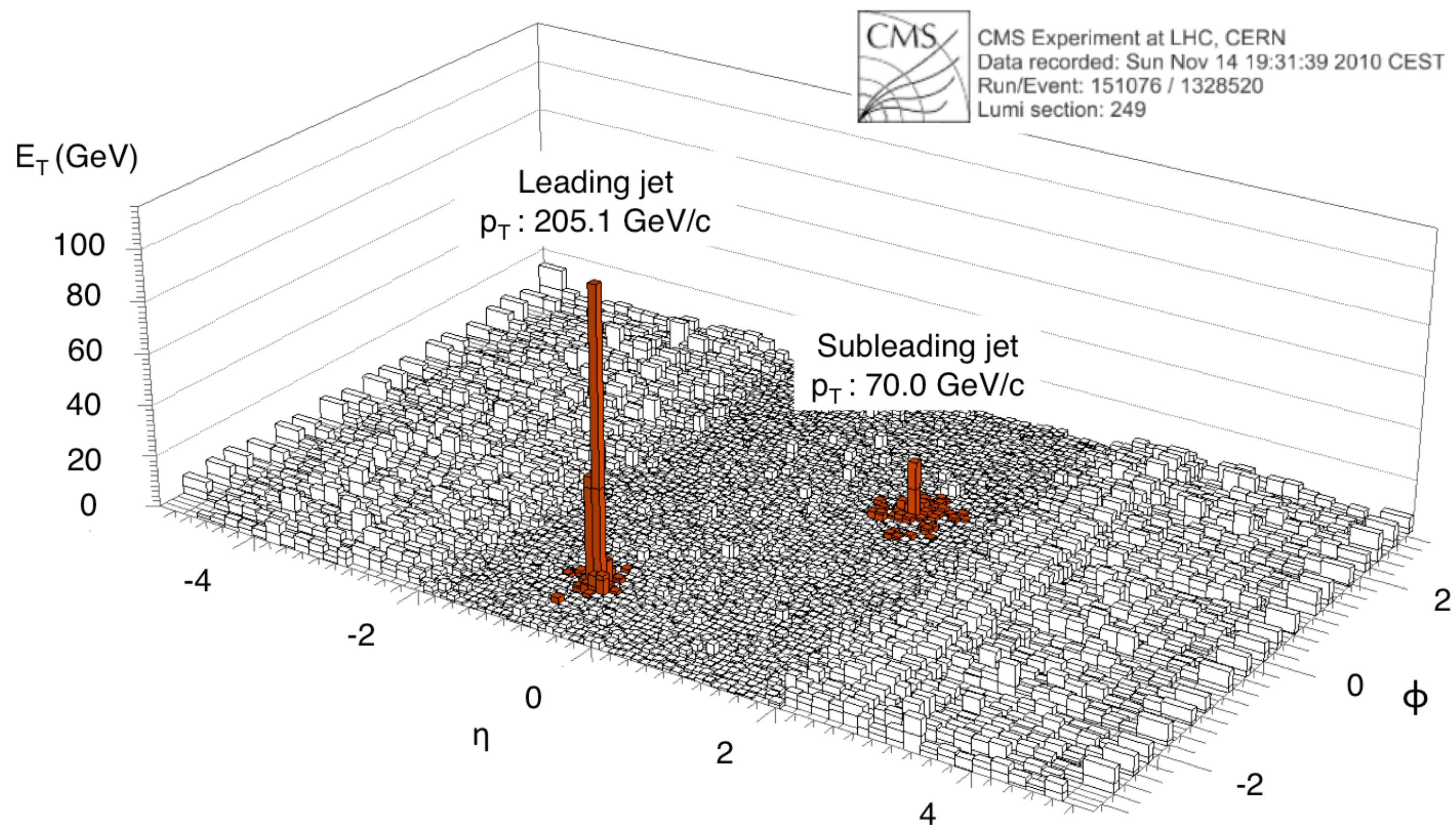
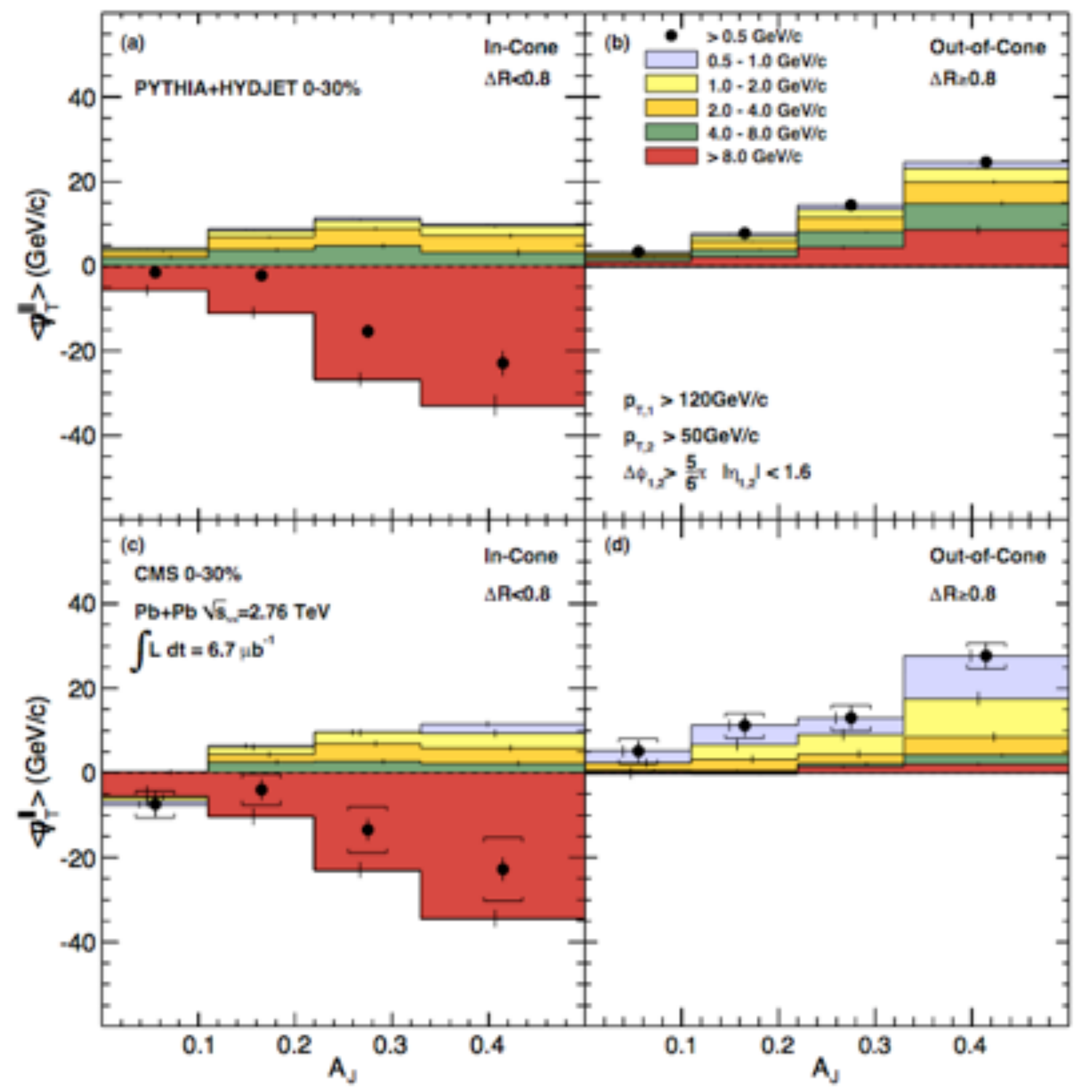
Rencontres Ions Lourds
IPN Orsay

Outline

- ✓ Motivation
- ✓ Jets in vacuum
- ✓ In medium single-gluon emission (BDMPS-Z)
- ✓ Decoherence and resummation scheme
- ✓ Generating functional and Master Equation
- ✓ Application: Evolution Equations

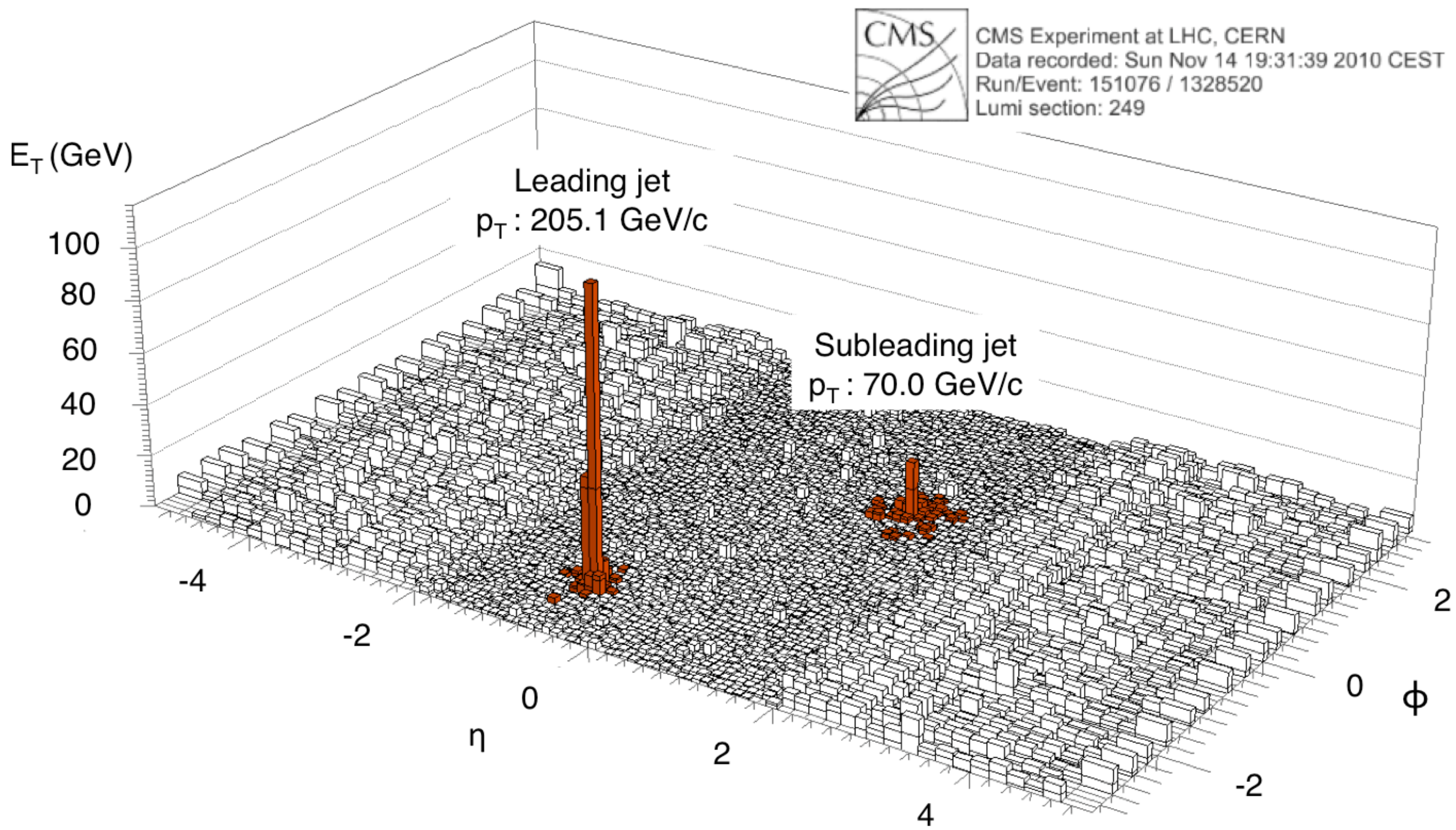
Motivation

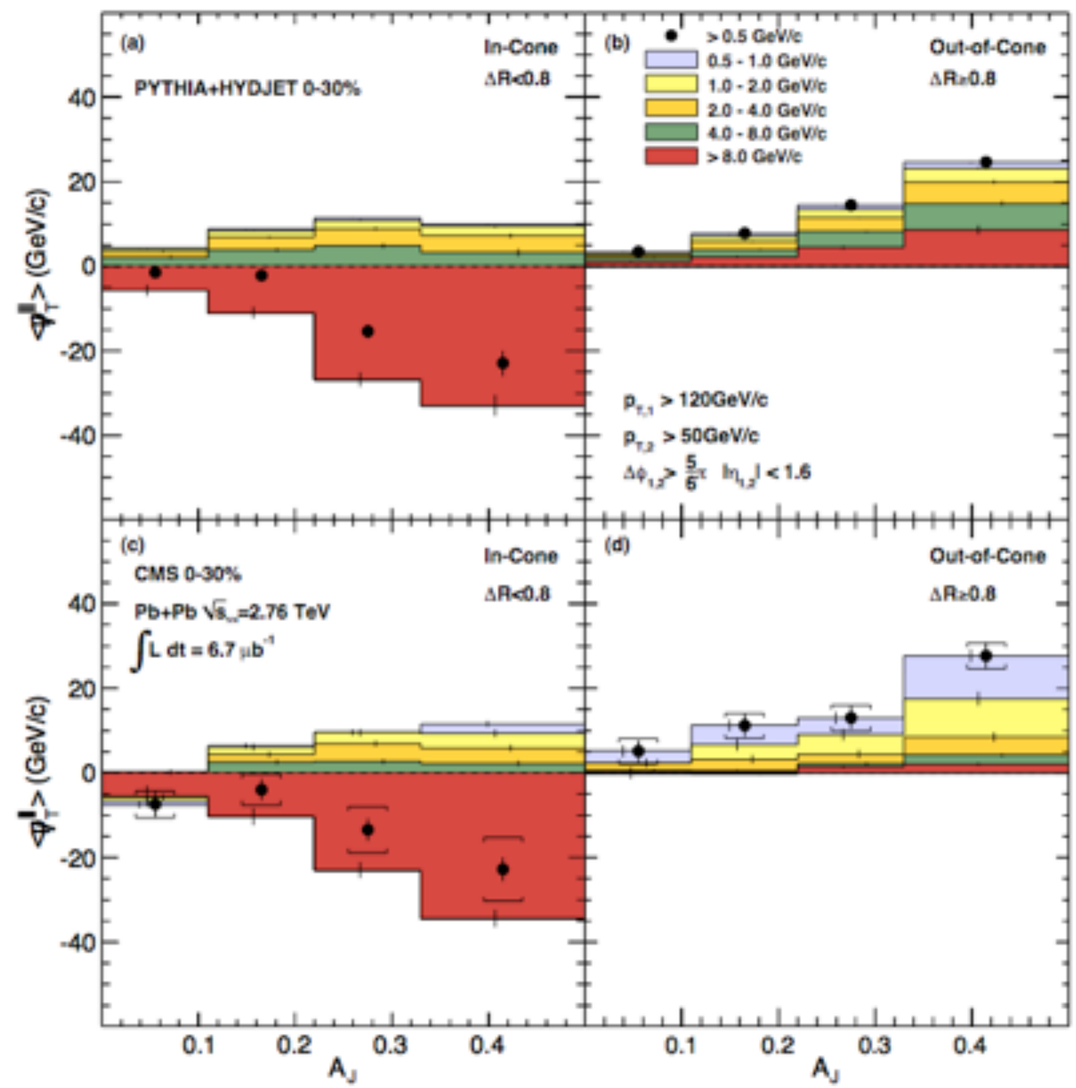




$$A_J = \frac{p_{\mathrm{T},1} - p_{\mathrm{T},2}}{p_{\mathrm{T},1} + p_{\mathrm{T},2}}$$

Motivation





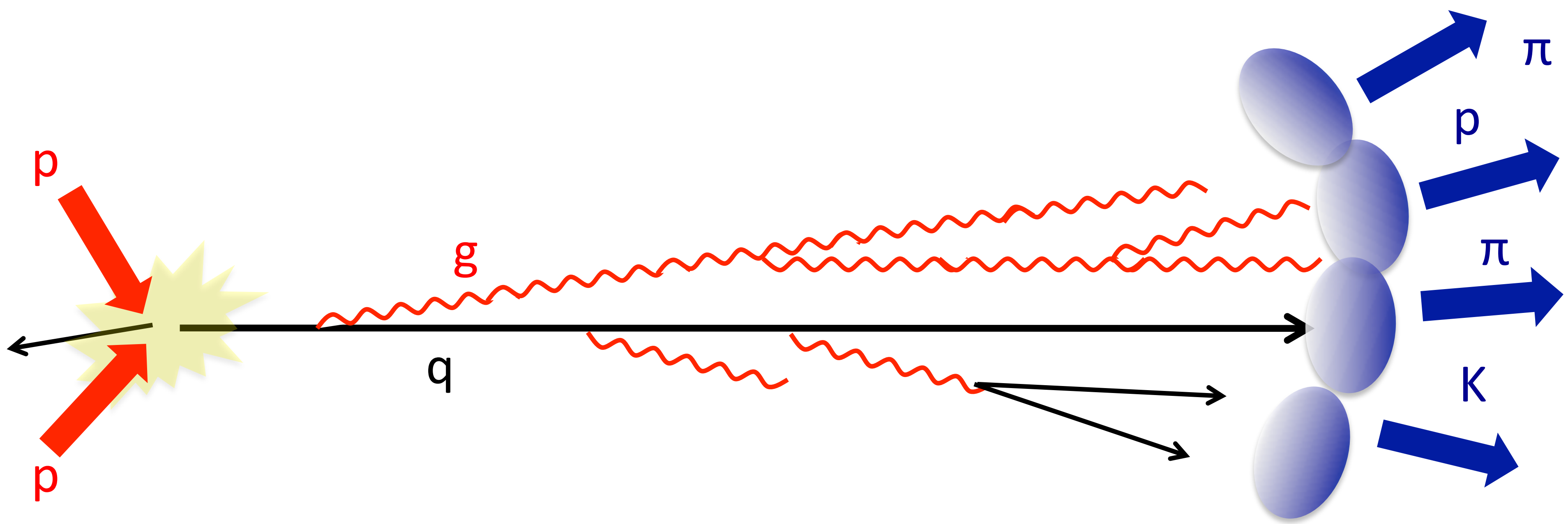
(I) Significant dijet energy asymmetry

(II) Soft particles at large angles

(III) Vacuum-like fragmentation

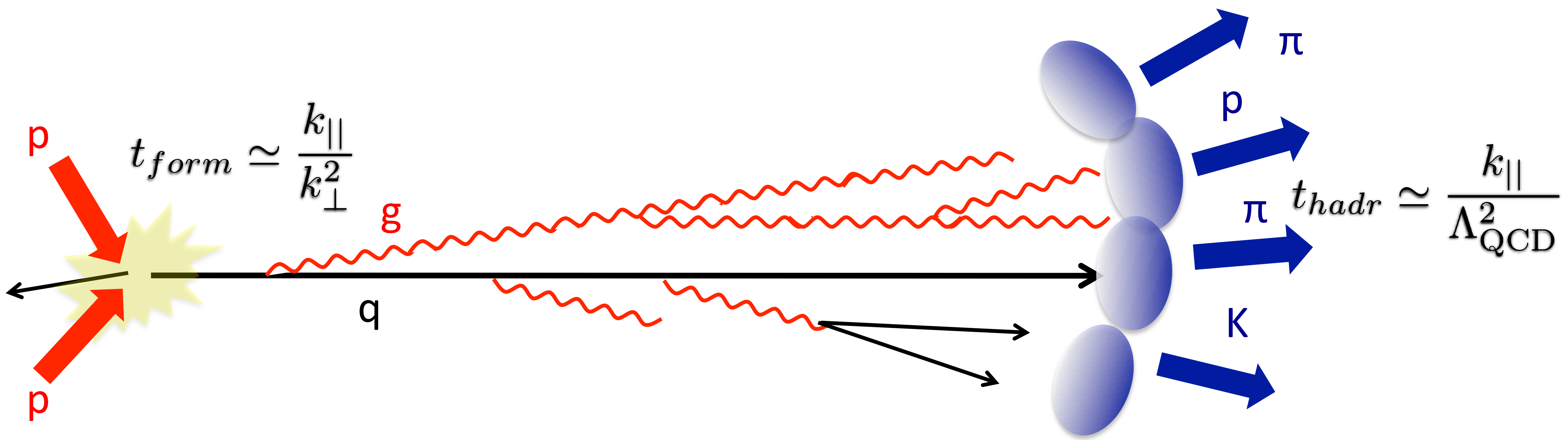
$$A_J = \frac{p_{T,1} - p_{T,2}}{p_{T,1} + p_{T,2}}$$

Jets in vacuum



- Originally a hard parton (quark/gluon) which fragments into many partons with virtuality down to a non-perturbative scale where it hadronizes
- LPHD: Hadronization does not affect inclusive observables (jet shape, energy distribution etc..)

Jets in vacuum



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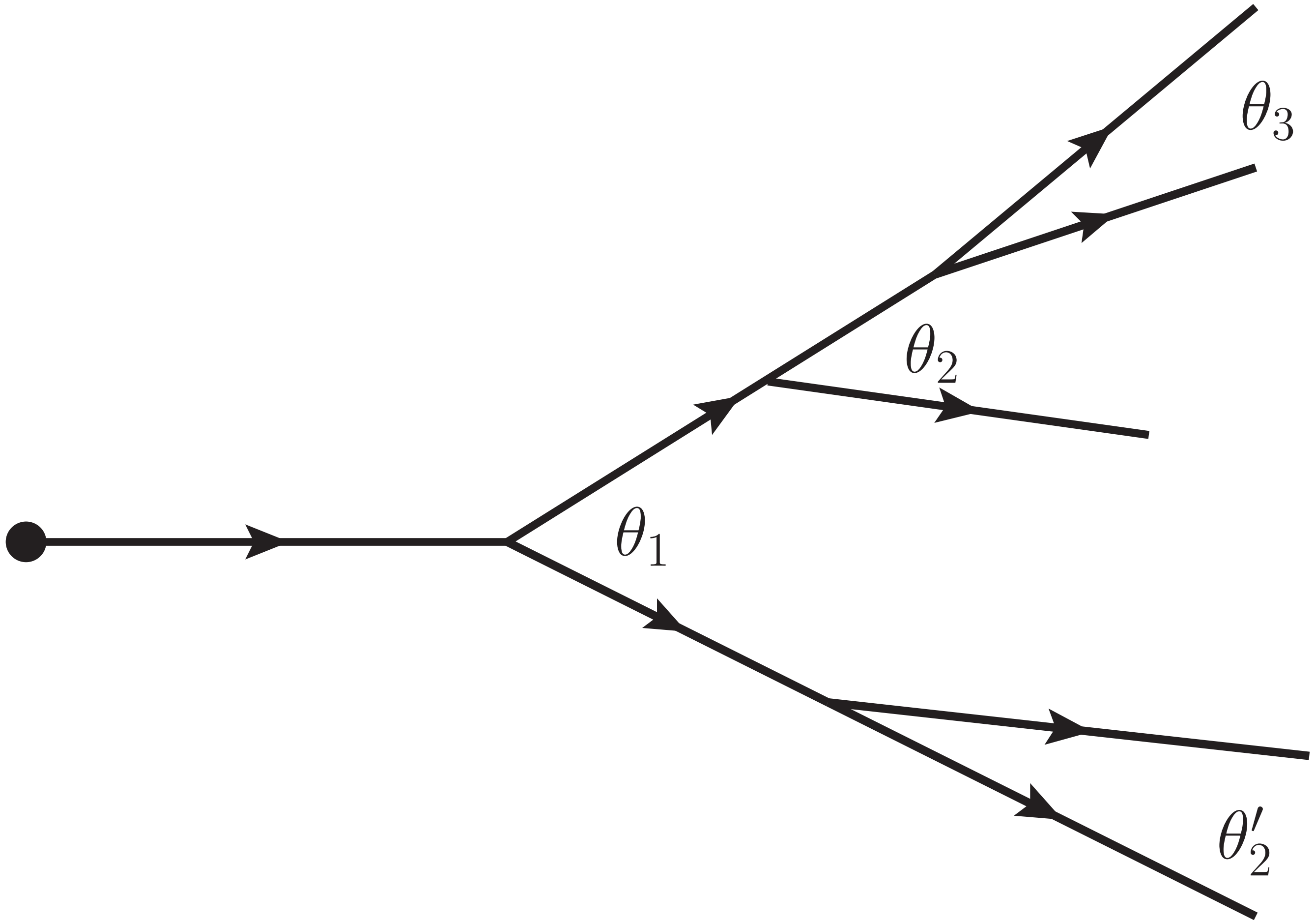
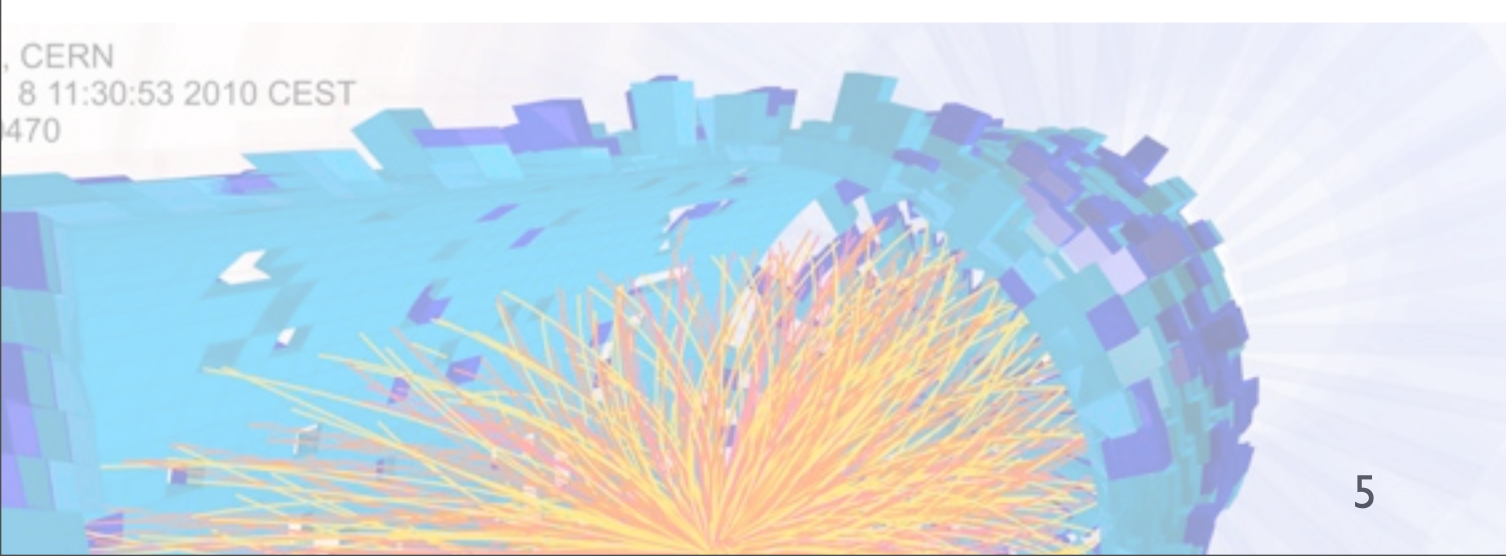
Large time domain for pQCD: $\frac{1}{\sqrt{s}} < t < \frac{\sqrt{s}}{\Lambda^2_{\rm QCD}}$

Jets in vacuum

[Bassetto, Mueller, Ciafaloni, Marchesini, Dokshitzer, Khoze, Troyan, Fadin, Lipatov, 80's]





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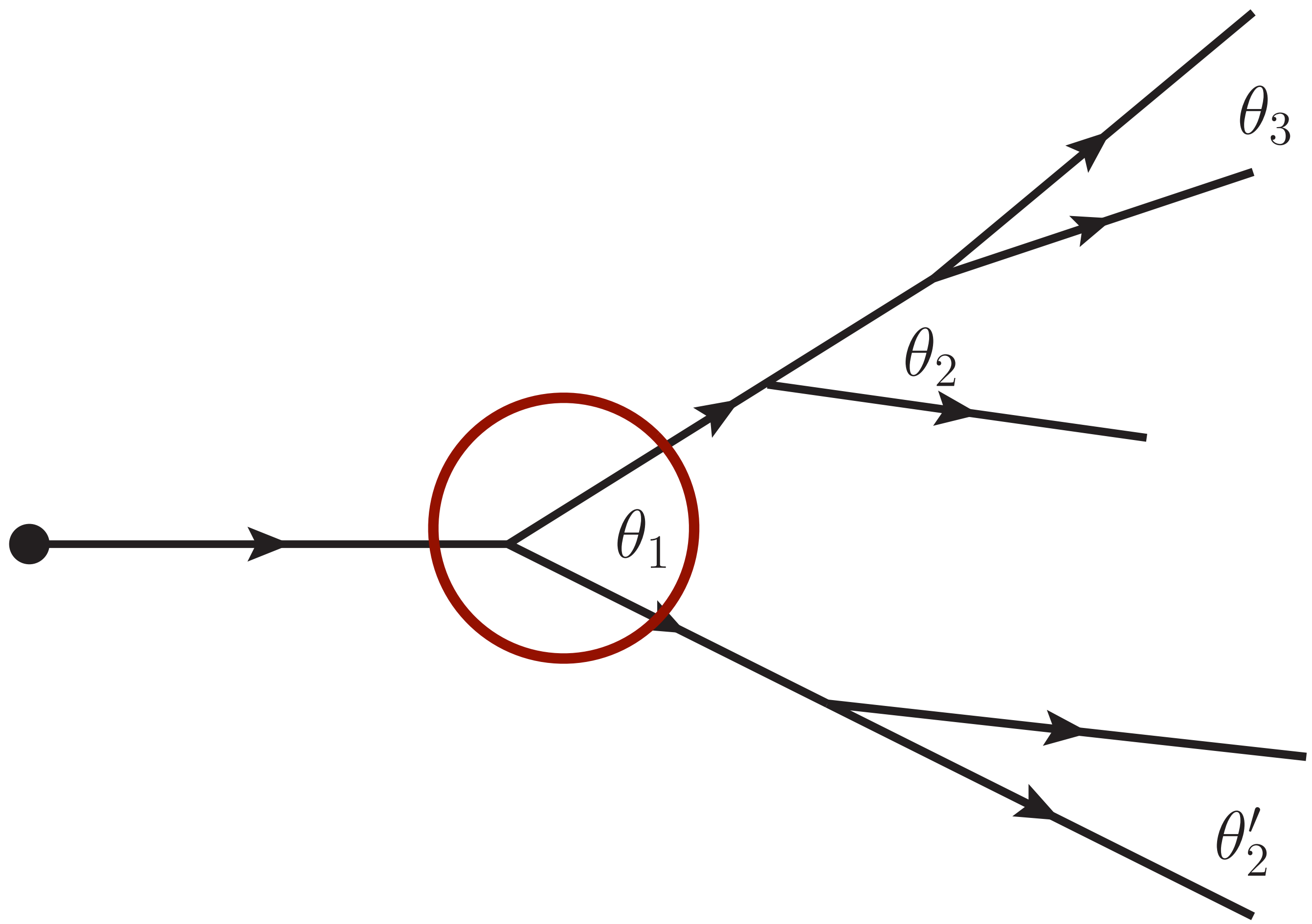
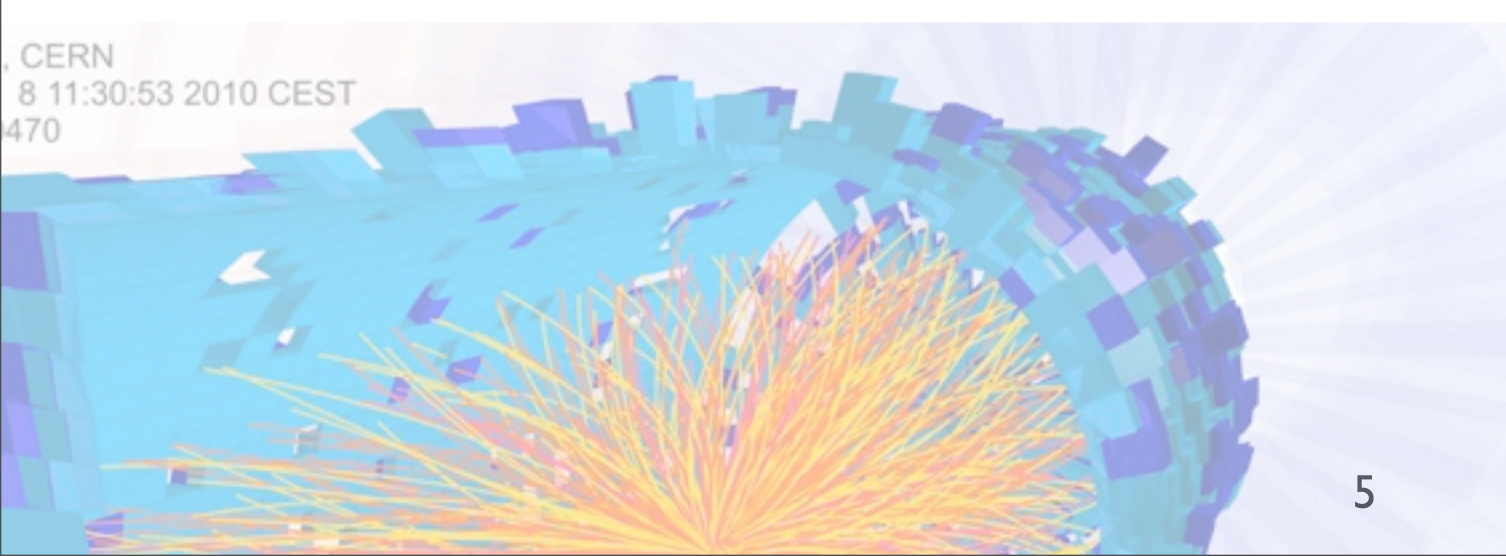


Leading Logarithms

$$dP \propto \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$



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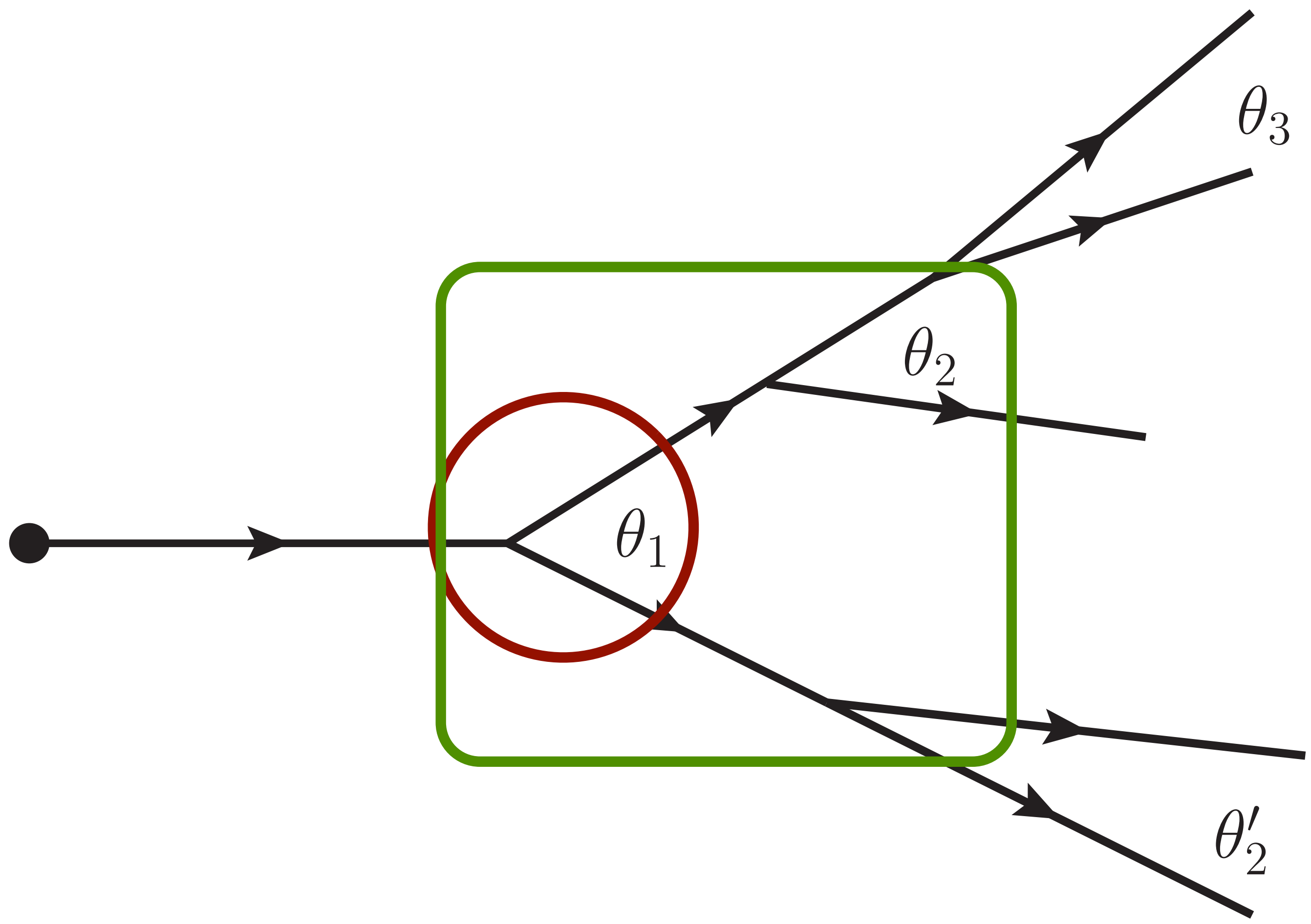

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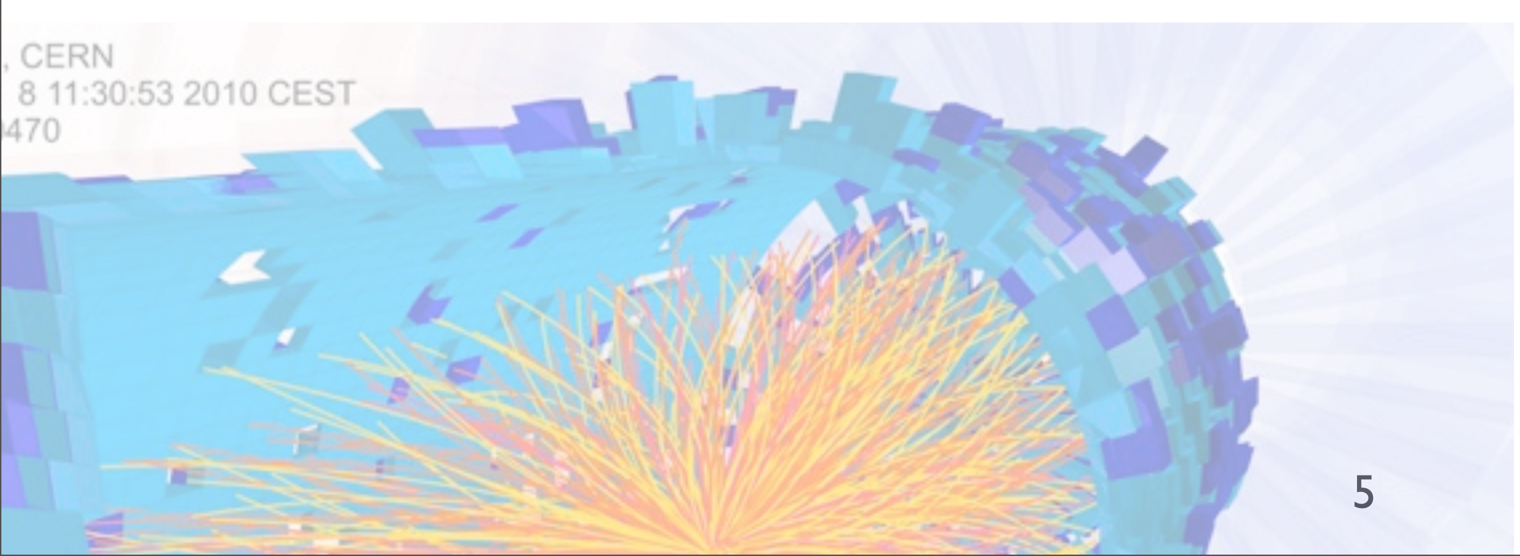
$$dP \propto \frac{\alpha_s C_R}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\theta}$$

Angular ordering

$$d^2P \propto \Theta(\theta_1 - \theta_2)\, dP_1\, dP_2$$



Jets in vacuum

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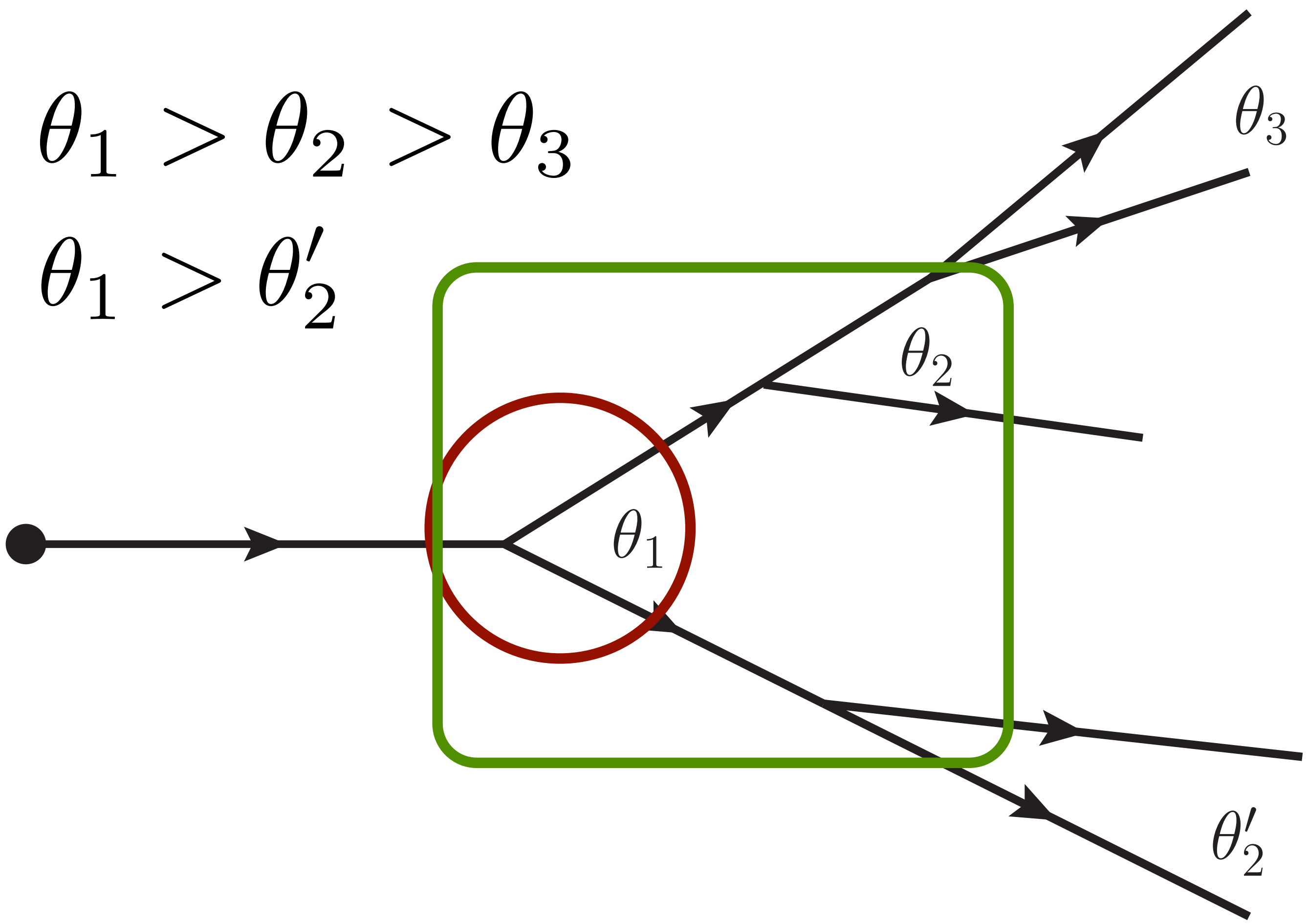
$$\theta_1 > \theta_2 > \theta_3$$

$$\theta_1 > \theta_2'$$



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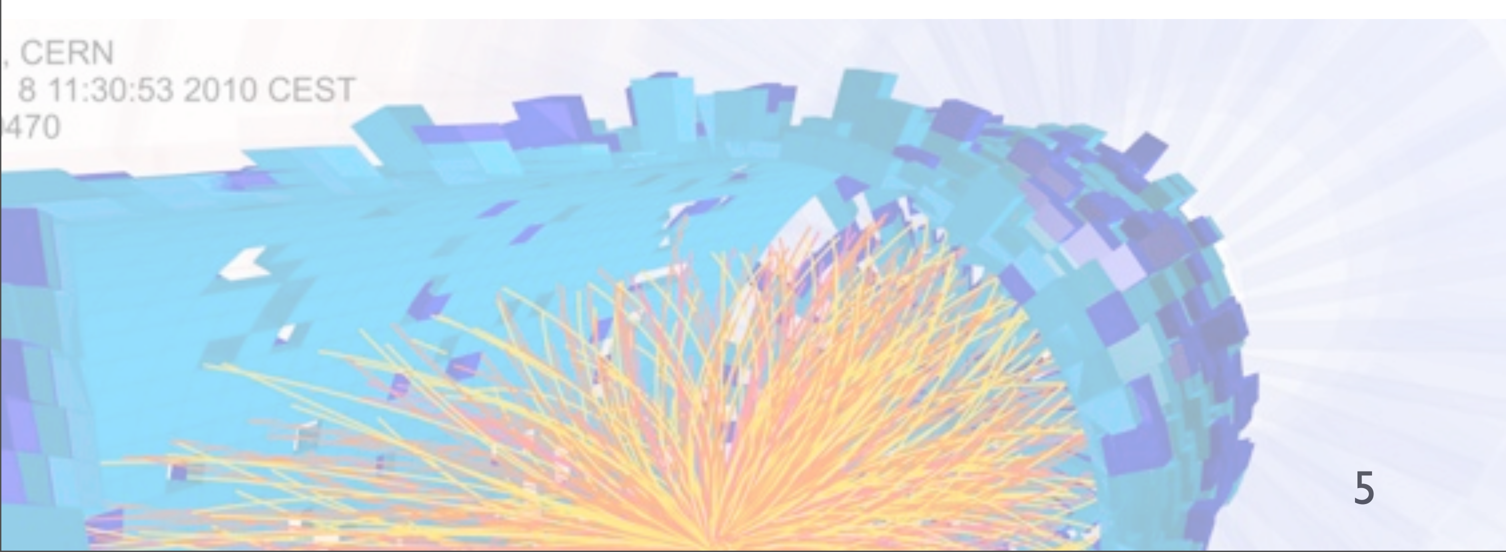

Markov process!

Jets in vacuum

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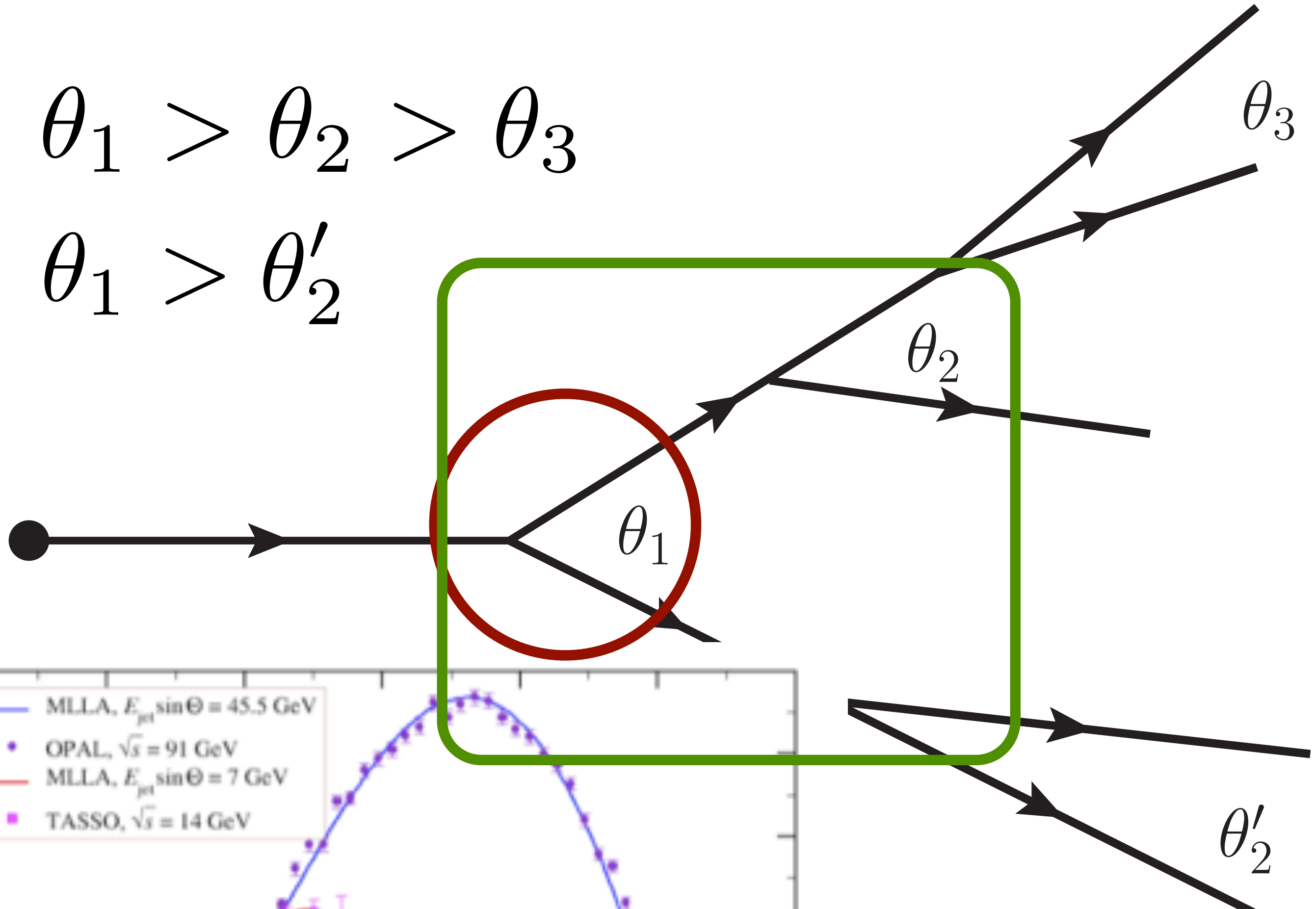

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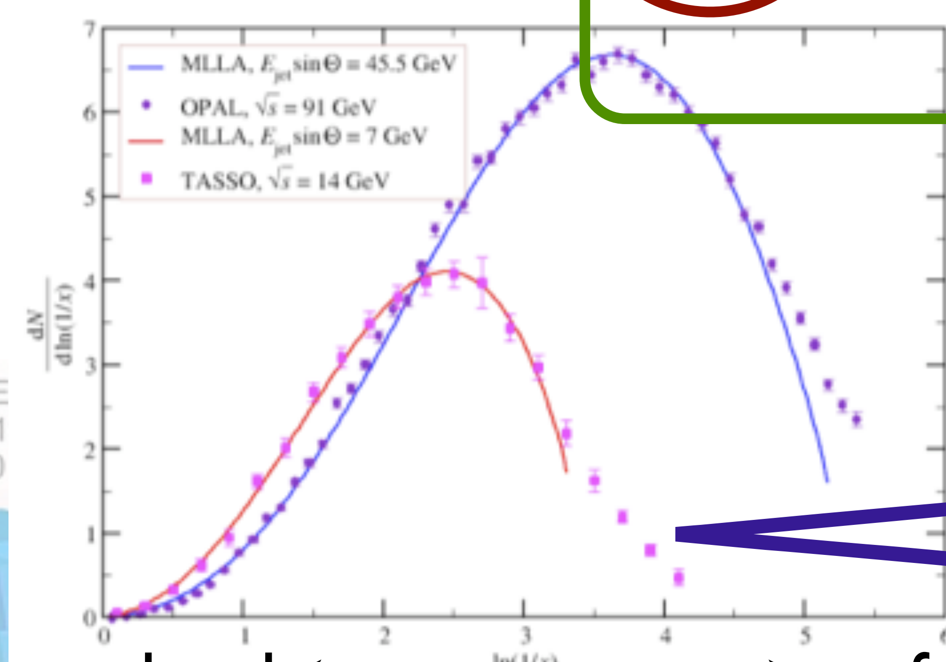


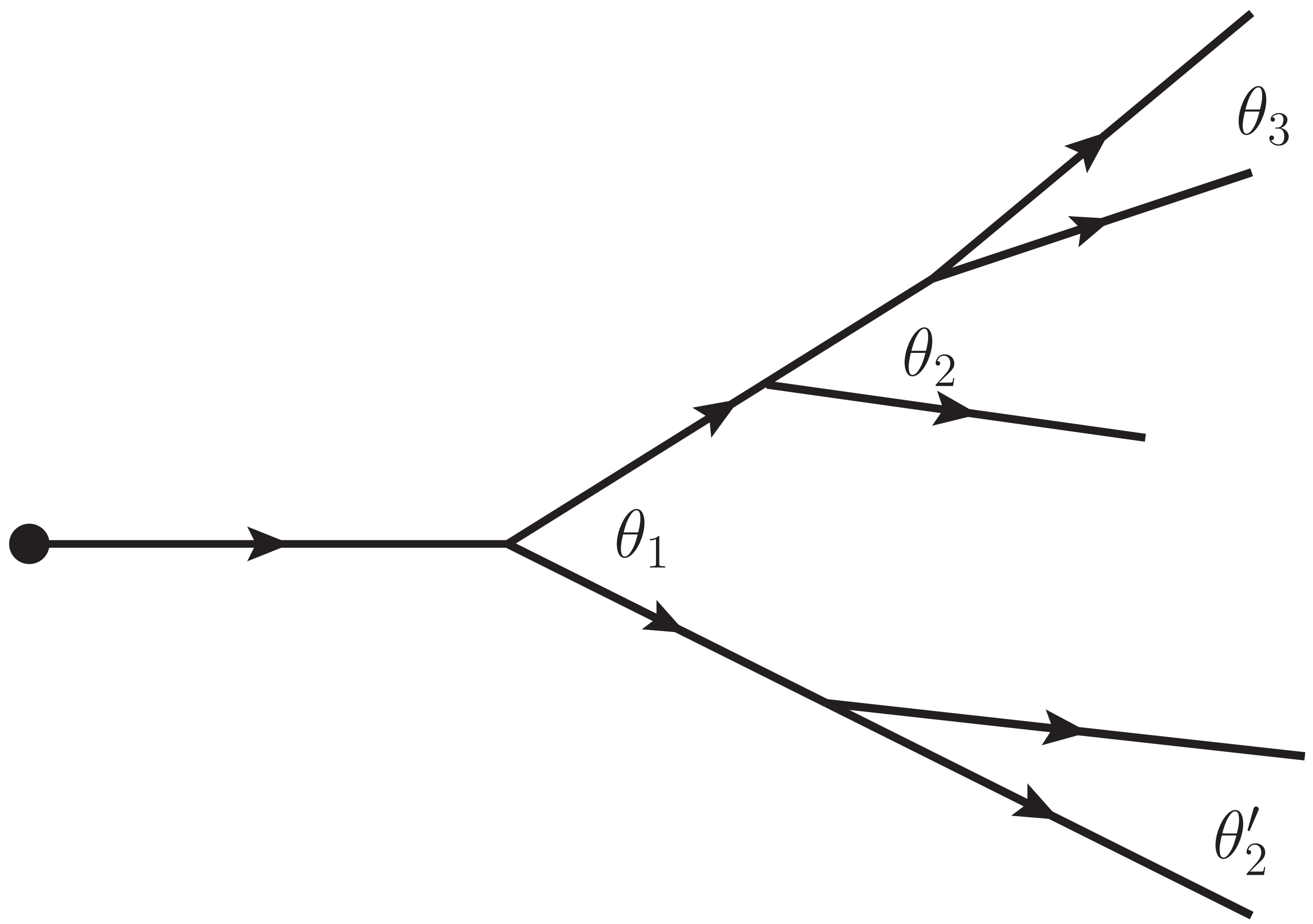
hard ⟷ soft

AO limits phase space for soft emissions!

Markov process!

TASSO Collaboration, Z. Phys. C 47 (1990)
OPAL Collaboration, Phys. Lett. B 247 (1990)

In-medium Jets



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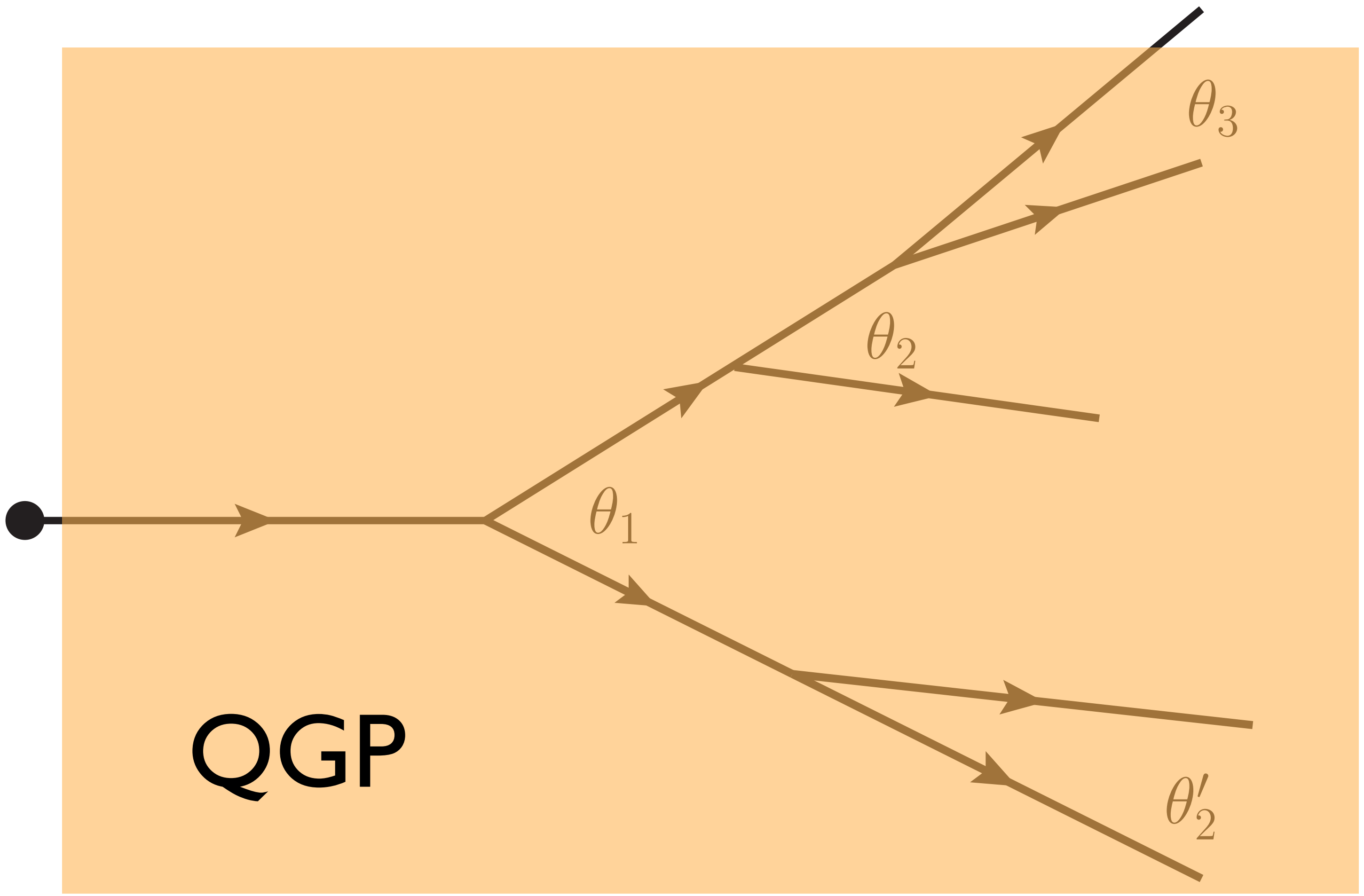
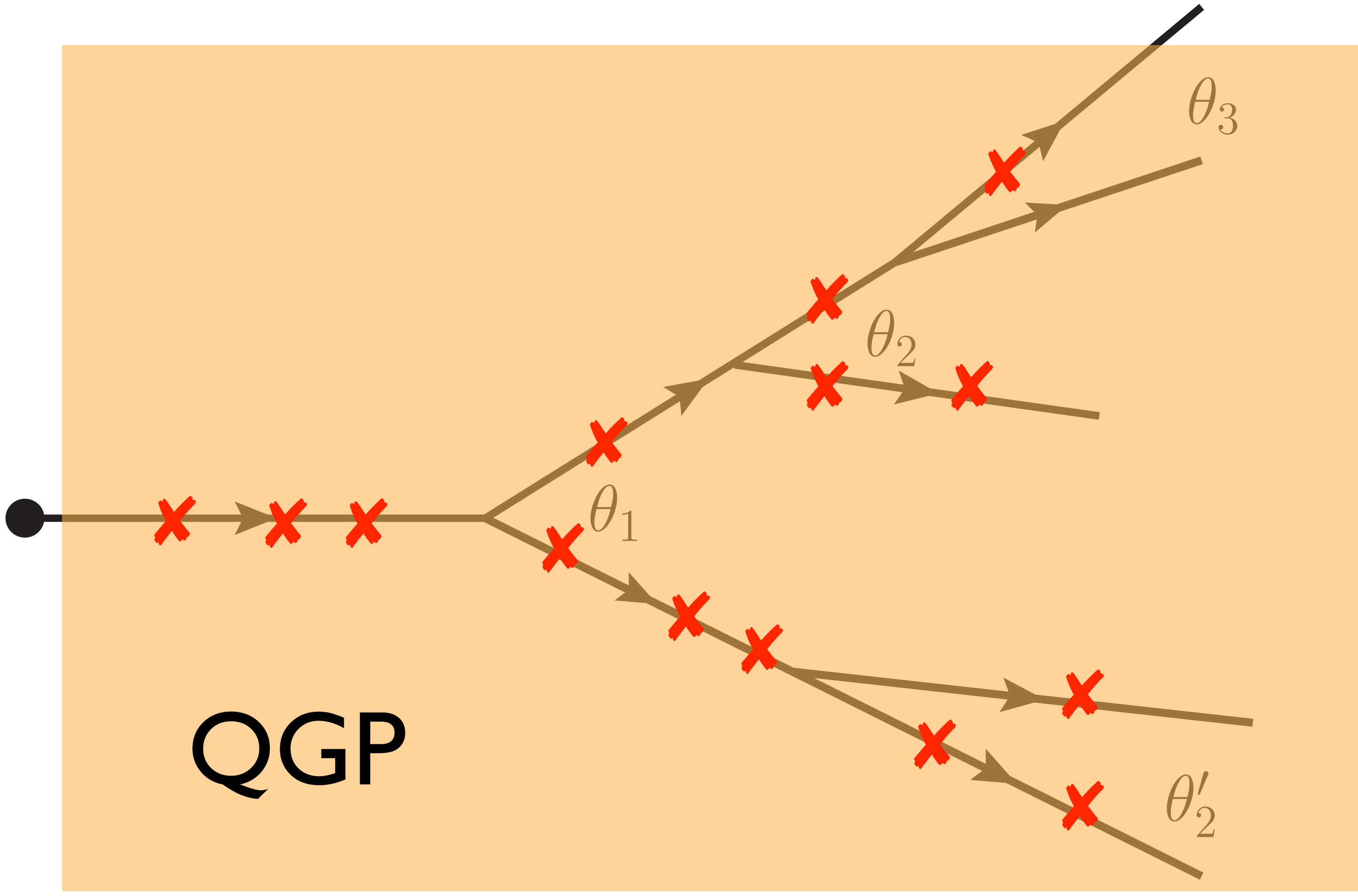


Q: How does the QGP alter QCD coherence in a jet?

In-medium Jets



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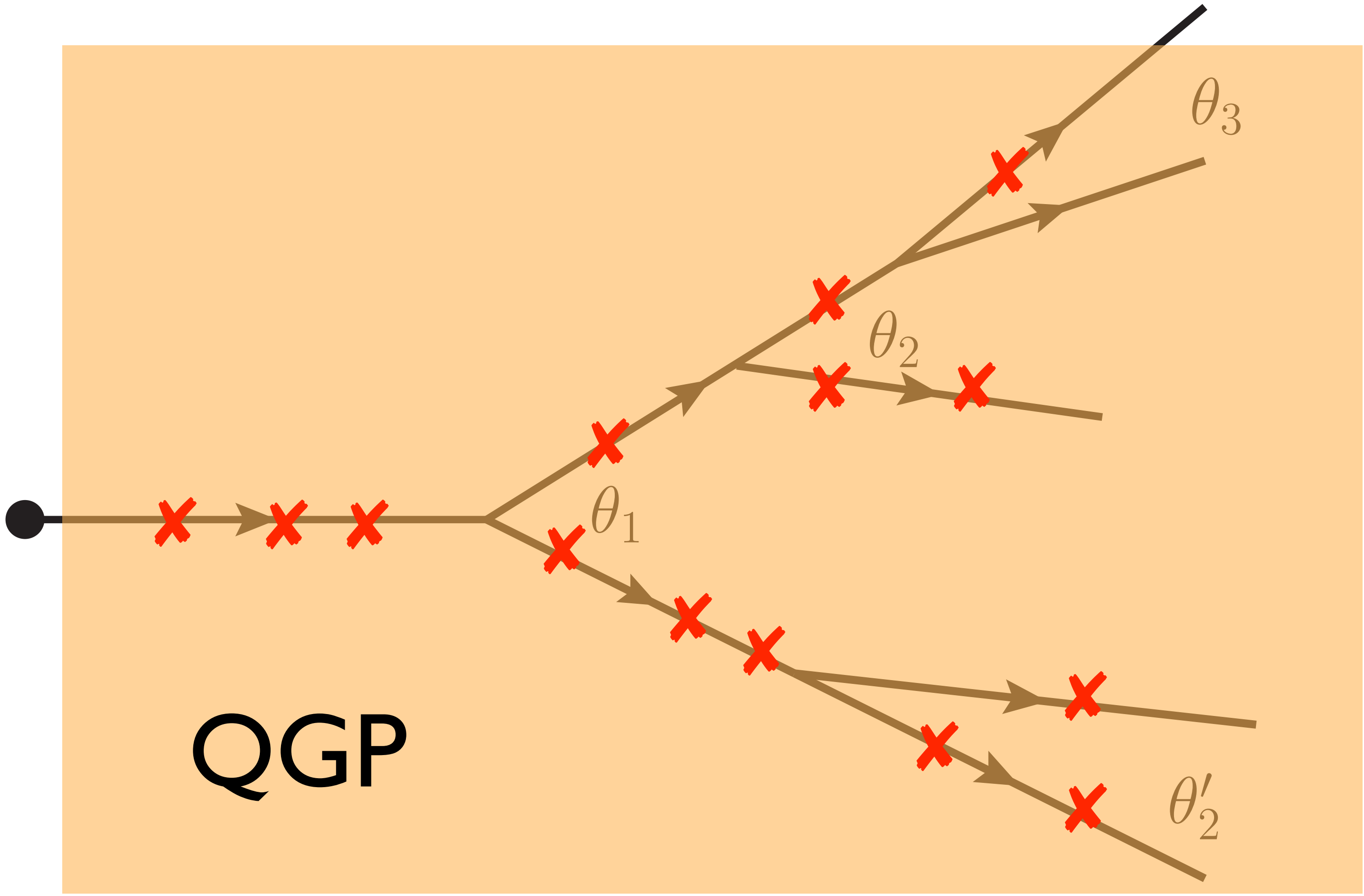


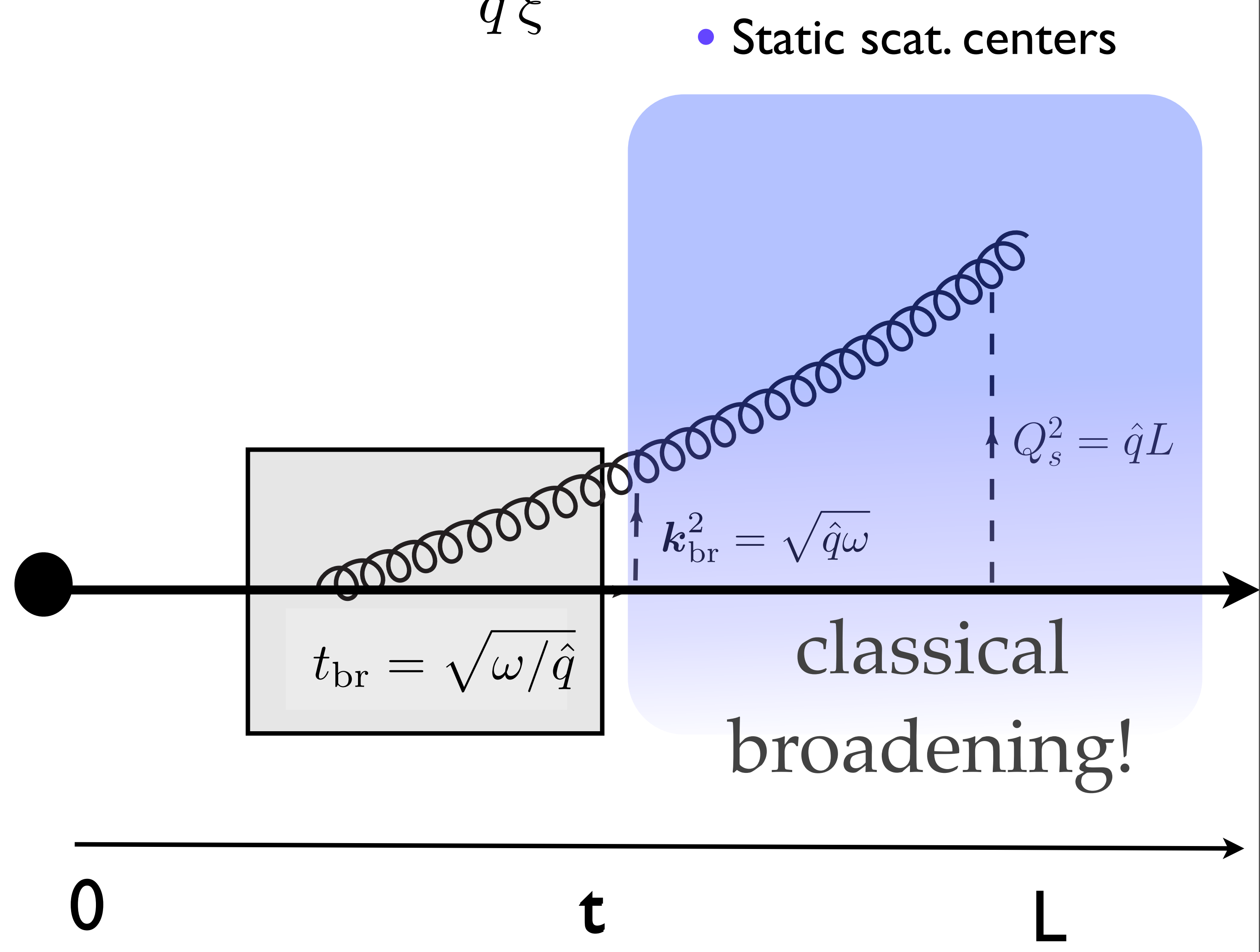
- AO?
- Ordering variable?

Q: How does the QGP alter QCD coherence in a jet?

1-gluon emission

$$(2\pi)^2 \omega \frac{dN}{d\omega d^2\boldsymbol{k}} = \frac{4C_F\alpha_s}{\omega} \int_0^L dt \int \frac{d^2\boldsymbol{q}}{(2\pi)^2} \mathcal{P}(\boldsymbol{k}-\boldsymbol{q}, L-t) \sin\left(\frac{\boldsymbol{q}^2}{2\boldsymbol{k}_{\rm br}^2}\right) \exp\left(-\frac{\boldsymbol{q}^2}{2\boldsymbol{k}_{\rm br}^2}\right)$$

- prob. of acquiring mom. k after ξ $\quad \mathcal{P}(\boldsymbol{k},\xi) = \frac{4\pi}{\hat{q}\,\xi} e^{-\frac{\boldsymbol{k}^2}{\hat{q}\xi}}$
- branching time $\quad t_{\rm br}$



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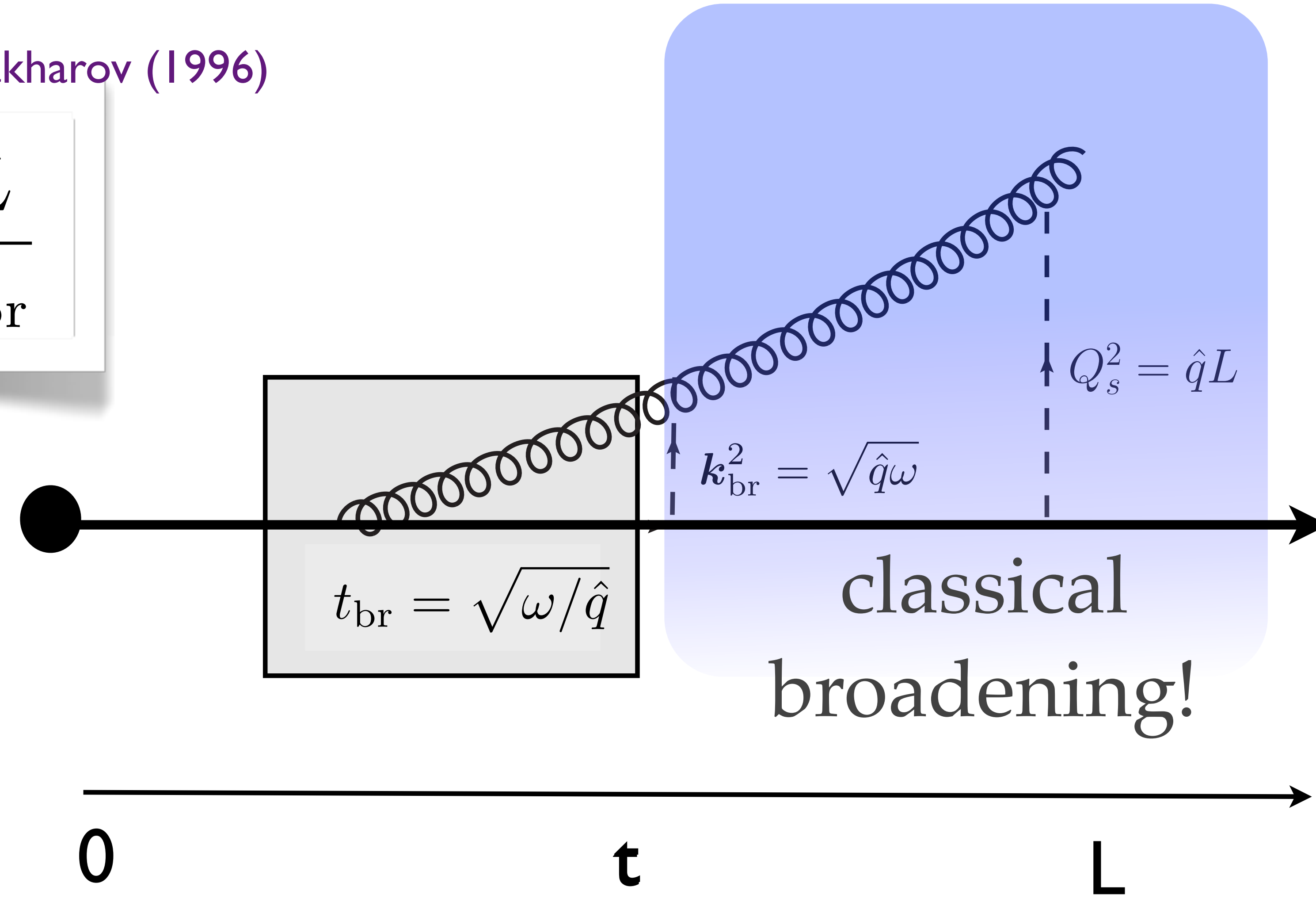
Baier, Dokshitzer, Mueller, Peigné, Schiff (1995-2000) Zakharov (1996)

$$\omega\frac{dN}{d\omega}=\frac{C_F\alpha_s}{\pi}\sqrt{\frac{\hat{q}L^2}{\omega}}\propto\alpha_s\frac{L}{t_{\rm br}}$$

- Static scat. centers



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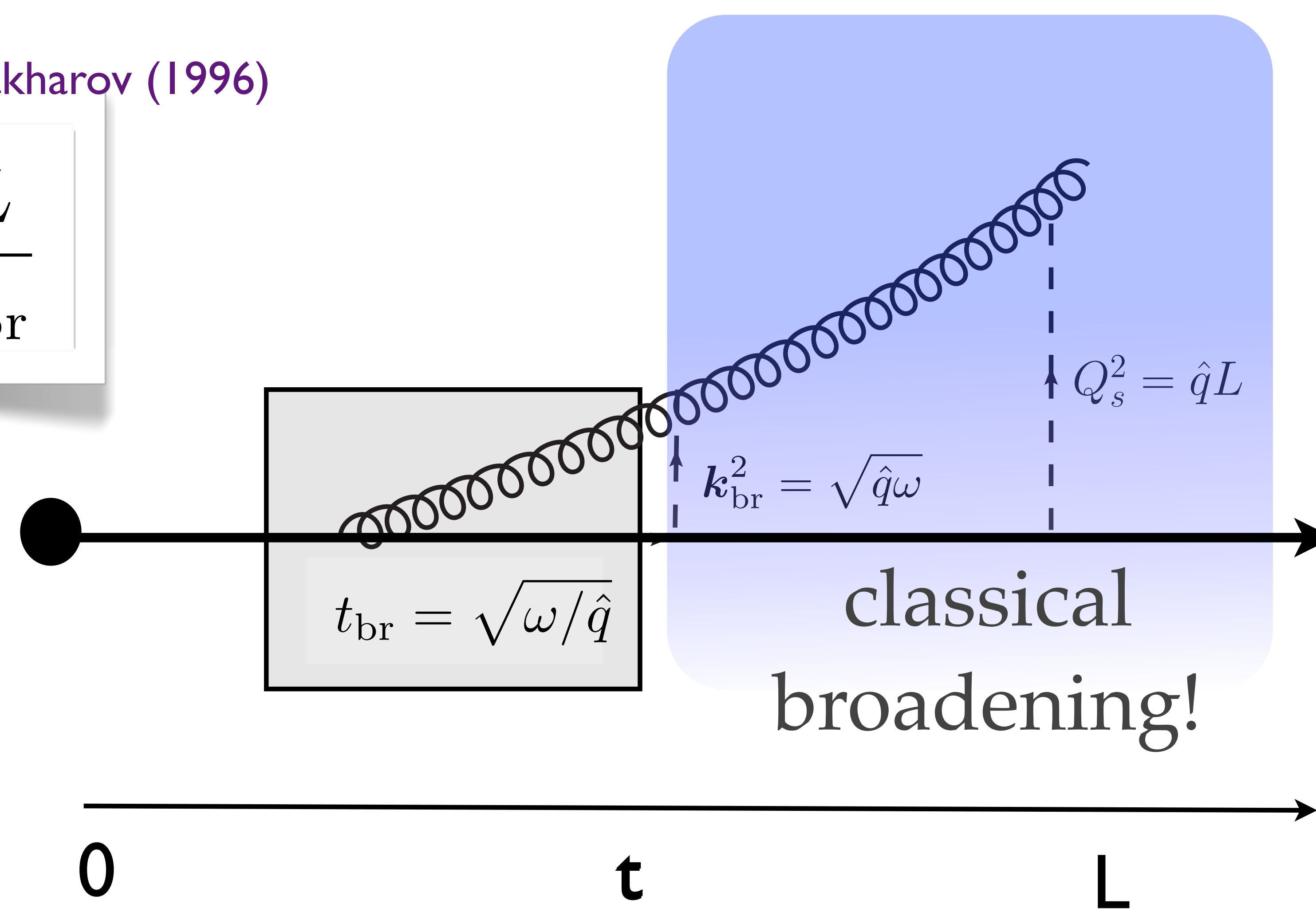
- 1-gluon ~ n-gluon when

$$\alpha_s\frac{L}{t_{\rm br}} \sim 1$$

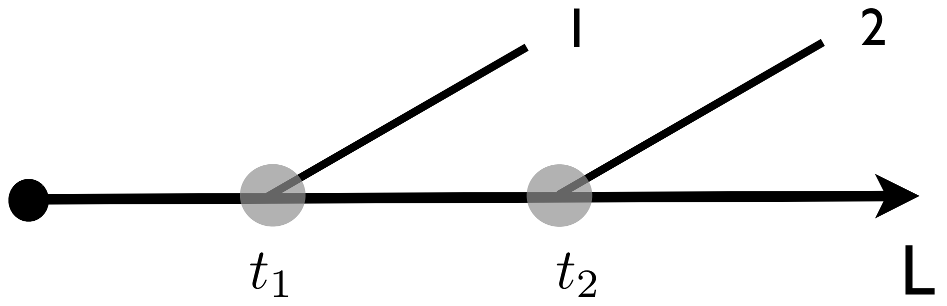
- Emission time $t \sim L \gg t_{\rm br}$

- Static scat. centers



N-gluon emissions

- Factorized contribution $t_1 \sim t_2 \sim L \gg t_{\rm br}$ or ($\omega_1 \sim \omega_2 \ll \omega_c = \hat{q}L^2$)



$$\propto \left(\alpha_s \frac{L}{t_{\rm br}} \right)^2$$

$$t_2 - t_1 \sim t_{\rm br}$$

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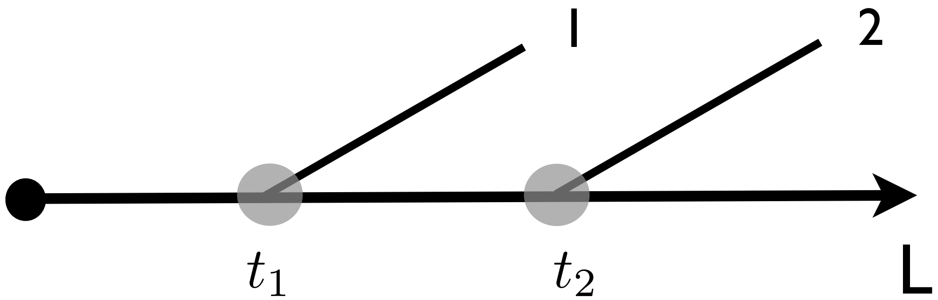
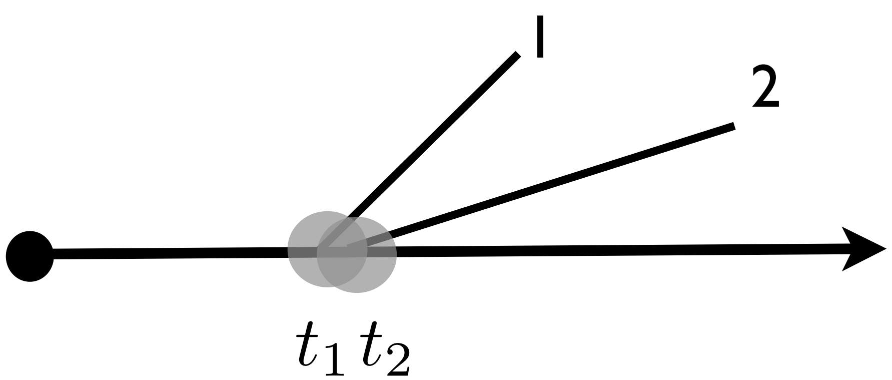


$$\propto \left(\alpha_s \frac{L}{t_{\rm br}} \right)^2$$

- Interferences $t_2 - t_1 \sim t_{\rm br}$



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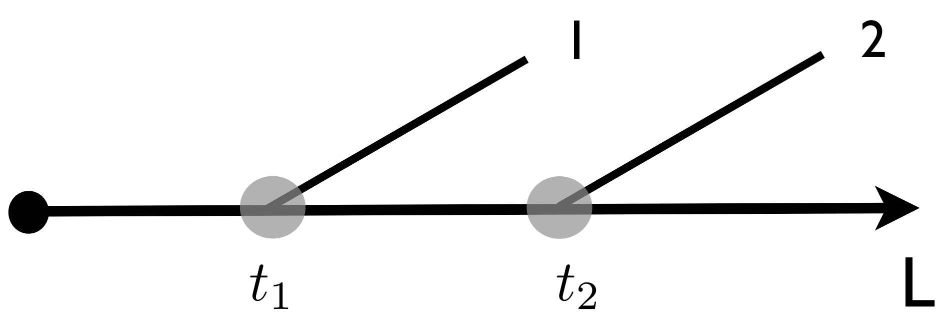

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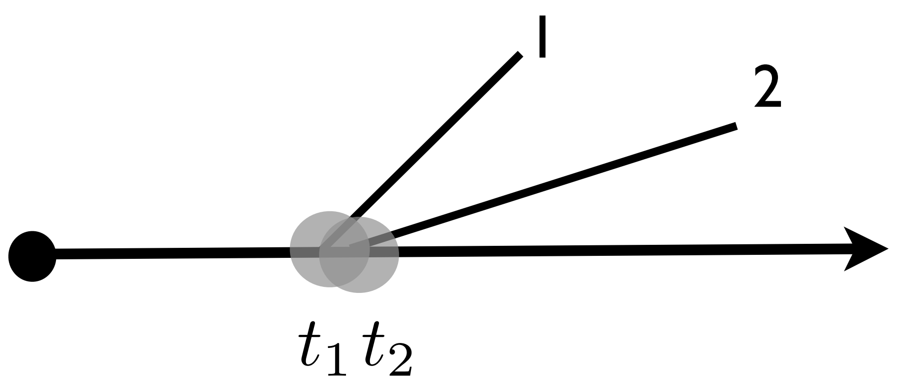

$$\propto \alpha_s \left(\alpha_s \frac{L}{t_{\rm br}}\right)$$

- Decoherence of successive splittings: Interferences are suppressed in a dense medium ⇒ No Angular Ordering!

Y. M.-T. , K. Tywoniuk, C. A. Salgado (2011)
E. Iancu, J. Casalderrey Solana (2011)

- Ordering variable: emission time $t_1 < t_2$

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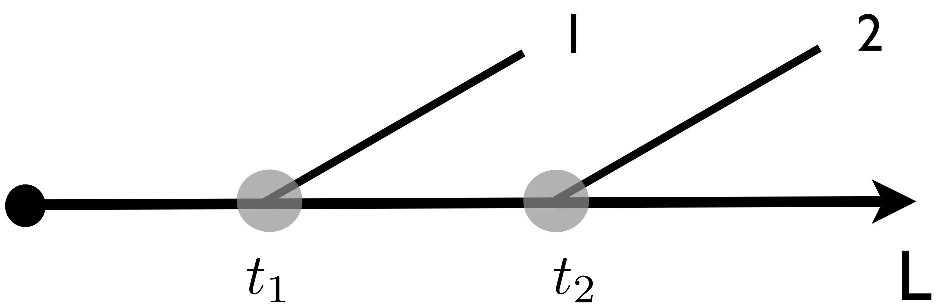

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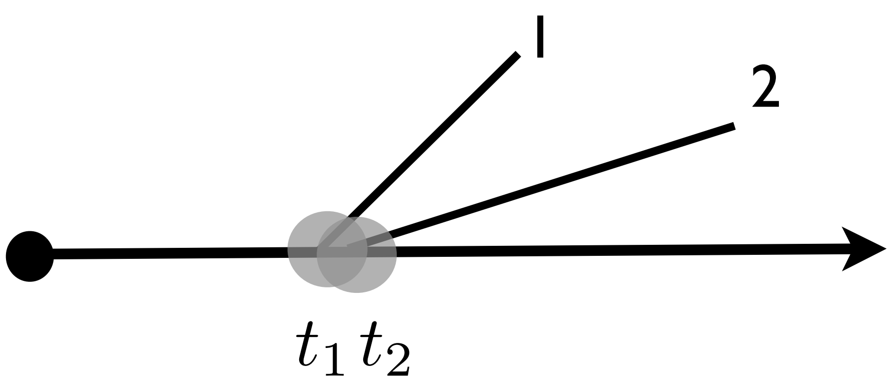

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- Decoherence of successive splittings: Interferences are suppressed in a dense medium $\Rightarrow$ No Angular Ordering!

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- Ordering variable: emission time $t_1 < t_2$

$\Rightarrow$ Probabilistic Scheme

- Resummation:

$$\sigma = \sum_n a_n \left(\alpha_s \frac{L}{t_{\rm br}} \right)^n$$

Generating Functional Method

- n-gluon probability P_n

$$\mathcal{Z}(u) = \sum_{n=1}^{\infty} P_n\, u^n$$

- Probability conservation

$$\mathcal{Z}(u=1) = 1$$

- Average gluon number $\langle n \rangle \equiv \frac{d}{du}\mathcal{Z}(u=1)$

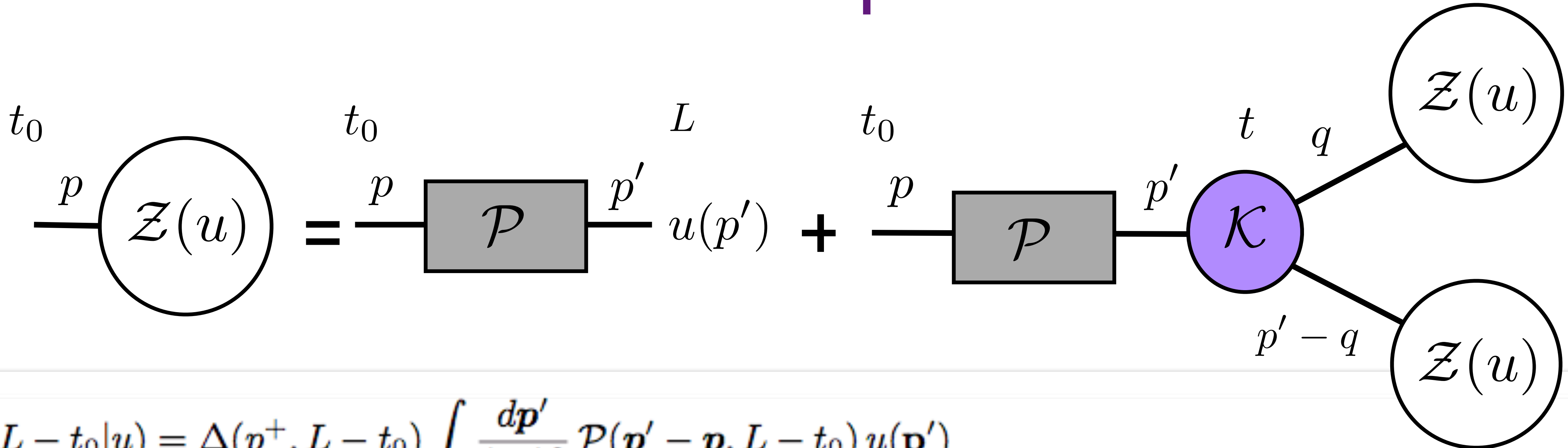
- Higher moments

$$\langle n(n-1)...(n-m+1)\rangle = \left(\frac{d}{du}\right)^m \mathcal{Z}(u=1)$$

- To compute differential distributions in k

$$u \rightarrow u(k) \qquad \frac{\delta u(k)}{\delta u(p)} = \delta^{(3)}(k-p)$$

Master Equation



$$\mathcal{Z}(\mathbf{p}, L-t_0|u) = \Delta(p^+, L-t_0) \int \frac{d\boldsymbol{p}'}{(2\pi)^2}\, \mathcal{P}(\boldsymbol{p}'-\boldsymbol{p}, L-t_0)\, u(\mathbf{p}')$$

$$+\alpha_s \int_{t_0}^{L} dt\, \Delta(p^+, t-t_0) \int_0^1 \frac{dz}{z} \int \frac{d\boldsymbol{p}'}{(2\pi)^2} \mathcal{P}(\boldsymbol{p}'-\boldsymbol{p}, L-t_0) \int \frac{d\boldsymbol{q}}{(2\pi)^2}\, \mathcal{K}(\boldsymbol{q}-z\boldsymbol{p}'|z)\, \mathcal{Z}(\mathbf{q}, L-t|u)\, \mathcal{Z}(\mathbf{p}'-\mathbf{q}, L-t|u)$$

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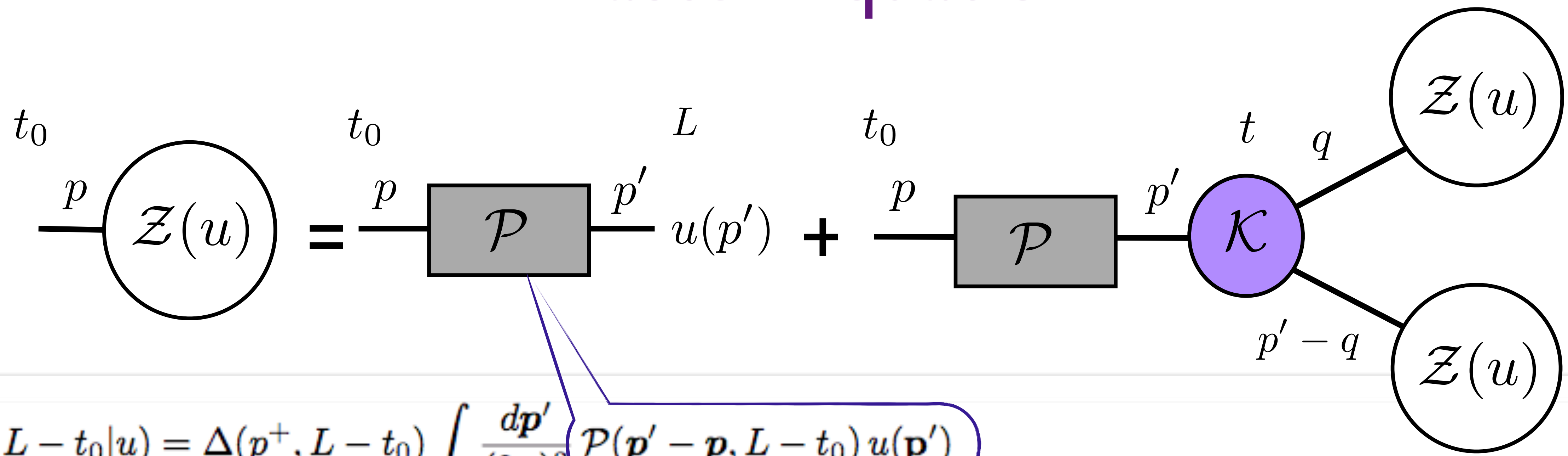

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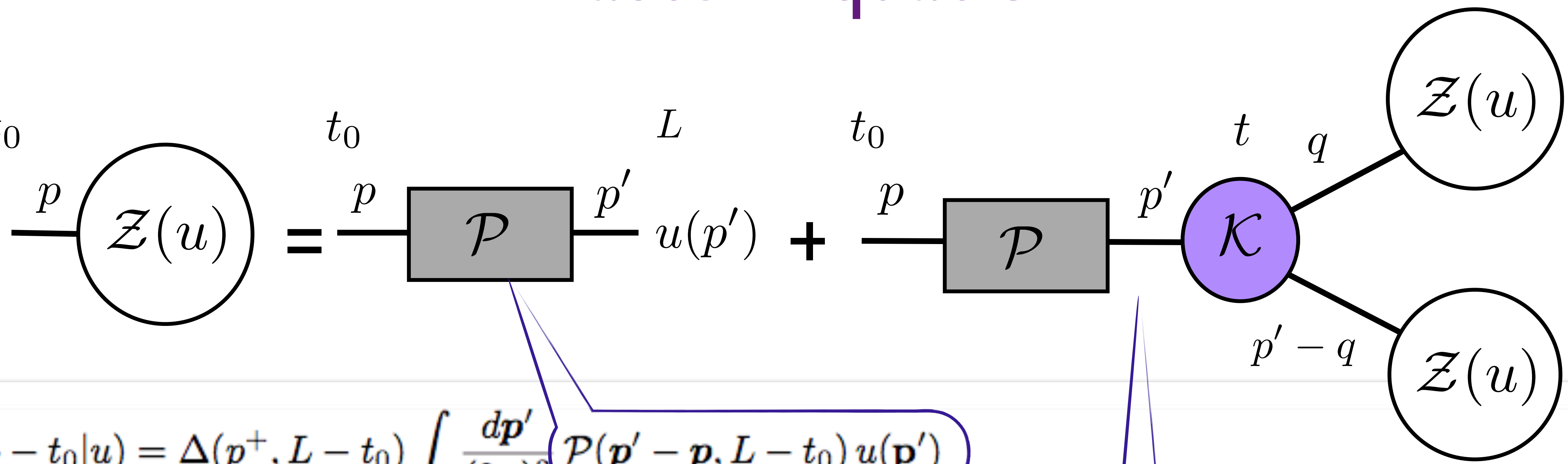

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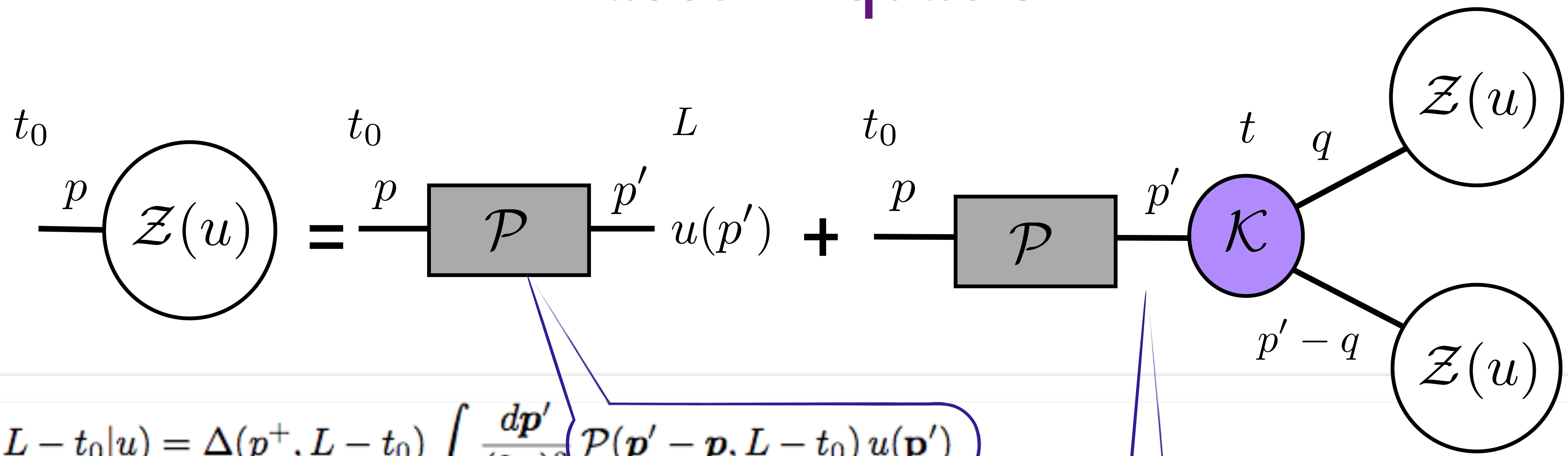

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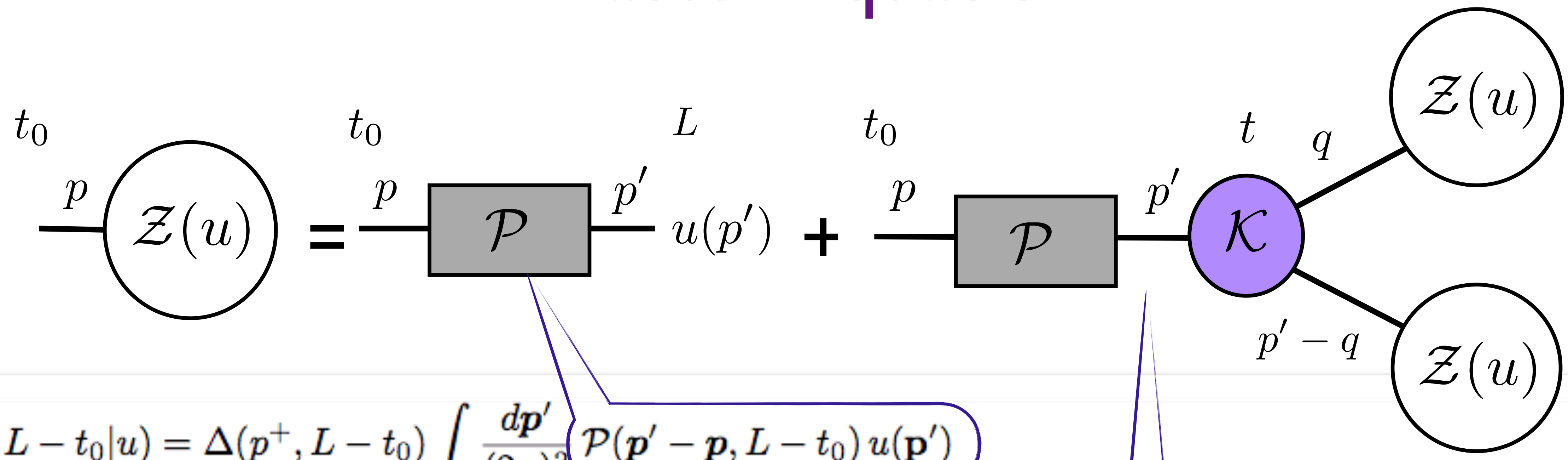
- In-medium splitting function

$$\mathcal{K}^A_{BC}(\boldsymbol{q}-z\boldsymbol{p},\, z) = \frac{2}{p^+}P_{AB}(z)\ \sin\left[\frac{(\boldsymbol{q}-z\boldsymbol{p})^2}{2\boldsymbol{k}_{\rm br}^2}\right]\exp\left[-\frac{(\boldsymbol{q}-z\boldsymbol{p})^2}{2\boldsymbol{k}_{\rm br}^2}\right]$$

- Relative pT at branching time

$$\boldsymbol{k}_{\rm br}^2 = \sqrt{z(1-z)p^+\hat{q}_{\rm eff}}$$

Master Equation



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- Relative pT at branching time

$$\boldsymbol{k}_{\rm br}^2 = \sqrt{z(1-z)p^+\hat{q}_{\rm eff}}$$

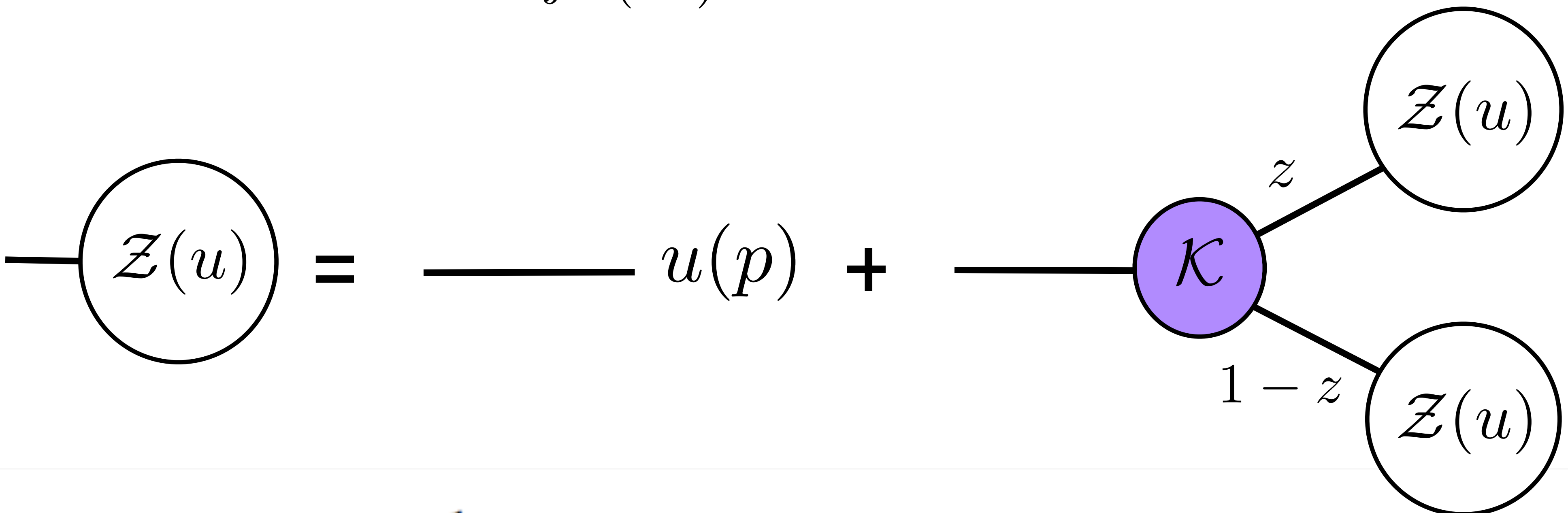
- Sudakov form factor:

Prob. not to emit (Unitarity)

$$\Delta(p^+, L-t_0) = \exp\left[-\alpha_s(L-t_0)\int_0^1 \frac{dz}{z}\mathcal{K}(z)\right]$$

Application 1: gluon distribution

- Integrating over transverse momenta: $$\int \frac{d^2\boldsymbol{k}}{(2\pi)^2}\,\mathcal{P}(\boldsymbol{k}-\boldsymbol{q},L-t)=1$$



$$\frac{\partial}{\partial L}\mathcal{Z}(E,u)=\alpha_s\int_0^1\frac{dz}{z}\mathcal{K}(z)\left[\mathcal{Z}(zE,u)\,\mathcal{Z}((1-z)E,u)-\mathcal{Z}(E,u)\right],$$

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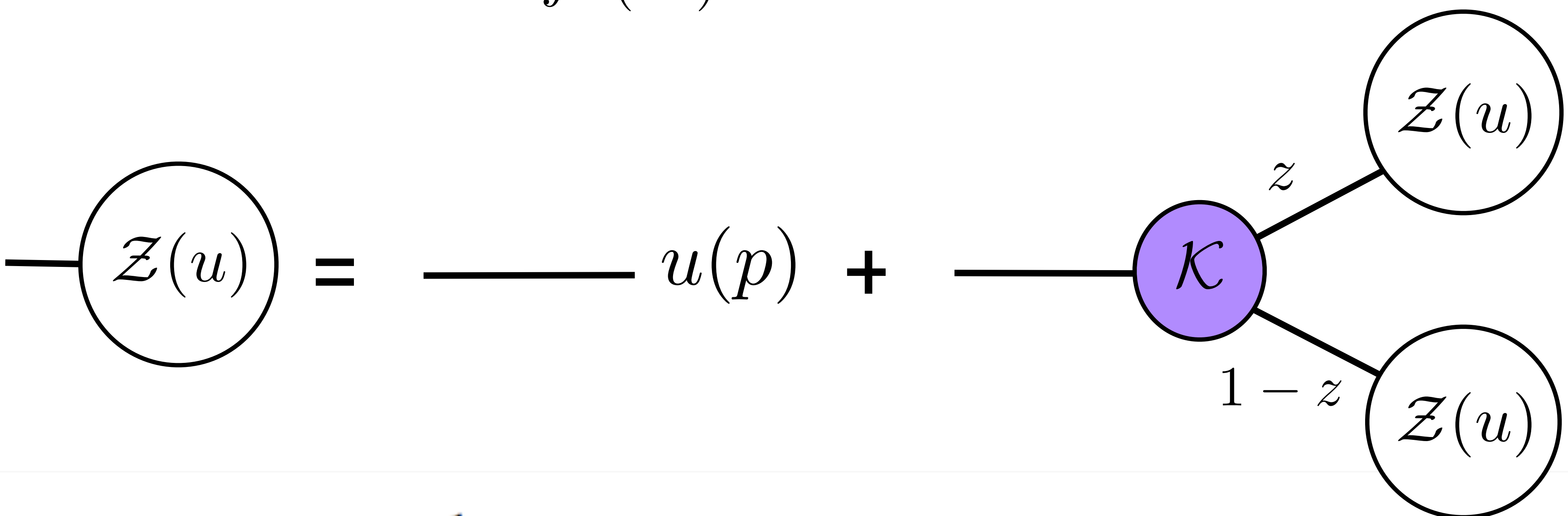

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- Gluon distribution $$D(x,E) \equiv x\frac{dN}{dx} = \omega\frac{\delta\mathcal{Z}(E,u)}{\delta u(\omega)}\Big|_{u=1}$$

$$x = \omega/E$$

$$\frac{\partial}{\partial L}D(x) = \alpha_s \int_0^1 \frac{dz}{z}\mathcal{K}(z)\left[D\left(\frac{x}{z}\right) + D\left(\frac{x}{1-z}\right) - D(x)\right]$$

S. Jeon, G. D. Moore(2003)

Application 1: gluon distribution

- Solution in the soft limit $\omega \ll \omega_c = \hat{q}L^2 < E$

$$\frac{\partial}{\partial L} D(x) \simeq 2\alpha_s \int_x^1 \frac{dz}{z} \mathcal{K}(z)\, D\left(\frac{x}{z}\right)$$

- Splitting function $\mathcal{K}(z) \approx \frac{C_A}{2\pi}\sqrt{\frac{\hat{q}}{zE}}$

Leading order BDMPS result

Exp increase of soft gluons (at large angles)

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$$\boxed{D(x) = x\delta(1-x) + \frac{1}{2}\left(\bar{\alpha}_s L\sqrt{\frac{\hat{q}}{xE}}\right)^{\frac{1}{2}} I_1\left[2\left(\bar{\alpha}_s L\sqrt{\frac{\hat{q}}{xE}}\right)^{\frac{1}{2}}\right]}$$

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$$D(\omega) = \bar{\alpha}_s\sqrt{\frac{\omega_c}{\omega}}$$

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- When $\omega < \alpha_s^2 \omega_c$

$$D(\omega) \sim \exp\left[2\left(\bar{\alpha}_s\sqrt{\frac{\omega_c}{\omega}}\right)^{1/2}\right]$$

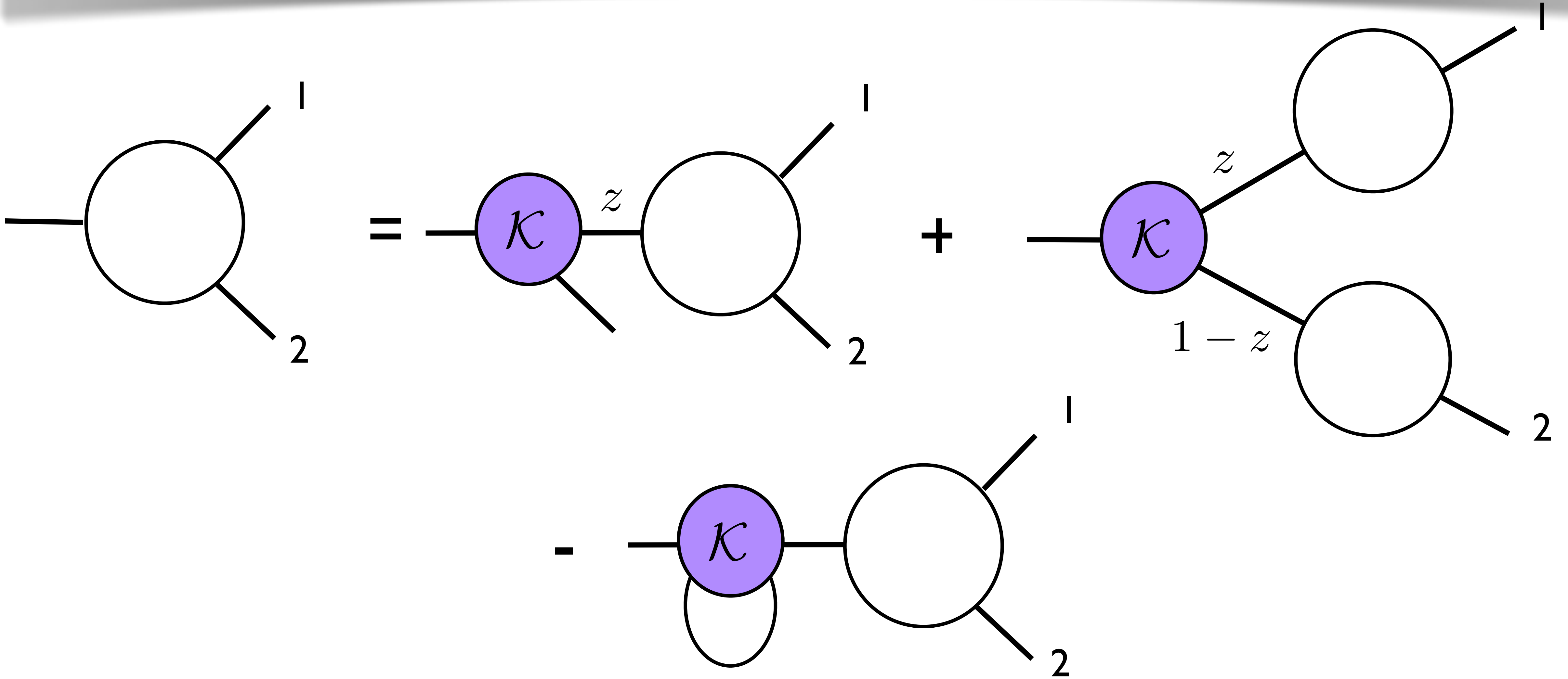
Exp increase of soft gluons (at large angles)

Application II: Correlations

- 2-particle correlations inside the jet

$$D(x_1, x_2) \equiv \omega_1 \omega_2 \frac{\delta^2 \mathcal{Z}(E,u)}{\delta u(\omega_1)\delta u(\omega_2)}\bigg|_{u=1}$$

$$\frac{\partial}{\partial L} D(x_1,x_2) = \alpha_s \int_0^1 \frac{dz}{z} \mathcal{K}(z) \left[D\left(\frac{x_1}{z}, \frac{x_2}{z}\right) + D\left(\frac{x_1}{z}\right) D\left(\frac{x_2}{1-z}\right) + \text{sym} - D(x_1,x_2) \right]$$



Summary

✓ In the limit of a dense medium, parton branchings are decoherent due to rapid color randomization.

✓ A probabilistic description of in-medium jet evolution is formulated in terms of a Master Eq. for Generating Functional

✓ $\Rightarrow$ Fully exclusive description of the jet including momentum broadening

✓ Possible implementation in a Monte Carlo generator