COLLECTIVE FLOW AND LONG-RANGE CORRELATIONS IN HEAVY-ION COLLISIONS

Matthew Luzum

based on: Phys.Lett.B696 499-504 (2011) (arXiv:1011.5773 [nucl-th])

Institut de physique théorique

Rencontre ions lourds 25 March, 2010

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LONG-RANGE CORRELATIONS

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OUTLINE

1 LONG-RANGE TWO-PARTICLE CORRELATIONS

- Azimuthal structure
- Recent STAR data (arXiv:1010.0690)

2 FOURIER DECOMPOSITION

- Expected contribution from collective flow
- $\langle \cos(n\Delta\phi) \rangle (\phi_s, p_t^{(a)})$

3 CONCLUSIONS

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TWO-PARTICLE CORRELATION MEASUREMENTS

 \sim # of particle pairs with relative azimuth $\Delta\phi$ and pseudorapidity $\Delta\eta$:



⁽PHOBOS, Phys. Rev. C75(2007)054913)

- Short range in rapidity
- Little azimuthal structure



- Long range in rapidity
- Distinct azimuthal structure

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LONG-RANGE CORRELATIONS

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LONG RANGE AZIMUTHAL STRUCTURE $(|\Delta \eta| > 0.7)$



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LONG RANGE AZIMUTHAL STRUCTURE ($|\Delta \eta| > 0.7$)



• Narrow "ridge" at $\Delta \phi = 0$



- Narrow "ridge" at $\Delta \phi = 0$
- Broad away side



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LONG-RANGE DIHADRON CORRELATIONS

QUESTION:

Can correlations at large $|\Delta \eta|$ be explained by collective flow alone?

- Can use new STAR data to test in detail.
- Idea: look at Fourier components of unsubtracted data



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Data well described by first 4 Fourier harmonics in all cases.

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LONG-RANGE CORRELATIONS



Collective flow: particles emitted according to 1-particle distribution

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Collective flow: particles emitted according to 1-particle distribution

$$\frac{dN}{dY d^2 p_t} \propto 1 + 2v_2 \cos 2(\phi - \psi_{RP}) + 2v_4 \cos 4(\phi - \psi_{RP}) + \dots$$



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LONG-RANGE CORRELATIONS

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$$\frac{dN}{dY d^2 p_t} \propto 1 + \sum_{n=1}^{\infty} 2v_n \cos n(\phi - \psi_n) = \sum_{n=-\infty}^{\infty} v_n e^{in\psi_n} e^{-in\phi}$$



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In a given event, independent particle emission:

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In a given event, independent particle emission:

$$V_{n\Delta} \equiv \langle \cos(n\Delta\phi) \rangle = \operatorname{Re} \langle e^{in(\phi_a - \phi_t)} \rangle$$

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angle \langle e^{-in\phi_t}
angle
ight) \end{aligned}$$

$$\equiv v_n^{(a)} v_n^{(t)}$$
COLLECTIVE FLOW CONTRIBUTION

Collective flow: particles emitted according to 1-particle distribution

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$$= \langle \cos(n\phi_a - n\psi_n) \rangle \langle \cos(n\phi_t - n\psi_n) \rangle$$
$$\equiv v_n^{(a)} v_n^{(t)}$$

 $\langle \cos(n\Delta\phi)\rangle(\phi_s, p_t^{(a)})$

$\langle \cos(2\Delta\phi) \rangle$ — Elliptic flow



Flow contribution to second harmonic:

•
$$V_{2\Delta} \equiv \langle \cos(2\Delta\phi) \rangle = v_2^{(a)} v_2^{(t,R)}$$

• $v_2^{(a)} = \langle \cos(2\phi_a - 2\psi_{EP}) \rangle$ = Standard elliptic flow
• $v_2^{(t,R)} \sim \cos(2\phi_s) = \cos(2\phi_t - 2\psi_{EP})$

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 $\langle \cos(n\Delta\phi) \rangle (\phi_s, p_t^{(a)})$

$\langle \cos(4\Delta\phi) angle$ — Quadrangular flow



Flow contribution to fourth harmonic:

- $V_{4\Delta} \equiv \langle \cos(4\Delta\phi) \rangle = v_4^{(a)} \{ EP \} v_4^{(t,R)} \{ EP \}$ + fluctuations
- $v_4^{(a)}{EP} = \langle \cos(2\phi_a 2\psi_{EP}) \rangle$ = Quadrangular flow with respect to v_2 event plane

•
$$v_4^{(t,R)}{EP} \simeq \cos(4\phi_s)$$

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 $\langle \cos(n\Delta\phi) \rangle (\phi_s, p_t^{(a)})$

$\langle \cos(3\Delta\phi) \rangle$ — Triangular flow



Flow contribution to third harmonic:

•
$$V_{3\Delta} \equiv \langle \cos(3\Delta\phi) \rangle = v_3^{(a)} v_3^{(t)}$$

- $v_3 = \langle \cos(3\phi_a 3\psi_3) \rangle$ = Triangular flow
- No dependence on ϕ_s

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$\langle \cos(\Delta \phi) angle$ — Momentum conservation and v_1



Flow contribution to first harmonic (plus p_t conservation):

•
$$V_{1\Delta} \equiv \langle \cos(1\Delta\phi) \rangle = (p_t \text{ cons.}) + v_1^{(a)} v_1^{(t)}$$

• $(p_t \text{ cons.}) = \frac{-p_t^{(a)} p_t^{(t)}}{\langle \sum P_t^2 \rangle}$
• $v_1 = \langle \cos(\phi_a - \psi_1) \rangle = \text{Directed flow}$

= nac

SUMMARY

To be consistent with data, a non-flow signal must have:

- **0** Odd harmonics with no dependence on ϕ_s
- **(3)** A fourth harmonic that decreases and then increases with ϕ_s
- *p_t* dependence that is identical to flow

SUMMARY

To be consistent with data, a non-flow signal must have:

- **0** Odd harmonics with no dependence on ϕ_s
- A second harmonic with monotonically decreasing dependence on ϕ_s
- **(3)** A fourth harmonic that decreases and then increases with ϕ_s
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- I.e., it must have all of the same properties as flow

SUMMARY

To be consistent with data, a non-flow signal must have:

- **0** Odd harmonics with no dependence on ϕ_s
- A second harmonic with monotonically decreasing dependence on ϕ_s
- ${f 0}$ A fourth harmonic that decreases and then increases with ϕ_s
- *p_t* dependence that is identical to flow
- I.e., it must have all of the same properties as flow

More likely: there are only collective flow correlations at large $\Delta \eta$

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- Entire long-range dihadron correlation can be explained by collective flow
- No compelling evidence of non-flow correlation
- Previous signals (mach-cones, etc.) are products of flawed background "subtraction" (flow fluctuations are important!)



(PHOBOS, Phys. Rev. C75(2007)054913)

Extra Slides:

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SHORT-RANGE "JET-LIKE" CORRELATION



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$\langle \cos(3\Delta\phi) \rangle$ — Triangular flow



(Recall that centrality dependence also follows hydro calculation)

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*V*₁ AT MIDRAPIDITY

Event-by-event fluctuations generate a dipole asymmetry which causes directed flow that is (roughly) independent of rapidity:



 $v_1 = \langle \cos(\phi_p) \rangle$

V₁ AT MIDRAPIDITY

By estimating momentum conservation term

$$(p_t \text{ cons.}) = rac{-p_t^{(a)}p_t^{(t)}}{\langle \sum p_t^2
angle},$$

can extract $v_1^{(a)}v_1^{(t)}$ from dihadron correlation:

$$oldsymbol{v}_1^{(a)}oldsymbol{v}_1^{(t)} = \langle \cos(\Delta \phi)
angle + rac{oldsymbol{p}_t^{(a)}oldsymbol{p}_t^{(t)}}{\langle \sum oldsymbol{p}_t^2
angle}$$

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V_1 AT MIDRAPIDITY

Choosing a value for $v_1(p_t^{(t)})$ then gives a curve for $v_1(p_t)$:



V_1 AT MIDRAPIDITY

Which agrees with hydrodynamic calculations!



*V*¹ AT MIDRAPIDITY

Can remove large uncertainties with a dedicated measurement:

$$Q\cos\psi_{EP,1}\equiv\sum w_j\cos\phi_j$$

 $Q\sin\psi_{EP,1}\equiv\sum w_j\sin\phi_j$

Usual choice w = y gives v_1 that is odd in rapidity y. To measure v_1 from fluctuations, choose w independent of y:

$$w = p_t - rac{\langle p_t^2
angle}{\langle p_t
angle}$$

Because $\langle w p_t \rangle = 0$, momentum conservation correlation is removed.

ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

2-particle correlations often analyzed using ZYAM:

$$\frac{dN^{pairs}}{d\Delta\phi} = B[F(\Delta\phi)] + NF(\Delta\phi)$$

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$$= B\left[1 + 2v_2^{(a)}v_2^{(t,R)}\cos(2\Delta\phi) + 2v_4^{(a)}v_4^{(t,R)}\cos(4\Delta\phi)\right] + NF(\Delta\phi)$$

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Fix *B* by assuming zero yield at minimum:

$$NF(\Delta \phi_{\min}) = NF'(\Delta \phi_{\min}) = 0$$

ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



 $\mathbf{B} = \mathbf{0}$

ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.1

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.2

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.3

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.4

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.5

ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.6

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.62

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.64

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.66

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.67

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Subtracted data from STAR:



B = 0.671314

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LONG-RANGE CORRELATIONS

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

2-hump structure depends on $\phi_s \Rightarrow$ not caused by (triangular) flow!

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LONG-RANGE CORRELATIONS

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

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 $V_{3\Delta}$ (and $V_{1\Delta}$) independent of ϕ_s , as predicted.

 ϕ_s dependence comes from even harmonics that aren't fully subtracted This is because of ZYAM prescription, not just because v_2 underestimated:

ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

Instead use
$$v_2^{(a)}v_2^{(t,R)} \equiv V_{2\Delta}$$
, $v_4^{(a)}v_4^{(t,R)} \equiv V_{4\Delta}$ — still depends on ϕ_s :



 $\mathbf{B} = \mathbf{0}$

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LONG-RANGE CORRELATIONS

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ZERO YIELD AT MINIMUM (ZYAM) PRESCRIPTION

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