Viscosity and the Quark Gluon Plasma

Viscous Hydrodynamic Simulations of Relativistic Heavy Ion Collisions

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Outline

1. Relativistic Heavy Ion Collisions
   - Motivation / Background
   - Soft Observables: Elliptic Flow
2. Theory
   - Hydrodynamics
   - Viscous Hydrodynamic Simulations
3. Analysis/Results
   - η/s from Elliptic Flow at RHIC
   - Elliptic Flow at LHC

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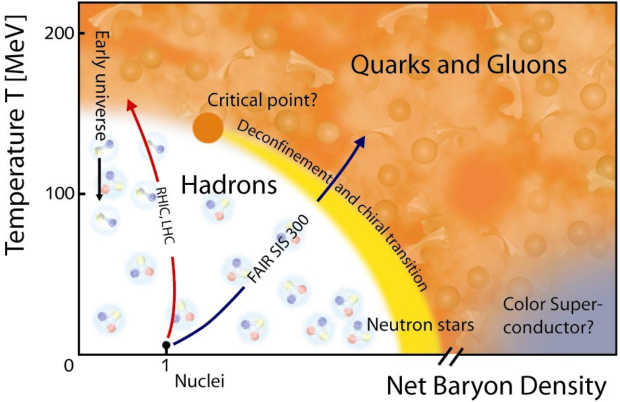
Why heavy ion collisions?

Goal:

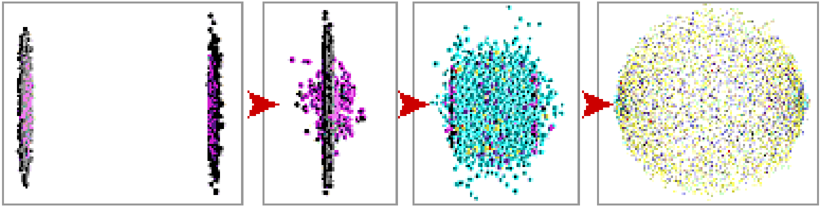
To investigate the high temperature regime of the strong interactions



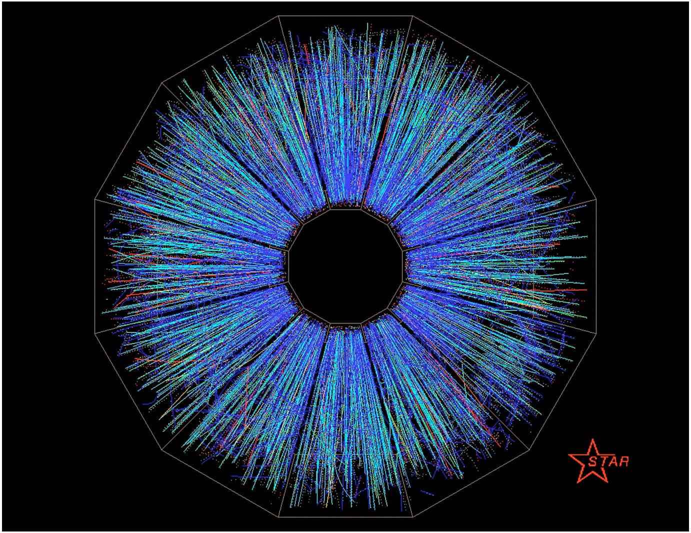
Creating a Quark Gluon Plasma

Can we make a QGP by colliding beams of heavy nuclei?

- Relativistic Heavy Ion Collider (RHIC) at BNL $\rightarrow$ Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- Large Hadron Collider (LHC) at CERN $\rightarrow$ Pb+Pb at $\sqrt{s_{NN}} = 5500$ GeV

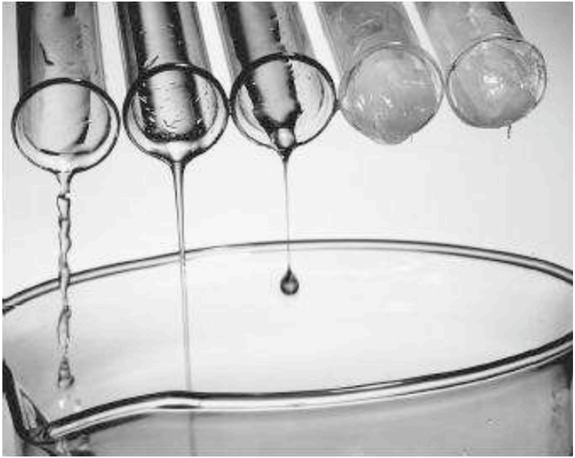


Task: learn as much as possible from analyzing what comes out

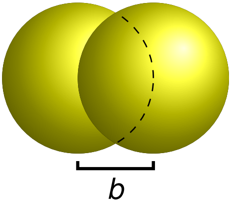


Viscosity

For example: what is the viscosity of a quark gluon plasma?



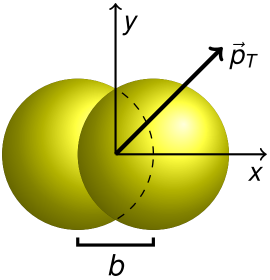
Elliptic Flow



Elliptic Flow

Single-particle momentum spectra:

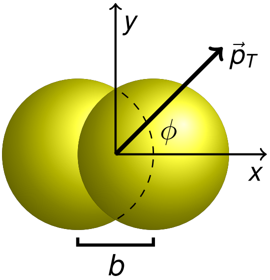
$$\frac{dN}{dY\, d^2 p_T}$$



Elliptic Flow

Single-particle momentum spectra:

$$\frac{dN}{dY\, d^2 p_T} = v_0 \left[1 + \sum_n 2 v_n \cos(n\,\phi) \right]$$

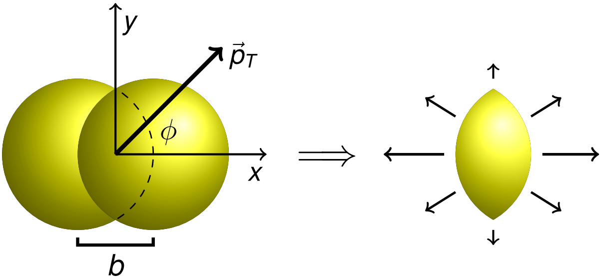


Elliptic Flow

Single-particle momentum spectra:

$$\frac{dN}{dY\, d^2 p_T} = v_0 \left[1 + \sum_n 2 v_n \cos(n\,\phi) \right]$$

Elliptic flow: $v_2 \equiv \langle \cos(2\,\phi) \rangle$



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Ideal (Relativistic) Hydrodynamic Equations

- Assume isotropic energy-momentum tensor in rest frame:

$$T^{\mu\nu} = T_0^{\mu\nu} = (\epsilon + p)\, u^\mu u^\nu - p\, g^{\mu\nu}$$

$$\Rightarrow T_{0_{rest}}^{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Plug in to conservation equations $\Rightarrow$ ideal hydrodynamics:

$$\partial_\mu T^{\mu\nu} = 0$$

- Equation of state closes the set of equations:

$$p = p(\epsilon)$$

- An additional relation for each additional conserved current (assumed unimportant for the following)

Viscosity at RHIC

- Ideal hydrodynamic models surprisingly successful at describing RHIC data:

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

April 18, 2005

- Kovtun, Son, Starinets (KSS) conjectured a universal lower bound on shear viscosity η.

$$\frac{\eta}{s} \geq \frac{1}{4\pi} \simeq 0.08$$

- $\Rightarrow$ Next step: add viscosity; use KSS bound as a yardstick

Viscous Hydrodynamics

- Add dissipative (viscous) effects—derivative expansion:

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

- To first order (Navier-Stokes):

$$\Pi^{\mu\nu} = \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \, \Delta^{\mu\nu} \nabla_\alpha u^\alpha$$

- Acausal signal propagation $\Rightarrow$ instabilities $\Rightarrow$ difficult to solve numerically.
- Can be fixed by adding second-order term(s).

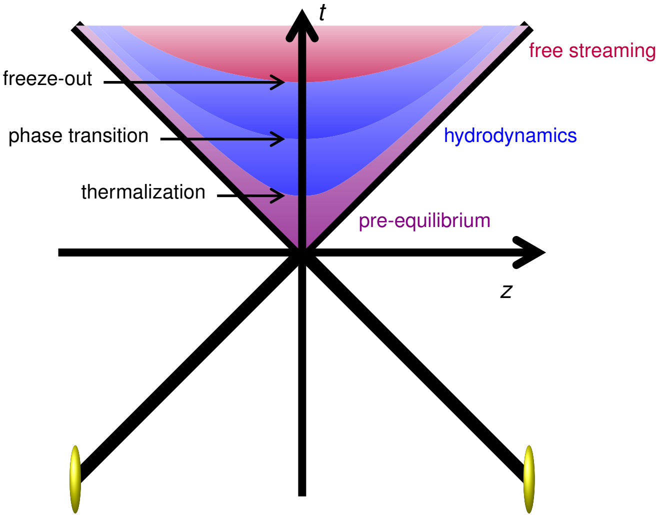
Causal Relativistic Viscous Hydrodynamics

- To start, set bulk viscosity to zero.
- $\Rightarrow$ most general form for a conformal fluid in flat space to second order [BRSSS]:

$$\Pi^{\mu\nu} = \eta\nabla^{\langle\mu}u^{\nu\rangle} - \tau_\pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta D\Pi^{\alpha\beta} + \frac{4}{3}\Pi^{\mu\nu}(\nabla_\alpha u^\alpha)\right] - \frac{\lambda_1}{2\eta^2}\Pi^{\langle\mu}{}_\lambda\Pi^{\nu\rangle\lambda} + \frac{\lambda_2}{2\eta}\Pi^{\langle\mu}{}_\lambda\omega^{\nu\rangle\lambda} - \frac{\lambda_3}{2}\omega^{\langle\mu}{}_\lambda\omega^{\nu\rangle\lambda}$$

- (Simulations insensitive to second-order transport coefficients $\Rightarrow$ can isolate effect of shear viscosity η.)

Anatomy of a Heavy Ion Collision

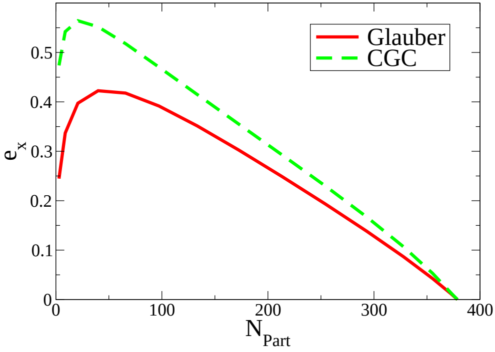


Initial Conditions

Two models are typically used for hydro initial conditions

- Glauber
- Color Glass Condensate (CGC)

Most important difference: initial eccentricity $e_x \equiv \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$



Freeze out

Cooper-Frye freeze out prescription:

- Fluid cell behaves hydrodynamically until it reaches temperature T_f, where it instantaneously "freezes out" into free hadrons

Or, operationally:

1. Allow system to evolve hydrodynamically indefinitely.
2. Go back and identify freeze out surface of constant temperature T_f and integrate over surface

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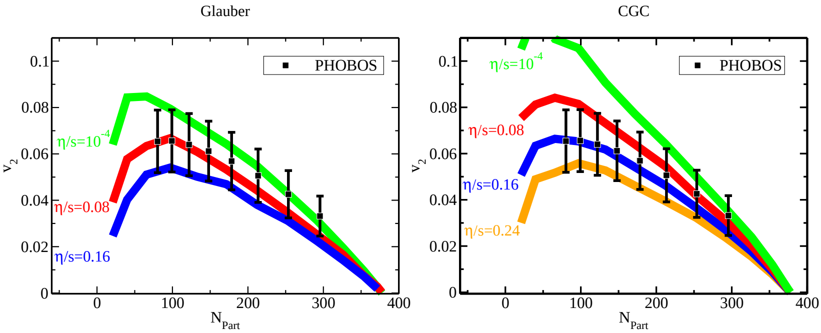
Analysis Procedure/Free Parameters

The procedure used is as follows:

1. Choose model for initial conditions (Glauber or CGC)
2. Choose value of η/s to study (set to a constant throughout the evolution)
3. Use mulipicity data to fix the energy density normalization (T_0) and thermalization time (τ_0)
4. Use $\langle p_T \rangle$ data to fix the freezeout temperature (T_F)
5. With all the parameters now fixed, compare v_2 to RHIC data

RHIC Results: Momentum Integrated v_2



$$\Rightarrow \frac{\eta}{s} \leq 0.5,$$

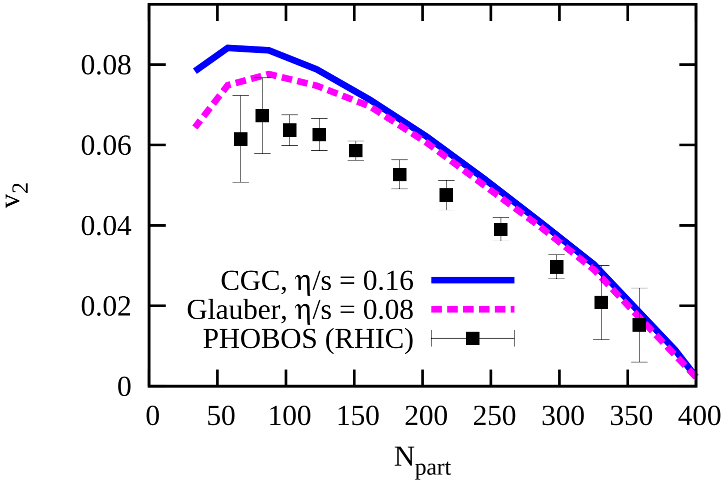
Smaller than any other known substance!

How to predict results at the LHC

Using the knowledge gained from RHIC, we can make a prediction for Pb-Pb collisions at top LHC energies.

1. Assume T_f, $\frac{\eta}{s}$, τ_0 do not change much $\Rightarrow$ use best RHIC values for each initial condition (Glauber and CGC)
2. Choose T_0 to match predicted multiplicity $\frac{dN_{ch}}{dY} \approx 1800$. (Can instead fix $\frac{dS}{dY} \simeq 7.85 \frac{dN_{ch}}{dY}$)
3. Make appropriate changes to the initial conditions (Pb instead of Au and increased collision energy)
4. $\Rightarrow v_2$ prediction!

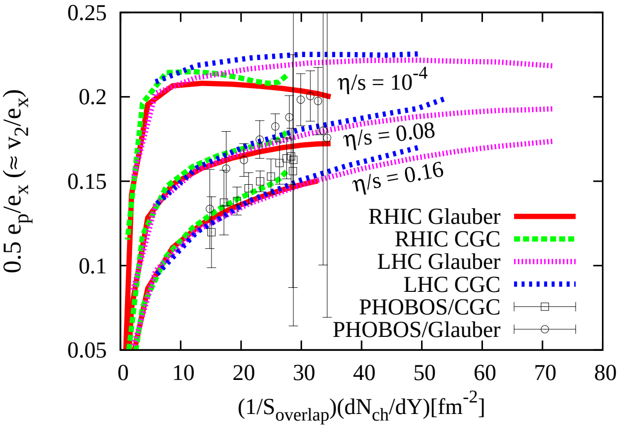
Momentum Integrated v_2 at RHIC and LHC



Summary/Conclusions

- Viscous hydrodynamic simulations of heavy ion collisions work well to describe single-particle observables at RHIC.
- At RHIC: $\frac{\eta}{s} \leq 0.5$
- At LHC: v_2 is predicted to be $\sim$ 10% larger than measured at RHIC

Universal curves



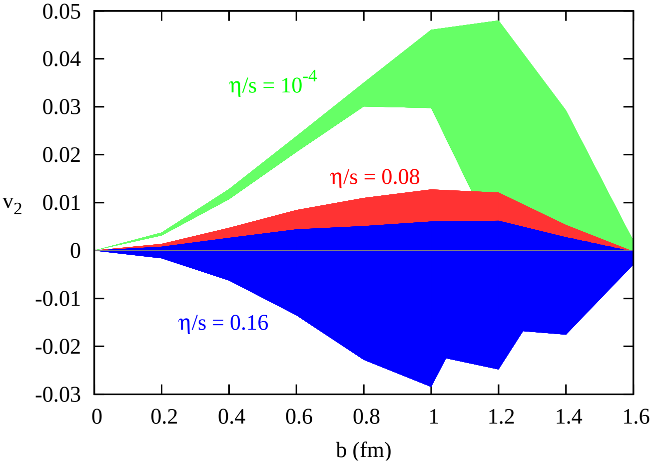
$$\left(S_{overlap} \equiv \pi \sqrt{\langle x^2 \rangle \langle y^2 \rangle} \right)$$

PP COLLISIONS AT LHC

One can perform the same procedure for proton-proton collisions at LHC:

- Assume $dN_{ch}/dY \approx 6$ for "central" collisions
- Glauber initial conditions, using the charge density of the proton.

Integrated elliptic flow



Parameter Values

Beam	Initial cond.	$\frac{dN_{\mathrm{ch}}}{dY}$	T_i [GeV]	$\sqrt{s}$ [GeV]	τ_0 [fm/c]
Gold	Glauber	800	0.34	200	1
Gold	CGC	800	0.31	200	1
Lead	Glauber	1800	0.42	5500	1
Lead	CGC	1800	0.39	5500	1

Table: Central collision parameters used for the viscous hydrodynamics simulations ($T_f = 0.14$ GeV for all).

Notation

$$\begin{aligned}
A_{\langle\mu}B_{\nu\rangle} &\equiv \left(\Delta^{\alpha}_{\mu}\Delta^{\beta}_{\nu} + \Delta^{\alpha}_{\nu}\Delta^{\beta}_{\mu} - \frac{2}{3}\Delta^{\alpha\beta}\Delta_{\mu\nu}\right) A_{\alpha}B_{\beta} \\
\Delta^{\mu\nu} &\equiv g^{\mu\nu} - u^{\mu}u^{\nu} \\
\nabla^{\mu} &\equiv \Delta^{\mu\alpha}D_{\alpha} \\
D &\equiv u_{\alpha}D^{\alpha} \\
\omega_{\mu\nu} &\equiv \frac{1}{2}\left[\nabla_{\nu}u_{\mu} - \nabla_{\mu}u_{\nu}\right]
\end{aligned}$$

e.g. Navier Stokes term:

$$\nabla^{\langle\mu}u^{\nu\rangle} = \nabla^{\mu}u^{\nu} + \nabla^{\nu}u^{\mu} - \Delta^{\mu\nu}\nabla_{\alpha}u^{\alpha}$$

Back