Applications of Lipatov's Action in High-Energy QCD

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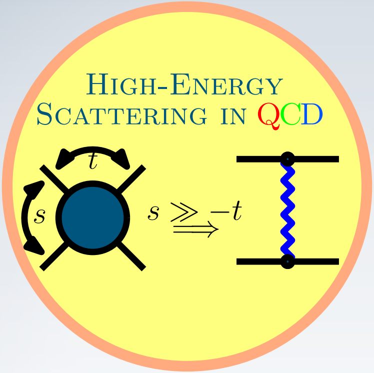
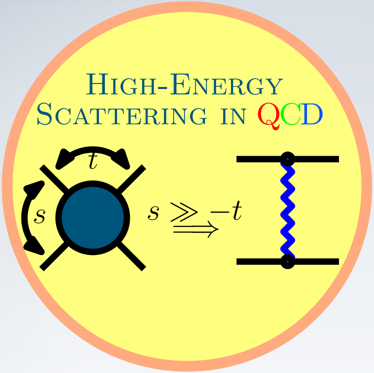
[†] Based on work in collaboration with G. Chachamis, M. Hentschinski, B. Murdaca and A. Sabio Vera [NPB**861** (2012) 133; PRD**87** (2013) 076009; NPB**876** (2013) 453, and work to appear soon]



- Processes with large spreads in rapidity ($y = \ln \frac{p^+}{p^-}; p^\pm = p^0 \pm p^3$)



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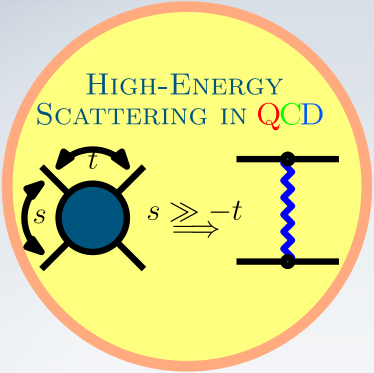
  - ☐ Small-angle elastic scattering
  - ☐ Diffractive processes



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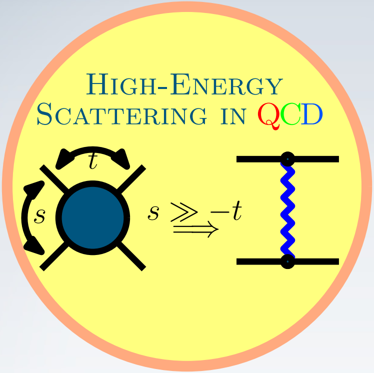

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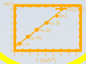
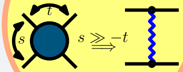
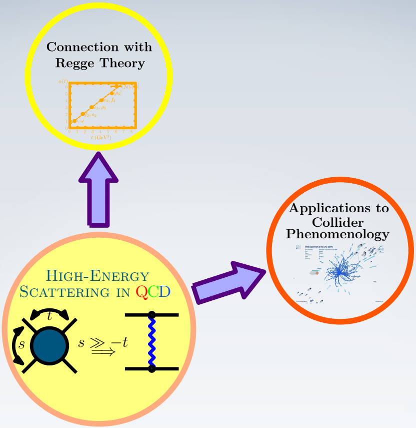
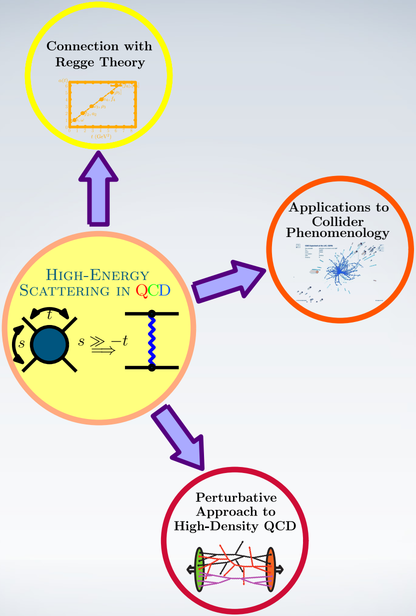
Connection with Regge Theory



High-Energy Scattering in QCD




Connection with Regge Theory
High-Energy Scattering in QCD
s
t
$s \gg -t$
Applications to Collider Phenomenology



Connection with Regge Theory
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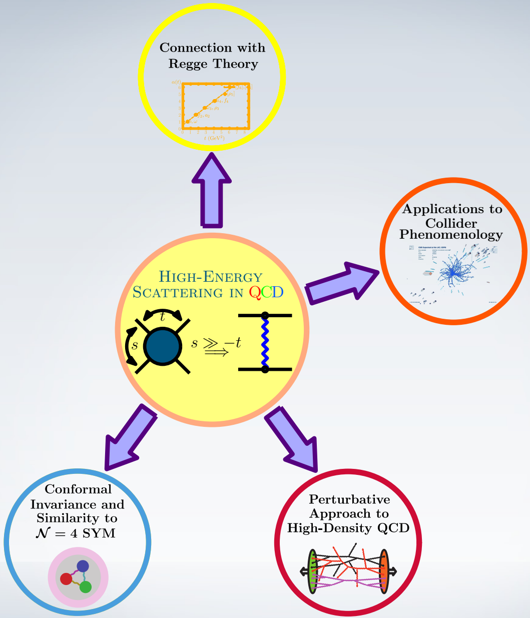
Perturbative Approach to High-Density QCD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Conformal Invariance and Similarity to $\mathcal{N}=4$ SYM
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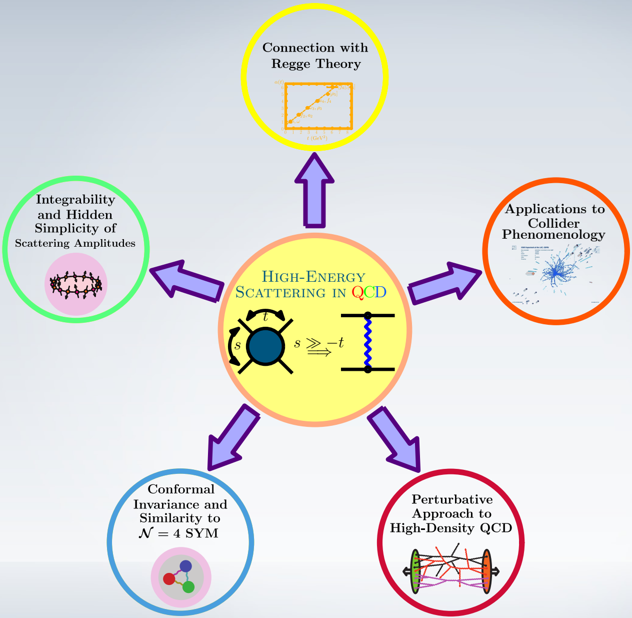

High-Energy Scattering in QCD
t
s
$s \gg -t$
$\Longrightarrow$
Connection with Regge Theory
Applications to Collider Phenomenology
Perturbative Approach to High-Density QCD
Conformal Invariance and Similarity to $\mathcal{N} = 4$ SYM
Integrability and Hidden Simplicity of Scattering Amplitudes

S-Matrix Approach [Regge'59,'60]

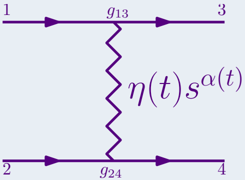
Amplitudes extremely constrained by

- *Lorentz invariance*
- *Unitarity*
- *Analyticity & Crossing*

Using complex angular momentum ℓ

$$\mathcal{A}(s,t) \xrightarrow{s \gg -t} \frac{\eta + e^{-i\pi\alpha(t)}}{2}\beta(t)s^{\alpha(t)}$$

Reggeon Exchange



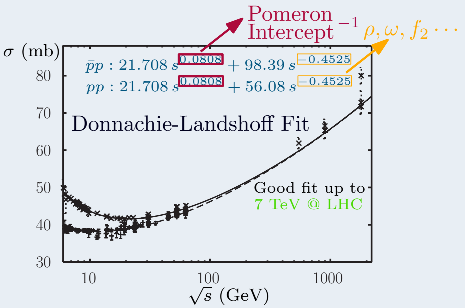
$\alpha(t)$: location in ℓ plane of poles of partial wave amplitude $\sim$ *effective spin*

Rising Cross-Section & Pomeron

Assuming Pole Dominance+Linear Regge Traj.

[Donnachie & Landshoff'90]

$$\sigma_{\text{tot}} \underset{s\to\infty}{\simeq} \frac{1}{s}\Im m\,\mathcal{A}(s,t=0) \sim \sum_i A_i s^{\alpha_i(0)-1}$$



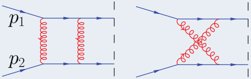
Rising σ_{tot}: Vacuum Quantum Numbers

POMERON [Pomeranchuk'56]

Reggeization in QCD & Leading-ln s Resummation

Corrections to Born Scattering

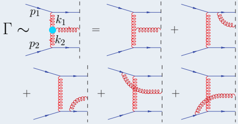
- **Virtual** ($\mathbf{8}_a$ Projected)



$$\simeq \text{Born} \times \omega(\boldsymbol{q}^2) \ln \frac{s}{s_0}$$

$$\omega(\boldsymbol{q}^2) = -\frac{g^2 N_c}{8\pi^2} \ln \frac{\boldsymbol{q}^2}{\mu^2}$$

- **Real** (Lipatov's Vertex)



$$\int d\Pi\, \Gamma\Gamma^* \sim \ln \frac{s}{s_0}$$

IR singularities cancel

High-Energy Factorization

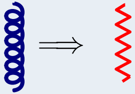
$$A^{\text{MRK}}_{2\to 2+n} = A^{\text{tree}}_{2\to 2+n} \prod_{i=1}^{n+1} s_i^{\omega(t_i)}, \qquad A^{\text{tree}}_{2\to 2+n} = 2gsT^{c_1}_{A'A}$$

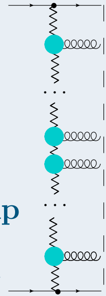
$$\times \Gamma_1 \frac{1}{t_1} gT^{d_1}_{c_2 c_1} \Gamma^1_{2,1} \frac{1}{t_2} \cdots gT^{d_n}_{c_{n+1} c_n} \Gamma^n_{n+1,n} \frac{1}{t_{n+1}} gT^{c_{n+1}}_{B'B} \Gamma_2$$

Lipatov's Ansatz

[Lipatov'76]

$$\frac{-\mathrm{i}}{\mathbf{k}_\mathrm{i}^2} \to \frac{-\mathrm{i}}{\mathbf{k}_\mathrm{i}^2} \left[-\frac{\mathrm{s_i}}{\mathbf{k}_\mathrm{i}^2} \right]^{\alpha(-\mathbf{k}_\mathrm{i}^2)}$$





The ansatz satisfies

- ☐ **Bootstrap**
- ☐ Consistency with Unitarity

Leading ln s terms captured by strong ordering in rapidity

BFKL Equation and Beyond

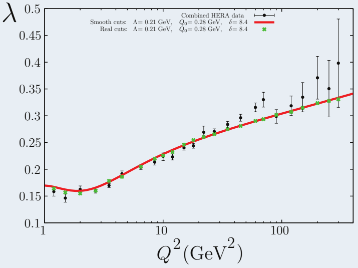
BFKL Equation [Fadin, Kuraev & Lipatov '75,'76,'77; Lipatov'76; Balitsky & Lipatov '78,'79]

$$\omega f_\omega(\boldsymbol{k},\boldsymbol{k}') = \delta^2(\boldsymbol{k}-\boldsymbol{k}') + \int d^2\boldsymbol{\kappa}\, \mathcal{K}(\boldsymbol{k},\boldsymbol{\kappa}) f_\omega(\boldsymbol{k},\boldsymbol{k}')$$

$$\mathcal{K}(\boldsymbol{k},\boldsymbol{\kappa}) = 2\omega(\boldsymbol{k}^2)\delta^2(\boldsymbol{k}-\boldsymbol{\kappa}) + \frac{N_c\alpha_s}{\pi^2}\frac{1}{(\boldsymbol{k}-\boldsymbol{\kappa})^2}$$

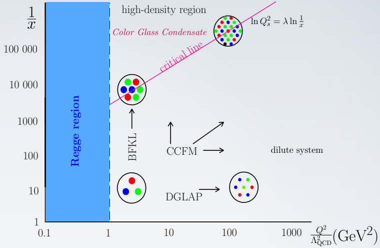
Forward Solution and Total Cross Section

$$\sigma_{\text{tot}}^{qq} = 4\alpha_s^2\mathcal{G}\iint d^2\boldsymbol{k}\, d^2\boldsymbol{k}'\, \frac{f(s,\boldsymbol{k},\boldsymbol{k}')}{\boldsymbol{k}^2\boldsymbol{k}'^2} \sim \frac{s^\lambda}{\ln s}$$

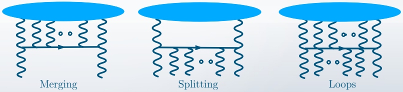


[Hentschinski, Salas & S. Vera'12]

- Evolution to high-energy $\Longrightarrow$ High partonic densities



Implementing *all* requirements of unitarity not easy in non-linear generalizations of BFKL (e.g. Balitsky-JIMWLK)



Lipatov's Action for High-Energy QCD

[Lipatov'91; Kirschner, Lipatov & Szymanowski'93,'94]
[Lipatov'95,'97]

$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}};$$

$$S_{\text{ind}} = \int d^4x \, \text{Tr}\left[(W_+[v(x)] - \mathscr{A}_+(x))\, \partial_\perp^2 \mathscr{A}_-(x)\right] + \int d^4x \, \text{Tr}\left[(W_-[v(x)] - \mathscr{A}_-(x))\, \partial_\perp^2 \mathscr{A}_+(x)\right];$$

$$W_\pm[v] = v_\pm \tfrac{1}{D_\pm} \partial_\pm = v_\pm - g v_\pm \frac{1}{\partial_\pm} v_\pm + \cdots$$

$\mathscr{A}_\pm$: reggeons, v_μ: gluons

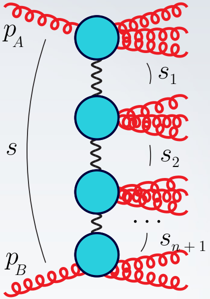
Kinematical Constraints

$\partial_\pm \mathscr{A}_\mp(x) = 0, \quad \sum_{i=0}^r k_i^\pm = 0$

Reggeon fields invariant under *local* gauge transformations

Generalized **Quasi-Multi-Regge Kinematics** (QMRK)

[Fadin & Lipatov'89]



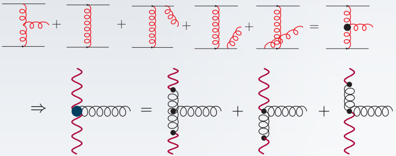
Strong rapidity ordering of clusters:
$y_0 \gg y_1 \gg \cdots \gg y_{n+1}$, $y_k = \frac{1}{2} \ln \frac{k^+}{k^-}$

Transverse **Reggeon propagators** with eikonal polarization tensor

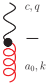
[Antonov, Cherednikov, Kuraev & Lipatov'05]

- Infinite number of new induced vertices (perturbative expansion of Wilson line) needed to reconcile gauge invariance and high-energy factorization

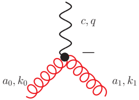
Generic Vertex = Projection + Induced Vertex



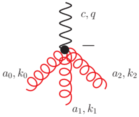
$$S_{\text{eff}} = S_{\text{QCD}} + S_{\text{ind}}$$



$$= \Delta^{\nu_0 -}_{a_0 c} = -i\boldsymbol{q}^2 \delta^{a_0 c}(n^-)^{\nu_0},$$



$$= g\boldsymbol{q}^2 f^{a_0 a_1 c}\frac{1}{k_0^-}(n^-)^{\nu_0}(n^-)^{\nu_1},$$



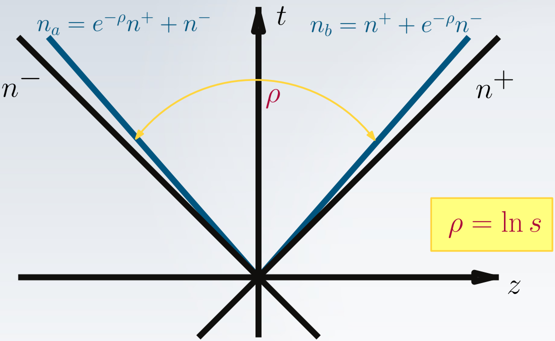
$$= \Delta^{\nu_0\nu_1\nu_2 -}_{a_0 a_1 a_2 c} = ig^2\boldsymbol{q}^2\left(\frac{f^{a_2 a_1 a} f^{a_0 a c}}{k_2^- k_0^-} + \frac{f^{a_2 a_0 a} f^{a_1 a c}}{k_2^- k_1^-}\right)(n^-)^{\nu_0}(n^-)^{\nu_1}(n^-)^{\nu_2},$$

$$= \frac{i}{2\boldsymbol{q}^2}.$$

Regularization & Scaling

Problem ♯1 Appearance of new rapidity divergences in longitudinal integrals

[Collins & Soper'81,'82]
[Korchemsky & Radyushkin'87]
[Balitsky'96]
[Hentschinski & Sabio Vera'11]



- Regularization needed to make sense of non-local $\frac{1}{\partial_\pm}$
- Rest of divergences $\Longrightarrow$ dimensional regularization
- $\rho \to \infty$ in high-energy limit
- Scaling arguments select small number of ρ-enhanced diagrams
- Pole prescription consistent with Hermiticity [Hentschinski'11]

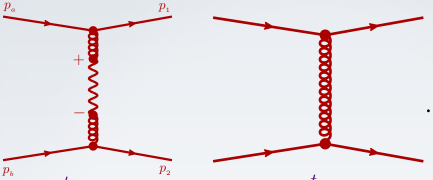
Tilting the light-cone vectors appearing in the induced vertices

The Subtraction Mechanism

Problem ♯2 Apparent overcounting of degrees of freedom

- Enforcing locality in rapidity with a cutoff manifestly breaks gauge invariance and makes computations unwieldy

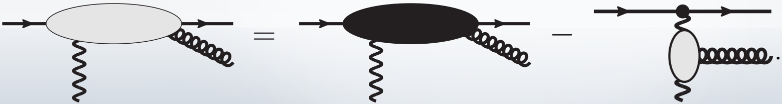
[Bartels, Hentschinski & Lipatov'08]



$\frac{-t}{s} < \eta$ $\frac{-t}{s} > \eta$

[IMPORTANT] Subtraction Mechanism respects gauge (and Lorentz) invariance *and agrees by construction with QCD amplitude*

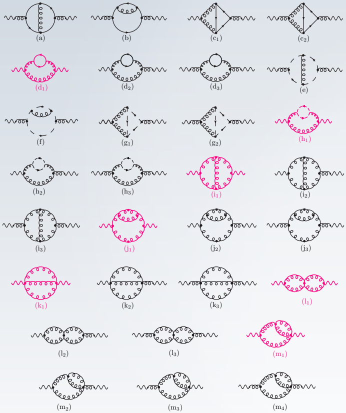
Subtract non-local contributions mediated by reggeon exchange



2-Loop Gluon Regge Trajectory

[Chachamis, Hentschinski, JDM & Sabio Vera'12,'13]

Computation of Reggeon Self-Energy



Subtractions ~ Iteration of 1-loop trajectory

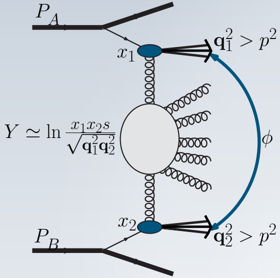
- We get trajectory from subtracted self-energy requiring ρ-independence of renormalized gluon propagator
- Ambiguities from mixed divergences fixed

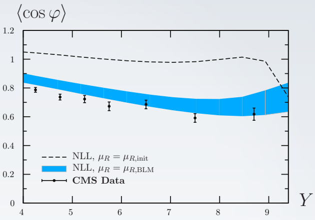
- Exact agreement with literature [Fadin, Fiore & Kotsky'95,'96; Fadin, Fiore & Quartarolo'96; Del Duca & Glover'01]

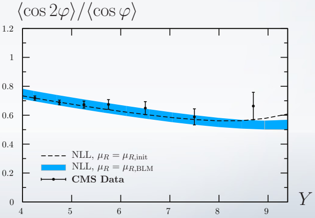
$$\omega^{(2)}(\boldsymbol{q}^2) = \frac{(\omega^{(1)}(\boldsymbol{q}^2))^2}{4}\left[\frac{11}{3} - \frac{2n_f}{3N_c} + \left(\frac{\pi^2}{3} - \frac{67}{9}\right)\epsilon + \left(\frac{404}{27} - 2\zeta(3)\right)\epsilon^2\right]$$

Mueller-Navelet Jets

[Mueller & Navelet'87]
[Bartels, Colferai & Vacca'02, '03]
[Schwennsen & Sabio Vera'06]
[Marquet & Royon'06,'08]
[Colferai, Schwennsen, Szymanowski & Wallon'10]
[Caporale, Ivanov, Murdaca, Papa & Perri'11]
[Caporale, Murdaca, Sabio Vera & Salas'13]
[Ducloué, Szymanowski & Wallon'13]







$$\frac{d\hat{\sigma}}{d^2\boldsymbol{q}_1^2 d^2\boldsymbol{q}_2^2} = \frac{\pi^2 \bar{\alpha}_s^2}{2} \frac{f(\boldsymbol{q}_1, \boldsymbol{q}_2, Y)}{\boldsymbol{q}_1^2 \boldsymbol{q}_2^2},$$

$$f(\boldsymbol{q}_1, \boldsymbol{q}_2, Y) = \int \frac{d\omega}{2\pi i} e^{\omega Y} f(\boldsymbol{q}_1, \boldsymbol{q}_2, \omega)$$

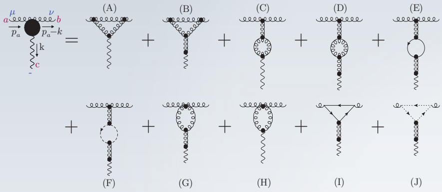
$$\frac{d\hat{\sigma}(\bar{\alpha}_s, Y, p_{1,2}^2)}{d\phi} = \frac{\pi^2 \bar{\alpha}_s^2}{4\sqrt{p_1^2 p_2^2}} \sum_{n=-\infty}^{\infty} e^{in\phi} \mathcal{C}_n(Y)$$

$$\frac{\langle \cos(m\phi) \rangle}{\langle \cos(n\phi) \rangle} = \frac{\mathcal{C}_m(Y)}{\mathcal{C}_n(Y)}$$

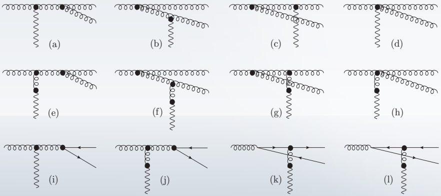
1-Loop Forward Jet Vertex

[Hentschinski & Sabio Vera'11; Chachamis, Hentshcinski, JDM & Sabio Vera'12]

Virtual Corrections to RGG Vertex

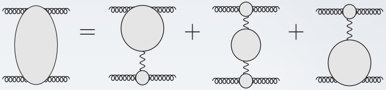


Real Emission Corrections

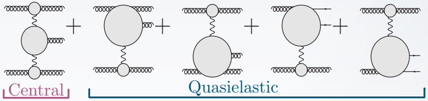


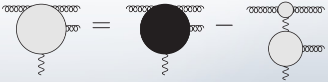
Subtracted 1-loop RGG Vertex

Full 1-loop $gg \to gg$ Amplitude



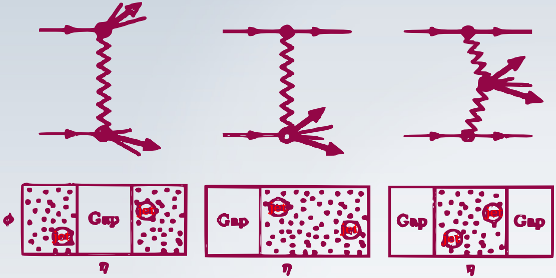
Contributions to the 1-loop Jet Vertex





NLO Mueller-Tang Vertex

[Hentschinski, JDM, Murdaca & Sabio Vera (arXiv:1402.xxxx, arXiv:1403.xxxx)]



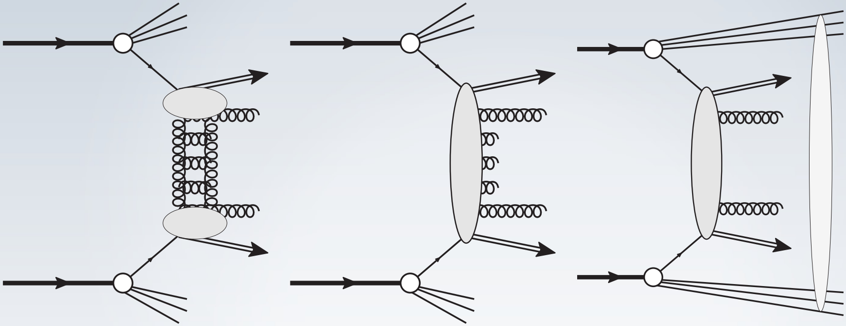
Forward-Backward Jets with Rapidity Gaps

Perturbative Diffraction

[Mueller & Tang'92]
[Cox, Forshaw & Lönnblad'99]
[Enberg, Ingelman, Motyka'01]
[Chevalier, Kepka, Marquet & Royon'09,'10]
[Marquet et al.'13]
[Hatta et al.'13]

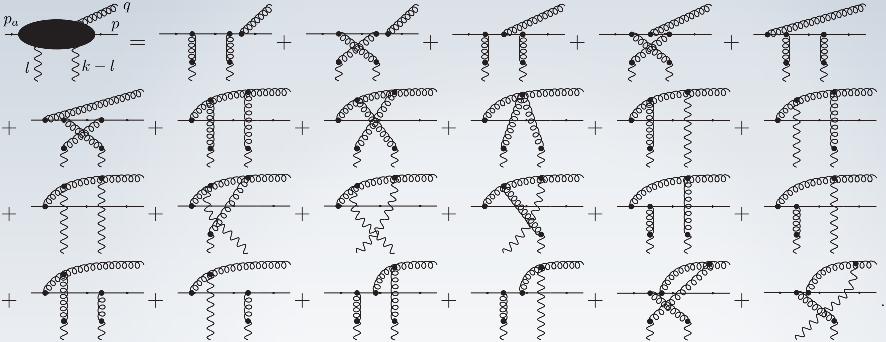
☐ No radiation $\Longrightarrow$ Singlet Exchange

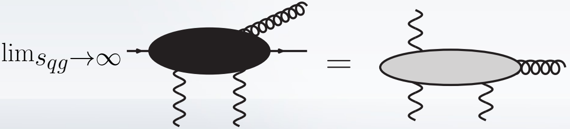
- Possibility to test *nonforward* NLO BFKL (pomeron slope)
- Expect large corrections, as in Mueller-Navelet case
  [Colferai et al.'11]
- Rapidity gap survival relatively under control



Singlet **Octet** **Residual Interaction**

Quasi-Elastic Corrections





$$d\sigma_{ab}^{\text{quasielastic}} = h_{a,\text{MT}}^{(0)} h_{b,\text{MT}}^{(0)} \left[\int \frac{d^{2+2\epsilon}\boldsymbol{l}}{\boldsymbol{l}^2(\boldsymbol{k}-\boldsymbol{l})^2} \right] \left[\int \frac{d^{2+2\epsilon}\boldsymbol{l}'}{\boldsymbol{l}'^2(\boldsymbol{k}-\boldsymbol{l}')^2} \right] \mathcal{F}_{qqg}^{\text{MT}}(\boldsymbol{l},\boldsymbol{l}') d[\boldsymbol{k}] d[\boldsymbol{p}] d\eta$$

$$\mathcal{F}_{qqg}^{\text{MT}}(\boldsymbol{l}_1,\boldsymbol{l}_2) = \frac{\alpha_s(\mu^2) P_{qg}(z,\epsilon)}{C_F^2 \pi^{1+\epsilon} \mu^{2\epsilon} \Gamma(1-\epsilon)}$$
$$\times \left[C_F \left(\frac{\boldsymbol{\Delta}}{\boldsymbol{\Delta}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) + C_A \left(\frac{\boldsymbol{p}}{\boldsymbol{p}^2} - \frac{1}{2} \frac{(\boldsymbol{l}_1 - \boldsymbol{q})}{(\boldsymbol{q}-\boldsymbol{l}_1)^2} - \frac{1}{2} \frac{(\boldsymbol{p}-\boldsymbol{l}_1)}{(\boldsymbol{p}-\boldsymbol{l}_1)^2} \right) \right]$$
$$\cdot \left[C_F \left(\frac{\boldsymbol{\Delta}}{\boldsymbol{\Delta}^2} - \frac{\boldsymbol{q}}{\boldsymbol{q}^2} \right) + C_A \left(\frac{\boldsymbol{p}}{\boldsymbol{p}^2} - \frac{1}{2} \frac{(\boldsymbol{l}_2 - \boldsymbol{q})}{(\boldsymbol{q}-\boldsymbol{l}_2)^2} - \frac{1}{2} \frac{(\boldsymbol{p}-\boldsymbol{l}_2)}{(\boldsymbol{p}-\boldsymbol{l}_2)^2} \right) \right]$$

$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}, \qquad P_{qg}(z,\epsilon) = C_F \frac{1 + (1-z)^2 + \epsilon z^2}{z}$$

Final Result for Mueller-Tang Jet Vertex

$$\frac{d\hat{\sigma}}{dJ_1\,dJ_2\;d^2\boldsymbol{k}} = \int \frac{d^2\boldsymbol{l}_1}{\pi}\int \frac{d^2\boldsymbol{l}_1'}{\pi}\int \frac{d^2\boldsymbol{l}_2}{\pi}\int \frac{d^2\boldsymbol{l}_2'}{\pi}\frac{d\hat{V}(\boldsymbol{l}_1,\boldsymbol{l}_2,\boldsymbol{k},\boldsymbol{p}_{J,1},y_1,s_0)}{dJ_1}$$
$$G\left(\boldsymbol{l}_1,\boldsymbol{l}_1',\boldsymbol{k},\frac{\hat{s}}{s_0}\right)G\left(\boldsymbol{l}_2,\boldsymbol{l}_2',\boldsymbol{k},\frac{\hat{s}}{s_0}\right)\frac{d\hat{V}(\boldsymbol{l}_1',\boldsymbol{l}_2',\boldsymbol{k},\boldsymbol{p}_{J,2},y_2,s_0)}{dJ_2},$$

$$\frac{d\hat{V}^{(1)}(x,\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2;x_J,\boldsymbol{k}_J;M_{X,\max},s_0)}{dJ} =$$
$$= v_q^{(0)}\frac{\alpha_s}{2\pi}\Bigg[S_J^{(2)}(\boldsymbol{k},x)\cdot\Bigg\{-\frac{\beta_0}{4}\ln\left(\frac{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2\boldsymbol{l}_2^2(\boldsymbol{k}-\boldsymbol{l}_2)^2}{\mu^4}\right)+C_f\left[4-\frac{\pi^2}{3}\right]$$
$$+\frac{C_a}{2}\Bigg[\frac{3}{2}\ln\left(\frac{\boldsymbol{k}^4}{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)+\frac{3}{2}\ln\left(\frac{\boldsymbol{k}^4}{\boldsymbol{l}_2^2(\boldsymbol{k}-\boldsymbol{l}_2)^2}\right)+\ln\left(\frac{\boldsymbol{k}^2}{\boldsymbol{l}_1^2}\right)\ln\left(\frac{(\boldsymbol{k}-\boldsymbol{l}_1)^2}{s_0}\right)$$
$$+\ln\left(\frac{\boldsymbol{k}^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)\ln\left(\frac{\boldsymbol{l}_1^2}{s_0}\right)+\ln\left(\frac{\boldsymbol{k}^2}{(\boldsymbol{k}-\boldsymbol{l}_1)^2}\right)\ln\left(\frac{\boldsymbol{l}_1^2}{s_0}\right)+\ln\left(\frac{\boldsymbol{k}^2}{\boldsymbol{l}_2^2}\right)\ln\left(\frac{(\boldsymbol{k}-\boldsymbol{l}_2)^2}{s_0}\right)$$
$$+\ln\left(\frac{\boldsymbol{k}^2}{(\boldsymbol{k}-\boldsymbol{l}_2)^2}\right)\ln\left(\frac{\boldsymbol{l}_2^2}{s_0}\right)-6\frac{\sqrt{\boldsymbol{l}_1^2(\boldsymbol{k}-\boldsymbol{l}_1)^2}}{\boldsymbol{k}^2}\phi_1\sin\phi_1-2\phi_1^2$$
$$-6\frac{\sqrt{\boldsymbol{l}_2^2(\boldsymbol{k}-\boldsymbol{l}_2)^2}}{\boldsymbol{k}^2}\phi_2\sin\phi_2-2\phi_2^2\Bigg]\Bigg\}+\ln\frac{\lambda^2}{\mu_F^2}\int_{z_0}^1 dz\;S_J^{(2)}(\boldsymbol{k},zx)\Bigg\{\Bigg[P_{qq}(z)$$
$$+C_f(1-z)+2C_f(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+\Bigg]+\frac{C_a^2}{C_f^2}\Bigg[P_{gq}(z)+2C_f\frac{z-1}{z}\Bigg]\Bigg\}$$
$$+\int_0^1 dz\int\frac{d^2\boldsymbol{q}}{\pi}S_J^{(3)}(\boldsymbol{p},\boldsymbol{q},zx,x)\Theta\left(M_{X,\max}^2-\frac{(\boldsymbol{q}-z\boldsymbol{k})^2}{z(1-z)}\right)$$
$$\Bigg\{P_{qq}(1-z)\cdot\frac{\boldsymbol{k}^2}{\boldsymbol{q}^2(\boldsymbol{q}-z\boldsymbol{k})^2}\Theta\left(\frac{|\boldsymbol{q}|}{1-z}-\lambda\right)+\frac{C_a}{C_f}\cdot\Bigg[P_{qg}(z)J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1)$$
$$+P_{qg}(z)J_{C_fC_a}(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_2)\Bigg]+\frac{C_a^2}{C_f^2}\cdot P_{qg}(z)J_2(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2)\;\cdot\Theta\left(\boldsymbol{p}^2-\lambda^2\right)\Bigg\}$$

$$J_1(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}) = \frac{1}{2}\frac{\boldsymbol{k}^2}{(\boldsymbol{q}-\boldsymbol{k})^2}\left(\frac{(1-z)^2}{(\boldsymbol{q}-z\cdot\boldsymbol{k})^2}-\frac{1}{\boldsymbol{q}^2}\right)-\frac{1}{4}\frac{1}{(\boldsymbol{q}-\boldsymbol{l})^2}\left(\frac{(\boldsymbol{l}-z\cdot\boldsymbol{k})^2}{(\boldsymbol{q}-z\cdot\boldsymbol{k})^2}-\frac{\boldsymbol{l}^2}{\boldsymbol{q}^2}\right)$$
$$-\frac{1}{4}\frac{1}{(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l})^2}\left(\frac{(\boldsymbol{l}-(1-z)\cdot\boldsymbol{k})^2}{(\boldsymbol{q}-z\cdot\boldsymbol{k})^2}-\frac{(\boldsymbol{l}-\boldsymbol{k})^2}{\boldsymbol{q}^2}\right);$$
$$J_2(\boldsymbol{q},\boldsymbol{k},\boldsymbol{l}_1,\boldsymbol{l}_2) = \frac{1}{4}\Bigg[\frac{\boldsymbol{l}_1^2}{(\boldsymbol{q}-\boldsymbol{k})^2(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l}_1)^2}+\frac{(\boldsymbol{k}-\boldsymbol{l}_1)^2}{(\boldsymbol{q}-\boldsymbol{k})^2(\boldsymbol{q}-\boldsymbol{l}_1)^2}$$
$$+\frac{\boldsymbol{l}_2^2}{(\boldsymbol{q}-\boldsymbol{k})^2(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l}_2)^2}+\frac{(\boldsymbol{k}-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{k})^2(\boldsymbol{q}-\boldsymbol{l}_2)^2}-\frac{1}{2}\Bigg(\frac{(\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{l}_2)^2}$$
$$+\frac{(\boldsymbol{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{l}_2)^2}+\frac{(\boldsymbol{k}-\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l}_2)^2(\boldsymbol{q}-\boldsymbol{l}_1)^2}$$
$$+\frac{(\boldsymbol{l}_1-\boldsymbol{l}_2)^2}{(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l}_1)^2(\boldsymbol{q}-\boldsymbol{k}+\boldsymbol{l}_2)^2}\Bigg)\Bigg].$$

- Finite result in $d=4$ after renormalization of coupling and PDFs

Outlook

New Applications of the Effective Action

- Full NLO Phenomenology of Jet-Gap-Jet Events
- Computation of NLO Reggeon-Reggeon-Higgs Vertex
- Independent Corroboration of NLO Photon Impact Factor

Many Issues Still To Be Understood within Lipatov's Action...

- Relation to other formalisms [JIMWLK Hamiltonian, Balitsky OPE & Wilson Lines]; Identification of the Reggeon
- Renormalization Group Equation for evolution with respect to *factorization scale* ρ
- Extension of Lipatov's action to dense regime: exchange of several reggeized gluons

Conclusions

- Consistency of Lipatov's action checked at loop level via explicit non-trivial computations (2-loop gluon Regge trajectory + 1-loop impact factors)
- General subtraction mechanism that allows for application of usual loop integration techniques
- Derivation of NLO effective vertex for jet-gap-jet events