Gluon saturation in high energy hadrons

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François Gelis
CEA, IPhT

Outline

1. **Deep Inelastic Scattering**
2. **Gluon saturation at small x**
3. **DIS in the CGC framework**
4. **Connection to Nucleus-Nucleus collisions**

1. **Deep Inelastic Scattering**
   - Inclusive DIS
   - Experimental results (I)
   - Experimental results (II)
2. **Gluon saturation at small x**
   - Structure of a nucleon
   - Gluon evolution
   - Saturation domain
   - Multiple scatterings
   - Color Glass Condensate effective theory
3. **DIS in the CGC framework**
   - Leading Order
   - NLO and Leading Logs
   - Inclusive DIS
   - Exclusive processes
   - EIC project
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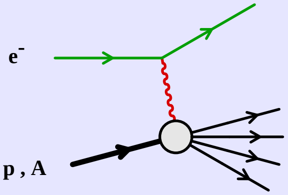
   - Stages of AA collisions
   - Energy-Momentum tensor
   - Correlations in rapidity

Introduction to DIS

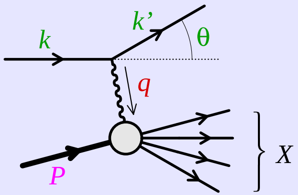
Basic idea : smash a well known probe on a nucleon or nucleus in order to try to figure out what is inside...

- Photons are very well suited for that purpose because their interactions are well understood
- Deep Inelastic Scattering : collision between an electron and a nucleon or nucleus, by exchange of a virtual photon



- Variant : collision with a neutrino, by exchange of $Z^0, W^\pm$

Kinematical variables



- Note : the virtual photon is space-like: $q^2 \leq 0$

Other invariants of the reaction :

$$
\begin{aligned}
\nu &\equiv P \cdot q \\
s &\equiv (P+k)^2 \\
M_X^2 &\equiv (P+q)^2 = m_N^2 + 2\nu + q^2
\end{aligned}
$$

- One uses commonly : $Q^2 \equiv -q^2$ and $x \equiv Q^2/2\nu$
- In general $M_X^2 \geq m_N^2$, and we have : $0 \leq x \leq 1$
  ($x = 1$ corresponds to the case of elastic scattering)

Structure functions

Inclusive cross-section :

$$E'\frac{d\sigma}{d^3\vec{k}'} = \frac{1}{32\pi^3(s-m_N^2)}\frac{e^2}{q^4}4\pi L^{\mu\nu}W_{\mu\nu}$$

$$4\pi W_{\mu\nu} = \int d^4y\; e^{iq\cdot y}\;\Big\langle\big\langle N\big|J_\nu^\dagger(y)J_\mu(0)\big|N\big\rangle\Big\rangle_{\rm spin}$$

- For DIS via photon exchange, the hadronic tensor reads

$$W_{\mu\nu} = -F_1\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)+\frac{F_2}{\nu}\left(P_\mu - q_\mu\frac{P\cdot q}{q^2}\right)\left(P_\nu - q_\nu\frac{P\cdot q}{q^2}\right)$$

Inclusive DIS cross-section in the nucleon rest frame

$$\frac{d\sigma_{e-N}}{dE'd\Omega} = \frac{\alpha_{\rm em}^2}{4m_N E^2\sin^4(\theta/2)}\left[2\sin^2(\theta/2)F_1 + \cos^2(\theta/2)\frac{m_N^2}{\nu}F_2\right]$$

where Ω is the solid angle of the scattered electron

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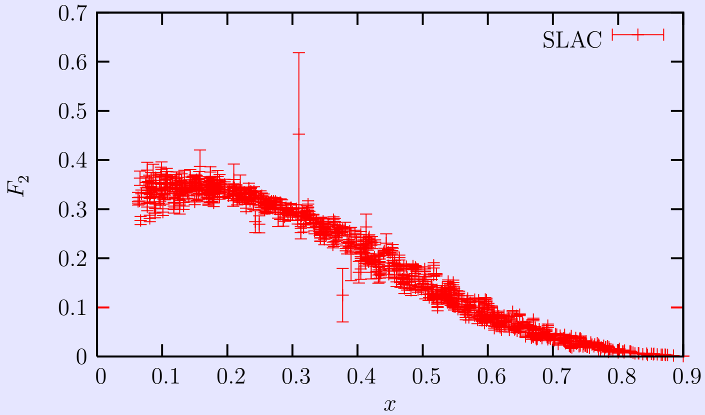
DIS: highlights on QCD

- Bjorken scaling
- Asymptotic freedom
- Scaling violations

Bjorken scaling

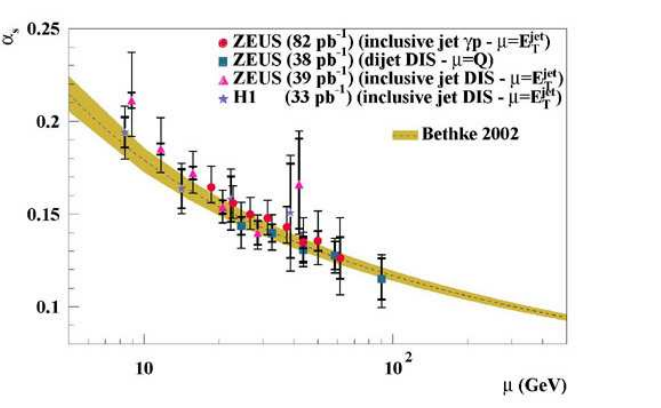
Bjorken scaling : F_2 depends very weakly on Q^2



- Bjorken scaling implies that the constituents are quasi-free

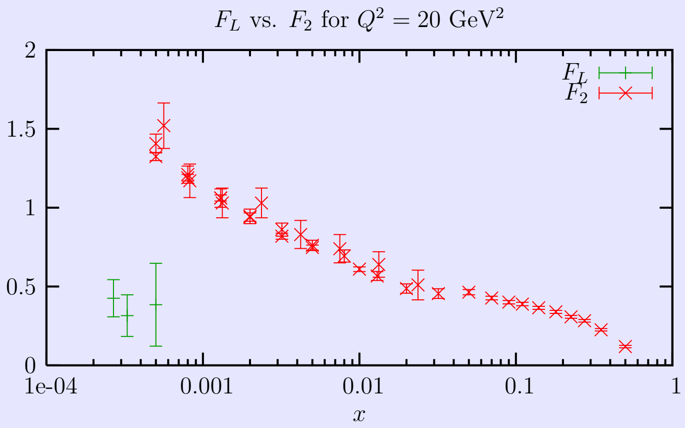
Asymptotic freedom

Asymptotic freedom



Longitudinal structure function

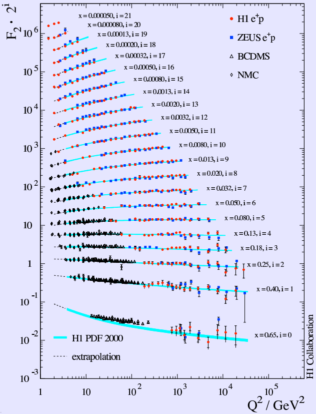
$F_L \equiv F_2 - 2xF_1$ is quite smaller than F_2



- The smallness of F_L implies that the struck partons are spin 1/2 point-like particles

Scaling violations

Scaling violations



- Scaling violations probe the interactions among quark and gluons

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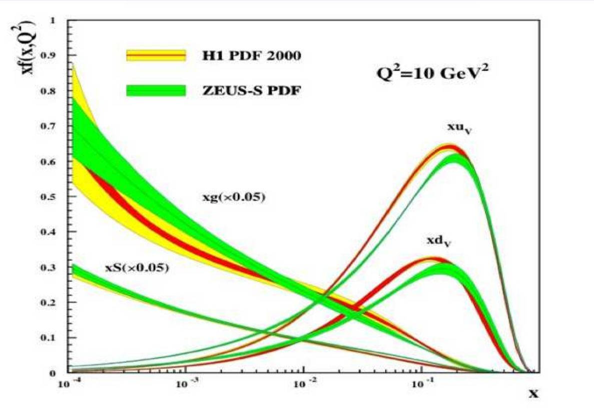
DIS: open issues at small x

- Gluon growth at small x
- Geometric scaling
- F_L at small x and small Q^2

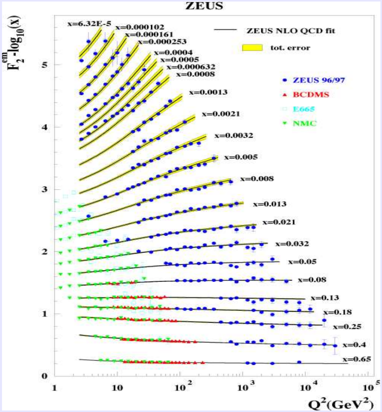
Growth of the gluon distribution at small x

Gluon distribution at small x



Geometric scaling

Geometric scaling : $\tau \sim Q^2 x^{0.3}$



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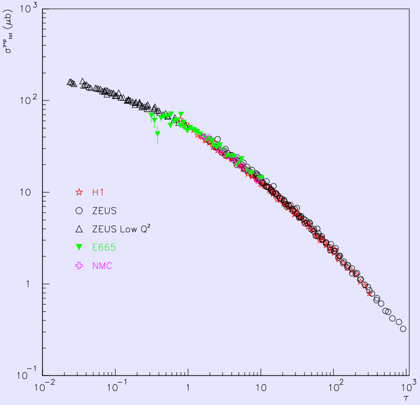
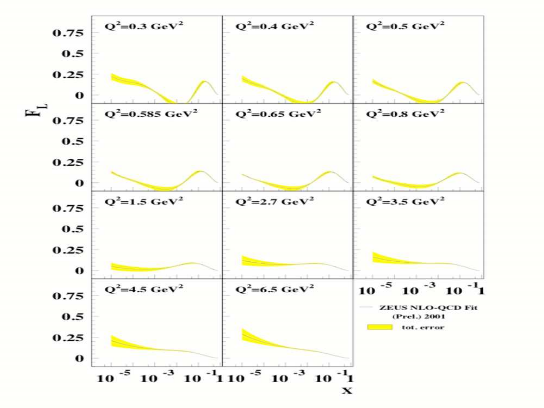


Some trouble with F_L at small Q^2

F_L from DIS fits



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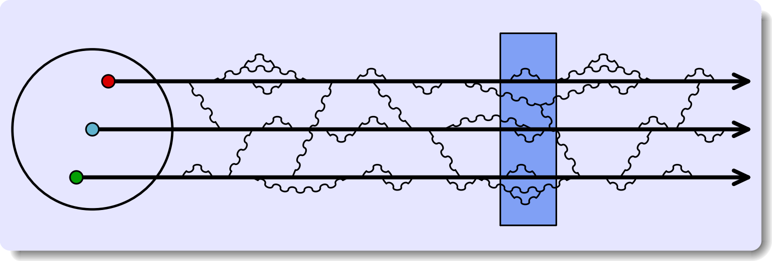
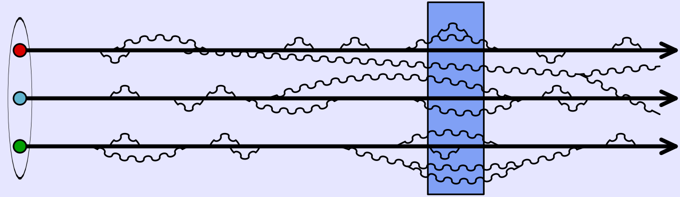
Nucleon partonic structure



At low energy:

- Fluctuations at all space-time scales smaller than its size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- Interactions are very complicated if the constituents of the nucleon have a non trivial dynamics over time-scales comparable to those of the probe

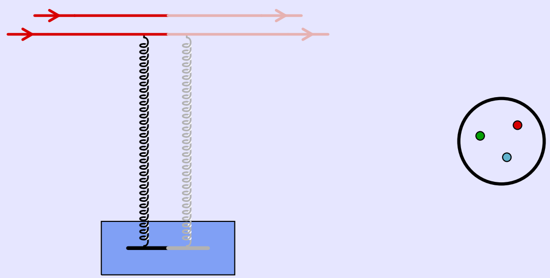
Nucleon partonic structure



At high energy:

- Dilation of all internal time-scales of the nucleon
- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  ▷ the constituents behave as if they were free
- Many fluctuations live long enough to be seen by the probe
  ▷ the nucleon appears denser at small x
- Pre-existing fluctuations are frozen over the time-scale of the probe, and act as static sources of new partons

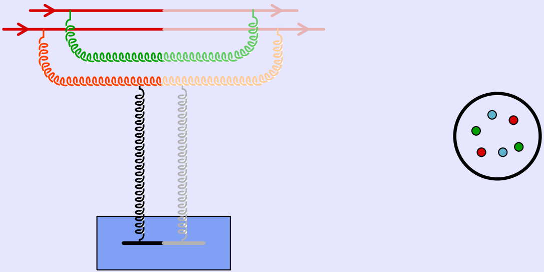
Gluon saturation



- at low energy, the probe sees mostly the valence quarks

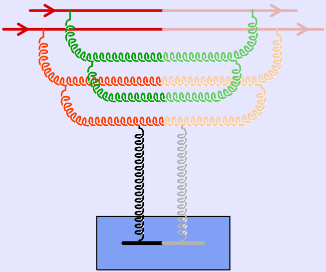
François Gelis

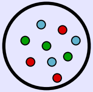
Gluon saturation



- when energy increases, new partons are emitted
- the emission probability goes like $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln(\frac{1}{x})$, with x the longitudinal momentum fraction of the gluon
- at small-x (i.e. high energy), these logs need to be resummed

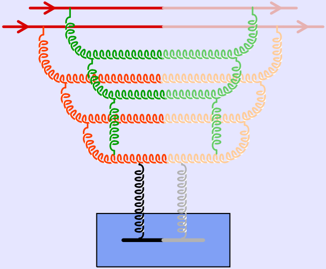
Gluon saturation

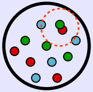




- as long as the density of constituents remains small, the evolution is linear: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

Gluon saturation





- eventually, the partons start overlapping in phase-space
- parton recombination becomes favorable
- after this point, the evolution is non-linear:
  the number of new partons depends non-linearly on the number of partons at the previous step

Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x,Q^2)}{\pi R_A^2}$$

Recombination cross-section :

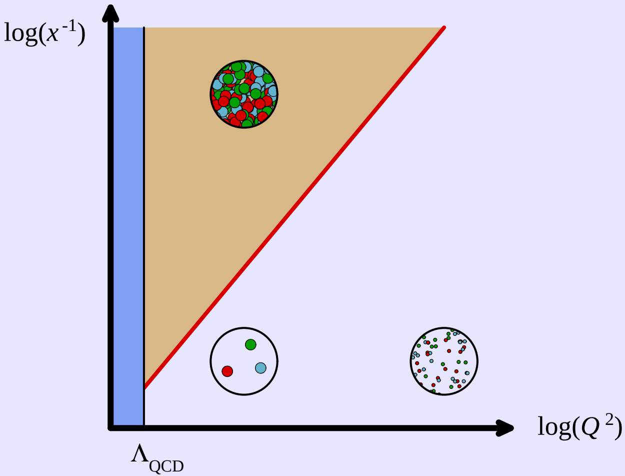
$$\sigma_{gg\to g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if $\rho\sigma_{gg\to g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x,Q_s^2)}{\pi R_A^2} \sim A^{1/3}\frac{1}{x^{0.3}}$$

Note: At a given energy, the saturation scale is larger for a nucleus (for $A = 200$, $A^{1/3} \approx 6$)

Saturation domain



Multiple scatterings

- Power counting :

$$\frac{\text{2 scatterings}}{\text{1 scattering}} \sim \frac{Q_s^2}{P_\perp^2} \quad \text{with} \quad Q_s^2 \sim \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2}$$

- When this ratio becomes ~ 1, all the rescattering corrections become important
  - ▷ one must resum all $\left[Q_s/P_\perp\right]^n$
- These effects are not accounted for in DGLAP or BFKL

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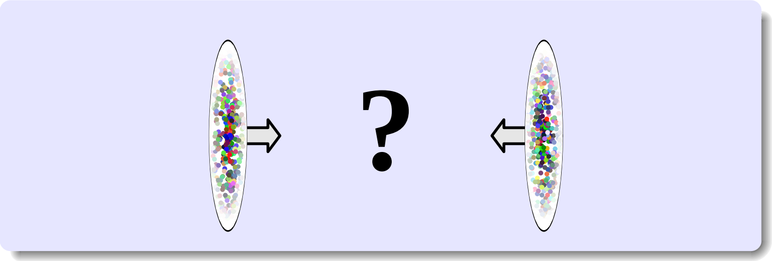
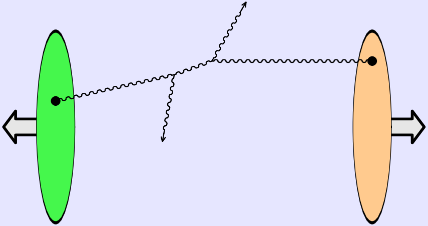
Implications for a QCD approach



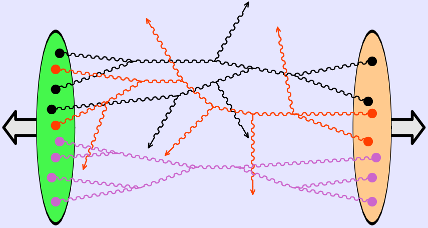
- Main difficulty: How to treat collisions involving a large number of partons?

Implications for a QCD approach



- Main difficulty: How to treat collisions involving a large number of partons?
- Dilute regime : one parton in each projectile interact (what the standard perturbative techniques are made for)

Implications for a QCD approach



- Main difficulty: How to treat collisions involving a large number of partons?
- Dense regime : multiparton processes become crucial
  ▷ new techniques are required

Color Glass Condensate: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons ($k^+ > \Lambda^+$) are frozen by time dilation
  ▷ described as static color sources on the light-cone :

$$J^\mu = \delta^{\mu +}\rho(x^-, \vec{\boldsymbol{x}}_\perp) \qquad (0 < x^- < 1/\Lambda^+)$$

- Slow partons ($k^+ < \Lambda^+$) cannot be considered static over the time-scales of the collision process
  ▷ they must be treated as standard gauge fields

  Eikonal coupling to the current J^μ : $A_\mu J^\mu$

- The color sources ρ are random, and described by a distribution functional $W_{\Lambda^+}[\rho]$, with Λ^+ the longitudinal momentum that separates "soft" and "hard"

Color Glass Condensate: RG evolution

Evolution equation (JIMWLK) :

$$\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H}\ \ W_{\Lambda^+}$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp} \frac{\delta}{\delta\alpha(\vec{\boldsymbol{y}}_\perp)} \eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp) \frac{\delta}{\delta\alpha(\vec{\boldsymbol{x}}_\perp)}$$

where $\alpha(\vec{\boldsymbol{x}}_\perp) = \frac{1}{\nabla_\perp^2} \rho(1/\Lambda^+, \vec{\boldsymbol{x}}_\perp)$

- $\eta(\vec{\boldsymbol{x}}_\perp, \vec{\boldsymbol{y}}_\perp)$ is a non-linear functional of ρ
- This evolution equation resums all the powers of $\alpha_s \ln(1/x)$ and of $Q_s/p_\perp$ that arise in loop corrections
- This equation simplifies into the BFKL equation when the source ρ is small (one can expand η in powers of ρ)

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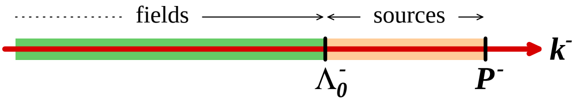
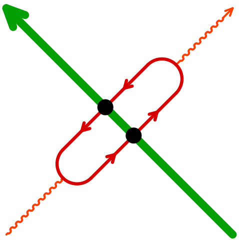
Inclusive DIS at Leading Order

- CGC effective theory with cutoff at the scale Λ_0^- :



- At Leading Order, DIS is an interaction between the target and a $q\bar{q}$ fluctuation of the virtual photon :



Inclusive DIS at Leading Order

- Forward dipole amplitude at leading order:

$$T_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp) = 1-\frac{1}{N_c}\,{\rm tr}\,\underbrace{\left(U(\vec{\boldsymbol{x}}_\perp)U^\dagger(\vec{\boldsymbol{y}}_\perp)\right)}_{\text{Wilson lines}}$$

$$U(\vec{\boldsymbol{x}}_\perp) = {\rm P}\,\exp i g\int^{1/xP^-} dz^+\,\mathcal{A}^-(z^+,\vec{\boldsymbol{x}}_\perp)$$

$$[\mathcal{D}_\mu,\mathcal{F}^{\mu\nu}] = \delta^{\nu-}\,\rho(x^+,\vec{\boldsymbol{x}}_\perp)$$

▷ at LO, the scattering amplitude on a saturated target is entirely given by classical fields

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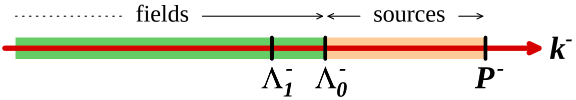
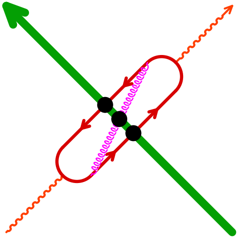
Inclusive DIS at NLO

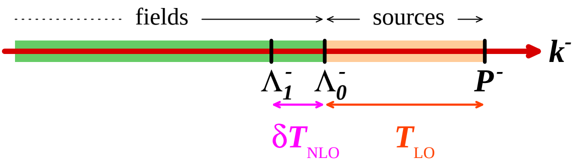
- Consider now quantum corrections to the previous result, restricted to field modes with $\Lambda_1^- < k^- < \Lambda_0^-$ (the upper bound prevents double-counting with the sources):



- At NLO, the $q\bar{q}$ dipole must be corrected by a gluon, e.g. :



Inclusive DIS at NLO



- At leading log accuracy, the contribution of the quantum modes in that strip is :

$$\delta T_{\rm NLO}(\vec{x}_\perp,\vec{y}_\perp) = \ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right)\ \mathcal{H}\ T_{\rm LO}(\vec{x}_\perp,\vec{y}_\perp)$$

$\mathcal{H}$ $=$ Hamiltonian of the JIMWLK evolution equation

- These NLO corrections can be absorbed in the LO result by a redefinition of the distribution of sources

Inclusive DIS at Leading Log

- By iterating the previous process to integrate out all the slow field modes at leading log accuracy:

Inclusive DIS at Leading Log accuracy

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \left|\psi(\boldsymbol{q}|z,\vec{\boldsymbol{r}}_\perp)\right|^2 \sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp)$$

$$\sigma_{\rm dipole}(x,\vec{\boldsymbol{r}}_\perp) \equiv 2\int d^2\vec{\boldsymbol{X}}_\perp \int [D\rho]\, W_{xP^-}[\rho]\, \boldsymbol{T}_{\rm LO}(\vec{\boldsymbol{x}}_\perp,\vec{\boldsymbol{y}}_\perp)$$

▷ the x dependence of the dipole cross-section can be predicted from the JIMWLK evolution equation

▷ one needs an initial condition at some x_0

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Geometric scaling

- In the saturated regime, the dipole cross-section depends on x and $\vec{\boldsymbol{r}}_\perp$ only through the combination

$$Q_s(x)|\vec{\boldsymbol{r}}_\perp|$$

- If one neglects the light quark masses, the photon wavefunction depends only on

$$Q|\vec{\boldsymbol{r}}_\perp|$$

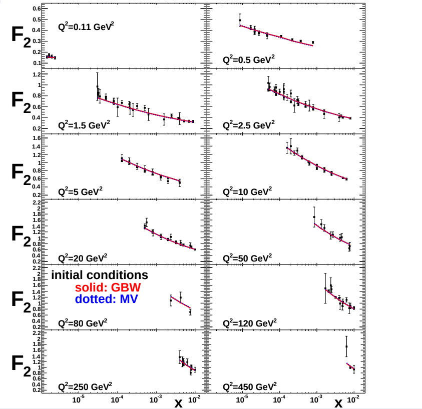
$\triangleright$ the $\gamma^* p$ cross-section depends only on

$$Q^2/Q_s^2(x)$$

DIS fit at small x based on the CGC

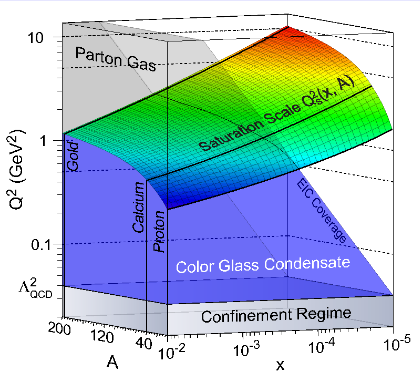
Albacete, Armesto, Milhano, Salgado (2009)

Inclusive F_2



Extraction of $Q_s(x)$

Kowalski, Lappi, Venugopalan (2007)



(figure from T. Ullrich)

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Exclusive processes

Kowalski, Motyka, Watt (2006)

- So far, we have only considered the total DIS cross-section, obtained from the forward dipole amplitude via the optical theorem
- In order to study more exclusive processes, one needs non-forward amplitudes. They read :

$$\left\langle\Omega\middle|\gamma^*\right\rangle = \int d^2\vec{\boldsymbol{r}}_\perp \int_0^1 dz\ \Psi_\Omega^*\psi\ \underbrace{\int d^2\vec{\boldsymbol{b}}\ e^{i\vec{\boldsymbol{q}}_\perp\cdot\vec{\boldsymbol{b}}}\left\langle \boldsymbol{T}\big(\vec{\boldsymbol{b}}-\frac{\vec{\boldsymbol{r}}_\perp}{2},\vec{\boldsymbol{b}}+\frac{\vec{\boldsymbol{r}}_\perp}{2}\big)\right\rangle}_{\text{non-forward dipole cross-section}}$$

Exclusive processes

- By squaring this amplitude, one gets the diffractive cross-section for the production of the state Ω with momentum transfer $\boldsymbol{q}_\perp$

$$\frac{d\sigma^{\rm diff}_{\gamma^* p\to\Omega p}}{d^2\vec{\boldsymbol{q}}_\perp} = \left|\left<\Omega\middle|\gamma^*\right>\right|^2$$

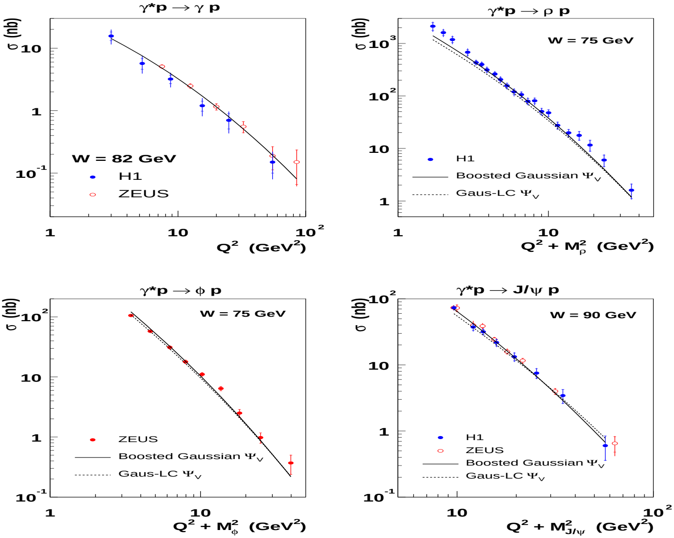
- The relationship to the inclusive DIS cross-section is

$$\sigma^{\rm tot}_{\gamma^* p}(Y, Q^2) = 2\,{\rm Im}\left<\gamma^*\middle|\gamma^*\right>_{\vec{\boldsymbol{q}}_\perp=0}$$

Note : inclusive DIS only constrains the dipole amplitude averaged over impact parameter. However, if one measures the $\boldsymbol{q}_\perp$ dependence in exclusive reactions, one obtains informations about the $\boldsymbol{b}$ dependence of the dipole amplitude

Exclusive reactions

- Exclusive photon and vector meson production :



Exclusive reactions

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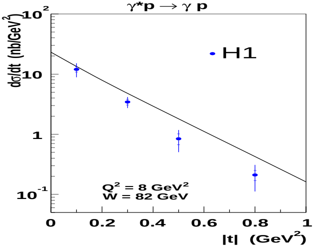
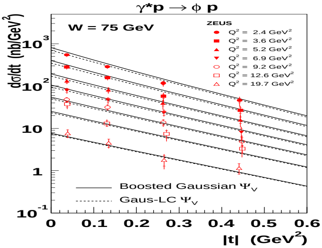
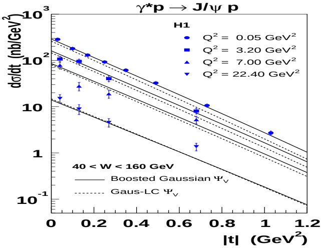
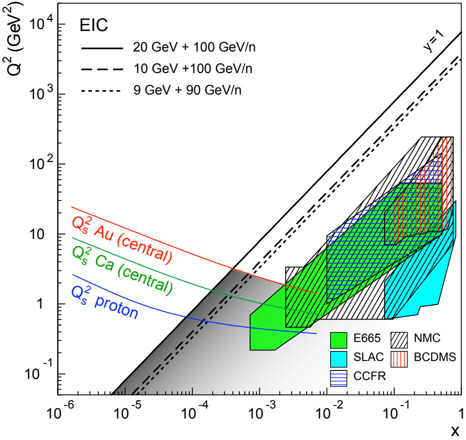




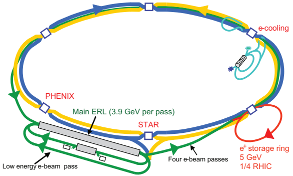


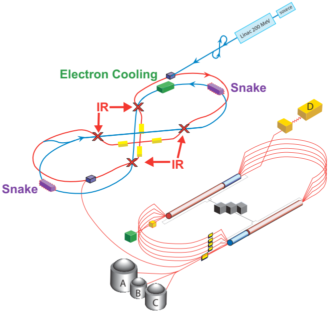
1. **Deep Inelastic Scattering**
   - Inclusive DIS
   - Experimental results (I)
   - Experimental results (II)
2. **Gluon saturation at small x**
   - Structure of a nucleon
   - Gluon evolution
   - Saturation domain
   - Multiple scatterings
   - Color Glass Condensate effective theory
3. **DIS in the CGC framework**
   - Leading Order
   - NLO and Leading Logs
   - Inclusive DIS
   - Exclusive processes
   - EIC project
4. **Connection to Nucleus-Nucleus collisions**
   - Stages of AA collisions
   - Energy-Momentum tensor
   - Correlations in rapidity

Kinematical coverage



EIC designs: BNL, JLab





Note: An EIC project is also being discussed at CERN (LHeC)

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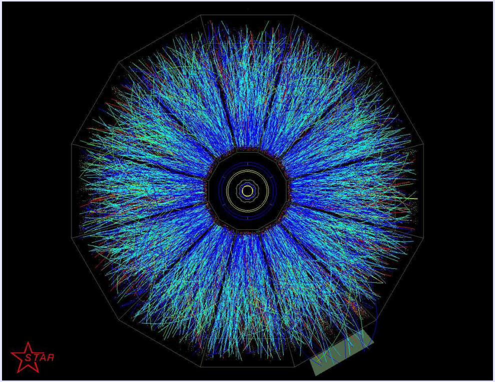
Longitudinal momentum fraction in AA collisions

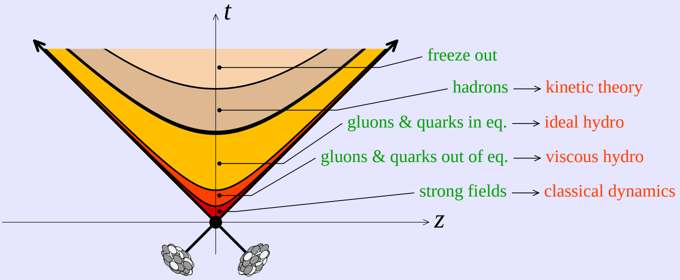
Nucleus-Nucleus collision



- 99% of the multiplicity below $p_\perp \sim 2$ GeV
- $x \sim 10^{-2}$ at RHIC ($\sqrt{s} = 200$ GeV)
- $x \sim 4.10^{-4}$ at the LHC ($\sqrt{s} = 5.5$ TeV)

▷ partons at small x are the most important

Stages of a nucleus-nucleus collision



- The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$
- Subsequent stages are usually described as fluid dynamics

Reminder on hydrodynamics

Equations of hydrodynamics = energy-momentum conservation:

$$\partial_\mu T^{\mu\nu} = 0$$

Inputs from the underlying microscopic theory :

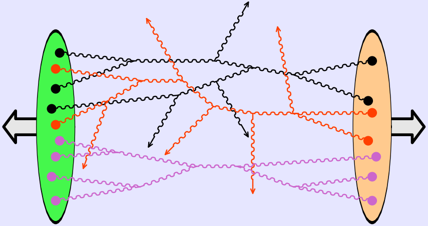
$$\text{EoS :} \quad p = f(\epsilon) \quad , \quad \text{Transport coefficients :} \quad \eta, \zeta, \cdots$$

- Required initial conditions : $T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp)$

Power counting

$$J^\mu \equiv \delta^{\mu+}\,\rho_1(x^-,\vec{\boldsymbol{x}}_\perp) + \delta^{\mu-}\,\rho_2(x^+,\vec{\boldsymbol{x}}_\perp)$$

$$\mathcal{S} = \underbrace{-\frac{1}{2}\int d^4x\ \mathrm{tr}\,F_{\mu\nu}F^{\mu\nu}}_{\text{gluon interactions}} + \int d^4x\ J^\mu A_\mu$$



Note: the dots denote insertions of the color current J^μ

François Gelis

Initial conditions from CGC: Leading Order

- Small coupling expansion for $T^{\mu\nu}$:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

- The Leading Order contribution is given by classical fields :

$$T^{\mu\nu}_{\rm LO} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu}\, \mathcal{F}^{\lambda\sigma}\mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda}\mathcal{F}^{\nu}{}_{\lambda}$$

$$\text{with} \quad \underbrace{\left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu}_{\text{Yang-Mills equation}} \quad , \quad \lim_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0$$

Initial conditions from CGC: Leading Log resummation

- The previous power counting implicitly assumes that the coefficients c_n are numbers of order one. However, large logarithms of the CGC cutoffs appear at NLO
- Like in DIS, the coefficients of the logs are given by the action of the JIMWLK Hamiltonian on the LO observable:

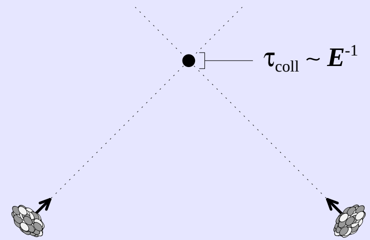
$$\delta T^{\mu\nu}_{\rm NLO} = \left[\ln\left(\frac{\Lambda_0^-}{\Lambda_1^-}\right) \mathcal{H}_1 + \ln\left(\frac{\Lambda_0^+}{\Lambda_1^+}\right) \mathcal{H}_2 \right] T^{\mu\nu}_{\rm LO}$$

- By resumming the leading logs, one obtains:

$$\left\langle T^{\mu\nu}(\tau,\eta,\vec{\boldsymbol{x}}_\perp)\right\rangle_{\rm LLog} = \int \left[D\rho_1\, D\rho_2\right]\, W_1\left[\rho_1\right]\, W_2\left[\rho_2\right]\, \underbrace{T^{\mu\nu}_{\rm LO}(\tau,\vec{\boldsymbol{x}}_\perp)}_{\text{for fixed } \rho_{1,2}}$$

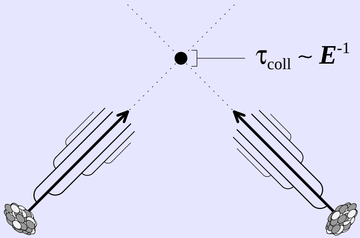
(FG, Lappi, Venugopalan (2008))

Factorization and causality



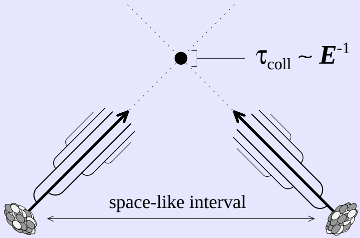
- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$

Factorization and causality



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we need to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision

Factorization and causality



- The duration of the collision is very short: $\tau_{\rm coll} \sim E^{-1}$
- The logarithms we need to resum arise from the radiation of soft gluons, which takes a long time
  ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

François Gelis

Correlations in η and $\vec{x}_\perp$

- The factorization valid for $\langle T^{\mu\nu}\rangle$ can be extended to multi-point correlations :

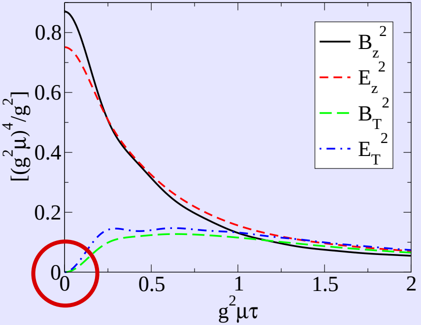
$$\left\langle T^{\mu_1\nu_1}(\tau,\eta_1,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}(\tau,\eta_n,\vec{x}_{n\perp})\right\rangle_{\rm LLog} =$$
$$= \int \big[D\rho_1\, D\rho_2\big]\; W_1\big[\rho_1\big]\; W_2\big[\rho_2\big]$$
$$\times\; T^{\mu_1\nu_1}_{\rm LO}(\tau,\vec{x}_{1\perp})\cdots T^{\mu_n\nu_n}_{\rm LO}(\tau,\vec{x}_{n\perp})$$

$\triangleright$ at leading log accuracy, all the correlations come from the distributions $W_{1,2}[\rho_{1,2}]$ (i.e. they pre-exist in the wavefunctions of the incoming projectiles)

Initial classical fields

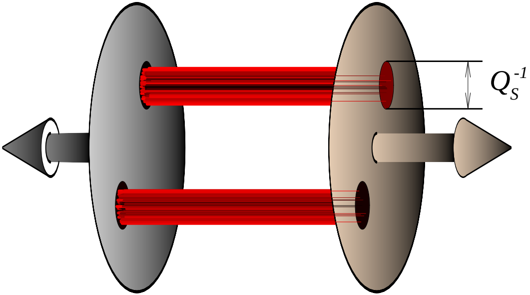
Lappi, McLerran (2006)

- Immediately after the collision, the chromo-$\vec{E}$ and $\vec{B}$ fields are purely longitudinal and boost invariant :



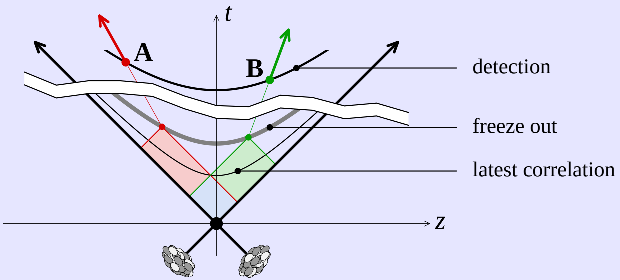
Color flux tubes

- The initial chromo-$\vec{E}$ and $\vec{B}$ fields form longitudinal "flux tubes" extending between the projectiles:



- The color correlation length in the transverse plane is Q_s^{-1}
  ▷ flux tubes of diameter Q_s^{-1}, filling up the transverse area
- The correlation length in the η direction is $\Delta\eta \sim \alpha_s^{-1}$
  ▷ long range rapidity correlations expected in the data

Importance of initial rapidity correlations

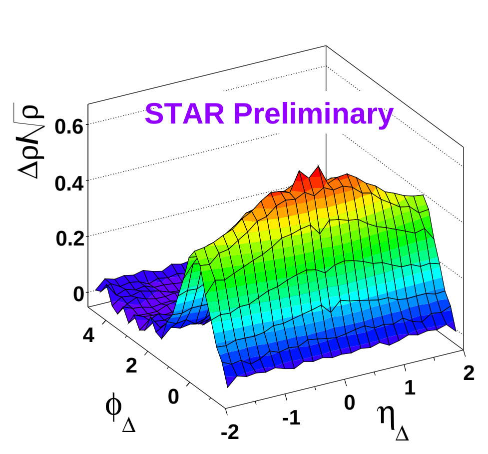


Long range rapidity correlations must be created early

$$t_{\text{correlation}} \le t_{\text{freeze out}}\ e^{-\frac{1}{2}|\eta_A - \eta_B|}$$

▷ the near η-independence of the initial color fields should induce a long range correlation in rapidity among the produced particles

2-hadron correlations at RHIC



- Long range correlation in $\Delta\eta$ (rapidity)
- Narrow correlation in $\Delta\varphi$ (azimuthal angle)

Summary

- The dynamics of gluons at small x is altered by high density effects $\triangleright$ saturation
- At a given energy, gluon saturation is more important for nuclei
- Saturation plays an important role in DIS at small x and in the description of nucleus-nucleus collisions
- A factorization theorem relates DIS and AA collisions in the saturated regime
- Design goals of an eA collider for saturation studies :
  - Energy comparable to that of HERA
  - Much higher luminosity than HERA
  - Variable $\sqrt{s}$ for direct measurement of F_L
  - Detector with good η coverage