CONFINEMENT FROM CORRELATION FUNCTIONS

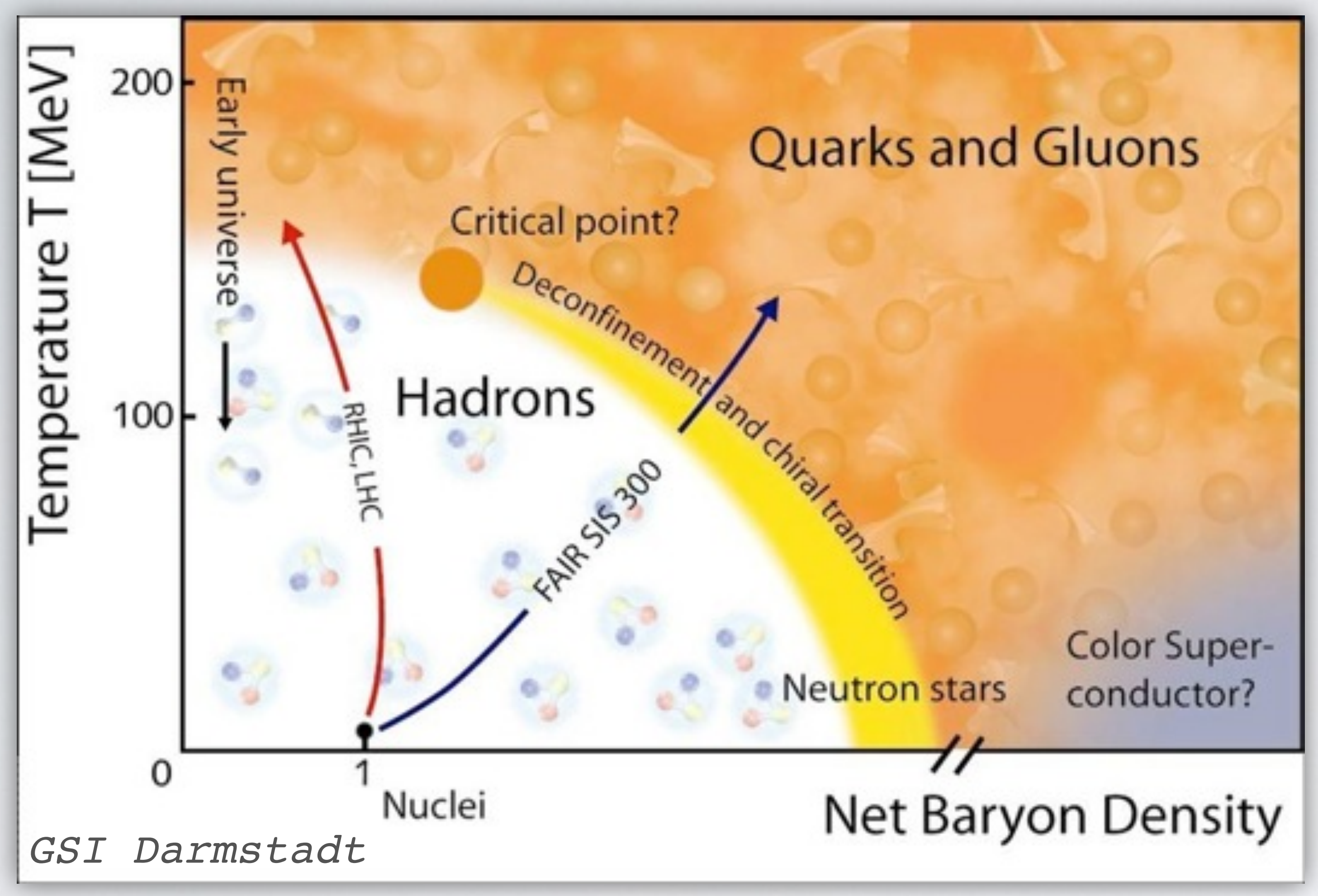
Leonard Fister
IPhT, CEA Saclay

LF, Pawlowski, Phys. Rev. D88 (2013).

Fischer, LF, Luecker, Pawlowski, 1306.6022.

heavy-ion meeting,
IPN Orsay, February 20, 2014

Motivation: QCD Phase Diagram



characteristic features at low energies

- confinement
- dynamical chiral symmetry breaking

non-perturbative computation of physical observables from microscopic dynamics

here: study aspects of the phase diagram with non-perturbative **functional continuum methods**

quark confinement via the Polyakov loop potential

phase transition order / temperature / density,
confinement criterion via (infrared) behaviour of propagators

OUTLOOK

- Yang–Mills Theory
- Non-Perturbative Functional Methods
- Quark Confinement

(PURE) YANG–MILLS THEORY

(field-strength tensor)²

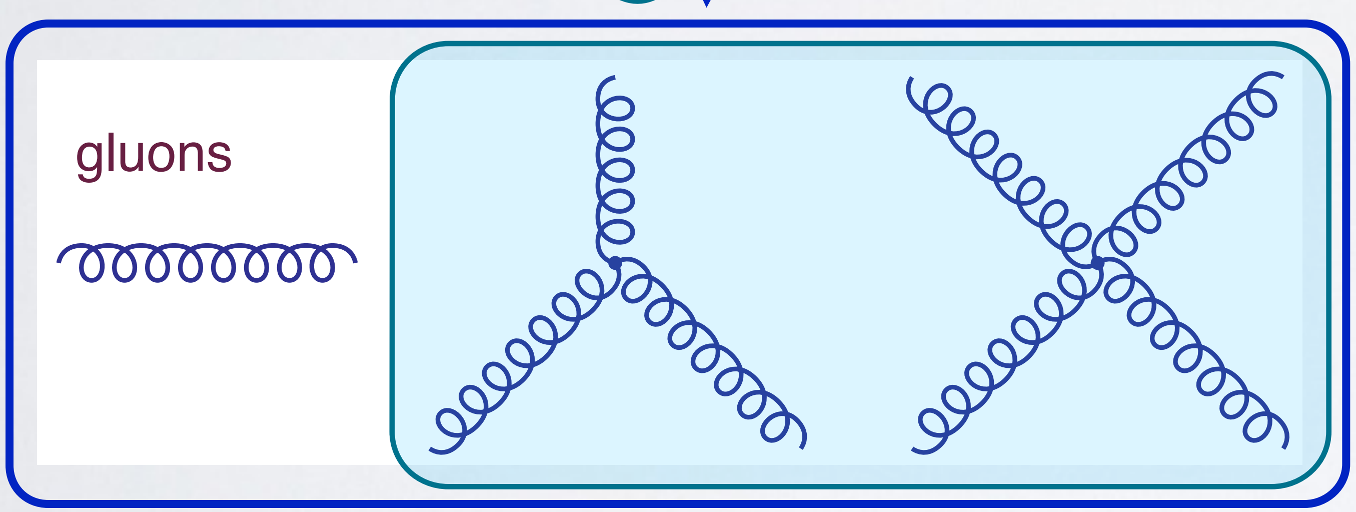
gauge-fixing

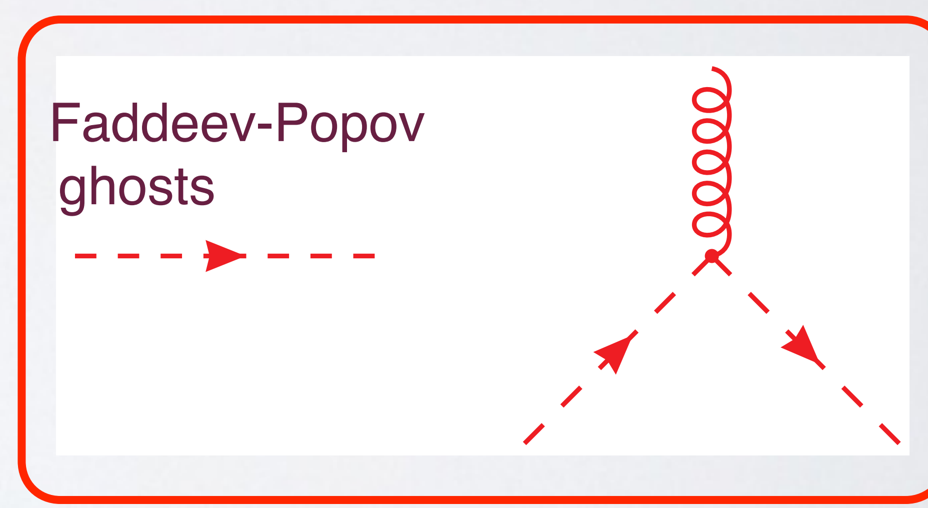
Faddeev–Popov determ.

action: $$S_{\mathrm{YM}} = \int_x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{\xi} \cancel{(\partial_\mu A^a_\mu)}^2 + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

Landau gauge $\xi \to 0$

non-Abelian





FUNCTIONAL METHODS

functional renormalisation group, Dyson–Schwinger equations, n-PI functionals, ...

pros

- continuum methods
- exact description via correlation functions

  (capture non-perturbative effects)
- no (conceptual) problem with fermions

  (chiral symmetry, no sign problem, ...)

links to
- confinement
- $D\chi SB$
- hadron phenomenology

… complementary to **lattice QCD**
- discretised space-time
- no approximations
- fermions are difficult
- pure gauge theory easy

cons

- approximations are inevitable
- pure gauge sector of QCD is difficult

FUNCTIONAL METHODS

? compute the **Gibbs free energy** Γ (= effective action)

$$\Gamma[\phi] = \sup_J \left(\int_x J \cdot \phi - \log \left\{ \int \mathcal{D}\varphi \, e^{-S[\varphi] + \int_x J \cdot \varphi} \right\} \right)$$

classical action, microscopic dynamics

integrate fluctuations

integrate **all** fluctuations **at once**

Dyson–Schwinger eqs. (DSEs)

$$\boxed{\int \mathcal{D}\phi \frac{\delta}{\delta\phi} e^{-S[\phi]+J\cdot\phi} = 0}$$

Dyson, Phys. Rev. 75, 1736 (1949).
Schwinger, Proc. Nat. Acad. Sci. 37 (1951).

integrate fluctuations **momentum-shell-wise**

functional renormalisation group

Wetterich, Phys. Lett. B301 (1993).

FUNCTIONAL RENORMALISATION GROUP (FRG)

UV: classical action S

↓

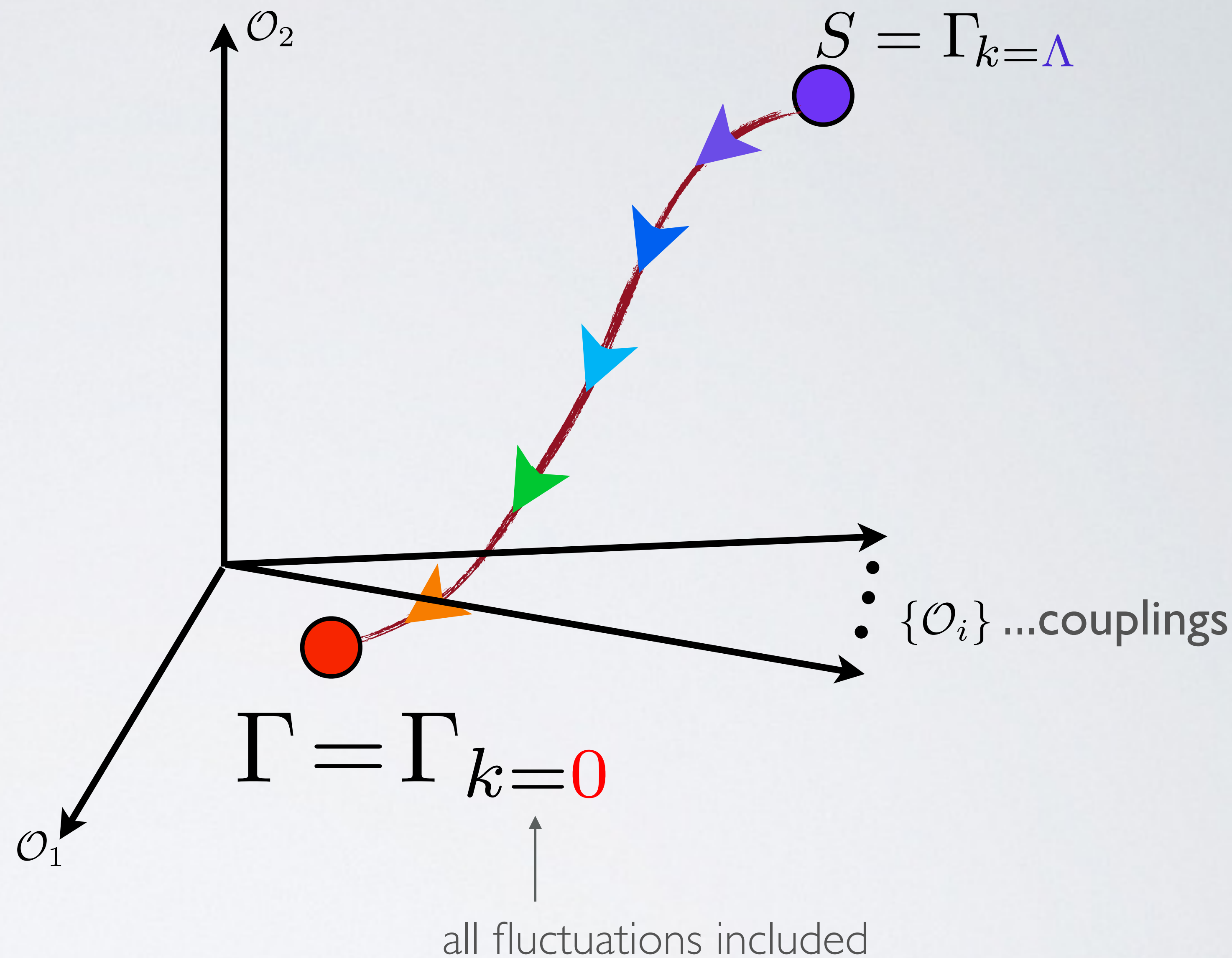
introduce energy scale k,
fluctuations $p^2 \sim k^2$

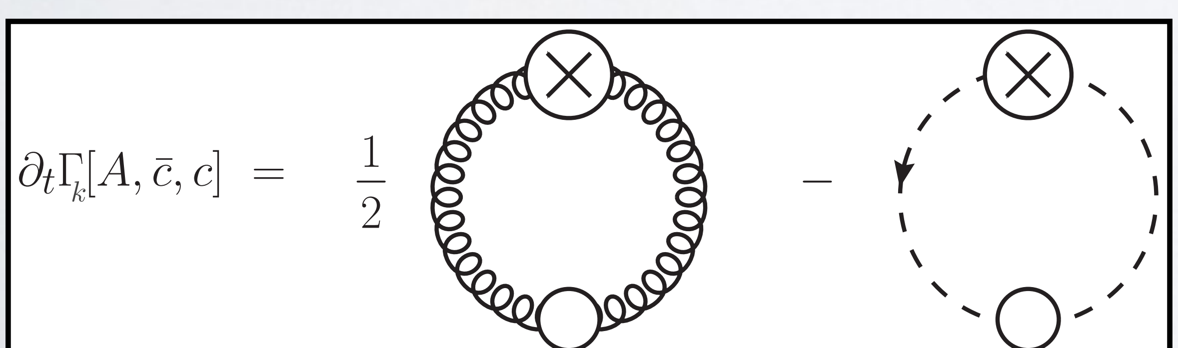
↓

from $k = \Lambda$ to $k = 0$

↓

effective action Γ

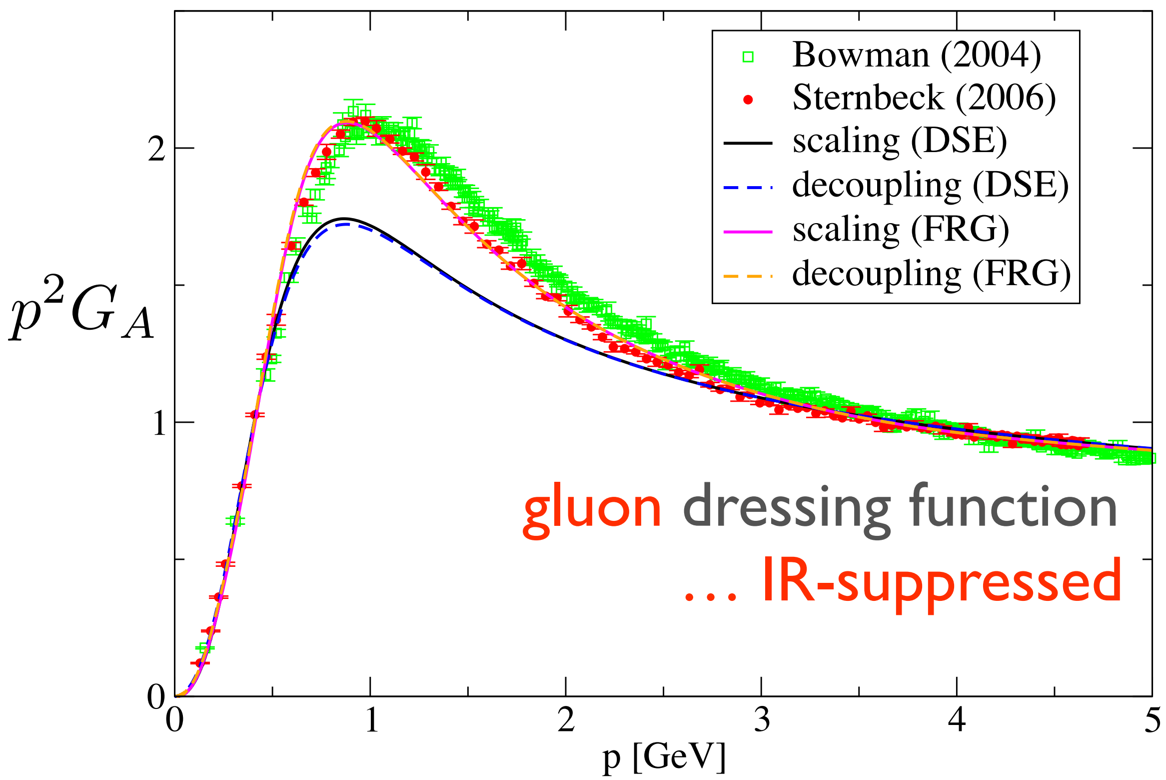


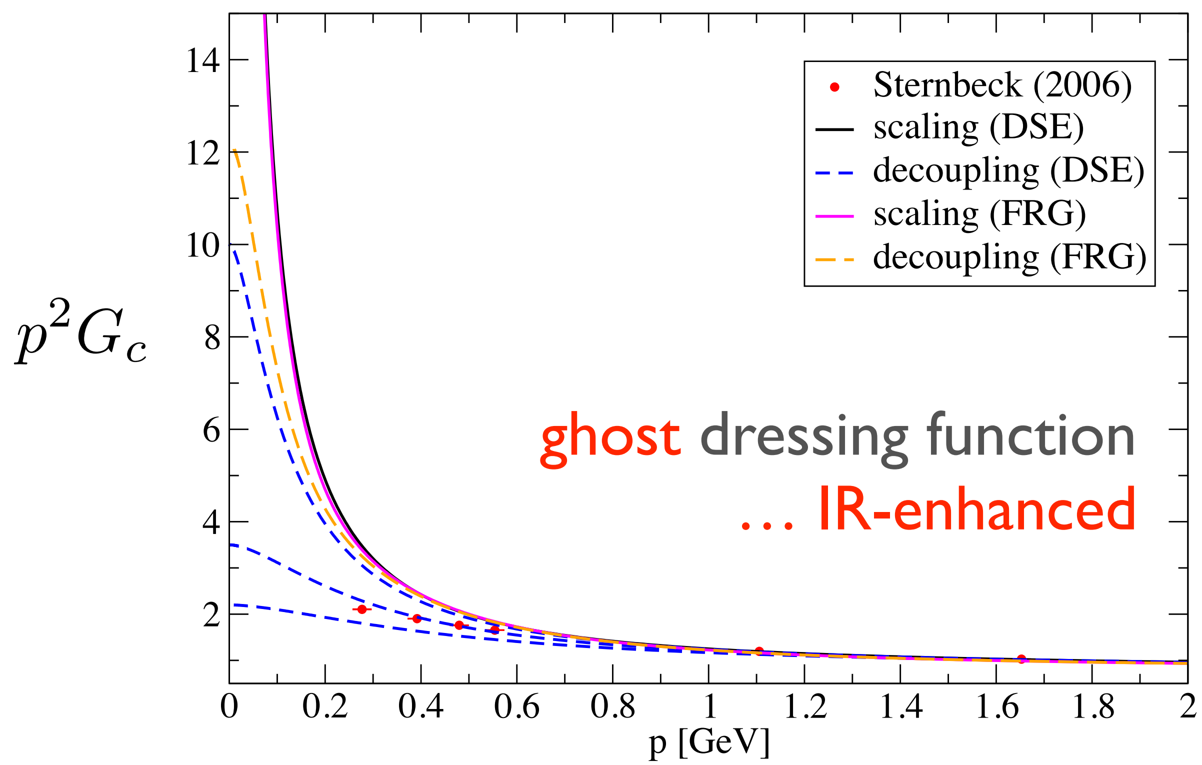


FRG equation depends on **full propagators**

YANG–MILLS PROPAGATORS

$G_{A/c}$ ... gluon / ghost propagator





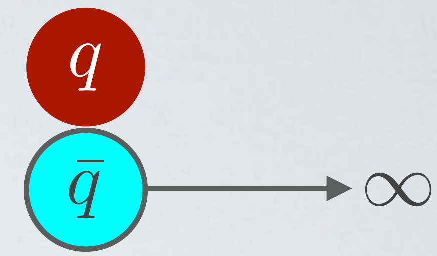
figures: Fischer, Maas, Pawlowski, Annals Phys. 324 (2009).

Quark Confinement

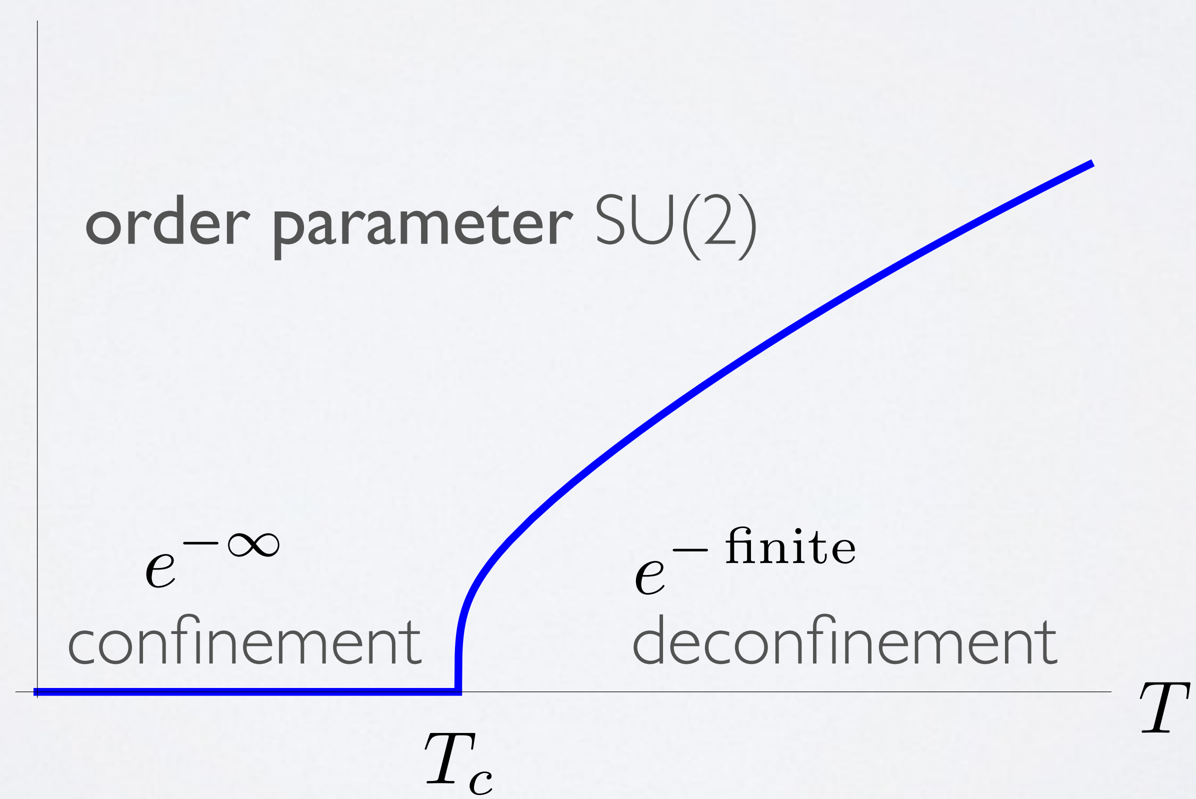
ORDER PARAMETER FOR CONFINEMENT

The expectation value of the **Polyakov loop**, $\langle L[A_0] \rangle$, relates to the free energy, F_q, of a single quark.

→ **order parameter** for static quark confinement



$$e^{-F_q/T} \begin{cases} = 0 \;\ldots\, \text{confinement} \\ > 0 \;\ldots\, \text{deconfinement} \end{cases}$$



ORDER PARAMETER FOR CONFINEMENT

$$L\left[\left\langle A_0\right\rangle\right] \sim \mathcal{P}\, e^{ig\int_0^{1/T} dt\, \langle A_0\rangle}$$

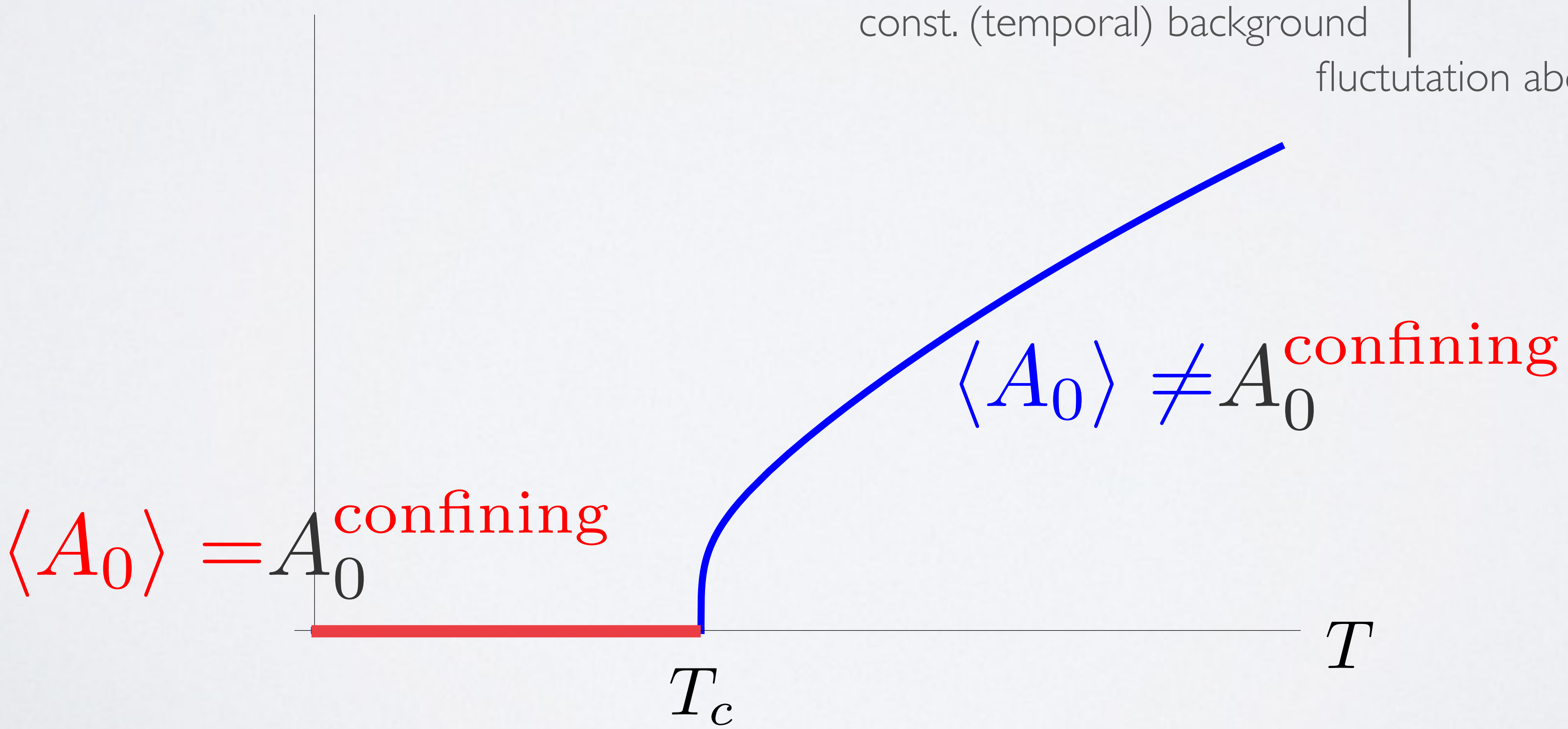
Braun, Gies, Pawlowski, Phys. Lett. B684 (2010).
Marhauser, Pawlowski, 0812.1144.

$$L\left[\langle A_0\rangle = A_0^{\text{confining}}\right] = 0$$

$\langle A_0\rangle$ minimum of **effective potential** $V\left[A_0\right] \sim \Gamma\left[A_0;0\right]$

const. (temporal) background

fluctutation about background



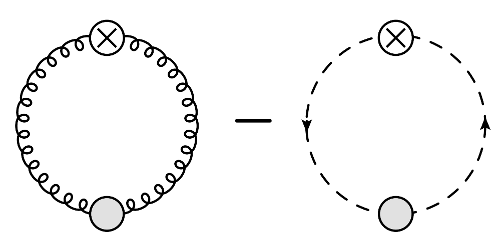
ORDER PARAMETER FOR CONFINEMENT

'Polyakov loop' potential: $V[A_0] \sim \Gamma[A_0; 0]$

Confinement, if **minima of** $V[A_0]$ at confining values.

FRG:

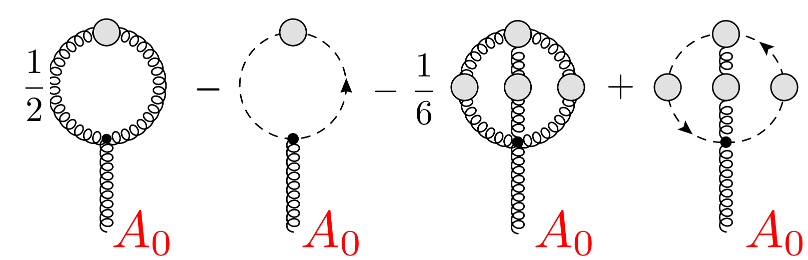
$$k\partial_k \Gamma_k [A_0; a, \bar{c}, c] = \frac{1}{2}$$



DSEs:

$$\int \mathcal{D}\phi \frac{\delta}{\delta\phi} e^{-S[\phi]+J\cdot\phi} = 0$$

$$\frac{\delta(\Gamma - S)}{\delta A_0} =$$

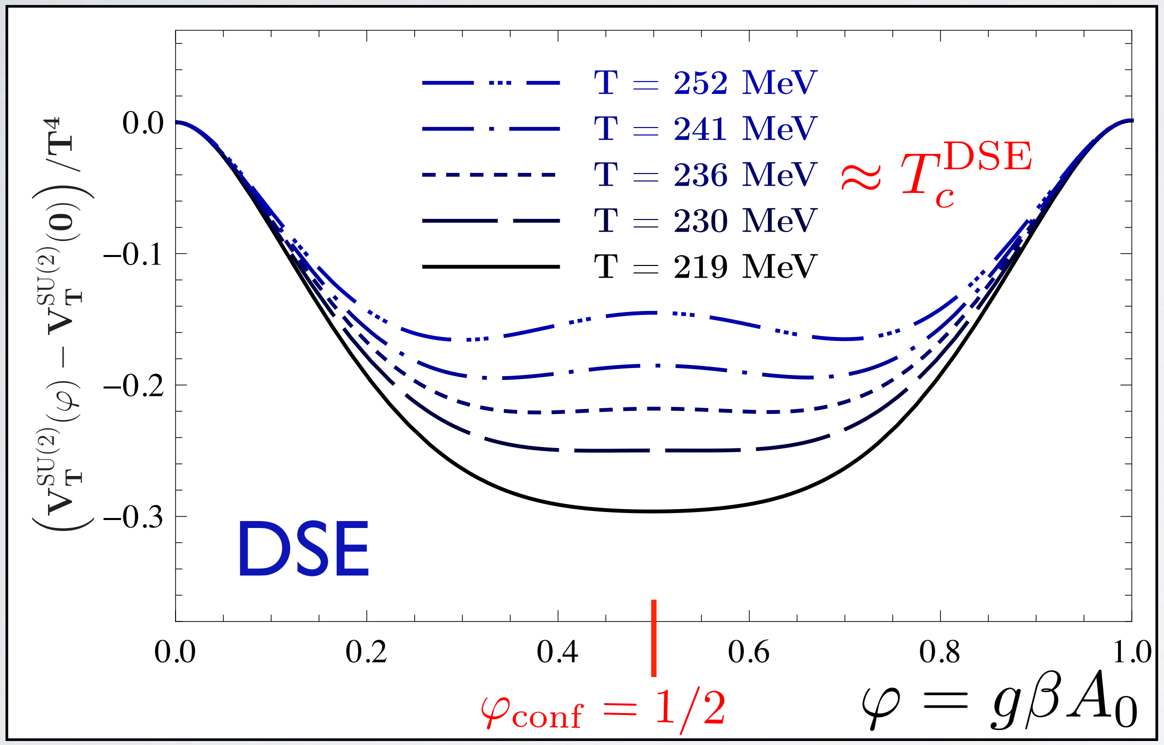


FRG: Braun, Gies, Pawlowski, Phys. Lett. B684 (2010).
Braun, Eichhorn, Gies, Pawlowski, Eur. Phys. J. C70 (2010).

FRG, DSE, 2PI: LF, Pawlowski, Phys. Rev. D88 (2013).
Fischer, LF, Luecker, Pawlowski, 1306.6022.

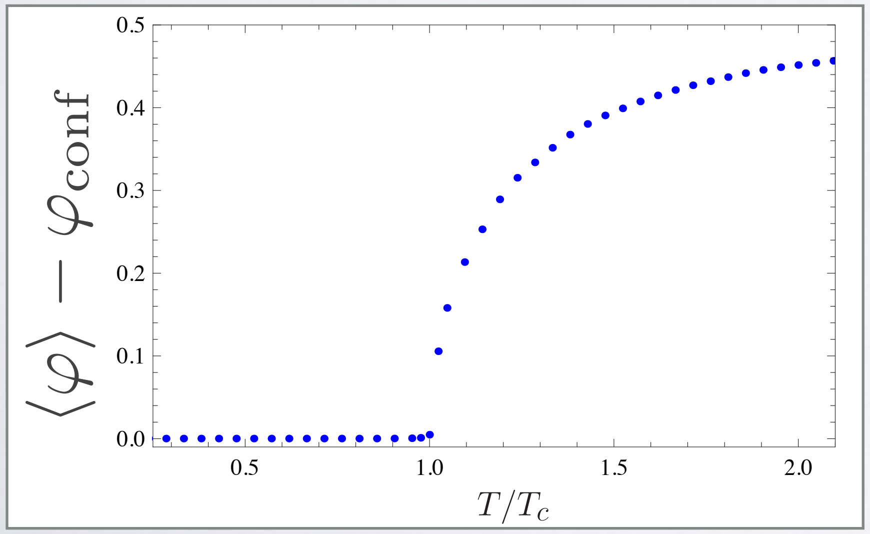
POLYAKOV POTENTIAL - SU(2) PURE YANG–MILLS

LF, Pawlowski, Phys. Rev. D88 (2013).



$$T_c^{\mathrm{DSE/FRG}}/\sqrt{\sigma} \approx .55$$
$$T_c^{\mathrm{lattice}}/\sqrt{\sigma} \approx .71$$

lattice: Lucini, Teper, Wenger, JHEP 01 (2004).



minimum moves smoothly

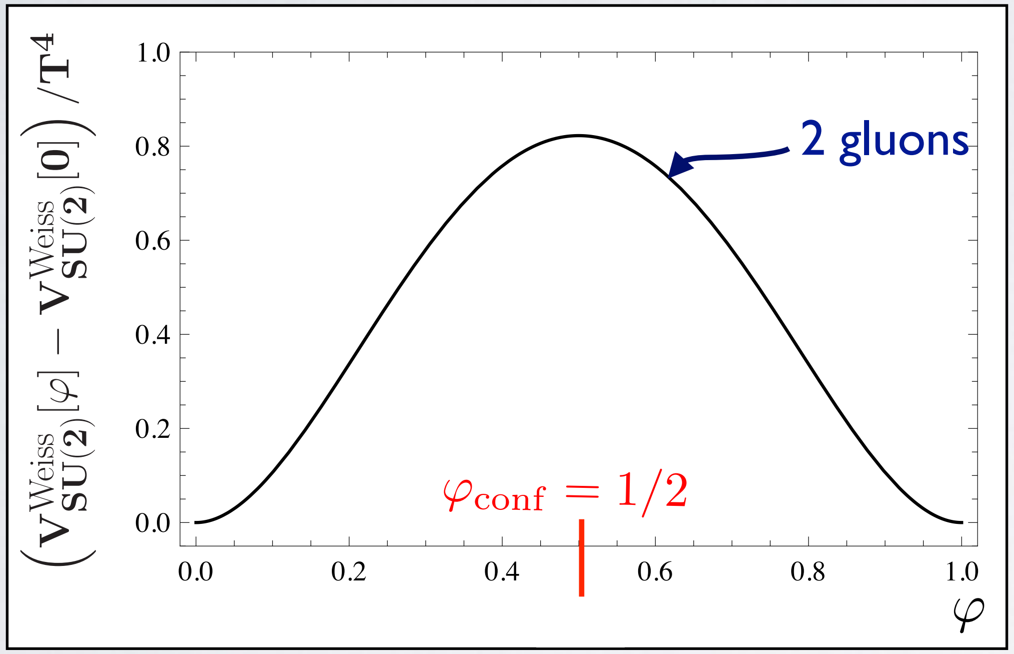
$\longleftrightarrow$ **second order** phase transition for SU(**2**)

CONFINEMENT CRITERION

LF, Pawlowski, Phys. Rev. D88 (2013).

perturbation theory → **Weiss potential**

Weiss '1981. Gross, Pisarski, Yaffe '1981.



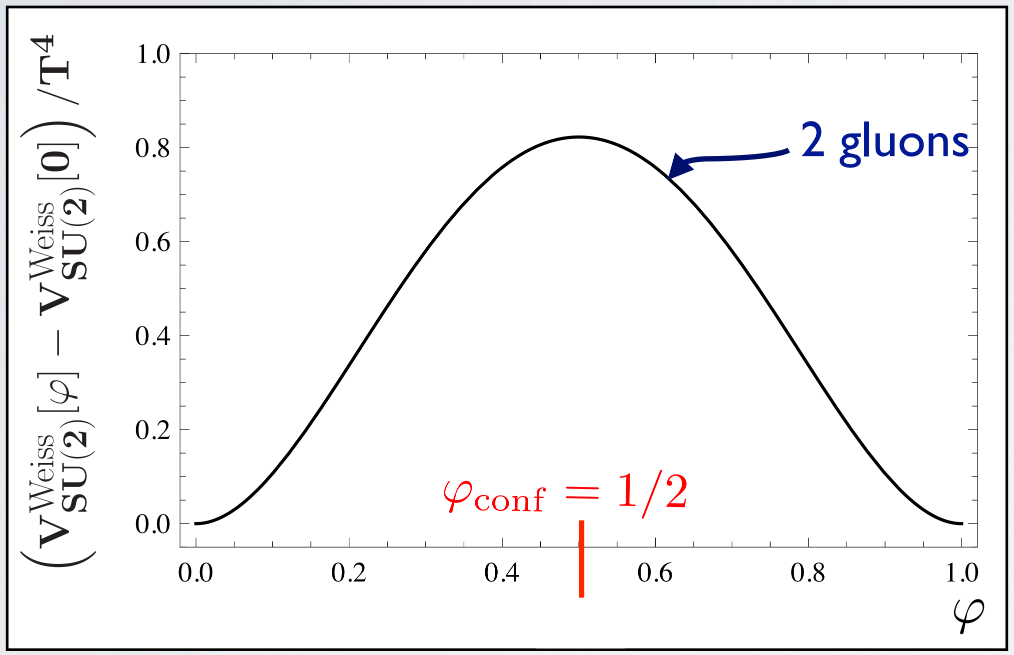
ghost confine, gluons deconfine,
two (transversal) gluonic modes remain,
others cancel exactly

→ **no confinement in perturbation theory**

CONFINEMENT CRITERION

perturbation theory → **Weiss potential**

Weiss '1981. Gross, Pisarski, Yaffe '1981.

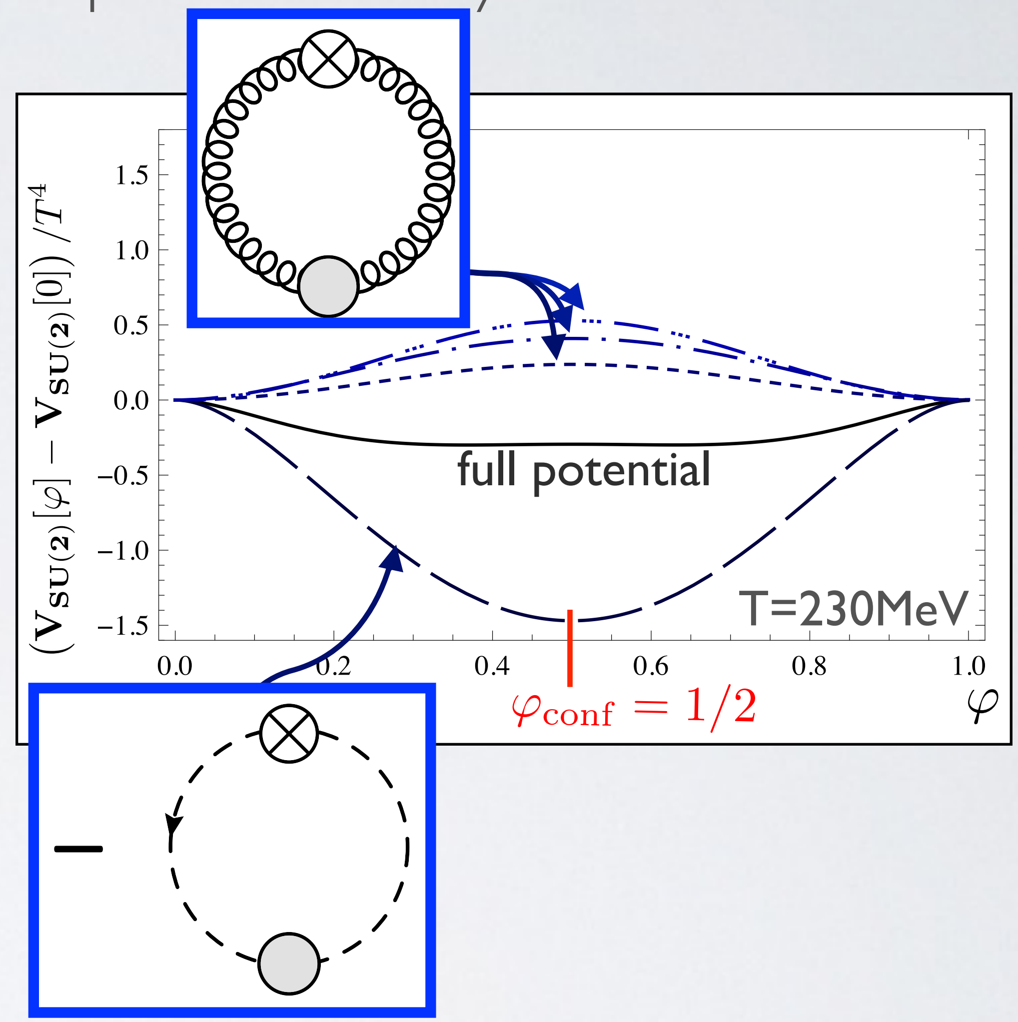


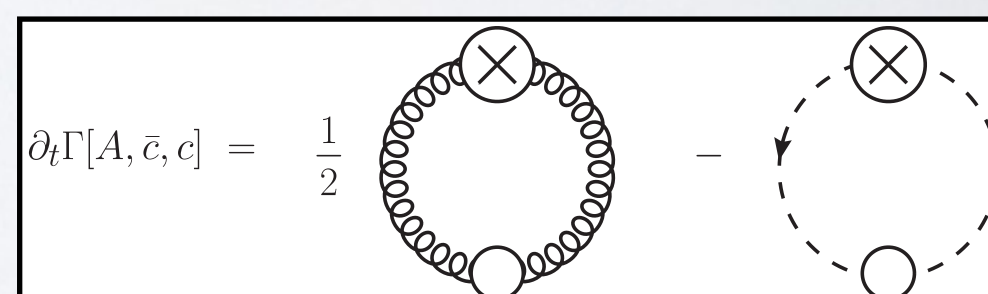
ghost confine, gluons deconfine,
two (transversal) gluonic modes remain,
others cancel exactly

→ **no confinement in perturbation theory**

LF, Pawlowski, Phys. Rev. D88 (2013).

non-perturbatively

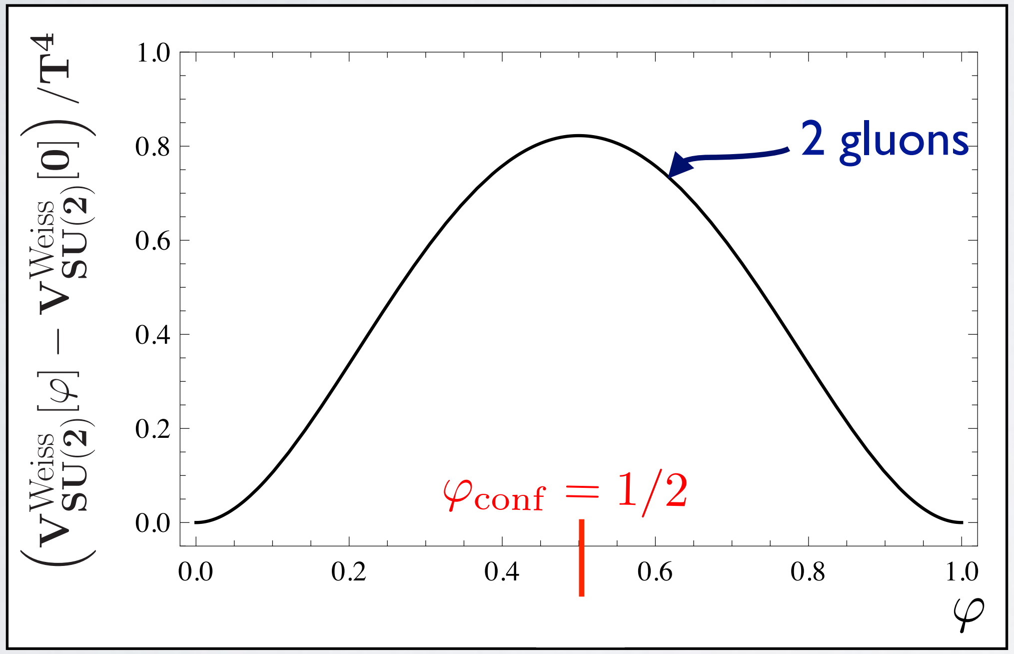




CONFINEMENT CRITERION

perturbation theory → **Weiss potential**

Weiss '1981. Gross, Pisarski, Yaffe '1981.

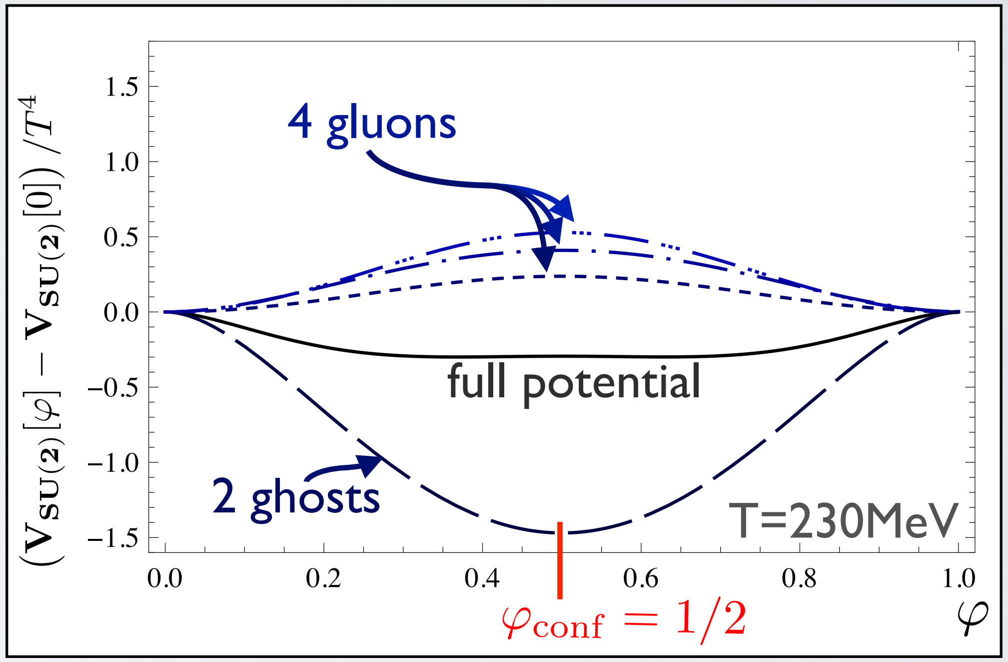


ghost confine, gluons deconfine,
two (transversal) gluonic modes remain,
others cancel exactly

→ **no confinement in perturbation theory**

LF, Pawlowski, Phys. Rev. D88 (2013).

non-perturbatively



gluon suppression, ghost enhancement

no exact cancellation of modes,
ghosts dominate at small temperatures

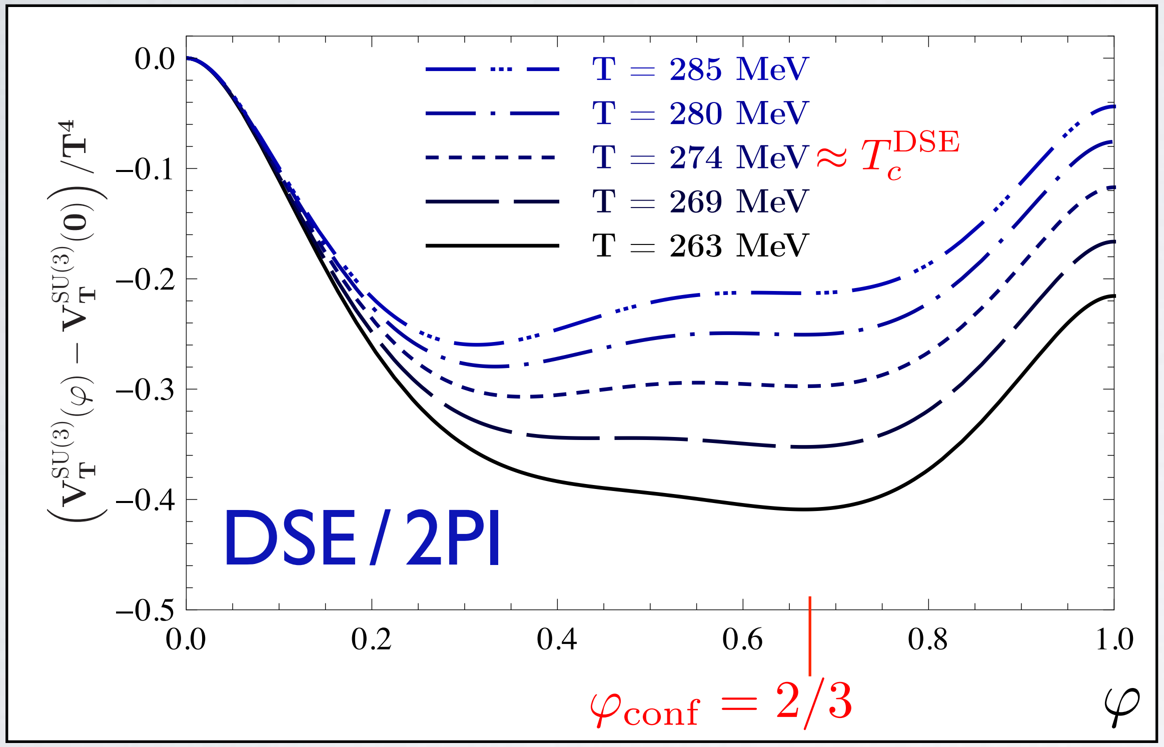
→ **confinement**

infrared suppressed gluons but non-suppressed ghosts → confinement

… applicable to Higgs–YM, adjoint quark-YM (, QCD)

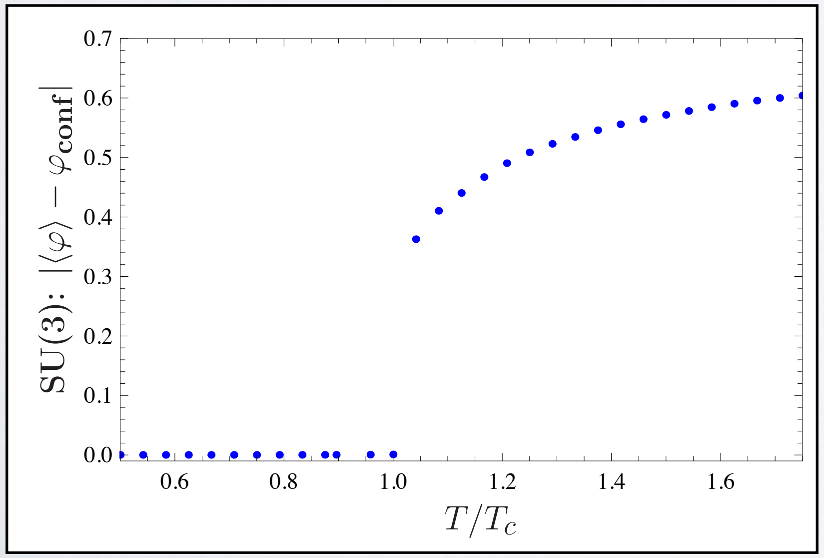
POLYAKOV POTENTIAL - SU(3) PURE YANG–MILLS

LF, Pawlowski, Phys. Rev. D88 (2013).



$$T_c^{\mathrm{DSE/FRG}}/\sqrt{\sigma} \approx .65$$
$$T_c^{\mathrm{lattice}}/\sqrt{\sigma} \approx .643$$

lattice: Lucini, Teper, Wenger, JHEP 01 (2004).



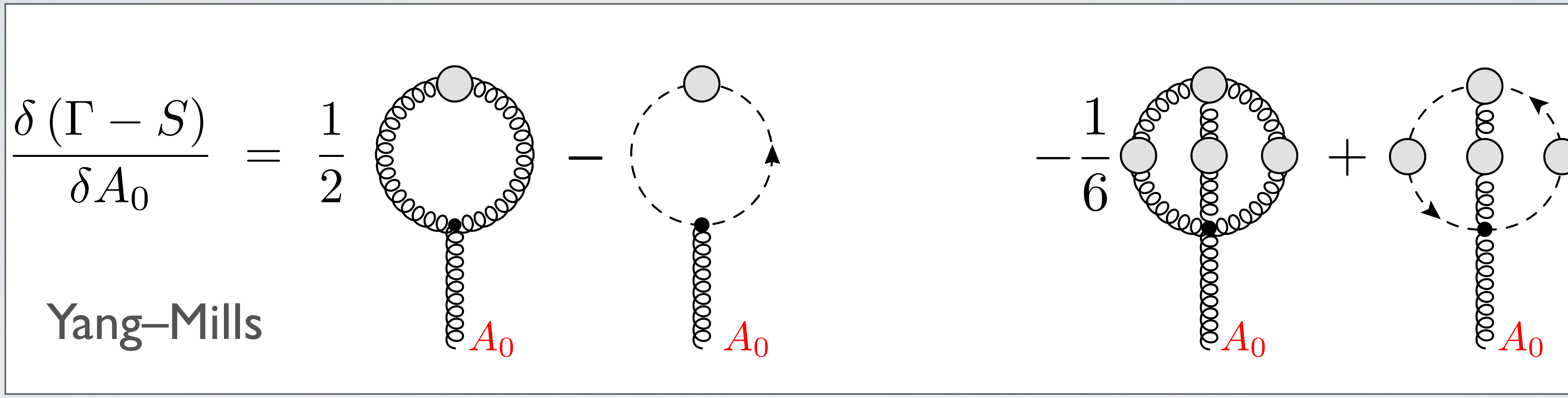
minimum jumps

$\longleftrightarrow$ **first order** phase transition for SU(**3**)

POLYAKOV POTENTIAL - QCD

`Fischer, LF, Luecker, Pawlowski, 1306.6022.`

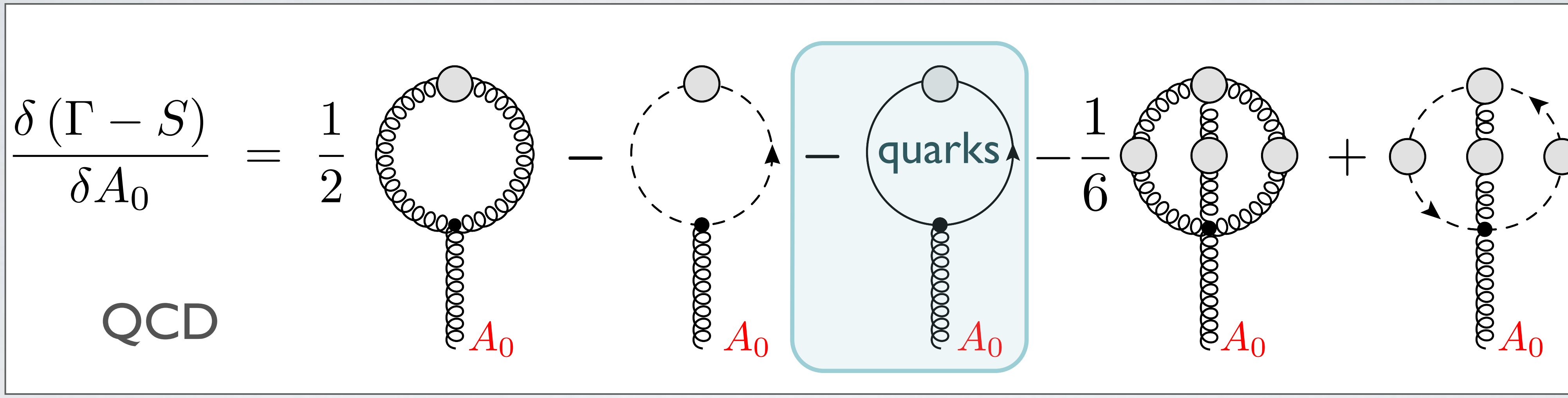
DSE

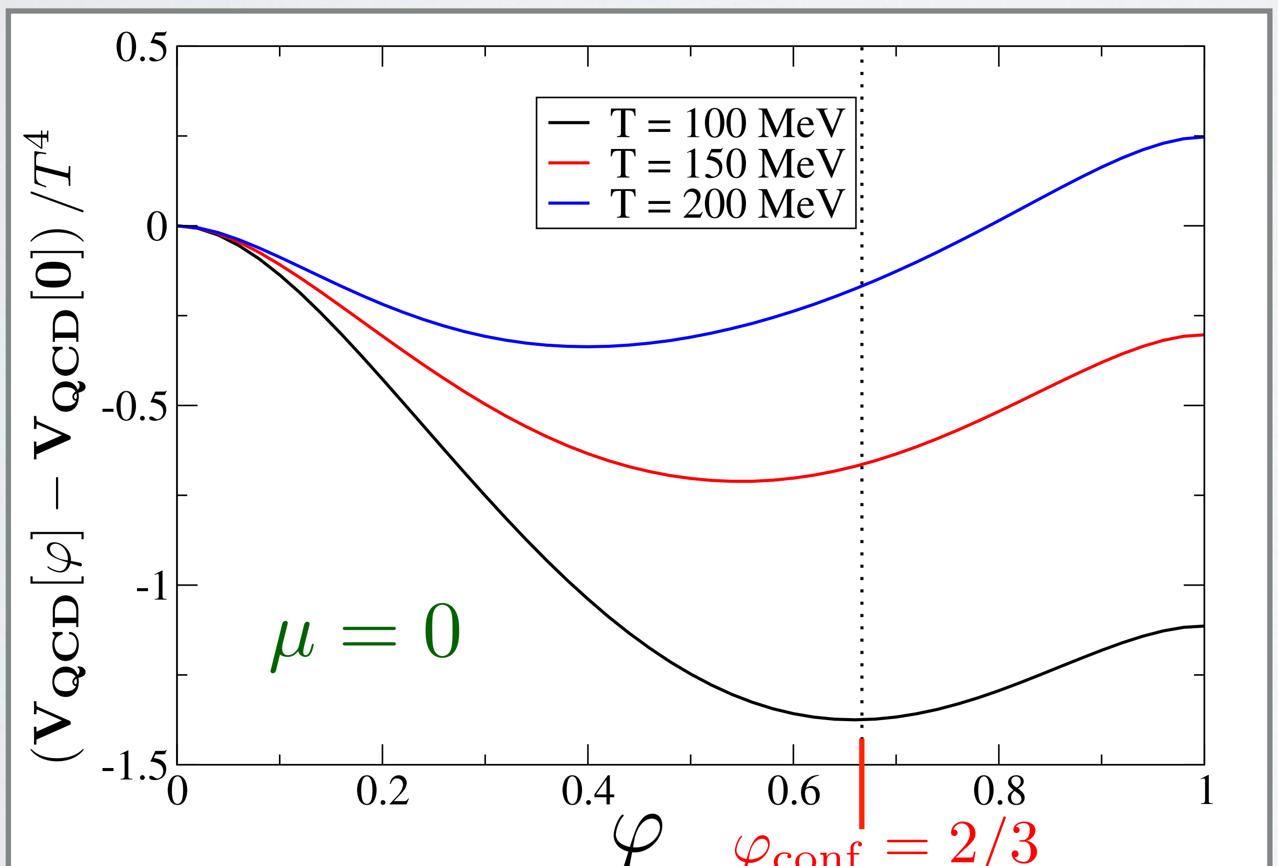


POLYAKOV POTENTIAL - QCD

Fischer, LF, Luecker, Pawlowski, 1306.6022.

DSE



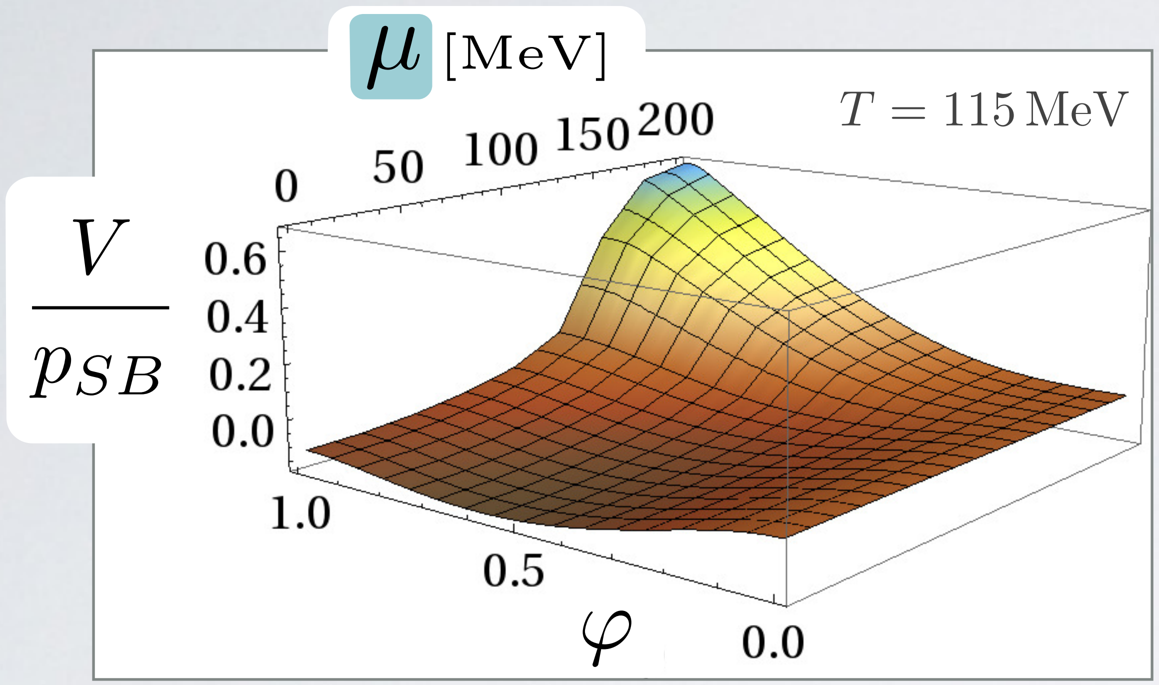


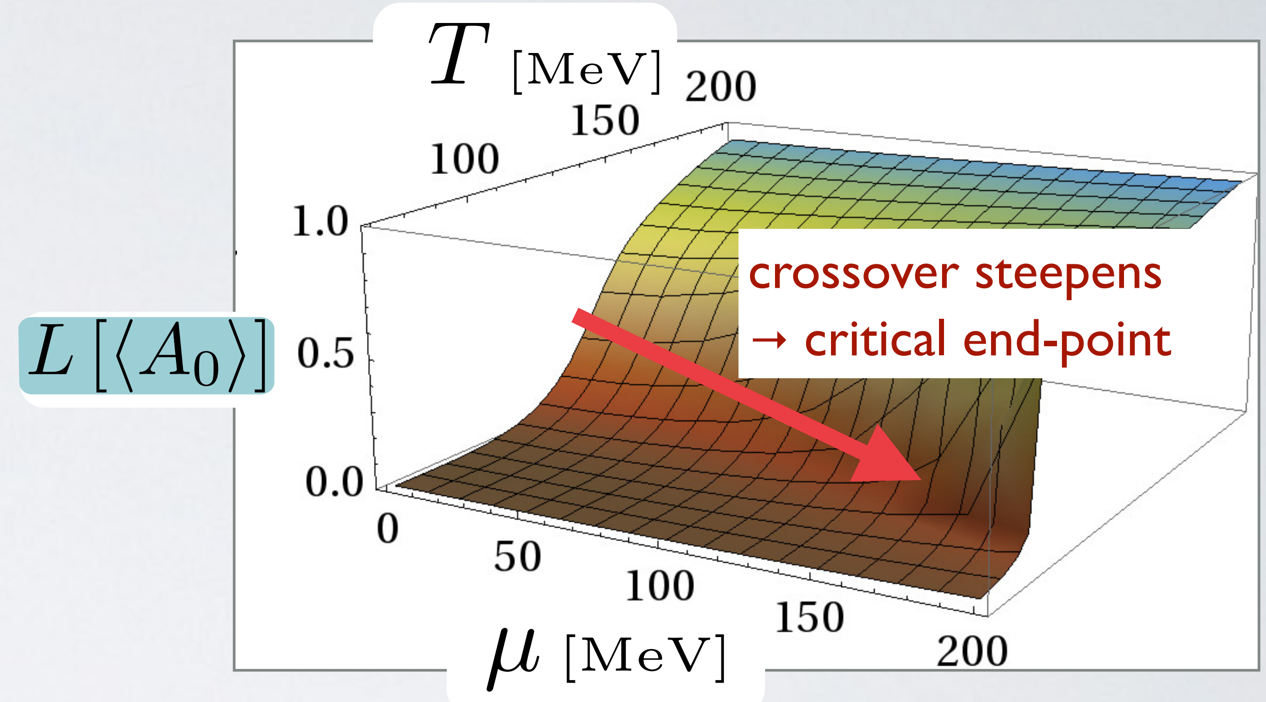
… smooth crossover transition
(quarks break centre symmetry)

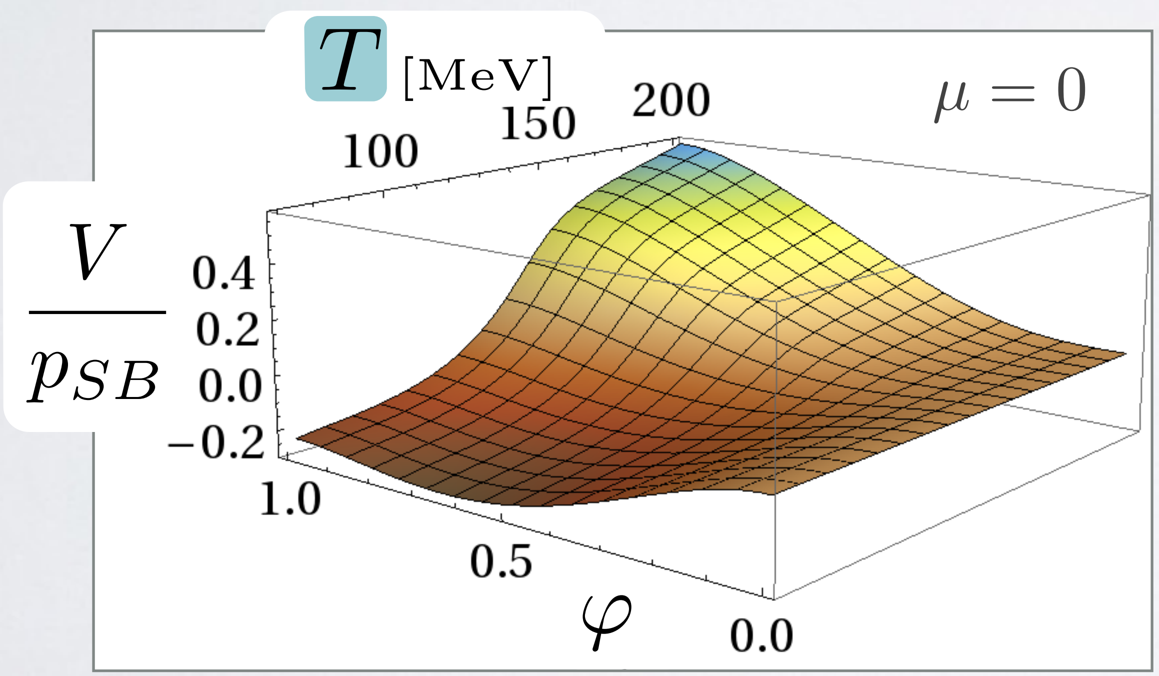
… quark chemical potential μ

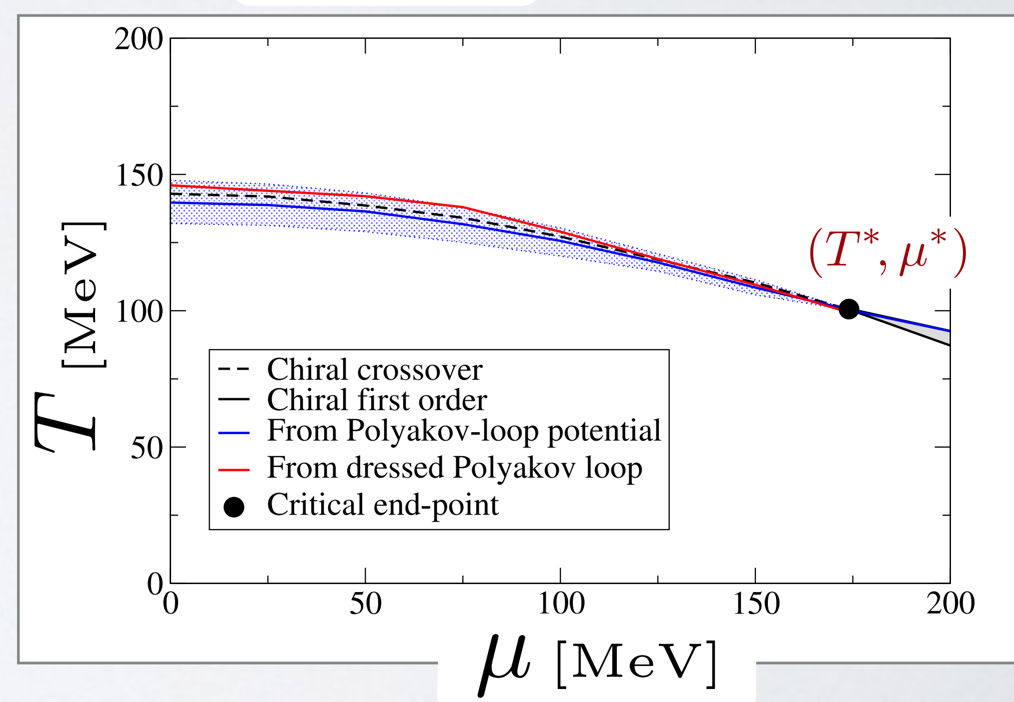
POLYAKOV POTENTIAL - QCD $N_f = 2+1$

Fischer, LF, Luecker, Pawlowski, 1306.6022.









$$T^* = 101\,\mathrm{MeV}$$
$$\mu^* = 174\,\mathrm{MeV}$$

CONCLUSIONS

Functional methods allow to study the confinement (quantitatively) via the **Polyakov loop potential** in terms of propagators.

- 2nd/1st order phase transition for SU(2)/SU(3), correct QCD pattern
- Criterion for confinement: gluons must be IR suppressed, while ghosts must *not.*
- critical temperatures for pure Yang–Mills $\frac{T_c^{\mathrm{SU}(2)|\mathrm{SU}(3)}}{\sqrt{\sigma}} \approx \begin{cases} .55 \mid .65 & \text{functional methods} \\ .709 \mid .646 & \text{lattice gauge theory} \end{cases}$
- QCD, $N_f = 2+1$, critical end-point at $(T^*,\, \mu^*) = (101,\, 174)$ MeV