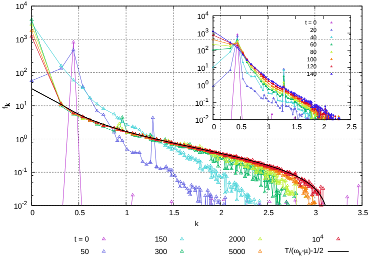
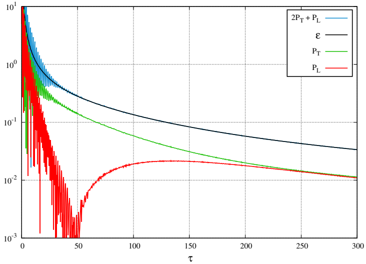
A First Step Towards Understanding Izotropization in HIC





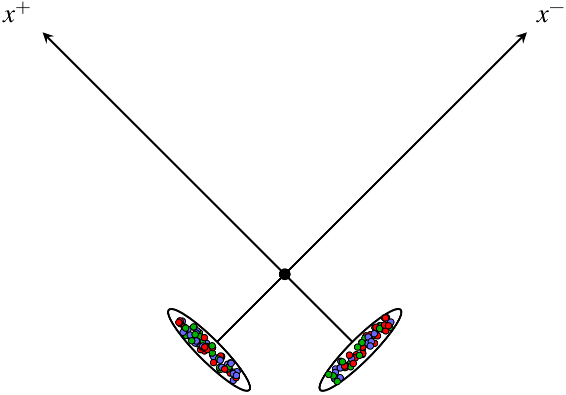
Orsay, 26 October 2012

Thomas EPELBAUM

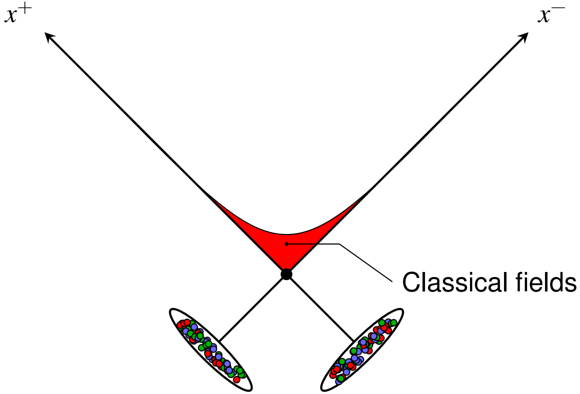
IPhT

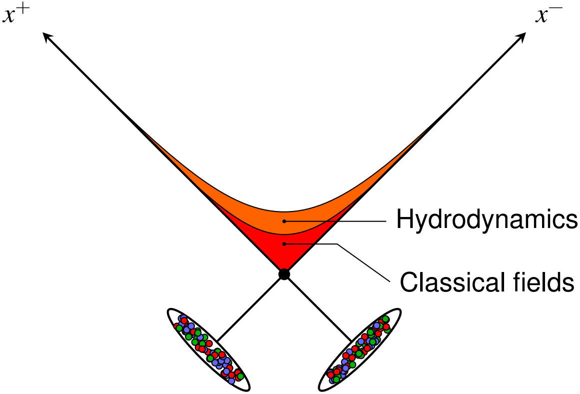
OUTLINE

THE GENERAL PICTURE

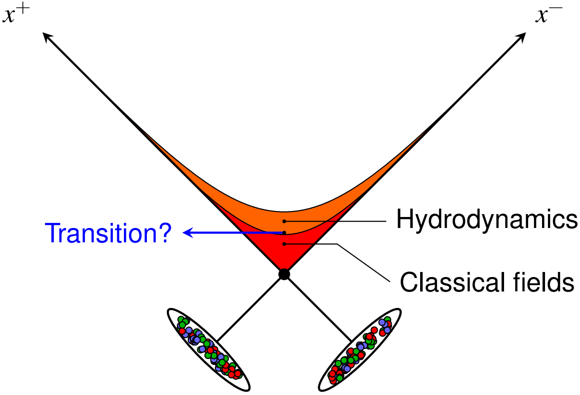


THE GENERAL PICTURE

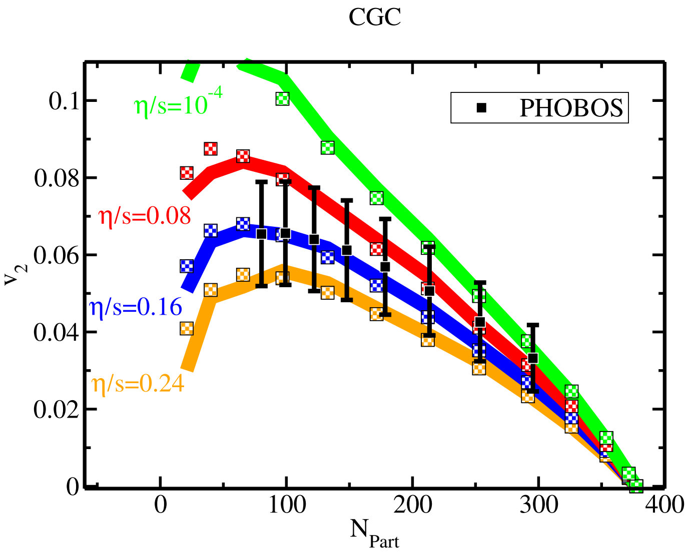



x^+
x^-
Hydrodynamics
Classical fields

The general picture



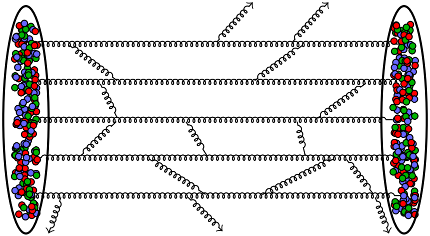
Hydrodynamics success



M. Luzum, P. Romatschke, Phys. Rev. **C 78**, (2008).

Dilute Regime

Dense regime



HYDRO PREREQUISITES

- Equation of state (EOS), small anisotropy
- Initial conditions: energy density, pressure
- Transport coefficients: viscosity...

THEORETICAL FRAMEWORK

- Color Glass Condensate (CGC)
- JIMWLK Equation
- Resummation Scheme

1. THEORETICAL FRAMEWORK
   - CGC and JIMWLK
   - Resummation Scheme

[MCLERRAN, VENUGOPALAN]
[JALILIAN-MARIAN, IANCU, MCLERRAN, WEIGERT, LEONIDOV, KOVNER]

Schematical vision of the degree of freedom

CUTOFF DEPENDANCE

Theory Λ dependant $\rightarrow \log(\Lambda)$ appears at NLO

[McLerran, Venugopalan]
[Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

Schematical vision of the degree of freedom

Cutoff dependance

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Renormalization group equation

Absorb the Λ dependance in the source

[McLerran, Venugopalan]
[Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner]

Schematical vision of the degree of freedom

Cutoff dependance

Theory Λ dependant $\rightarrow \log(\Lambda)$ appears at NLO

Renormalization group equation

Absorb the Λ dependance in the source

Issues:

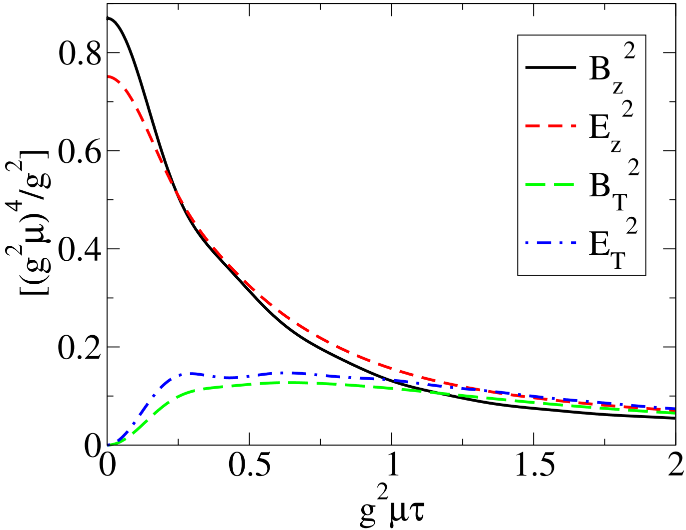
- Very Anisotropic system at $\tau = 0^+$
- Secular divergences.

$$\epsilon = E_T^2 + B_T^2 + E_L^2 + B_L^2$$

$$P_T = E_L^2 + B_L^2$$

$$P_L = E_T^2 + B_T^2 - E_L^2 - B_L^2$$

Anisotropy



T. Lappi, L. McLerran Nucl. Phys. A**772**, (2006).

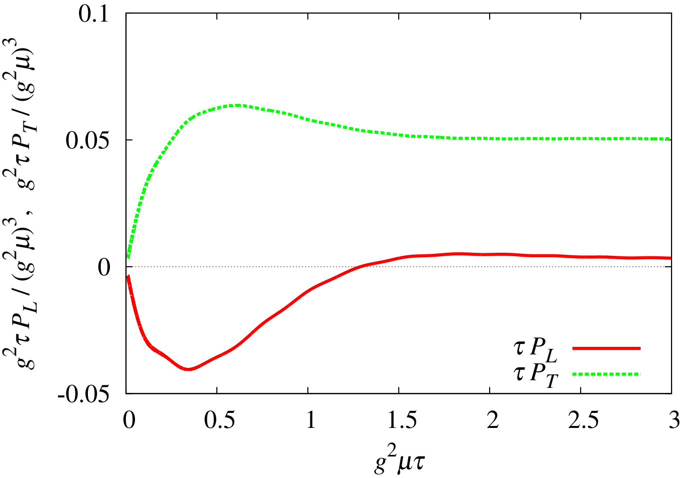
$$\epsilon = \underbrace{E_T^2}_{0} + \underbrace{B_T^2}_{0} + E_L^2 + B_L^2$$

$$P_T = E_L^2 + B_L^2$$

$$P_L = \underbrace{E_T^2}_{0} + \underbrace{B_T^2}_{0} - E_L^2 - B_L^2$$

Initial $T^{\mu\nu}$ is $(\epsilon, \epsilon, \epsilon, -\epsilon)$!

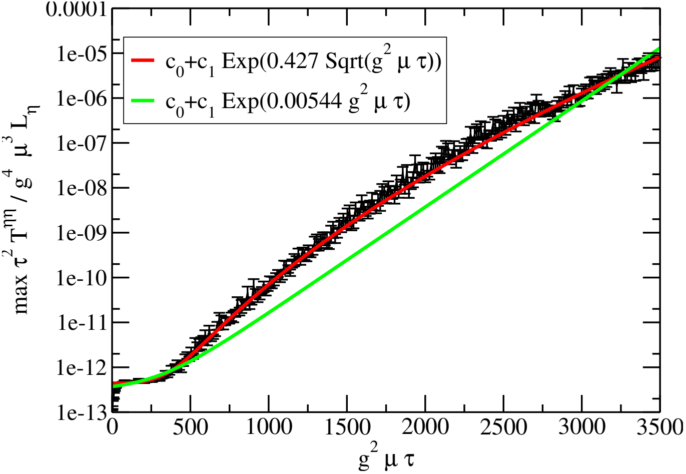
Anisotropy



K. Fukushima, F. Gelis Nucl. Phys. A**874**, (2012).

WEIBEL INSTABILITIES → SECULAR DIVERGENCES

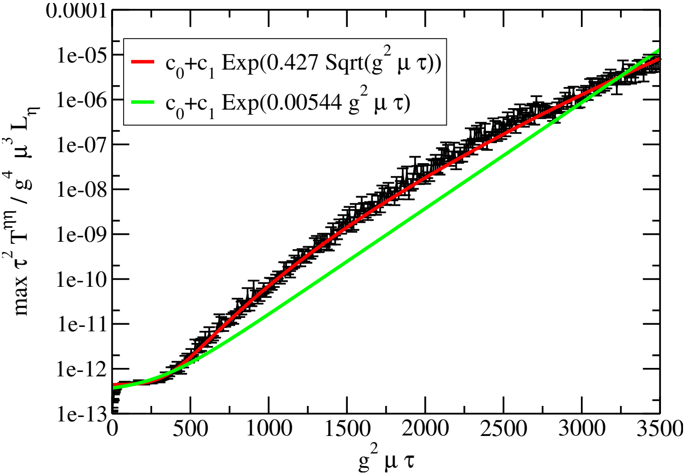
[MROWCZYNSKI]



P. Romatschke, R. Venugopalan Phys. Rev. Lett. **96**, (2006).

Fluctuations grows like $e^{\sqrt{\mu\tau}}$!

[MROWCZYNSKI]



P. Romatschke, R. Venugopalan Phys. Rev. Lett. **96**, (2006).

when $g\, e^{\sqrt{\mu\tau}} \approx 1$, perturbative expansion breaks down!

1 THEORETICAL FRAMEWORK

CGC and JIMWLK

Resummation Scheme

RESUMMATION

$$T^{\mu\nu}_{\text{resum}} = e^{\widehat{O}}\, T^{\mu\nu}_{\text{LO}}[\varphi_0] = T^{\mu\nu}_{\text{LO}}[\varphi_0] + T^{\mu\nu}_{\text{NLO}}[\varphi_0] + \ldots$$

New perturbative development parameter $g\, e^{\sqrt{\mu\tau}}$

INITIAL ϕ

$$\phi = \varphi_0 + \int_k c_k\, a_k + c_k^*\, a_{-k}$$

with c_k random gaussian numbers.

2 FIXED VOLUME

The Model

Energy-Momentum Tensor

Distribution Function

Bose-Einstein Condensation

LAGRANGIAN OF THE THEORY

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \underbrace{\frac{g^2}{4!}\phi^4}_{V(\phi)}$$

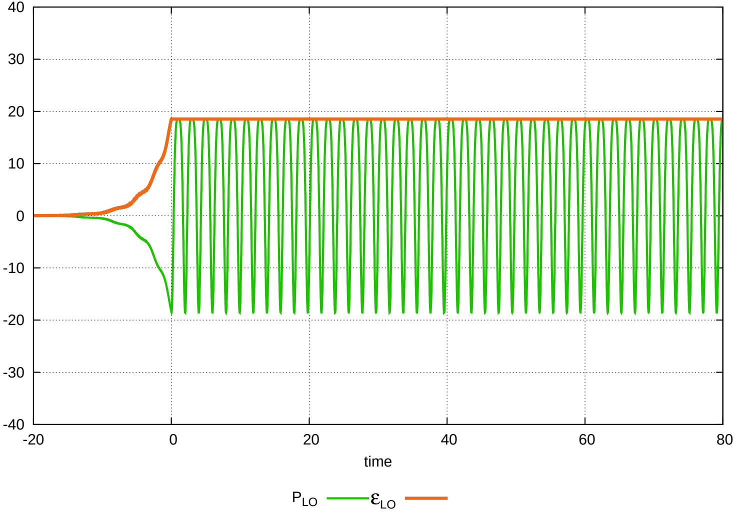
WHY DO WE USE THIS MODEL?

- Scale invariance in $3+1$ dimensions
- Parametric resonance
- A lot simpler!

2 Fixed volume

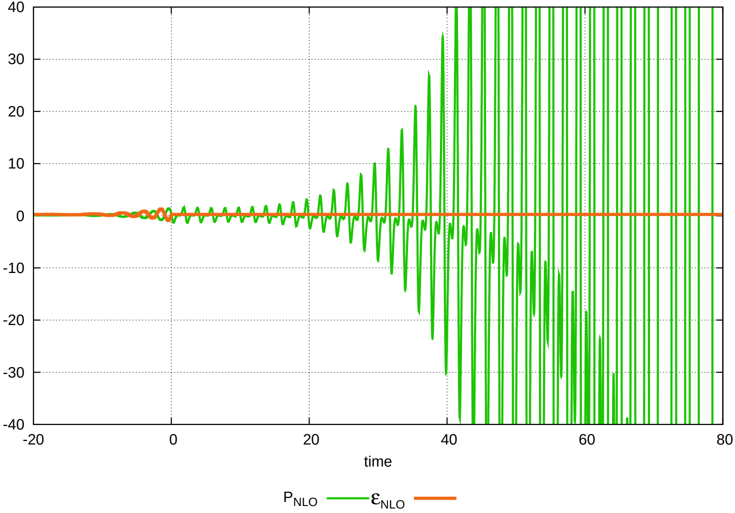
[Dusling, TE, Gelis, Venugopalan (2010)]

$T^{\mu\nu}_{\text{LO}}$



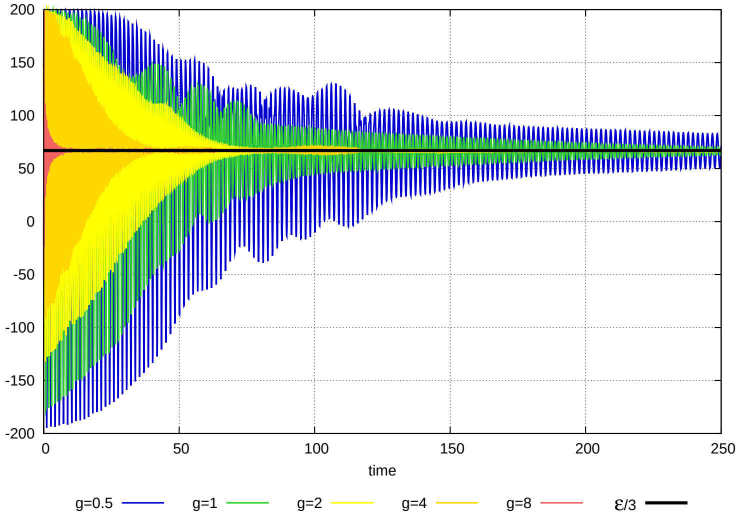
SECULAR DIVERGENCES

$T^{\mu\nu}_{\text{NLO}}$



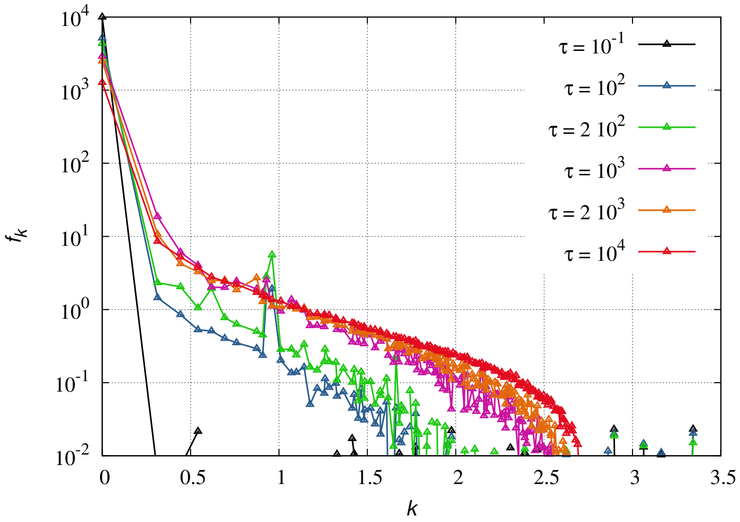
Pressure equilibration

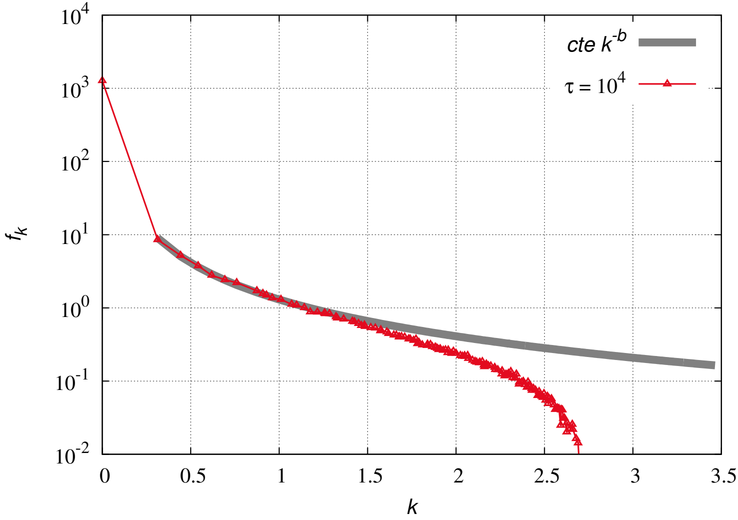
$T^{\mu\nu}_{\text{resum}}$



2 FIXED VOLUME

TIME EVOLUTION OF THE OCCUPATION NUMBER
[TE, GELIS (2011)]

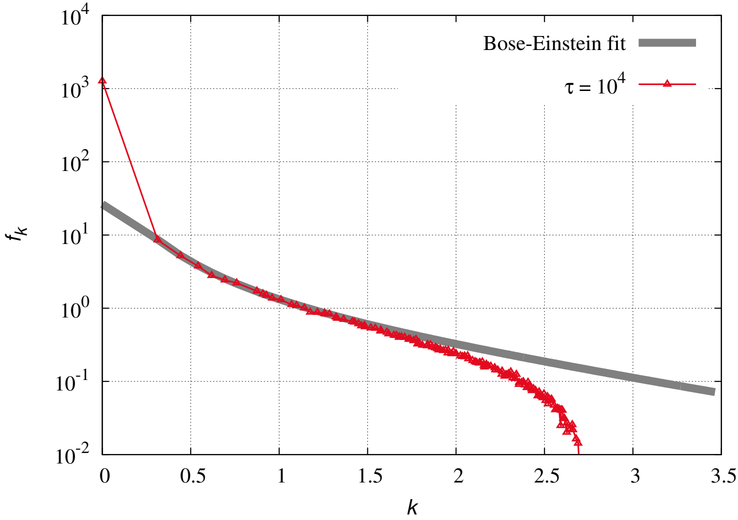



10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
f_k
0
0.5
1
1.5
2
2.5
3
3.5
k
$cte\ k^{-b}$
$\tau = 10^4$

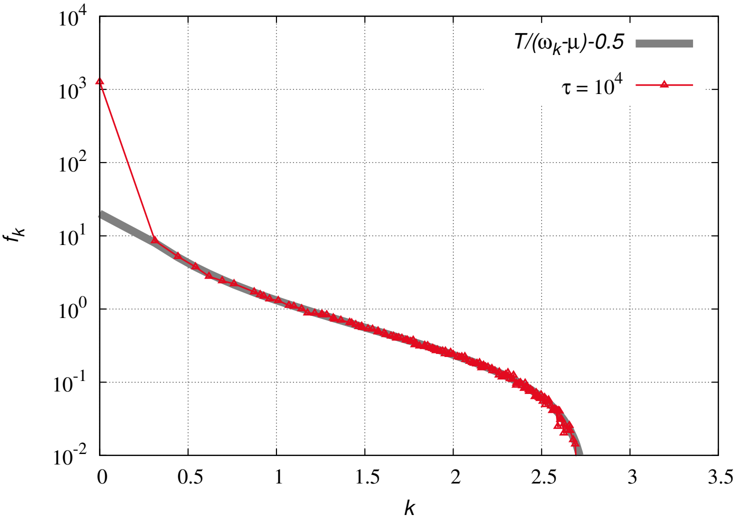
See also:

- R. Micha, I. Tkachev, Phys. Rev. **D 70**, (2004).
- J. Berges, D. Sexty, Phys. Rev. Lett. **108**, (2012).

BOSE-EINSTEIN EQUILIBRIUM DISTRIBUTION?

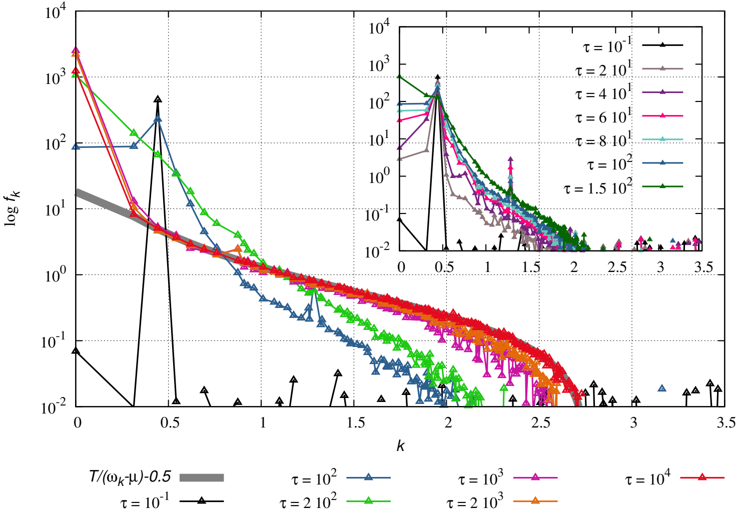


"Classical" equilibrium distribution



❷ FIXED VOLUME

NON-ZERO INITIAL MODE: $\varphi_0 \sim \cos \boldsymbol{k}.\boldsymbol{x}$

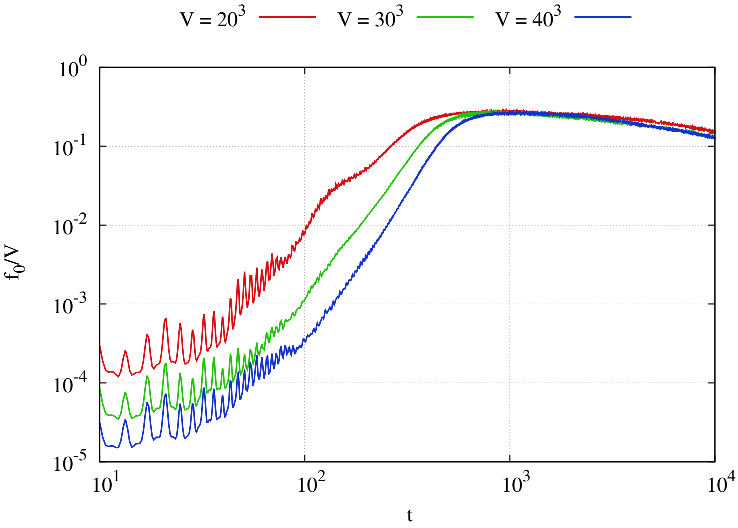


$$f_{\boldsymbol{k}} = \frac{T}{\omega_{\boldsymbol{k}} - \mu} - \frac{1}{2} + n_0 \delta(\boldsymbol{k})$$

implies

$$\frac{f_0}{V} = \text{cte}$$

Evolution of the condensate



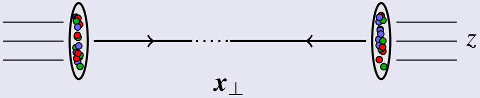
3 EXPANDING VOLUME

The model

Izotropization

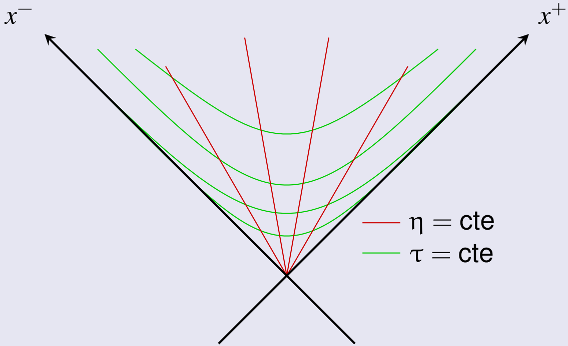
Comparison with Hydro

ADAPTED COORDINATE SYSTEM TO DESCRIBE A HEAVY ION COLLISION?

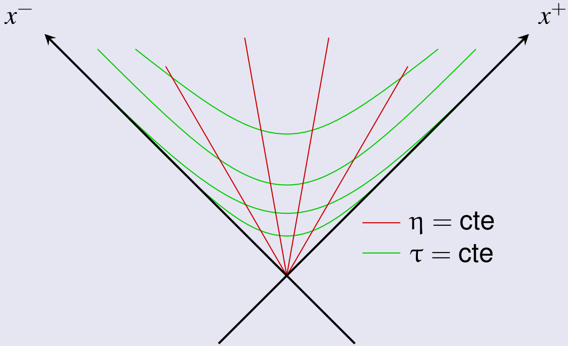


System boost invariant in z direction

PROPER TIME/RAPIDITY COORDINATE SYSTEM



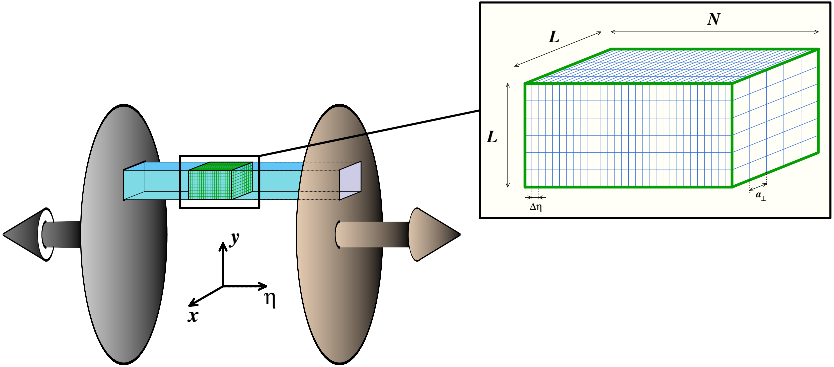
PROPER TIME/RAPIDITY COORDINATE SYSTEM



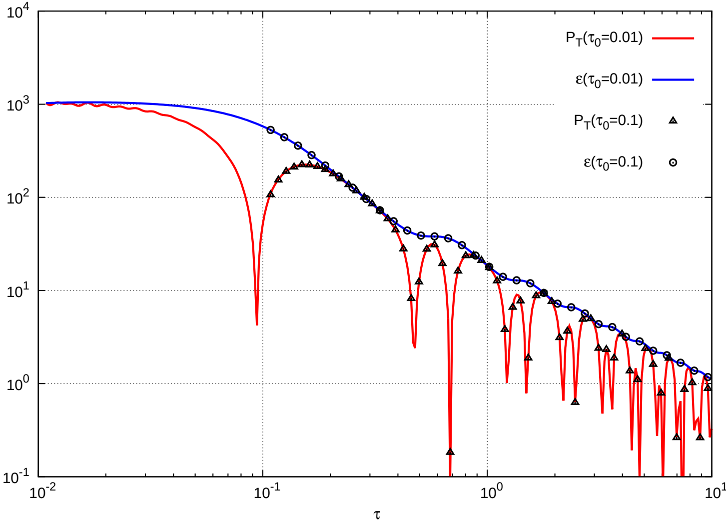
EOM FOR A BOOST-INVARIANT FIELD

$$\left[\frac{\partial^2}{\partial\tau^2} + \frac{1}{\tau}\frac{\partial}{\partial\tau} - \boldsymbol{\nabla}_\perp^2\right] \varphi_0(\tau, \boldsymbol{x}_\perp) + V'(\varphi_0(\tau, \boldsymbol{x}_\perp)) = 0$$

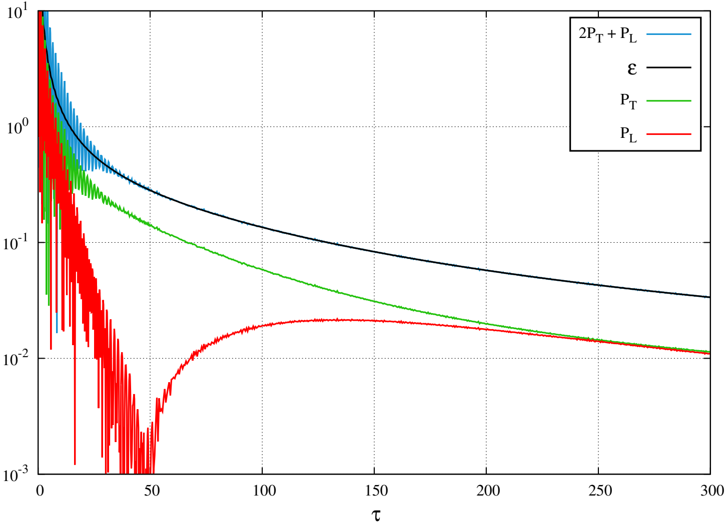
SCALAR FIELD THEORY



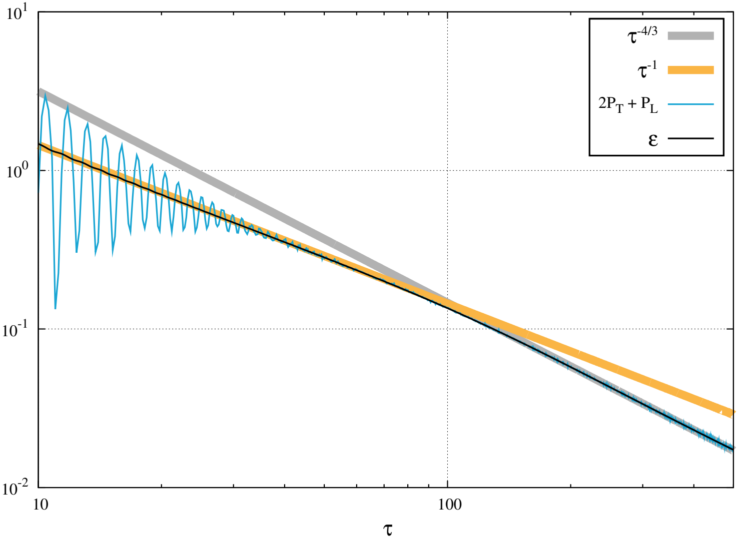
$T^{\mu\nu}_{\text{RESUM}}$
[Dusling, TE, Gelis, Venugopalan (2012)]



$T^{\mu\nu}_{\text{RESUM}}$
[Dusling, TE, Gelis, Venugopalan (2012)]



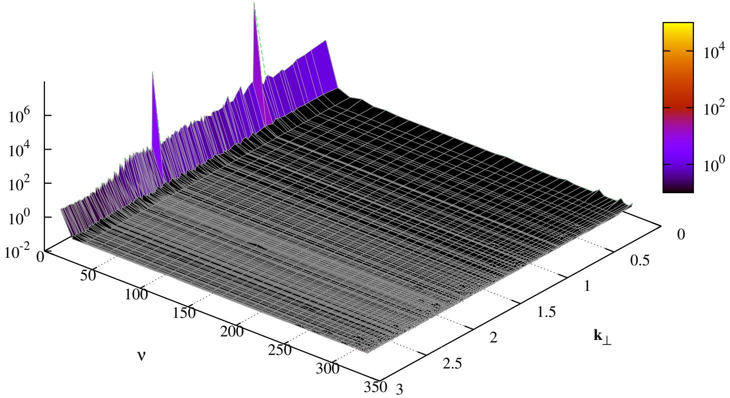
ϵ BEHAVIOUR



Bjorken Law: $\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0$

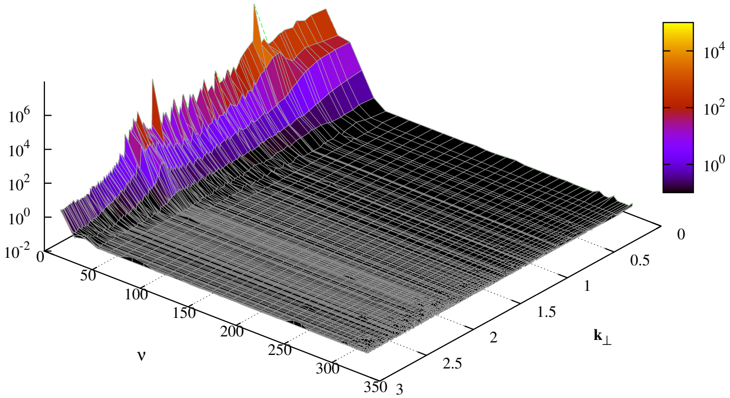
EVOLUTION OF $f_{\nu \boldsymbol{k}_\perp}$

$\tau = 10 \quad [40 \times 40 \times 320]$



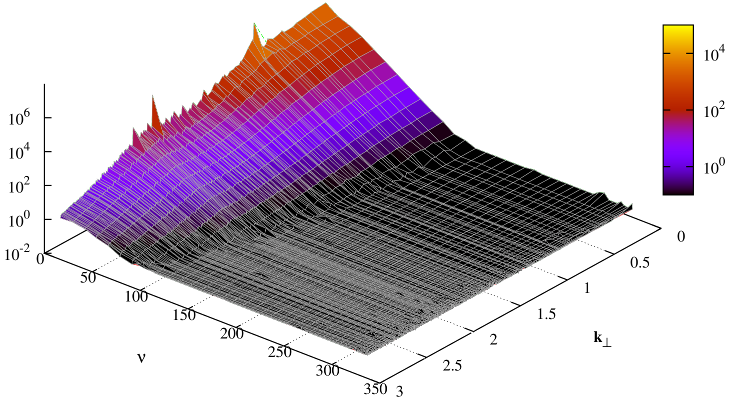
Evolution of $f_{\nu \boldsymbol{k}_\perp}$

$\tau = 50 \quad [40 \times 40 \times 320]$



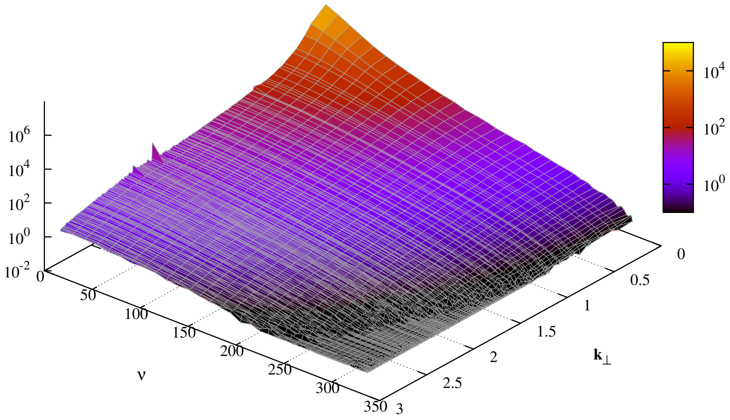
EVOLUTION OF $f_{\nu \boldsymbol{k}_\perp}$

$\tau = 100 \quad [40 \times 40 \times 320]$



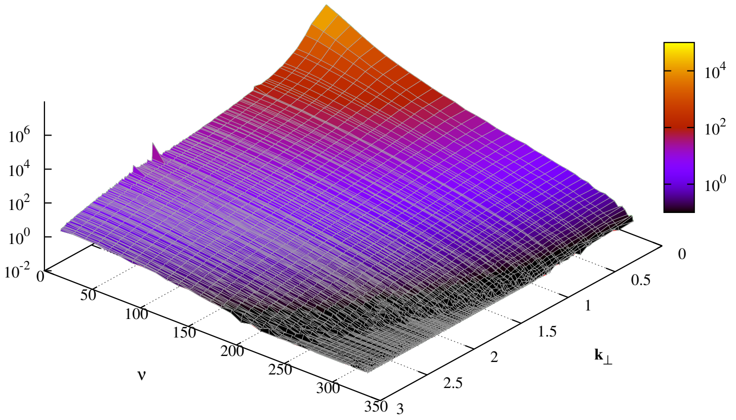
Evolution of $f_{\nu \boldsymbol{k}_\perp}$

$\tau = 300 \quad [40 \times 40 \times 320]$



EVOLUTION OF $f_{\nu k_\perp}$

$\tau = 300 \quad [40 \times 40 \times 320]$



$$\nu \propto \tau^{2/3}$$

3 EXPANDING VOLUME

The model

Izotropization

Comparison with Hydro

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

Equation of state: $\epsilon = 2P_L + P_T$

IDEAL HYDRO

Isotropic system

$$T^{\mu\nu}_{\text{ideal}} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

VISCOUS HYDRO

Anisotropic system

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \eta\, \pi^{\mu\nu}$$

In our case

$$P_T = \frac{\epsilon}{3} + \frac{2\eta}{3\tau} \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \;\rightarrow \partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2}$$

FIRST ORDER VISCOUS HYDRODYNAMICS

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \;\rightarrow \partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2}$$

assuming $\eta = \frac{\eta_0}{\tau}$ and STEFAN-BOLTZMANN entropy $s \approx \epsilon^{\frac{3}{4}}$

$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\underbrace{\frac{\eta}{s}}_{\text{cte}}\frac{\epsilon^{\frac{3}{4}}}{\tau^2}$$

BJORKEN's Law (coming from $\partial_\mu T^{\mu\nu} = 0$):

$$\partial_\tau \epsilon + \frac{\epsilon + P_L}{\tau} = 0 \ \rightarrow \partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3}\frac{\eta}{\tau^2}$$

assuming $\eta = \frac{\eta_0}{\tau}$ and STEFAN-BOLTZMANN entropy $s \approx \epsilon^{\frac{3}{4}}$

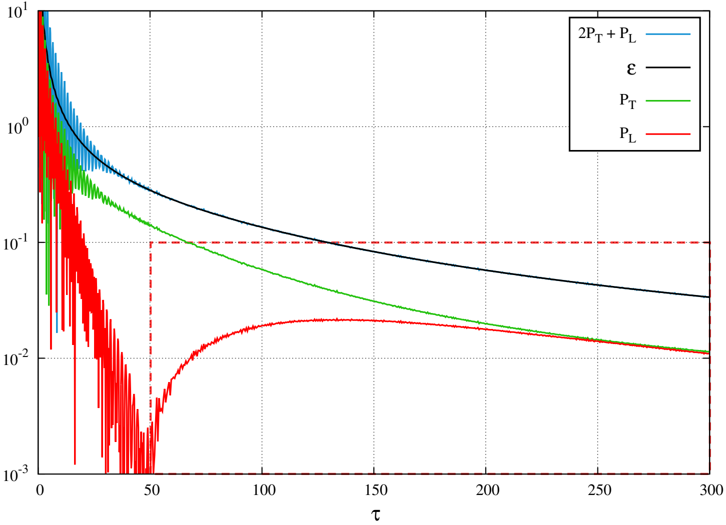
$$\partial_\tau \epsilon + \frac{4}{3}\frac{\epsilon}{\tau} = \frac{4}{3} \underbrace{\frac{\eta}{s}}_{\text{cte}} \frac{\epsilon^{\frac{3}{4}}}{\tau^2}$$

At a given time, knowing ϵ, P_T, P_L and assuming

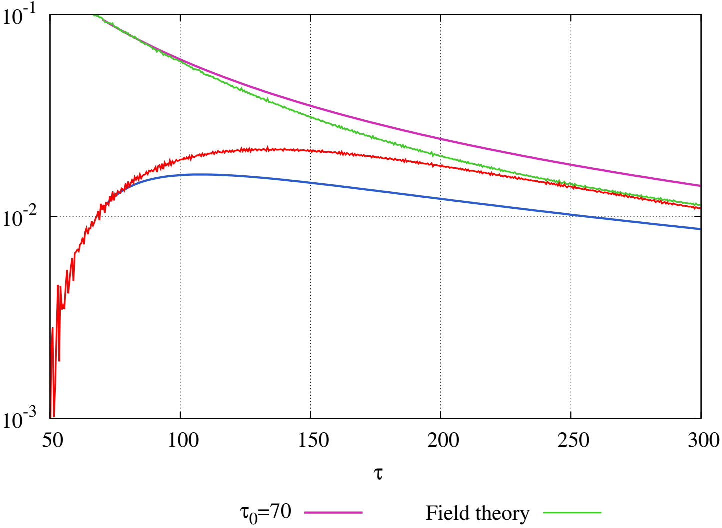
- an EOS
- STEFAN-BOLTZMANN entropy
- $\eta = \frac{\eta_0}{\tau}$
- $\frac{\eta}{s} =$ cte

gives a very simple hydro model.

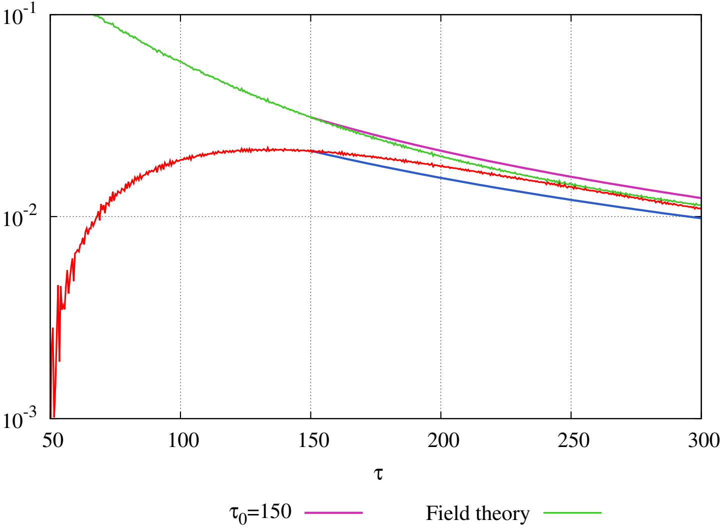
Comparison with Hydro



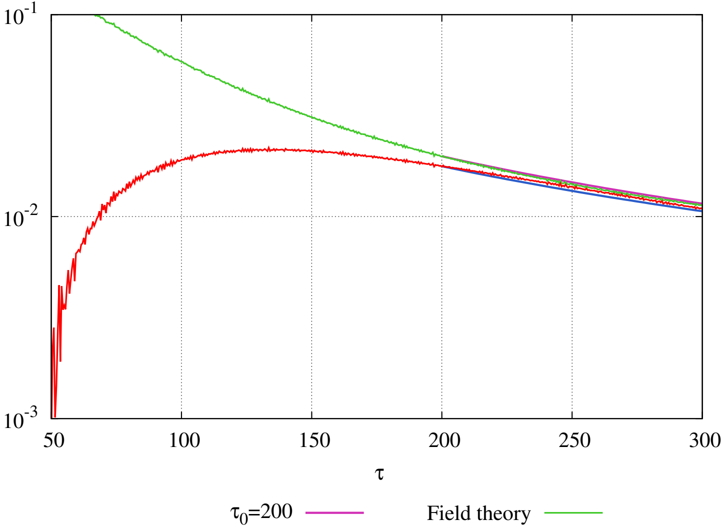
Comparison with Hydro



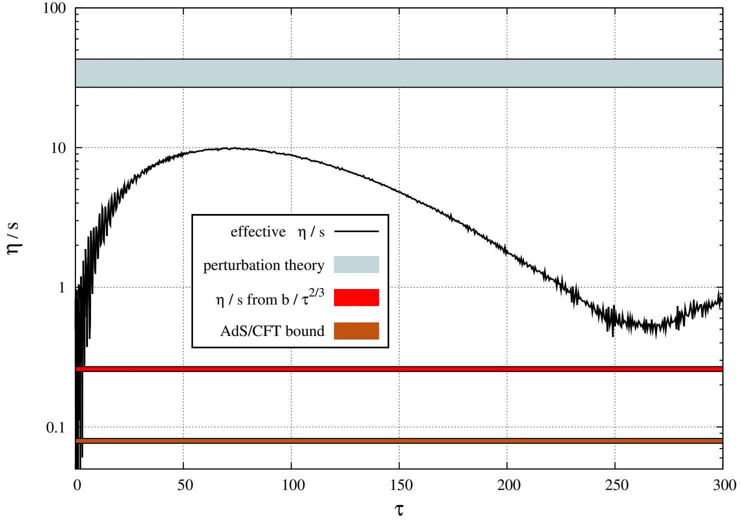
Comparison with Hydro



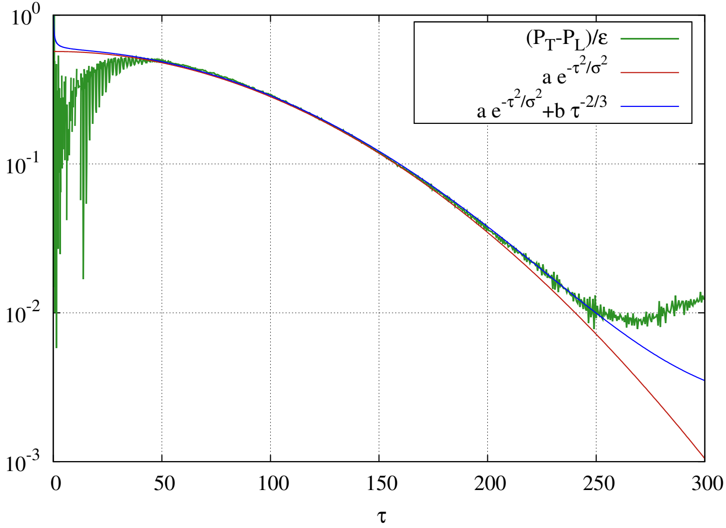
Comparison with Hydro



Comparison with Hydro



Comparison with Hydro



GAUGE CASE
[DUSLING, GELIS, VENUGOPALAN (2011)]

- EOM for the field $\rightarrow$ Yang-Mills Equations known

GAUGE CASE
[DUSLING, GELIS, VENUGOPALAN (2011)]

- EOM for the field $\rightarrow$ Yang-Mills Equations known
- Linearized EOM for the small fluctuations $\rightarrow$ known

GAUGE CASE
[DUSLING, GELIS, VENUGOPALAN (2011)]

- EOM for the field $\rightarrow$ Yang-Mills Equations known
- Linearized EOM for the small fluctuations $\rightarrow$ known
- Final form of the solution $\rightarrow$ known

Gauge case
[Dusling, Gelis, Venugopalan (2011)]

- EOM for the field → Yang-Mills Equations known
- Linearized EOM for the small fluctuations → known
- Final form of the solution → known
- initial A_μ^a

$$A_\mu^a(\tau_0, \eta, \boldsymbol{x}_\perp) = \mathcal{A}_\mu^a(\boldsymbol{x}_\perp) + \mathrm{Re} \int_{\boldsymbol{k}} c_{\boldsymbol{k}} \, a^a{}_{\mu \boldsymbol{k}}(\tau_0, \eta, \boldsymbol{x}_\perp)$$

Gauge case
[Dusling, Gelis, Venugopalan (2011)]

- EOM for the field → Yang-Mills Equations known
- Linearized EOM for the small fluctuations → known
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$$A_\mu^a(\tau_0, \eta, \boldsymbol{x}_\perp) = \mathcal{A}_\mu^a(\boldsymbol{x}_\perp) + \text{Re} \int_{\boldsymbol{k}} c_{\boldsymbol{k}}\, a^a{}_{\mu \boldsymbol{k}}(\tau_0, \eta, \boldsymbol{x}_\perp)$$

- Lattice implementation → Work in progress

Decoherence

Decoherence

$\downarrow$

Equation of state $\epsilon = 3P$ (isotropy by construction)

Decoherence

$\downarrow$

Equation of state $\epsilon = 3P$ (isotropy by construction)

$\downarrow$

BOSE-EINSTEIN condensate

Decoherence

$\downarrow$

Equation of state $\epsilon = 3P$ (isotropy by construction)

$\downarrow$

BOSE-EINSTEIN condensate

$\downarrow$

$$f_{\boldsymbol{k}} \propto \frac{T}{\omega_{\boldsymbol{k}} - m} - \frac{1}{2}$$

Decoherence

Decoherence

$\downarrow$

Equation of state $\epsilon = 2P_T + P_L$

Decoherence

$\downarrow$

Equation of state $\epsilon = 2P_T + P_L$

$\downarrow$

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

Decoherence

$\downarrow$

Equation of state $\epsilon = 2P_T + P_L$

$\downarrow$

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

$\downarrow$

Isotropization

Decoherence

$\downarrow$

Equation of state $\epsilon = 2P_T + P_L$

$\downarrow$

Instabilities [BERGES, BOGUSLAVSKI, SCHLICHTING (2012)]

$\downarrow$

Isotropization

$\downarrow$? Thermal distribution

$\downarrow$? BOSE-EINSTEIN condensate?

Thank You!