Di-gluon production at the LHC.

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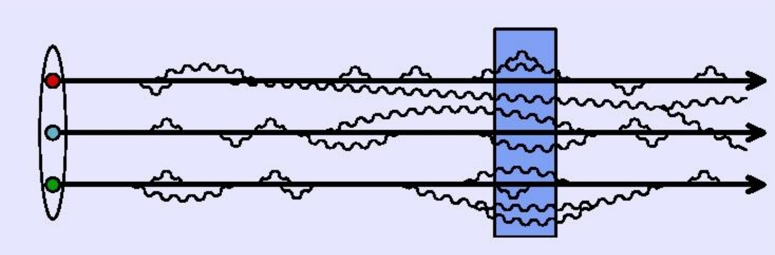
April 18th, 2013

Paper to come soon in collaboration with Edmond Iancu.

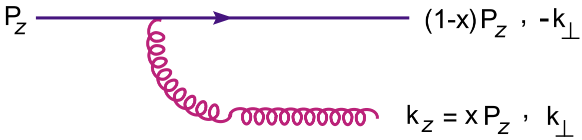
Plan

1. Why double gluon production ?
   - Why saturation ?
   - Why p-A ?
   - Why di-hadron correlations ?
   - Saturation and experiments : knowledge and expectation
2. General results
3. Hard transverse gluons
4. Summary and outlook



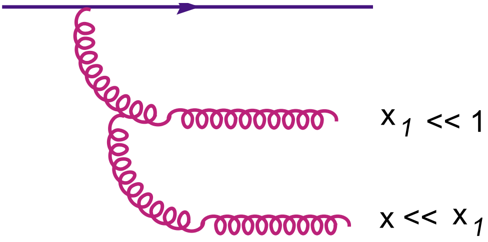
- Infinite momentum frame : $P^0 \simeq P^3 \rightarrow P^+$ very large and $P^- = P_\perp = 0$.
- Parton with momentum p carries a fraction x of longitudinal momentum : $x = p^+/P^+$.

Bremsstrahlung :



$$d\mathcal{P}_{\rm brem} \sim \alpha_s(\mathbf{k}_\perp^2)\frac{d^2k_\perp}{\mathbf{k}_\perp^2}\frac{dx}{x}$$

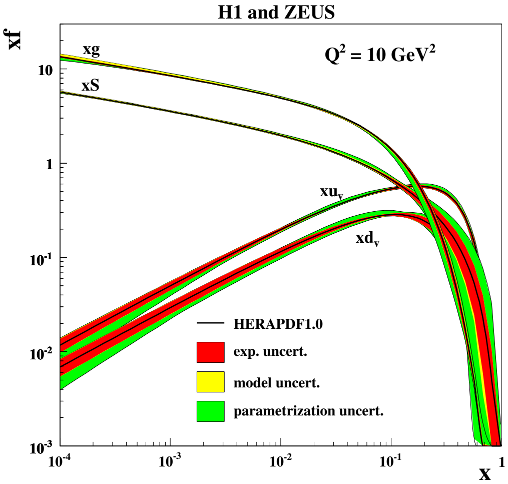
Collinear ($k_\perp \to 0$) and soft ($x \to 0$) divergences.



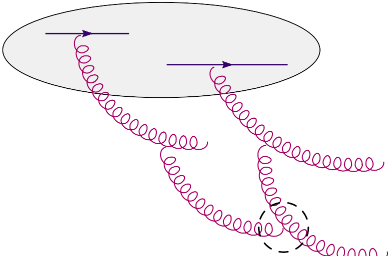
- The emitted gluon can in turn emit another (softer) gluon.
- The additional gluon brings a factor $\alpha_s \ln \frac{1}{x}$.

$\ln \frac{1}{x} \equiv Y$ is known as the rapidity.
Requires a full resummation as $\alpha_s Y \sim 1$ (Balitsky, Fadin, Kuraev, Lipatov, 75-78).

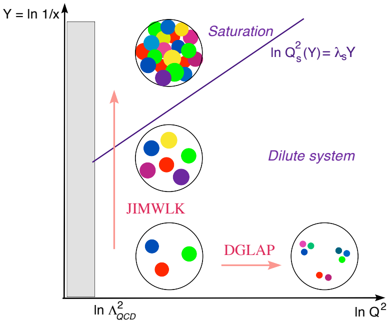
Raise of the parton distribution at small x.



Low x $\rightarrow$ high density $\rightarrow$ recombination effects



Saturation scale Q_s



High energy $\longrightarrow$ perturbative description of saturation

The Color Glass Condensate :

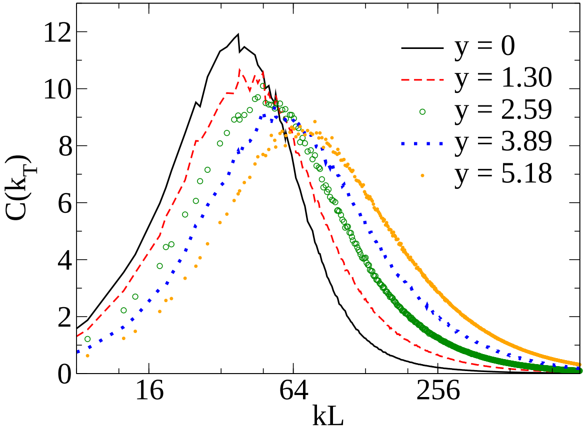
- Effective field theory
- Semi-hard gluons = gluons with $x' \gg x$ considered = classical field $\mathcal{A}$ radiated by classical sources ρ
- Source configuration ρ randomly frozen during the process
- CGC weight function $\mathcal{W}_Y[\rho]$ = probability of occurrence of a given source configuration
- Independance of physical observables on Y $\rightarrow$ renormalization group equation known as the JIMWLK equation (Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner; 1997-2000) :

$$\frac{\partial \mathcal{W}_Y}{\partial Y} = \mathcal{H}_{\mathrm{JIMWLK}} \mathcal{W}_Y$$

CGC requires to perform averages over the classical field weighted with the CGC weight function $W_Y[\mathcal{A}^-]$ for observables :

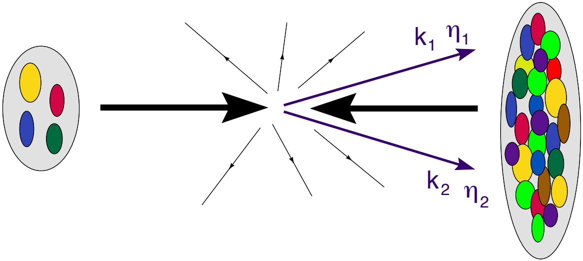
$$\langle \mathcal{O} \rangle_Y = \int \mathcal{D}[\rho]\ \mathcal{W}_Y[\rho]\mathcal{O}[\rho].$$

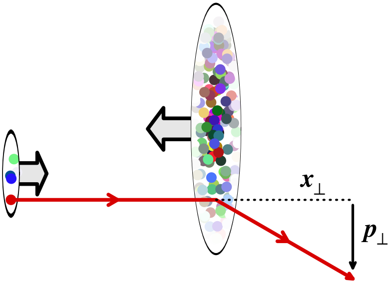
CGC predicts that partons within a dense hadron typically carry transverse momenta of order Q_s.



source T. Lappi (numerical solution to JIMWLK)

Aim : clear probe of a saturated medium, the target, with a dilute projectile.



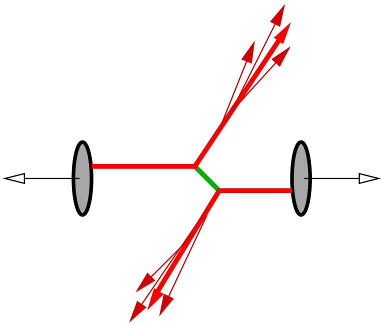


Single scattering :

$$x_1 = \frac{p_\perp}{\sqrt{s}} e^{Y} \sim 1 \qquad x_2 = \frac{p_\perp}{\sqrt{s}} e^{-Y} \ll 1$$

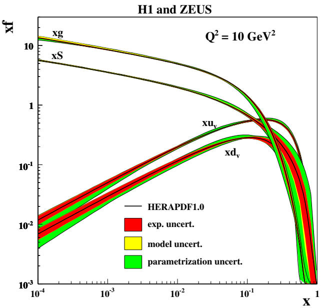
One has to look at forward rapidities ($Y > 0$).

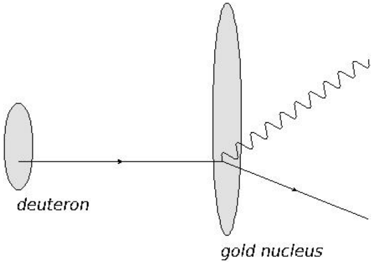
dilute-dilute collision :



dilute-dense colision : we expect that multiple scattering broaden the final state distribution in the transverse plane.

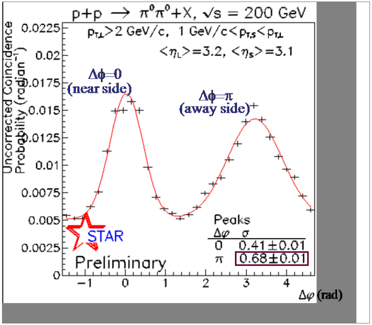
At RHIC, saturation has been marginally reached in d-Au collisions. Deuteron probed at $x_1 \sim 10^{-1}$.

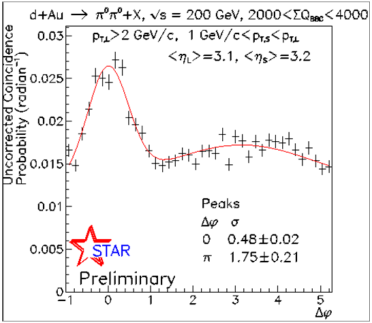




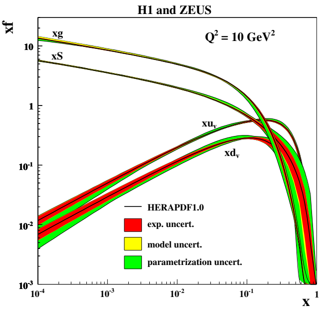
CGC-based predictions provide a good description for the $Aq \to qgX$ inclusive cross-section (Albacete, Marquet, 2010 - Stasto, Xiao, Yuan, 2012).

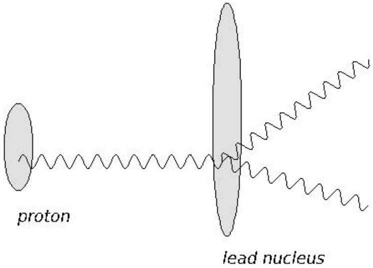
Evidences of saturation : pp vs. pA





At the LHC, the first p-Pb runs just occurred.





Proton probed at $x_1 \sim 10^{-2} - 10^{-3}$. Dominant process : $Ag \to ggX$.

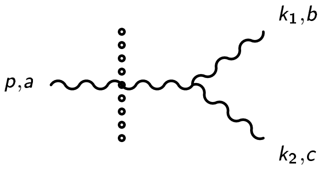
Plan

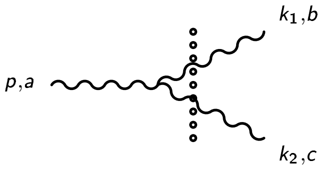
Collinear factorization at the projectile (proton) level :

$$\frac{\mathrm{d}\sigma(pA \to ggX)}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2k_{1,\perp}\,\mathrm{d}^2k_{2,\perp}} = \int \mathrm{d}x_1\, G(x_1,\mu^2)\frac{\mathrm{d}\sigma(gA \to ggX)}{\mathrm{d}y_1\,\mathrm{d}y_2\,\mathrm{d}^2k_{1,\perp}\,\mathrm{d}^2k_{2,\perp}}$$
$$= \frac{1}{256\pi^5(p^+)^2}x_1\,G(x_1,\mu^2)\left\langle\overline{|\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2}\right\rangle_Y .$$

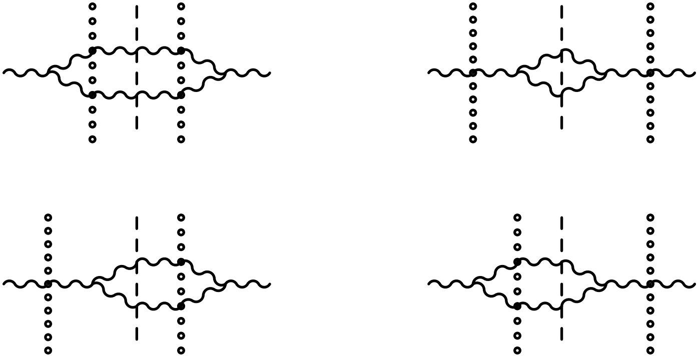
$\left\langle\overline{|\mathcal{M}|^2}\right\rangle_Y$ will be computed thank to the CGC effective theory.

Proton = dilute medium → collinear factorization ∼ the gluon is an in state.
Nucleus = dense medium = CGC. It is flat by Lorentz length contraction in the lab frame (doted line). Referred to as a shockwave.



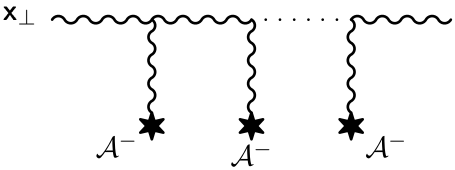


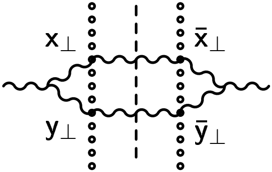
$\overline{|\mathcal{M}|^2}$ receives 4 contributions :



The interaction with the background field is encoded into Wilson lines (eikonal approximation) :

$$U(\mathbf{x}_\perp) = \mathcal{P}\exp\left[ig\int \mathrm{d}x^+ \mathcal{A}_a^-(x^+,\mathbf{x}_\perp)T^a\right].$$

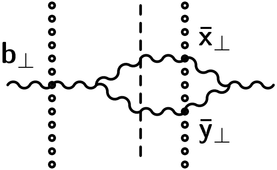




$$\sim \mathrm{Tr}\left[\tilde{U}(\mathbf{x}_\perp)\,\tilde{T}^a\,\tilde{U}^\dagger(\mathbf{y}_\perp)\tilde{U}(\bar{\mathbf{y}}_\perp)\,\tilde{T}^a\,\tilde{U}^\dagger(\bar{\mathbf{x}}_\perp)\right]$$

4 points intersecting the shockwave $\rightarrow$ four adjoint Wilson lines $\rightarrow$ quadrupole operator

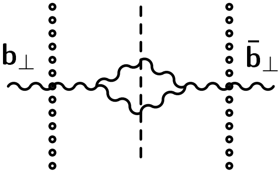
$$\tilde{S}^{(4)}(\mathbf{x}_\perp,\mathbf{y}_\perp,\bar{\mathbf{x}}_\perp,\bar{\mathbf{y}}_\perp) = \frac{1}{N_c(N_c^2-1)} f^{aef} f^{ae'f'}\,\tilde{U}_{be}(\mathbf{x}_\perp)\tilde{U}_{cf}(\mathbf{y}_\perp) \times \tilde{U}_{be'}(\bar{\mathbf{x}}_\perp)\tilde{U}_{cf'}(\bar{\mathbf{y}}_\perp).$$



$\sim \tilde{U}_{ab}(\mathbf{b}_\perp)\mathrm{Tr}\left[\tilde{T}^a\tilde{U}(\bar{\mathbf{y}}_\perp)\tilde{T}^b\tilde{U}^\dagger(\bar{\mathbf{x}}_\perp)\right]$

with $\mathbf{b}_\perp = z\mathbf{x}_\perp + (1-z)\mathbf{y}_\perp$.
3 points intersecting the shockwave $\rightarrow$ three adjoint Wilson lines

$$\tilde{S}^{(3)}(\mathbf{b}_\perp, \bar{\mathbf{x}}_\perp, \bar{\mathbf{y}}_\perp) = \frac{1}{N_c(N_c^2-1)} f^{dbc} f^{aef}\,\tilde{U}_{da}(\mathbf{b}_\perp)\tilde{U}_{be}(\bar{\mathbf{x}}_\perp)\tilde{U}_{cf}(\bar{\mathbf{y}}_\perp).$$



$$\sim \tilde{U}_{ac}\tilde{U}^{\dagger}_{cb}(\bar{\mathbf{b}}_\perp)(\mathbf{b}_\perp)\mathrm{Tr}\left[\tilde{T}^a\tilde{T}^b\right]$$

2 points intersecting the shockwave $\rightarrow$ two adjoint Wilson lines $\rightarrow$ dipole operator

$$\begin{aligned}\tilde{S}^{(2)}(\mathbf{b}_\perp,\bar{\mathbf{b}}_\perp) &= \frac{1}{N_c(N_c^2-1)}f^{dbc}f^{d'bc}\tilde{U}_{da}(\mathbf{b}_\perp)\tilde{U}_{d'a}(\bar{\mathbf{b}}_\perp)\\ &= \frac{1}{N_c^2-1}\mathrm{Tr}\left[\tilde{U}(\mathbf{b}_\perp)\tilde{U}^{\dagger}(\bar{\mathbf{b}}_\perp)\right].\end{aligned}$$

The squared averaged $\mathcal{M}$-matrix for the $gA \to ggX$ process reads :

$$\left\langle \overline{|\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2} \right\rangle_Y = \frac{4g^2 N_c}{\pi^2} (p^+)^2 z(1-z) P_{g\leftarrow g}(z)$$
$$\times \int \mathrm{d}^2 x_\perp \mathrm{d}^2 y_\perp \mathrm{d}^2 \bar{x}_\perp \mathrm{d}^2 \bar{y}_\perp \frac{(\mathbf{x}_\perp - \mathbf{y}_\perp)\cdot(\bar{\mathbf{x}}_\perp - \bar{\mathbf{y}}_\perp)}{(\mathbf{x}_\perp - \mathbf{y}_\perp)^2 (\bar{\mathbf{x}}_\perp - \bar{\mathbf{y}}_\perp)^2}$$
$$\times e^{-i\mathbf{k}_{1,\perp}\cdot(\mathbf{x}_\perp - \bar{\mathbf{x}}_\perp) - i\mathbf{k}_{2,\perp}\cdot(\mathbf{y}_\perp - \bar{\mathbf{y}}_\perp)} \Big\langle \tilde{S}^{(2)}(\mathbf{b}_\perp, \bar{\mathbf{b}}_\perp)$$
$$-\tilde{S}^{(3)}(\mathbf{b}_\perp, \bar{\mathbf{x}}_\perp, \bar{\mathbf{y}}_\perp) - \tilde{S}^{(3)}(\bar{\mathbf{b}}_\perp, \mathbf{x}_\perp, \mathbf{y}_\perp) + \tilde{S}^{(4)}(\mathbf{x}_\perp, \mathbf{y}_\perp, \bar{\mathbf{x}}_\perp, \bar{\mathbf{y}}_\perp) \Big\rangle_Y .$$

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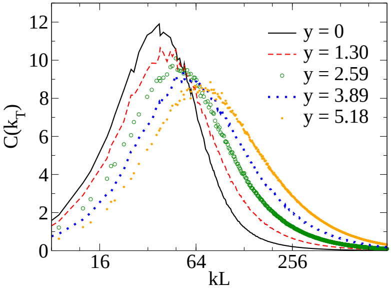
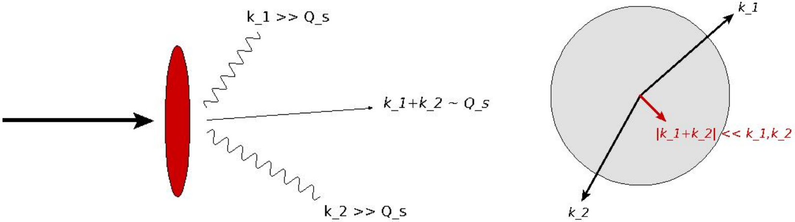


Transverse momenta of the two final gluons :

- both very large w.r.t. $Q_s \rightarrow$ hard process
- both smaller or of order $Q_s \rightarrow$ semi-hard process

cannot get one $\gg Q_s$ and one $\lesssim Q_s$.

Hard regime :



- The transverse momentum distribution of the final gluons is centered around a relative angle $\Delta\phi = \pi$.
- One nevertheless has to consider multiple scatterings, i.e. non linear effects, since $|\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}| \sim Q_s$.

This is the back-to-back regime.

- The fine detail of the $\Delta\phi$ distribution is still sensitive to saturation.
- The back-to-back limit allows generalizations of unintegrated distribution functions in presence of non-linear effects $\longrightarrow$ effective gluon distributions that take saturation into account.

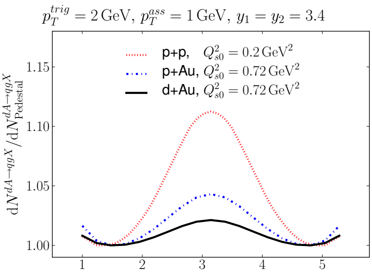
$f_Y^{\rm dip}$ is the ordinary unintegrated distribution function associated to the dipole but including non linear effects :

$$S_\perp f_Y^{dip}(\mathbf{K}_\perp) \equiv \frac{\mathbf{K}_\perp^2}{g^2 N_c} \int \mathrm{d}^2 b_\perp \mathrm{d}^2 \bar{b}_\perp e^{-i\mathbf{K}_\perp\cdot(\mathbf{b}_\perp - \bar{\mathbf{b}}_\perp)} \times \left\langle \tilde{S}^{(2)}(\mathbf{b}_\perp, \bar{\mathbf{b}}_\perp) \right\rangle_Y .$$

This already appears in the single gluon production.
$f_Y^{\rm quad}$ is a new distribution function associated with the quadrupole :

$$S_\perp f_Y^{\rm quad}(\mathbf{K}_\perp) \equiv \frac{1}{g^2 N_c} \int \mathrm{d}^2 b_\perp \mathrm{d}^2 \bar{b}_\perp e^{-i\mathbf{K}_\perp\cdot(\mathbf{b}_\perp - \bar{\mathbf{b}}_\perp)} \times \left\langle \partial_x^i \partial_u^i \tilde{S}^{(4)}(\mathbf{x}_\perp, \mathbf{b}_\perp, \mathbf{u}_\perp, \bar{\mathbf{b}}_\perp)\Big|_{\mathbf{b}_\perp \mathbf{b}_\perp \bar{\mathbf{b}}_\perp \bar{\mathbf{b}}_\perp} \right\rangle_Y .$$

The semi-hard regime : we expect a broadening of the $\Delta\phi = \pi$ peak as the momenta of final gluons approach Q_s. This has been shown by T. Lappi for qg production.

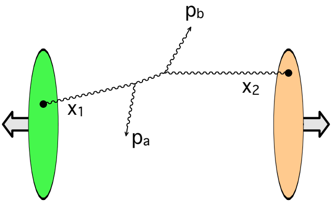


We have the master formula, the gg case is right now a matter of programming.

Plan

- We obtained a general formula for the di-gluon production cross-section that stands for a large range of kinematics encountered at the LHC.
- The back-to-back limit is interesting in the sense it is expected for hard processes : hard transverse gluons are strongly correlated up to $\sim Q_s$ momentum imbalance that is very small w.r.t their intrinsic transverse momentum. We now have a quantitative description of this regime.
- The semi-hard regime is the signature of saturation but its quantitative predictions requires numerical devices for computing the averages of color operators (mean field approximation to JIMWLK : Iancu, Triantafyllopoulos - 2011).

Thanks for your attention.



$$x_1 = \frac{p_{a,\perp}}{\sqrt{s}} e^{y_1} + \frac{p_{b,\perp}}{\sqrt{s}} e^{y_2}$$

$$x_2 = \frac{p_{a,\perp}}{\sqrt{s}} e^{-y_1} + \frac{p_{b,\perp}}{\sqrt{s}} e^{-y_2}.$$

In the back-to-back limit, the squared averaged $\mathcal{M}$-matrix reads :

$$\left\langle |\mathcal{M}(g(p)A \to g(k_1)g(k_2))|^2 \right\rangle_Y = 16g^4 N_c^2 S_\perp \frac{(p^+)^2 z(1-z)}{((1-z)\mathbf{k}_{1,\perp} - z\mathbf{k}_{2,\perp})^4}$$
$$\times P_{g\leftarrow g}(z) \left[f_Y^{\mathrm{quad}}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}) - z(1-z) f_Y^{\mathrm{dip}}(\mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp}) \right].$$