

Strongly coupled QGP or turbulent thermalization ?

François Gelis

CERN and CEA/Saclay



Outline

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

Summary

- What have we learned from RHIC?
- AdS/CFT duality and the strongly coupled QGP
- Ab initio perturbative approach and turbulent thermalization



RHIC results on flow

- Collective flow
- Is the QGP a perfect fluid?

AdS/CFT duality and the sQGP

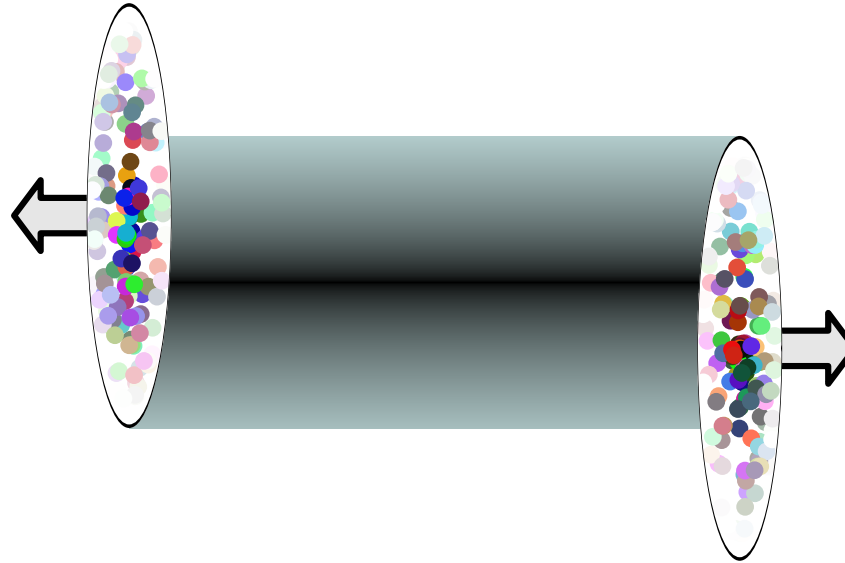
Ab initio perturbative approach

Summary

What have we learned from RHIC?

Collective flow

- Consider a non-central collision :



RHIC results on flow

● Collective flow

● Is the QGP a perfect fluid?

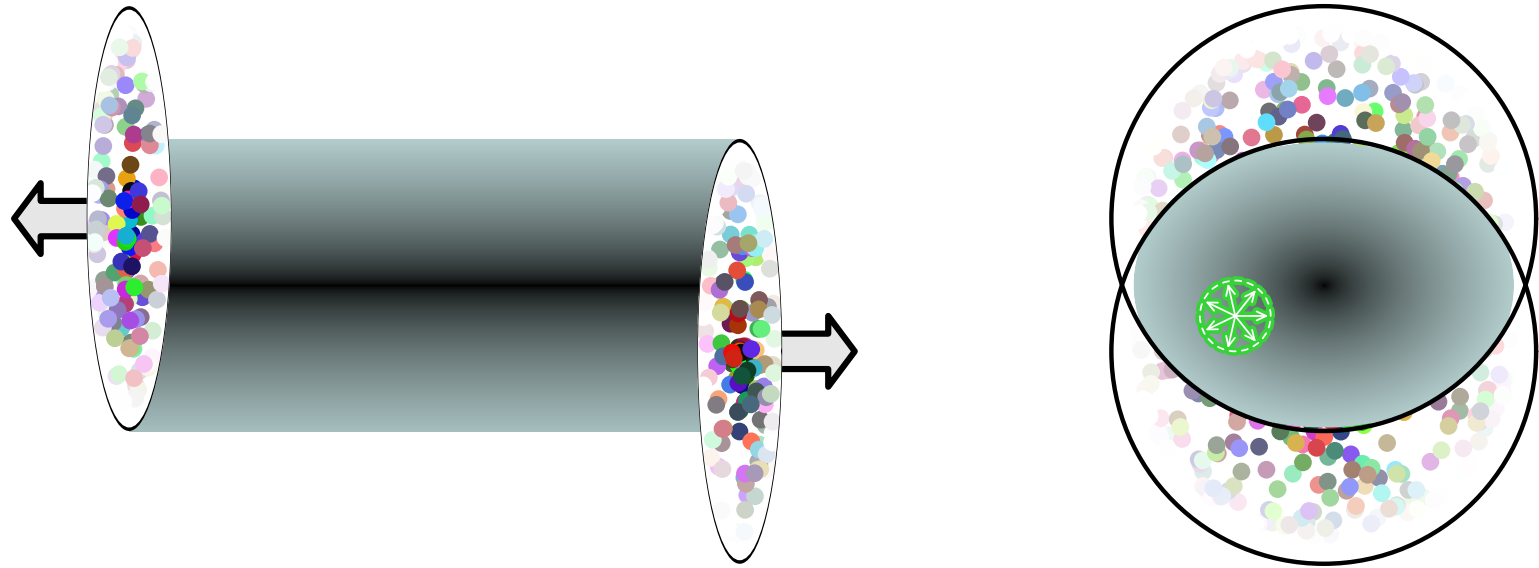
AdS/CFT duality and the sQGP

Ab initio perturbative approach

Summary

Collective flow

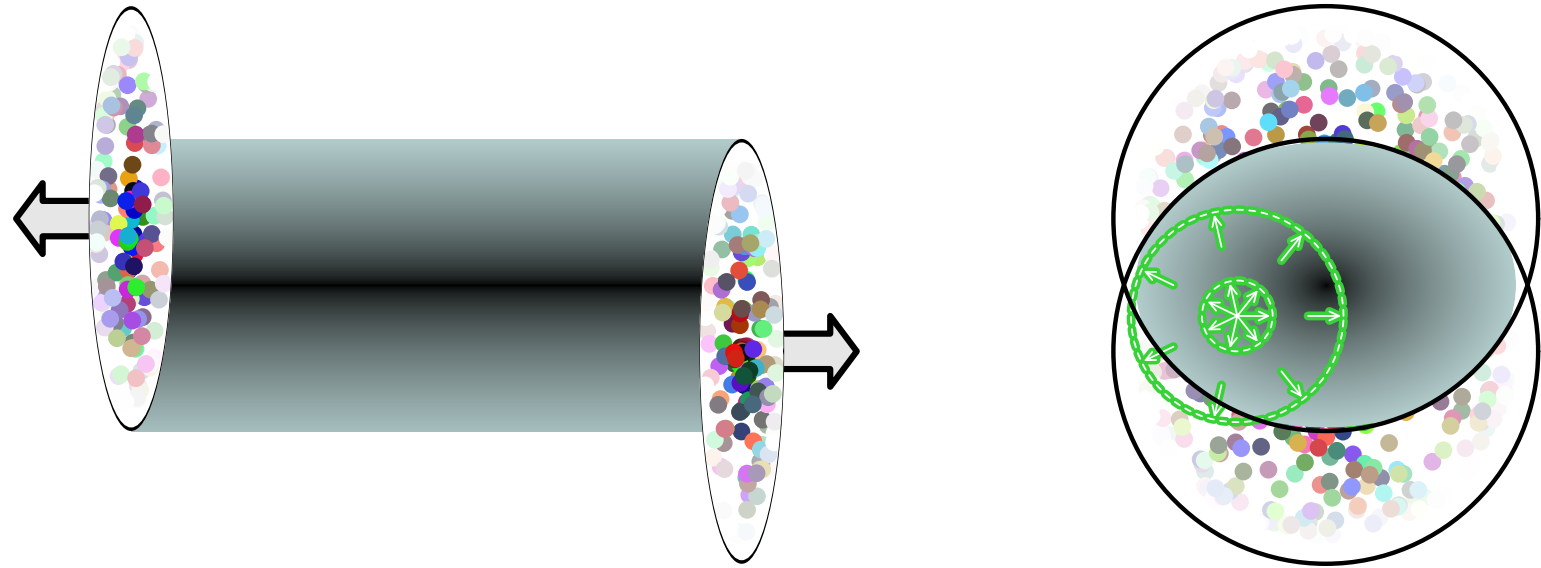
- Consider a non-central collision :



- ◆ Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from **local partonic interactions**

Collective flow

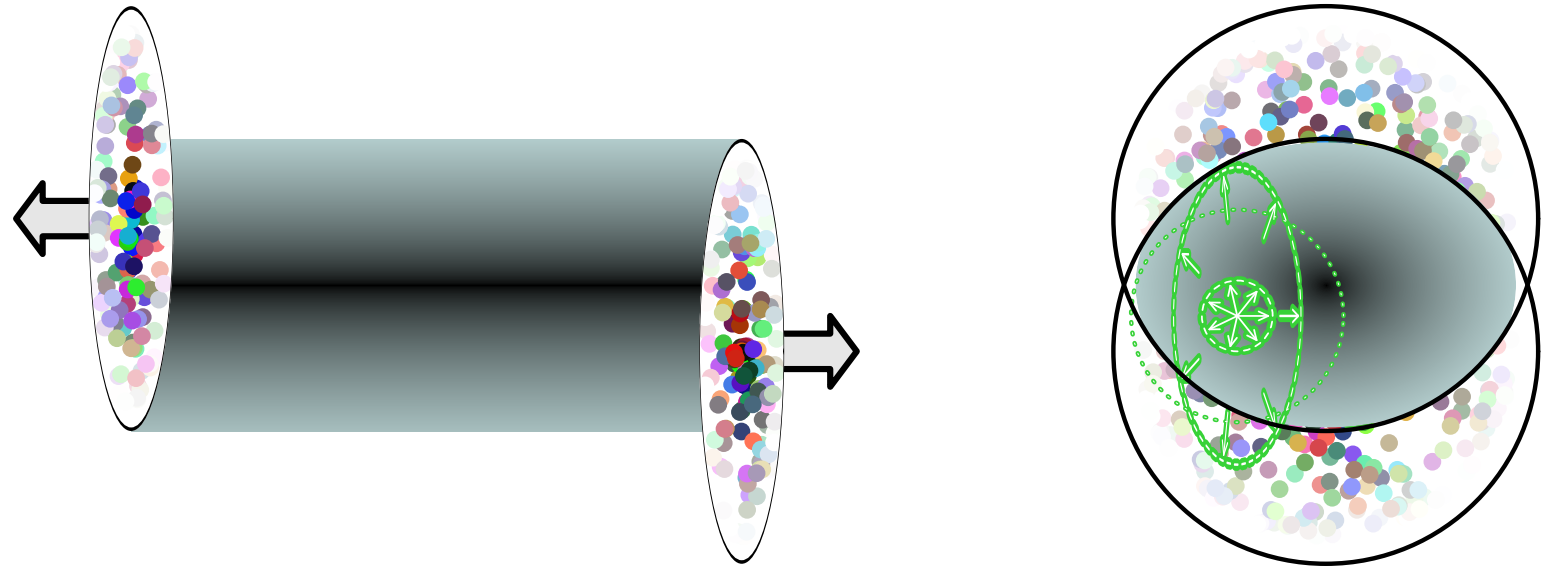
■ Consider a non-central collision :



- ◆ Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from **local partonic interactions**
- ◆ If these particles were escaping freely, the distribution would remain isotropic at all times

Collective flow

■ Consider a non-central collision :



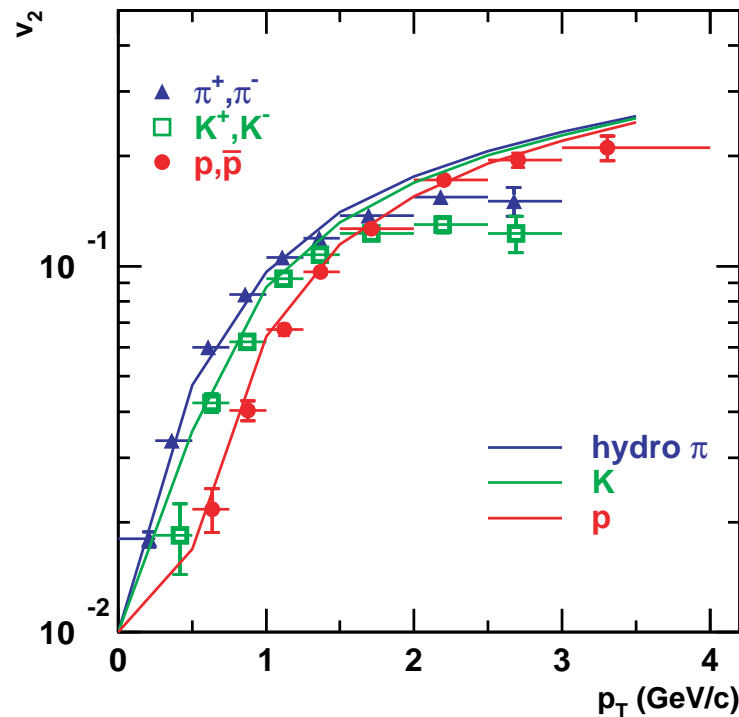
- ◆ Initially, the momentum distribution of particles is isotropic in the transverse plane, because their production comes from **local partonic interactions**
- ◆ If these particles were escaping freely, the distribution would remain isotropic at all times
- ◆ If the system has a small mean free path, pressure gradients are anisotropic and induce an anisotropy of the distribution

Collective flow and ideal hydrodynamics

- Observable: 2nd harmonic of the azimuthal distribution

$$dN/d\varphi \sim 1 + 2v_1 \cos(\varphi) + 2v_2 \cos(2\varphi) + \dots$$

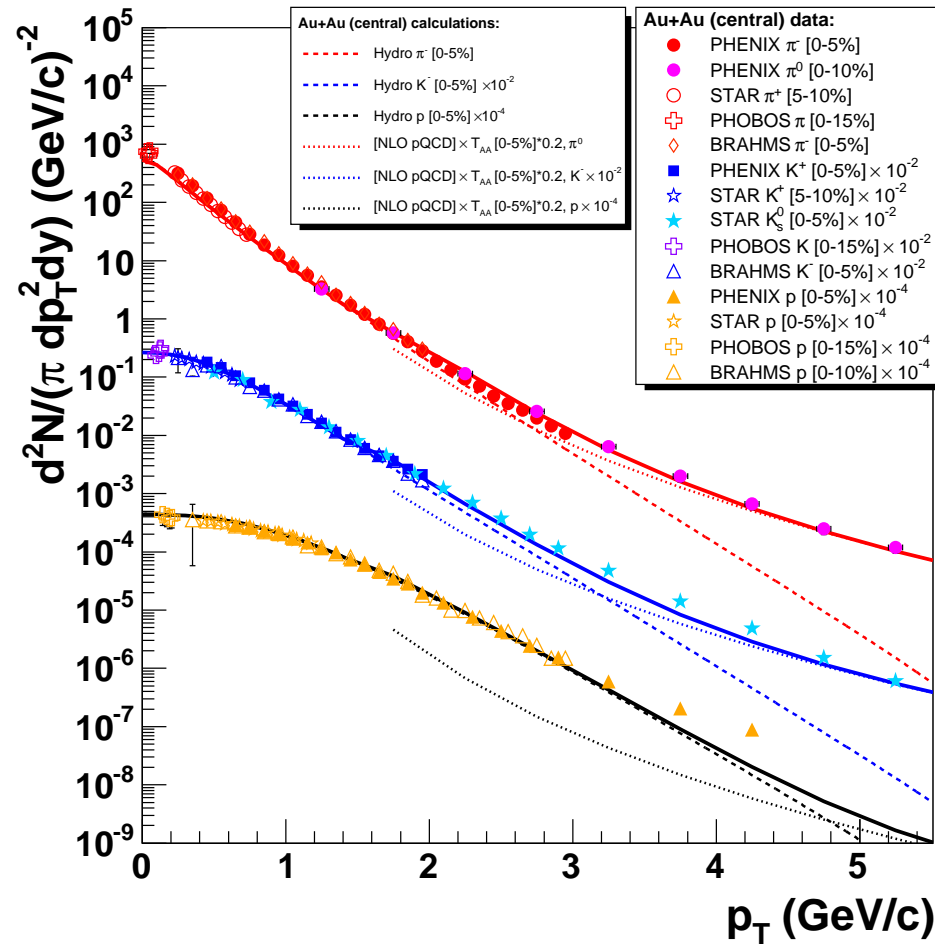
- ▷ v_2 measures the ellipticity of the momentum distribution



- Note : even heavy quarks seem to follow this flow

Another success of hydrodynamics

- Hydrodynamics reproduces the hadron spectra at low p_{\perp}





Is the QGP a perfect fluid?

RHIC results on flow

● Collective flow

● Is the QGP a perfect fluid?

AdS/CFT duality and the sQGP

Ab initio perturbative approach

Summary

- Note: a **perfect fluid** is a fluid with a **very small viscosity**, that can be described with Euler equations (**ideal hydrodynamics**)
- The elliptic flow coefficient v_2 measured at RHIC **is well reproduced by ideal hydrodynamics**, that has no viscosity
 - ◆ In hydrodynamics, **the relevant parameter is the dimensionless ratio η/s** of the shear viscosity to the entropy density
 - ◆ It has been concluded from there that **the QGP must have a very small ratio η/s**
- In the weakly coupled QGP, η/s is all but small...



RHIC results on flow

AdS/CFT duality and the sQGP

- Weak coupling viscosity
- Uncertainty bound on η/s
- Viscosity in SUSY Yang-Mills
- Limitations of AdS/CFT

Ab initio perturbative approach

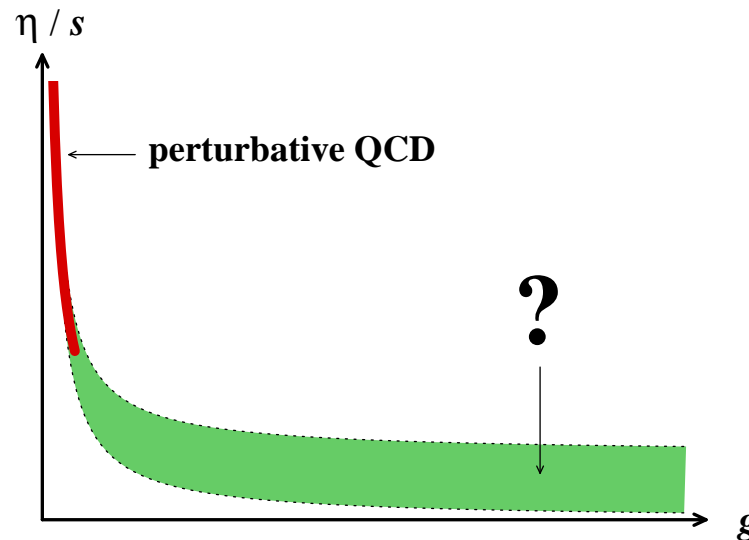
Summary

AdS/CFT duality and the sQGP

Weak coupling viscosity

- The shear viscosity has been calculated in QCD at weak coupling ($g \rightarrow 0$), and it is quite large :

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln\left(\frac{2.42}{g}\right)}$$



- However, η/s decreases quickly when the coupling increases \triangleright one way to have a small viscosity is to have a large coupling. Problem : how to calculate it?

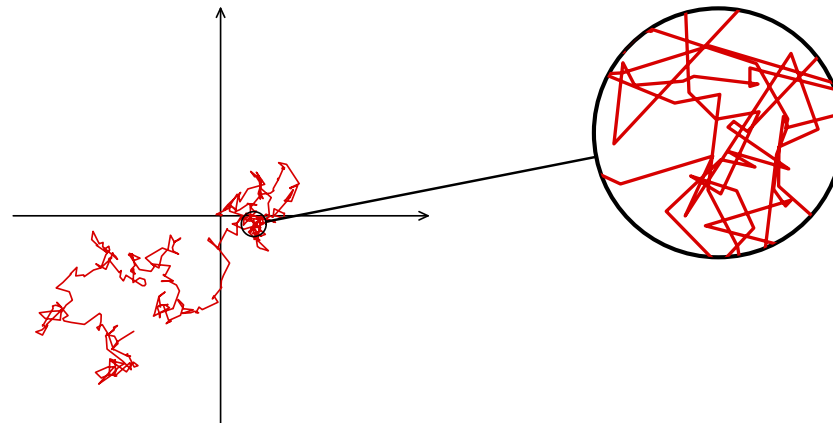
Uncertainty bound on η/s

- $\eta \sim \lambda \epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}$$

energy per particle

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every λ_{Broglie} at most :



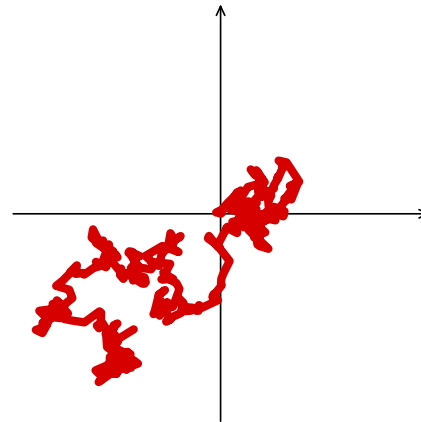
Uncertainty bound on η/s

- $\eta \sim \lambda \epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}$$

energy per particle

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every λ_{Broglie} at most :



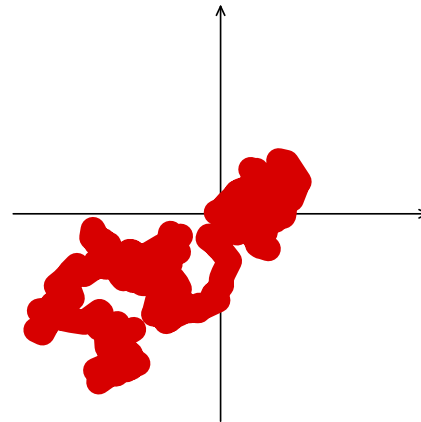
Uncertainty bound on η/s

- $\eta \sim \lambda \epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}$$

energy per particle

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every λ_{Broglie} at most :



- Hence, $\frac{\eta}{s} \geq \mathcal{O}(1)$



AdS/CFT duality at T=0

RHIC results on flow

AdS/CFT duality and the sQGP

- Weak coupling viscosity
- Uncertainty bound on eta/s
- Viscosity in SUSY Yang-Mills
- Limitations of AdS/CFT

Ab initio perturbative approach

Summary

- In QCD, we cannot compute the strong coupling limit
- **Maximally super-symmetric $SU(N)$ Yang-Mills theories** in the limit $g^2 N \rightarrow +\infty$ are **dual to classical super-gravity** on an $AdS_5 \times S_5$ manifold with metric

$$ds^2 = \frac{R^2}{z^2} \underbrace{(-dt^2 + d\vec{x}^2)}_{\text{we live here... (at } z=0)} + R^2 d\Omega_5^2$$

we live here... (at $z=0$)

- If an operator \mathcal{O} of our world is coupled on the boundary to a field ϕ that lives in the bulk, the duality states that :

$$e^{-S_{\text{cl}}[\phi]} = \langle e^{\int_{\text{boundary}} \mathcal{O} \phi(z=0)} \rangle$$

- ◆ The right hand side is a generating functional for the correlators of operators \mathcal{O} in the 4-dim super Yang-Mills theory
- ◆ The left hand side is calculable in the gravity dual (solve the classical EOM for ϕ with the boundary condition $\phi(z=0)$)

AdS/CFT duality at high T

RHIC results on flow

AdS/CFT duality and the sQGP

- Weak coupling viscosity
- Uncertainty bound on eta/s
- Viscosity in SUSY Yang-Mills
- Limitations of AdS/CFT

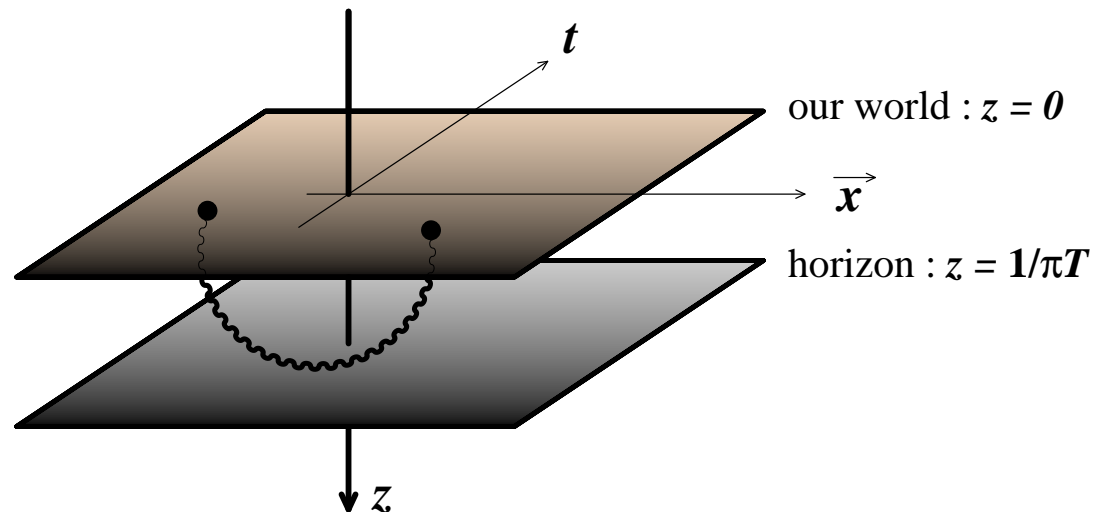
Ab initio perturbative approach

Summary

- At finite temperature T :

$$-dt^2 + dz^2 \rightarrow -f(z)dt^2 + dz^2/f(z) \quad \text{with} \quad f(z) = 1 - (\pi z T)^4$$

- $f(z) = 0$ at $z = 1/\pi T \Rightarrow$ **black hole horizon**



- Ordinary particles in 4-dimensions are the end points of open strings living in the bulk



Viscosity in SUSY Yang-Mills

RHIC results on flow

AdS/CFT duality and the sQGP

- Weak coupling viscosity
- Uncertainty bound on η/s
- Viscosity in SUSY Yang-Mills
- Limitations of AdS/CFT

Ab initio perturbative approach

Summary

- The shear viscosity can be obtained from correlations of the energy-momentum tensor :

$$\eta \propto \int dt d^3 \vec{x} \left\langle T_{xy}(t, \vec{x}) T_{xy}(0, \vec{0}) \right\rangle$$

(linear response theory)

- In the dual theory, T_{xy} couples to metric perturbations, i.e. to the graviton. The above correlation function is also the absorption cross-section of a graviton (of zero frequency) by the black hole. Hence :

$$\eta \propto \sigma_{\text{abs}}$$

- In the classical limit, σ_{abs} is the area of the horizon. Moreover, the area of a black-hole horizon is its entropy
- Combining everything, one obtains $\eta/s = 1/4\pi$



Viscosity in SUSY Yang-Mills

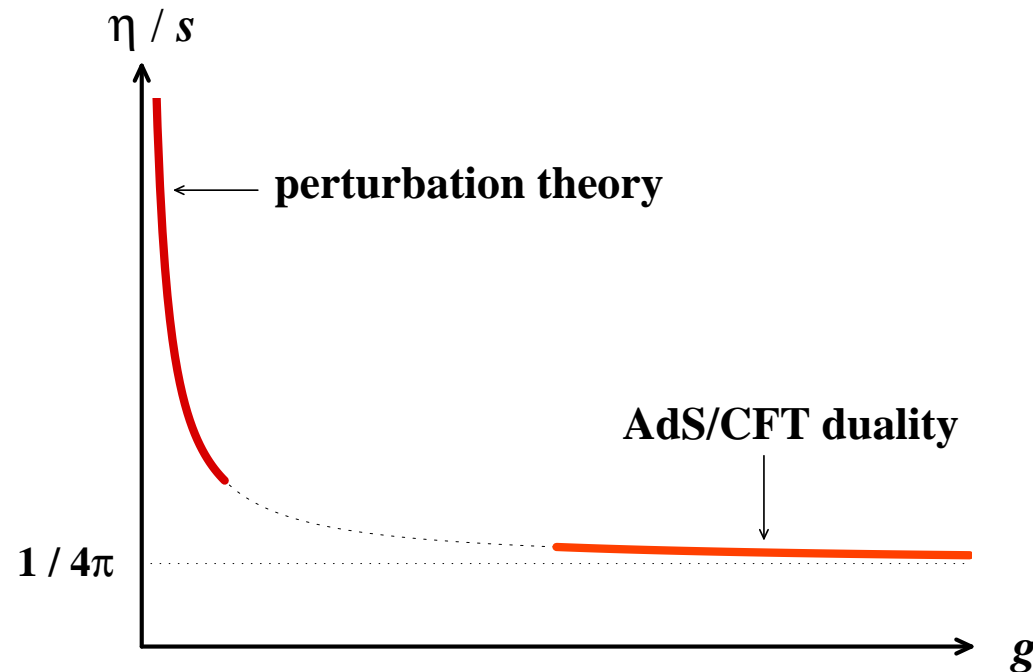
RHIC results on flow

AdS/CFT duality and the sQGP

- Weak coupling viscosity
- Uncertainty bound on η/s
- Viscosity in SUSY Yang-Mills
- Limitations of AdS/CFT

Ab initio perturbative approach

Summary



- Conjecture : $1/4\pi$ is the lowest possible value for η/s
- Note: all the known substances have a viscosity to entropy ratio (much) larger than the bound
 - ▷ led to the idea that the QGP may be the “most perfect fluid”



Caveats of AdS/CFT: SUSY YM \neq QCD

RHIC results on flow

AdS/CFT duality and the sQGP

- Weak coupling viscosity
- Uncertainty bound on η/s
- Viscosity in SUSY Yang-Mills
- Limitations of AdS/CFT

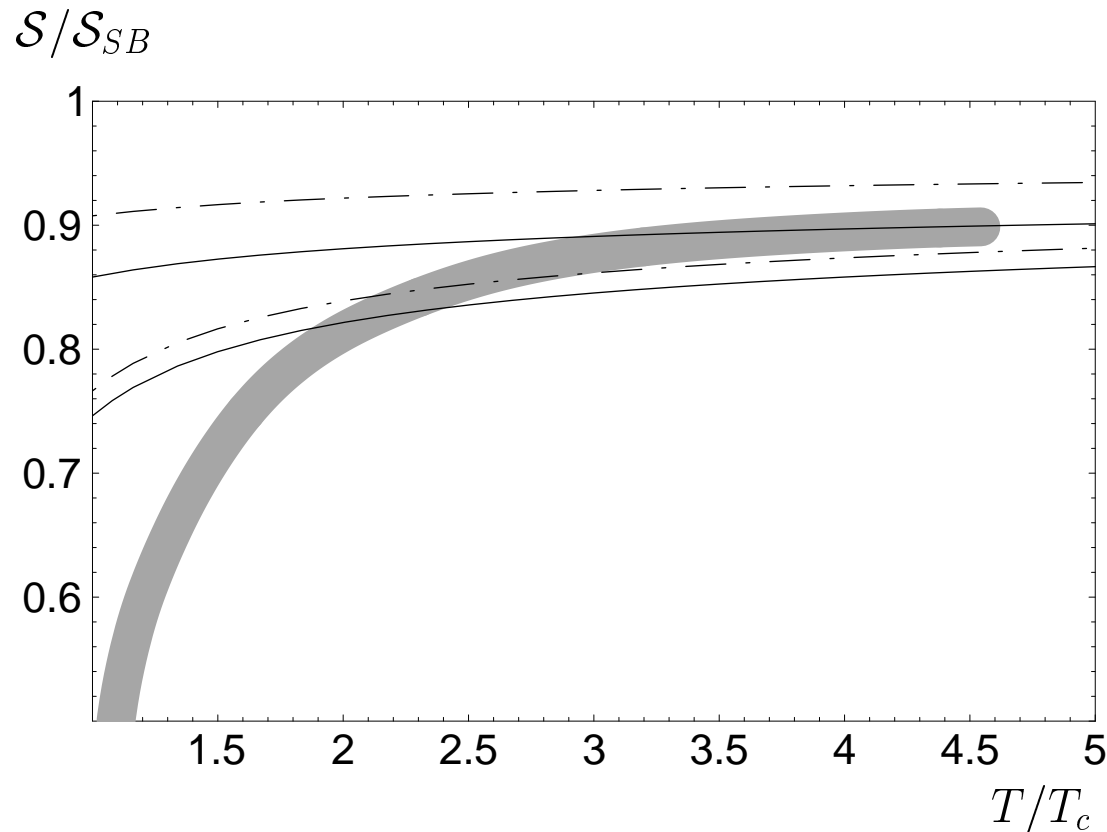
Ab initio perturbative approach

Summary

- AdS/CFT only applies to maximally super-symmetric Yang-Mills theories. Such theories are **scale invariant**, have **no running coupling**, **no chiral symmetry breaking**, and **no confinement**
- Whether what we learn about these theories is accurate for QCD (that has broken scale invariance, running coupling, chiral symmetry breaking, confinement, and quite different matter fields...) is at best a wishful thinking
- **Nevertheless an interesting playground in order to realize how wrong one's weak coupling prejudices may be...**
- Note : in the strong coupling limit of any sensible field theory, η/s is probably close to the uncertainty principle limit

Caveats of AdS/CFT: is g really large?

- There are some dissenting views about whether the physics of the QGP at $T/T_{\text{crit}} \sim 2 - 3$ is really strongly coupled. For quantities such as the entropy, perturbative techniques (+resummations) lead to accurate results in this region
[Blaizot, Iancu, Rebhan \(1999-2000\)](#)





RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η 's in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

Ab initio perturbative approach

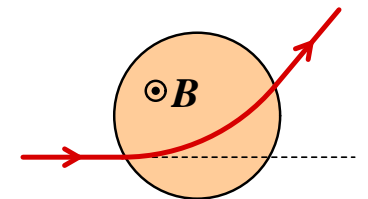
Small η/s in weak coupling ?

Asakawa, Bass, Muller (2006)

- Assume that $\alpha_s = \frac{g^2}{4\pi} \ll 1$
- Consider a domain of size Q_s^{-1} , in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain. The Lorenz force deflects its trajectory by an angle of order unity :

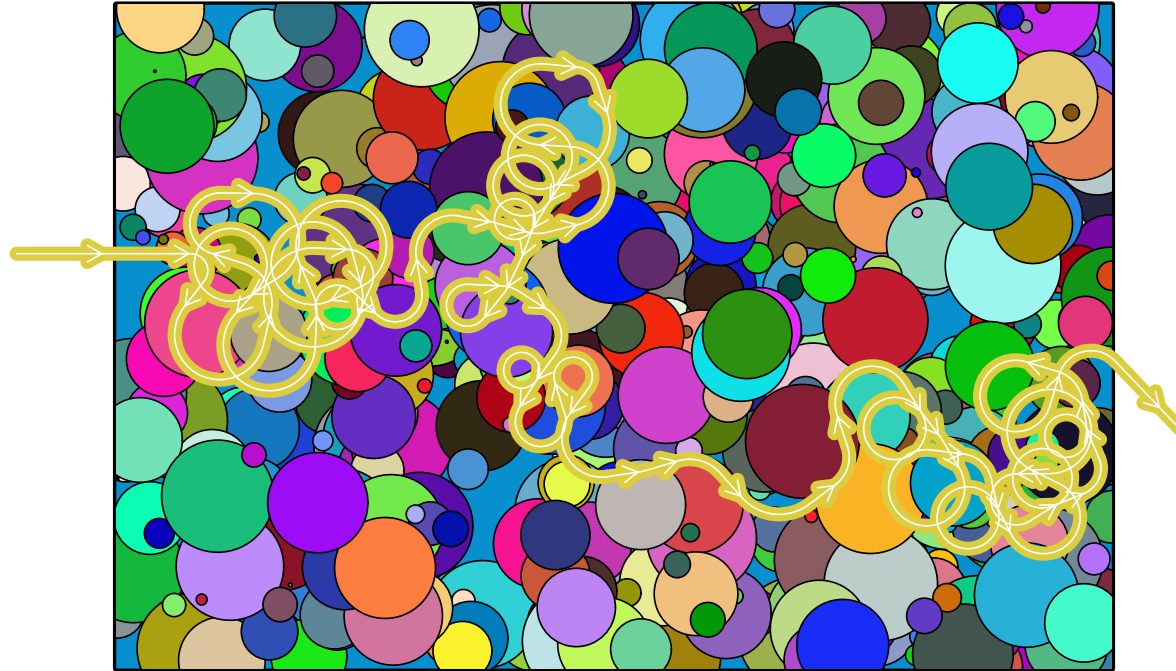
$$\frac{d\vec{p}}{dt} = g \vec{v} \times \vec{B} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

$$\text{time spent in the domain : } \delta\tau \sim Q_s^{-1}$$



Small η/s in weak coupling ?

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



- ▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1} , i.e. as low as permitted by the uncertainty principle
- ▷ η/s must be close to the lower bound

Ab initio perturbative approach

RHIC results on flow

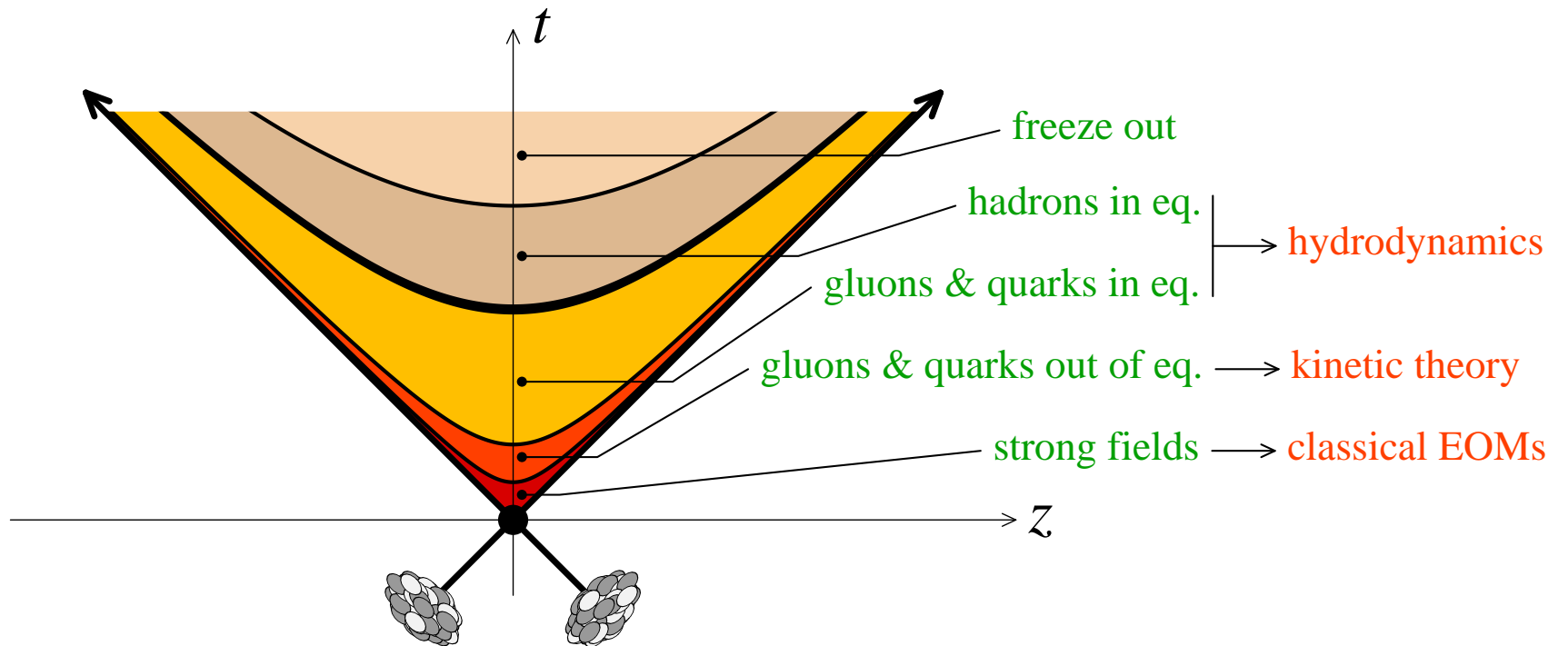
AdS/CFT duality and the sQGP

Ab initio perturbative approach

● Small η 's in weak coupling

- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



- Start from the beginning with perturbative QCD, and see whether large random magnetic fields are produced

Initial state: gluon saturation

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

● Small eta/s in weak coupling

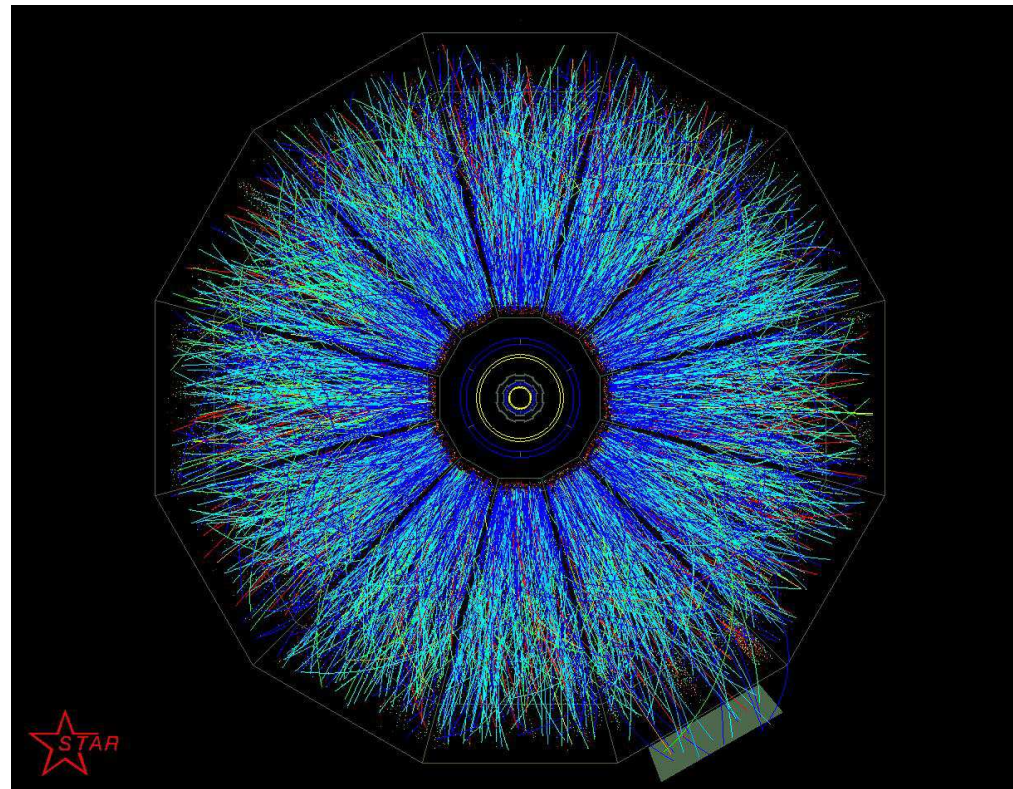
● Gluon saturation

● Initial particle production

● Initial state factorization

● Glasma instability

Summary



- 99% of the multiplicity below $p_{\perp} \sim 2$ GeV
- the bulk of of particle production comes from (very) low x
 - ▷ high gluon density (even more so in nuclei : $G_A/G_P \approx A$)



Criterion for gluon recombination

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

● Small eta/s in weak coupling

● Gluon saturation

● Initial particle production

● Initial state factorization

● Glasma instability

Summary

Gribov, Levin, Ryskin (1983)

- Number of gluons per unit area:

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

- Recombination cross-section:

$$\sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2}$$

- Recombination happens if $\rho\sigma_{gg \rightarrow g} \gtrsim 1$, i.e. $Q^2 \lesssim Q_s^2$, with:

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}}$$

Saturation domain

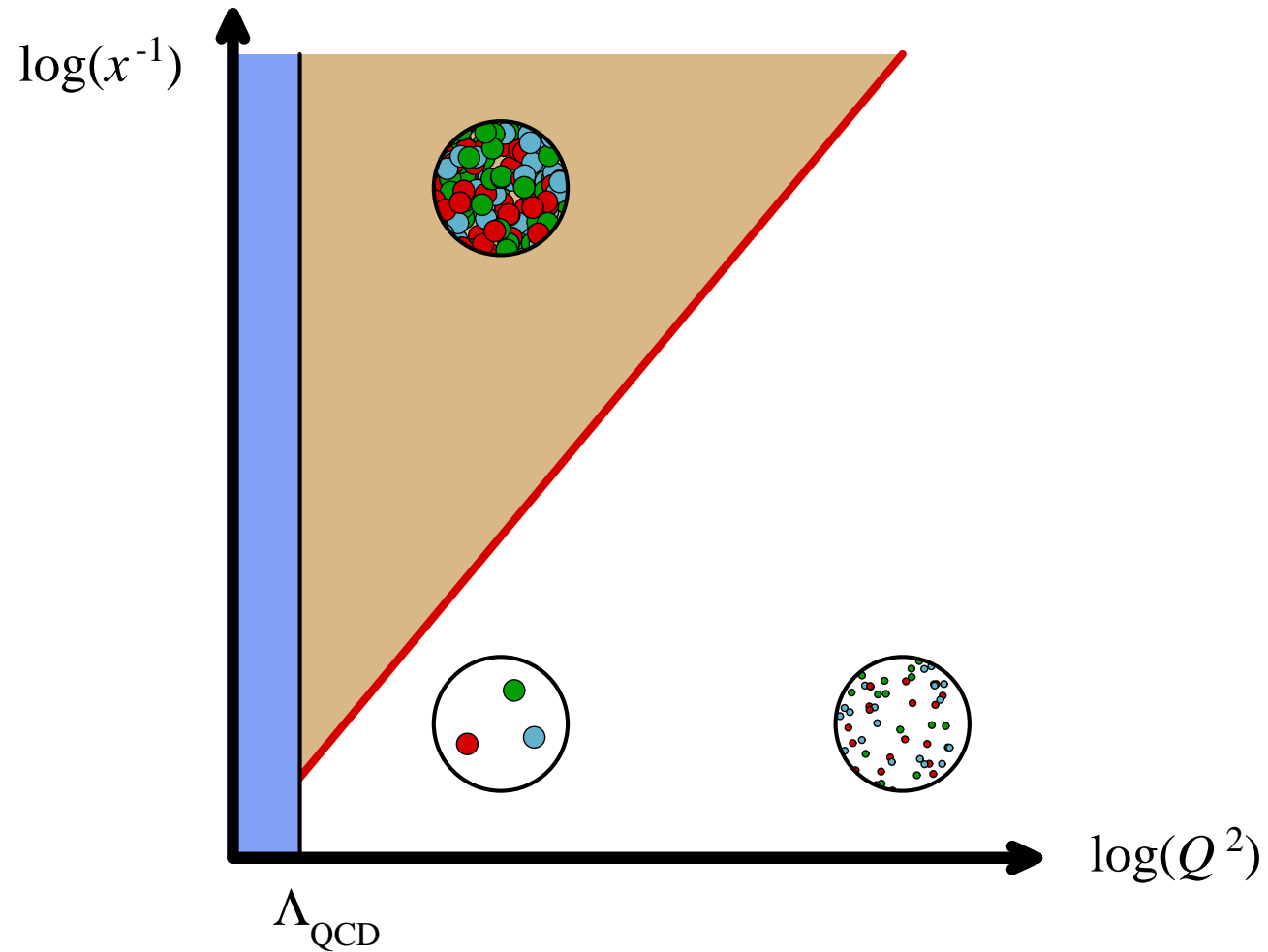
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



Saturation domain

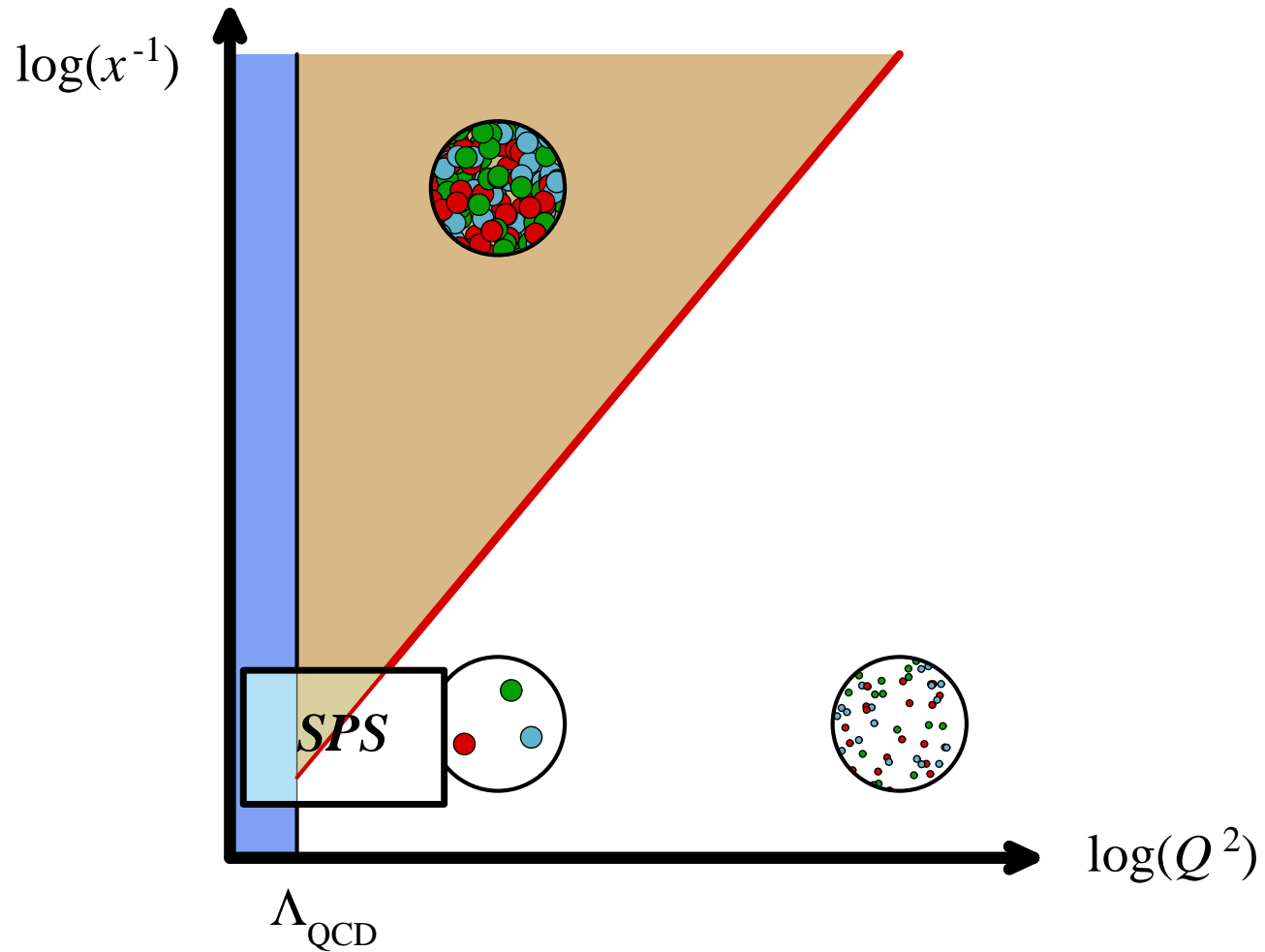
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



Saturation domain

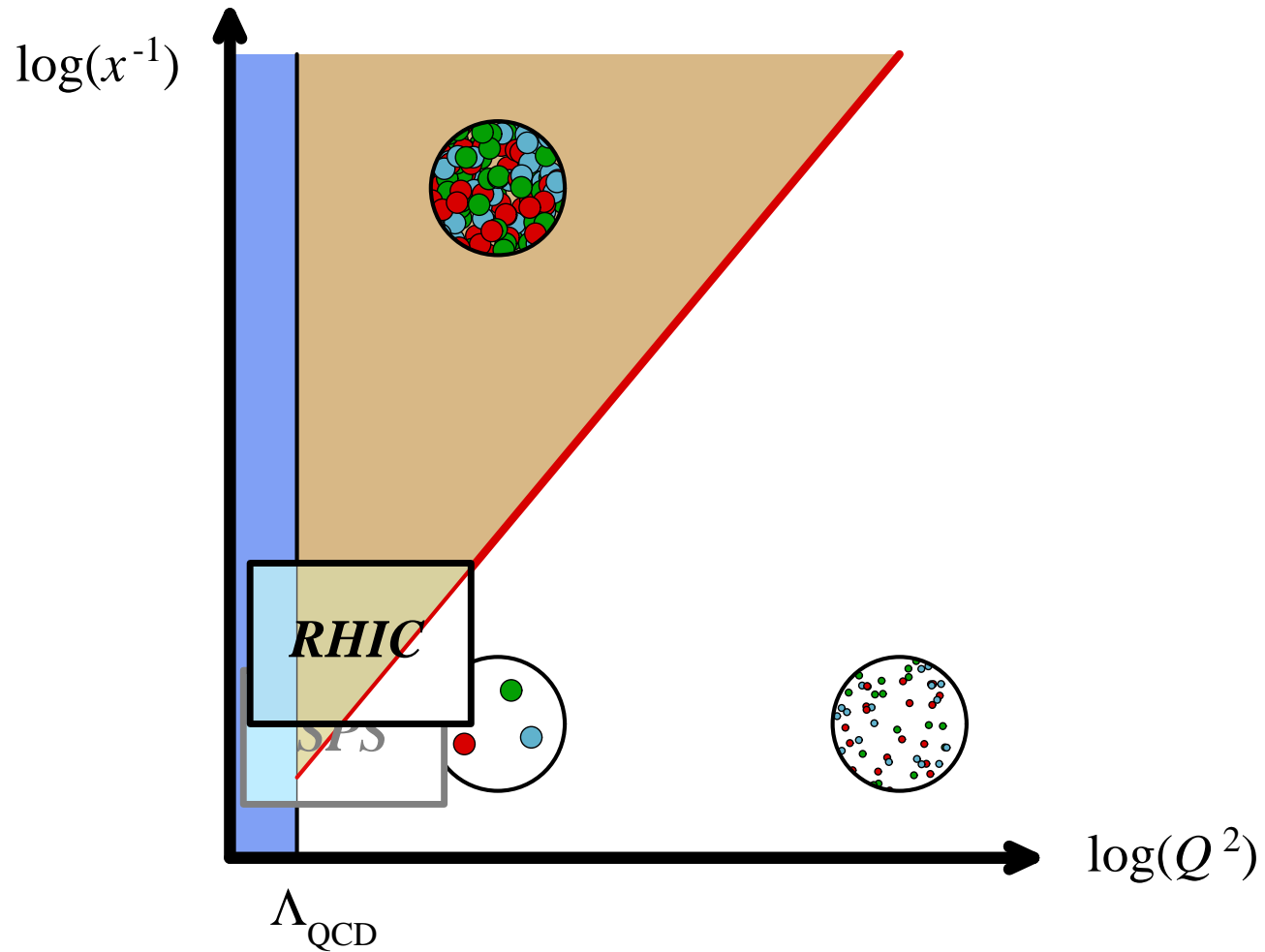
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



Saturation domain

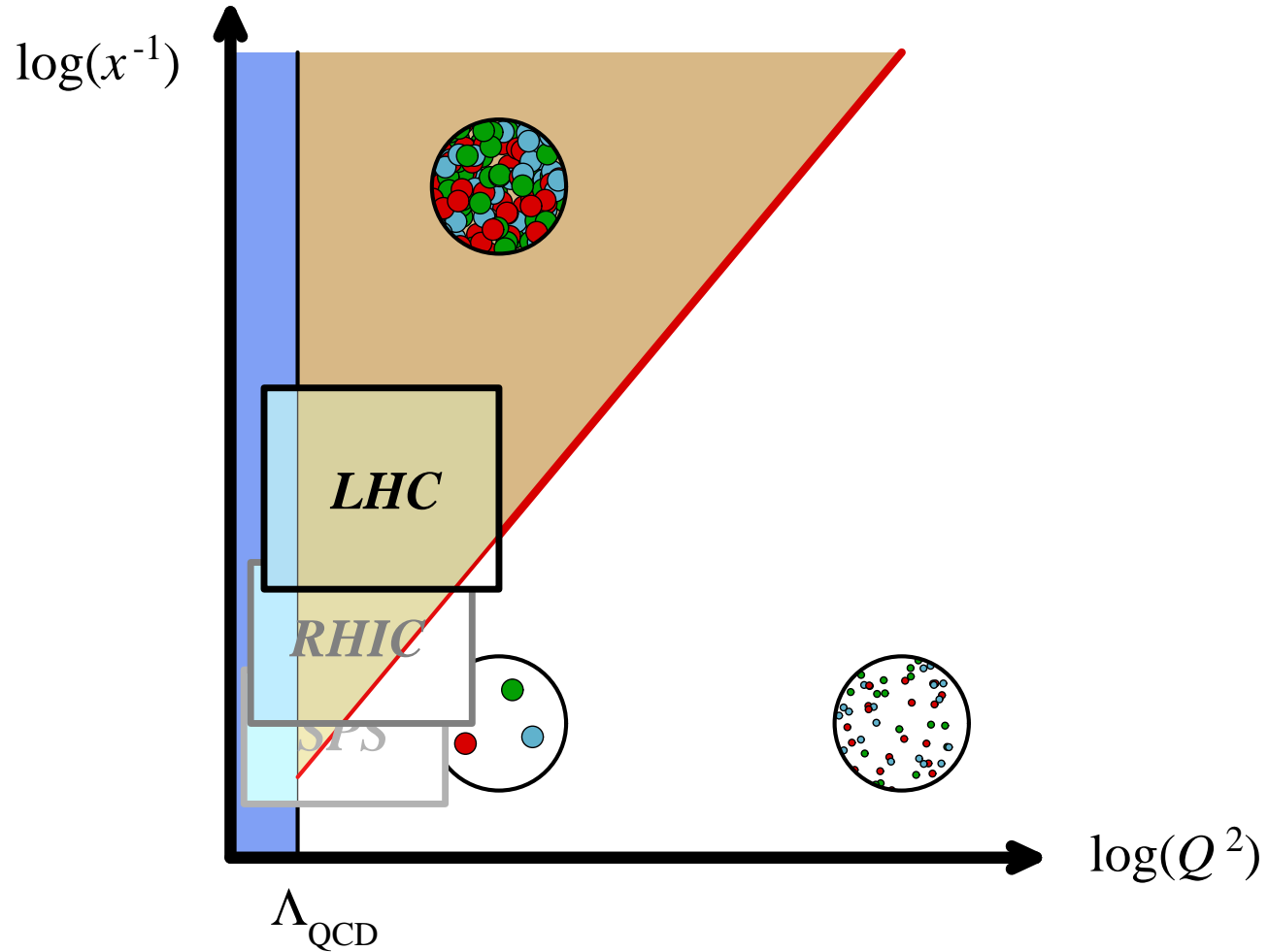
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



Saturation domain

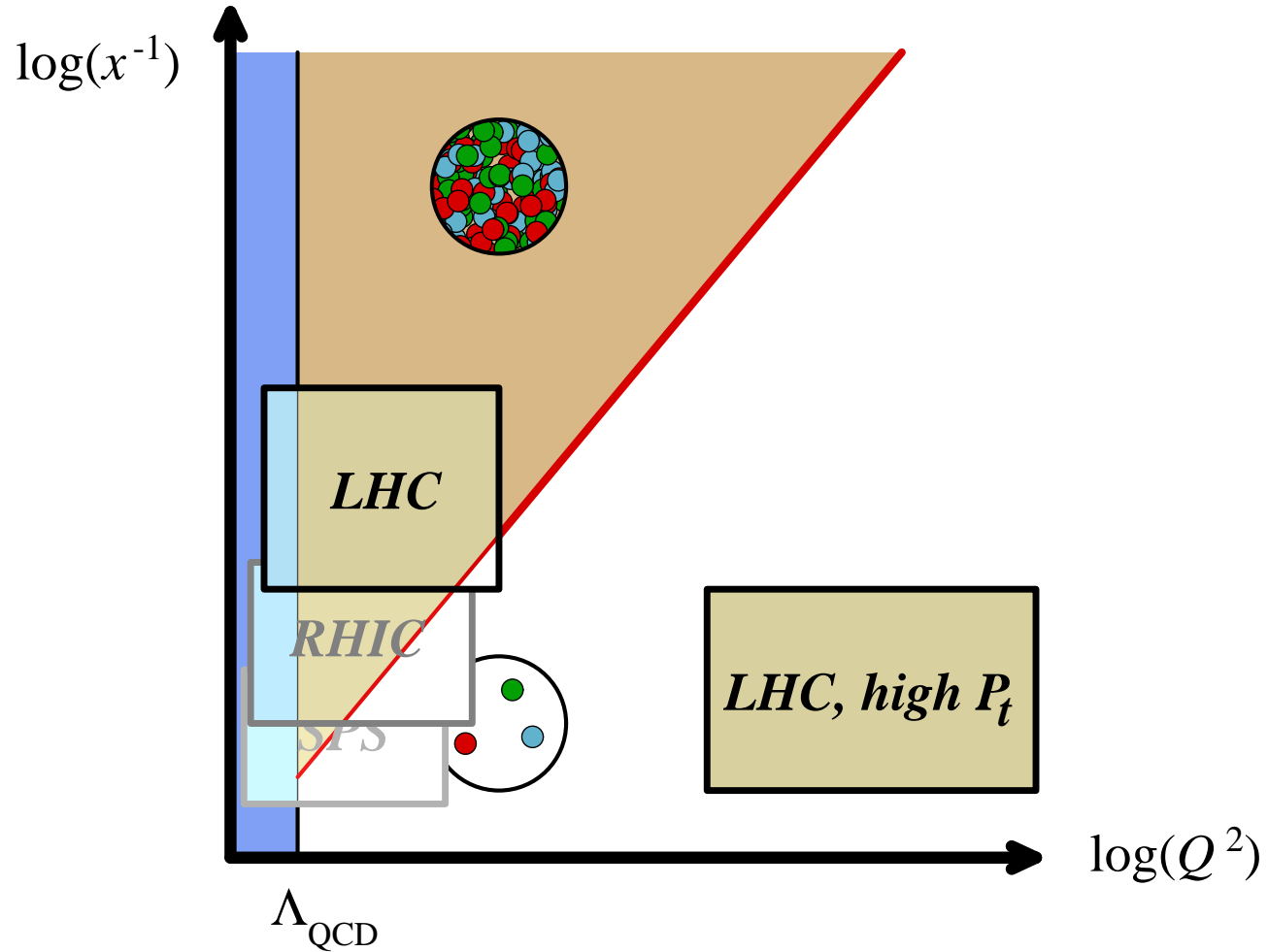
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



Initial particle production

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

- Main difficulty : the formalism for studying the collision of two densely occupied projectiles is quite involved



Initial particle production

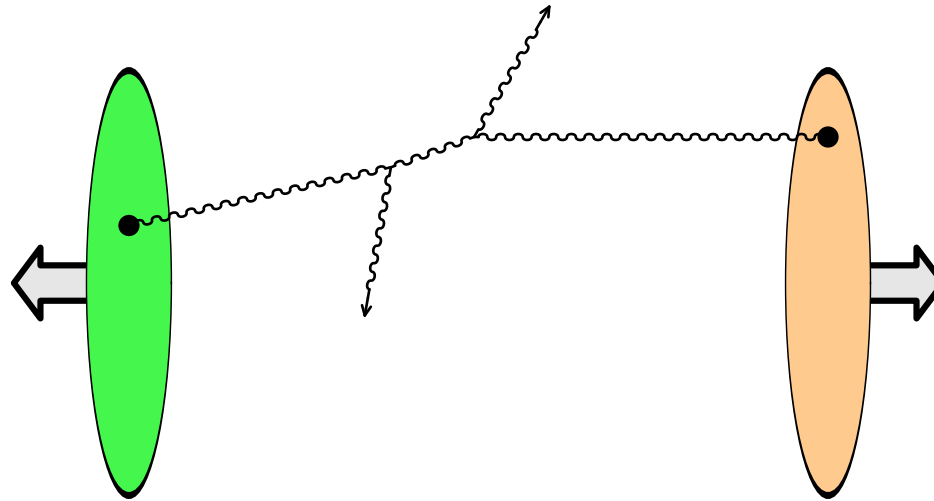
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η 's in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



- Dilute regime : one parton in each projectile interact

Initial particle production

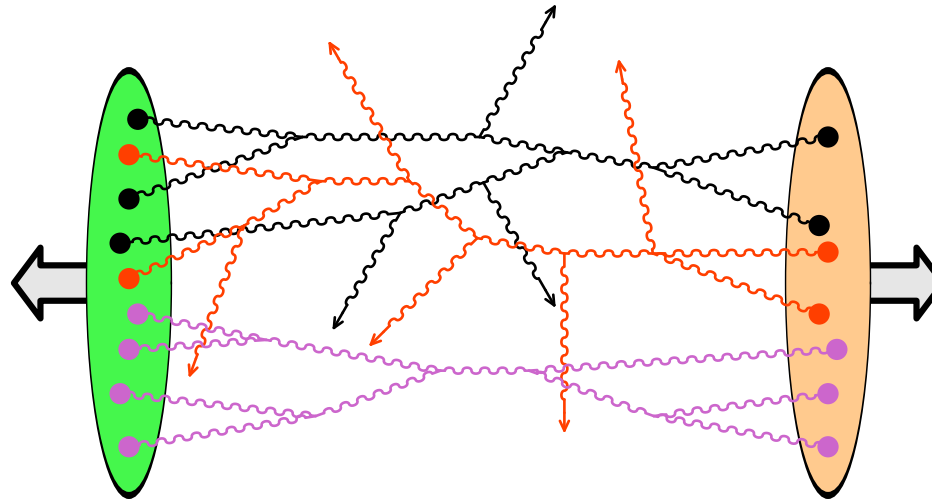
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η 's in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary



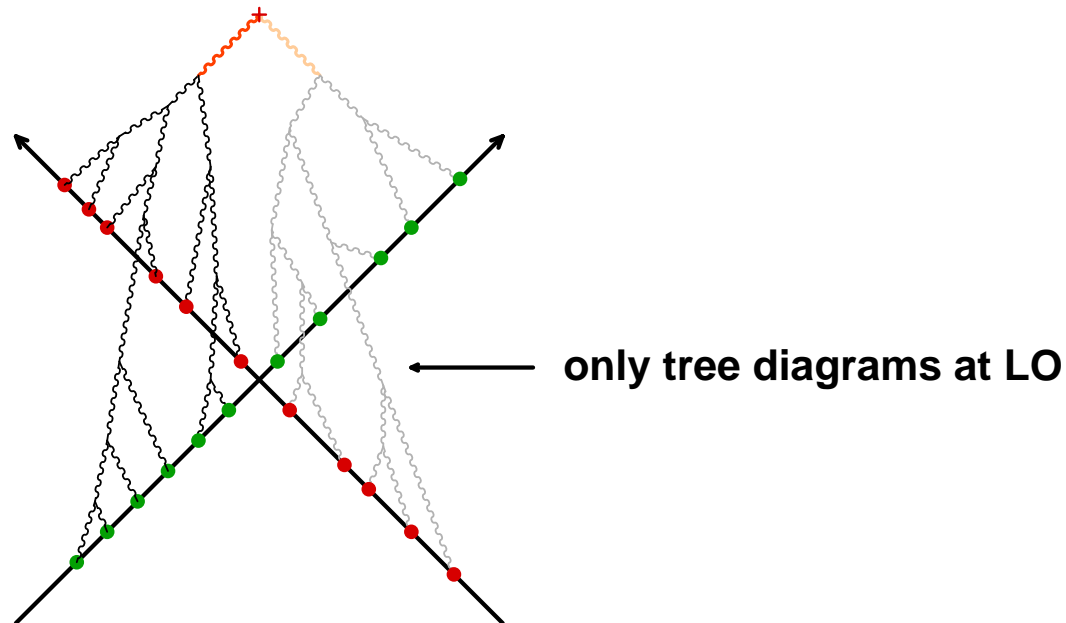
- **Dilute regime** : one parton in each projectile interact
- **Dense regime** : **multiparton processes** become crucial
+ pileup of many partonic scatterings in every AA collision

Initial particle production - LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

$$\frac{d\bar{N}_{LO}}{d^3\vec{p}} \propto \int_{x,y} e^{ip \cdot (x-y)} \dots \mathcal{A}_\mu(x) \mathcal{A}_\nu(y)$$

- $\mathcal{A}^\mu(x)$ = **classical** solution of **Yang-Mills equations** with color sources ρ_1 and ρ_2 on the light-cone



RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

Initial particle production - LO

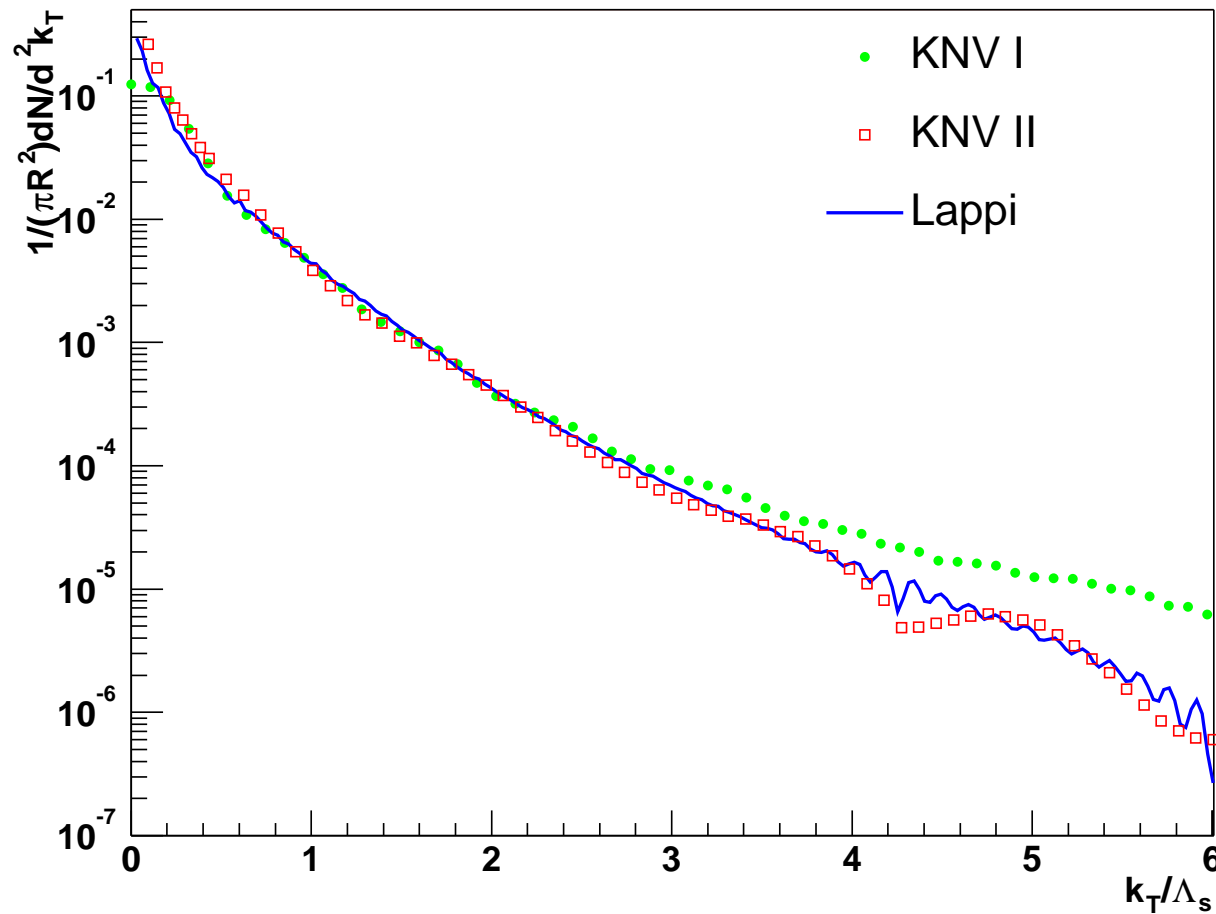
RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

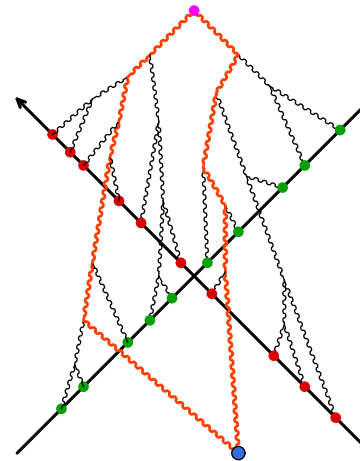


- Important softening at small k_{\perp} compared to pQCD (saturation)
- Quark production has also been computed (FG, Kajantie, Lappi (2005))

Initial particle production - NLO

FG, Venugopalan (2006), FG, Lappi, Venugopalan (work in progress)

■ Typical graph :



■ Why is it important ?

- ◆ Questions such as factorization can only be answered by looking at loop corrections
- ◆ Instabilities in the classical solutions may inflate the effect of small perturbations

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η 's in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

Initial particle production - NLO

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

- The sum of all the 1-loop contributions can be written in terms of classical fields, and small field fluctuations above the classical field
- The expressions can be rearranged in a way that clearly separates the initial and final state :

$$\delta \bar{N} = \underbrace{\left[\int_{\vec{u} \in \text{light cone}} \delta \mathcal{A}_{\text{in}}(\vec{u}) T_{\vec{u}} + \int_{\vec{u}, \vec{v} \in \text{light cone}} \frac{1}{2} \Sigma(\vec{u}, \vec{v}) T_{\vec{u}} T_{\vec{v}} \right]}_{\text{operator that depends only on the initial state}} \underbrace{\bar{N}_{LO}[\mathcal{A}_{\text{in}}]}_{\text{final state}}$$

- Notes :

\mathcal{A}_{in} = value of the classical field at $\tau = 0^+$

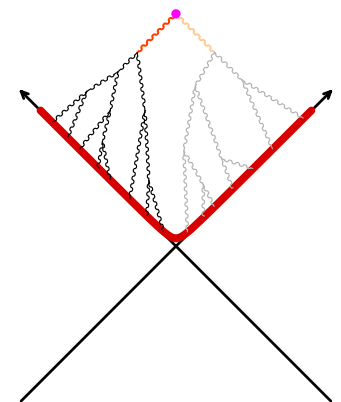
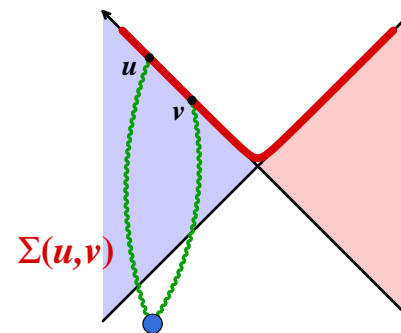
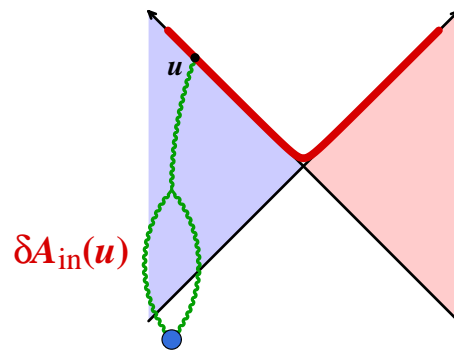
$\delta \mathcal{A}_{\text{in}}$ and Σ depend only on the dynamics before the collision

$$T_{\vec{u}} \sim \frac{\delta}{\delta \mathcal{A}_{\text{in}}(\vec{u})} \quad (\text{generator of shifts of } \mathcal{A}_{\text{in}}(\vec{u}))$$

Initial particle production - NLO

- The sum of all the 1-loop contributions can be written in terms of classical fields, and small field fluctuations above the classical field
- The expressions can be rearranged in a way that clearly separates the initial and final state :

$$\delta \bar{N} = \underbrace{\left[\int_{\vec{u} \in \text{light cone}} \delta \mathcal{A}_{\text{in}}(\vec{u}) T_{\vec{u}} + \int_{\vec{u}, \vec{v} \in \text{light cone}} \frac{1}{2} \Sigma(\vec{u}, \vec{v}) T_{\vec{u}} T_{\vec{v}} \right]}_{\text{operator that depends only on the initial state}} \underbrace{\bar{N}_{LO}[\mathcal{A}_{\text{in}}]}_{\text{final state}}$$





Initial state factorization

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

- The coefficients $\delta\mathcal{A}_{\text{in}}$ and Σ in the initial state factor contain divergences, that manifest themselves as large logarithms $\log(1/x_{1,2})$
- These large logs invalidate the naive perturbative expansion, because $\alpha_s \log(1/x_{1,2})$ may be large even if α_s is small
 - ▷ All the terms in $[\alpha_s \log(\cdot)]^n$ should be collected and resummed
- It is expected that these large logs can be **factorized** in the distributions of color sources $W[\rho_{1,2}]$:

$$\frac{d\bar{N}}{dY d^2\vec{p}_\perp} = \int \underbrace{[D\rho_1] [D\rho_2] W_{Y_{\text{beam}-Y}}[\rho_1] W_{Y_{\text{beam}+Y}}[\rho_2]}_{\text{projectiles source distributions}} \underbrace{\frac{d\bar{N}[\mathcal{A}_{\text{in}}(\rho_1, \rho_2)]}{dY d^2\vec{p}_\perp}}_{\text{“event-by-event” gluon spectrum}}$$

Glasma instability

RHIC results on flow

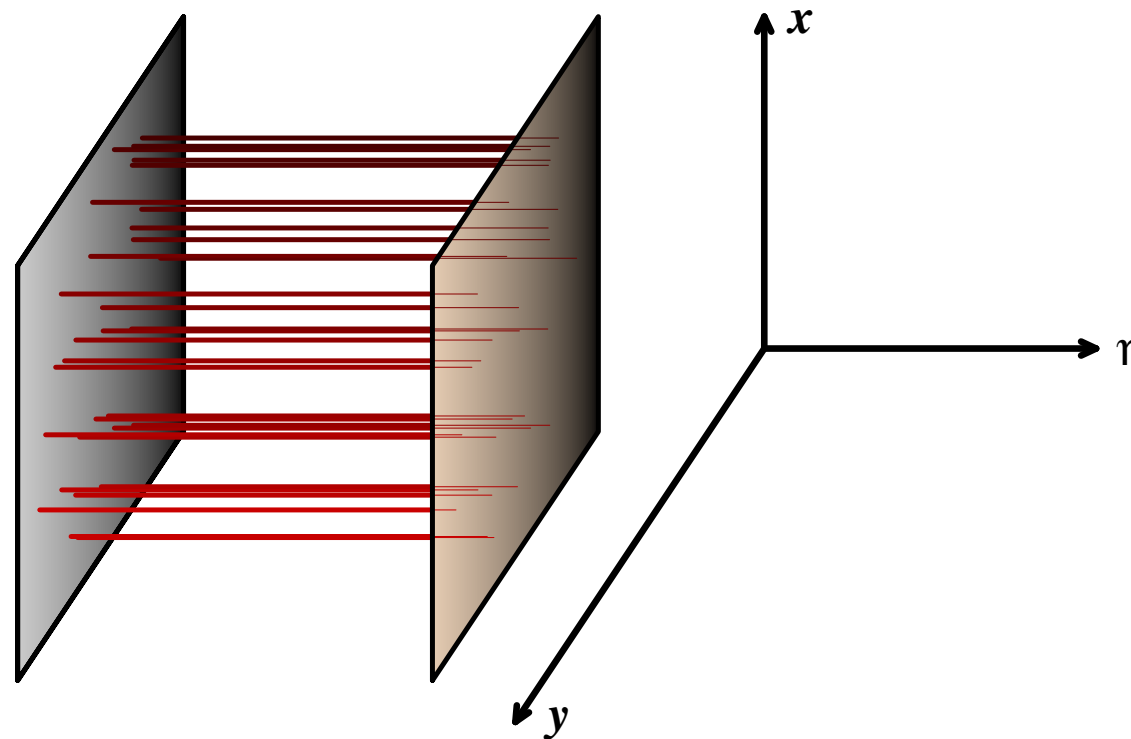
AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η 's in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

- Leading order magnetic fields at $\tau = 0^+$:



- ◆ At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal (Lappi, McLerran (2006))
- ◆ They are also boost invariant (independent of η)

Glasma instability

RHIC results on flow

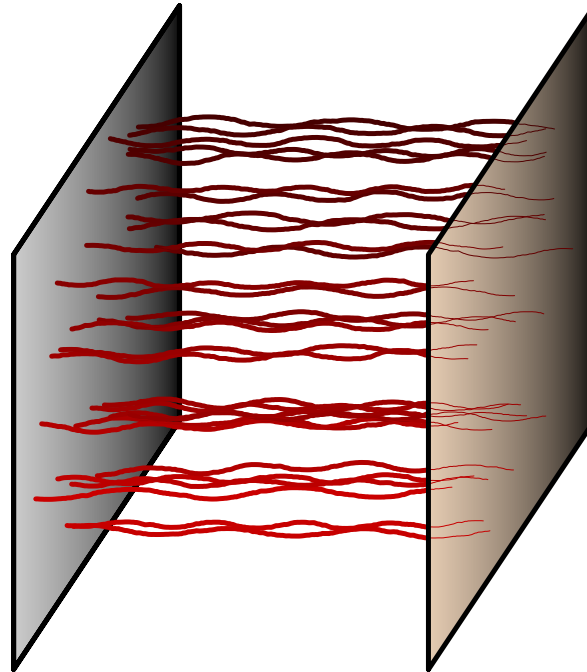
AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

- Leading order + quantum fluctuations at $\tau = 0^+$:



- ◆ Loop corrections bring quantum fluctuations in this picture
- ◆ In the weak coupling regime, they are small corrections
- ◆ The spectrum of fluctuations is encoded in $\delta\mathcal{A}_{\text{in}}$ and Σ

Glasma instability

RHIC results on flow

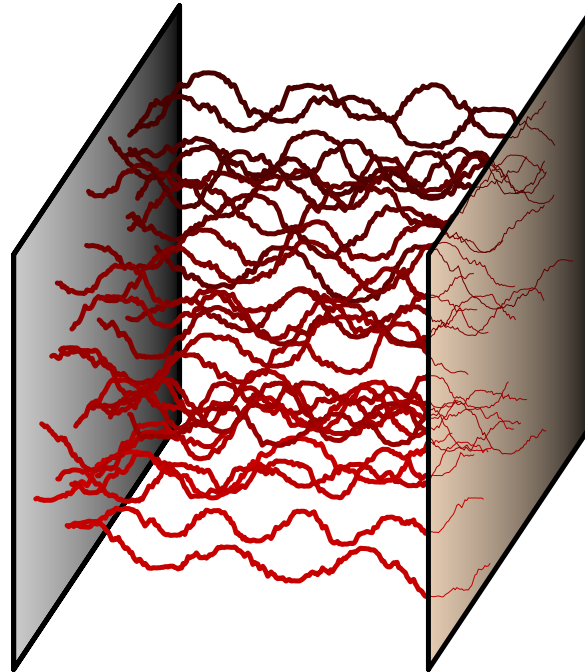
AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small η 's in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

■ Effect of the instability :



- ◆ η -dependent perturbations grow quickly in time, like $\exp(\sqrt{\mu\tau})$
- ◆ Breakdown of the CGC approach at $\tau_{\max} \sim Q_s^{-1} \ln^2(1/\alpha_s)$?
- ◆ At $\tau \sim \tau_{\max}$, one gets patches where \vec{B} is large and random



Glasma instability

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

- Small eta/s in weak coupling
- Gluon saturation
- Initial particle production
- Initial state factorization
- Glasma instability

Summary

- In order to push the CGC description beyond τ_{\max} , one must resum all the corrections in $[\alpha_s e^{\sqrt{\mu\tau}}]^n$
- This resummation amounts to add fluctuations to the color fields at $\tau = 0$. The gluon spectrum becomes :

$$\frac{d\bar{N}}{dY d^2\vec{p}_\perp} = \int [D\rho_1] [D\rho_2] W_{Y_{\text{beam}}-Y}[\rho_1] W_{Y_{\text{beam}}+Y}[\rho_2] \\ \times \int \underbrace{[Da] Z[a]}_{\text{fluctuation spectrum}} \frac{d\bar{N}[\mathcal{A}_{\text{in}}(\rho_1, \rho_2) + a]}{dY d^2\vec{p}_\perp}$$

- The spectrum of fluctuations $Z[a]$ has been calculated (Fukushima, FG, McLerran (2006)). Open questions :
 - ◆ Does the instability make the spectrum locally isotropic?
 - ◆ Does this system have a small η/s ?



RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

Summary

Summary



Summary

RHIC results on flow

AdS/CFT duality and the sQGP

Ab initio perturbative approach

Summary

- The data seems to indicate that the matter formed at RHIC has a small η/s and thermalizes early
- The uncertainty principle gives a lower bound to η/s
- In the strong coupling limit of gauge theories, η/s is close to the minimal value

For super-symmetric Yang-Mills theory, one can compute it explicitly in this limit by using the AdS/CFT correspondence

- One can also have a small η/s if the system has large random magnetic fields, even if the coupling is small

In the (perturbative) CGC framework, the Glasma instability enhances quantum noise, which leads to such magnetic fields