The Color Glass Condensate:

Forward Physics at the LHC

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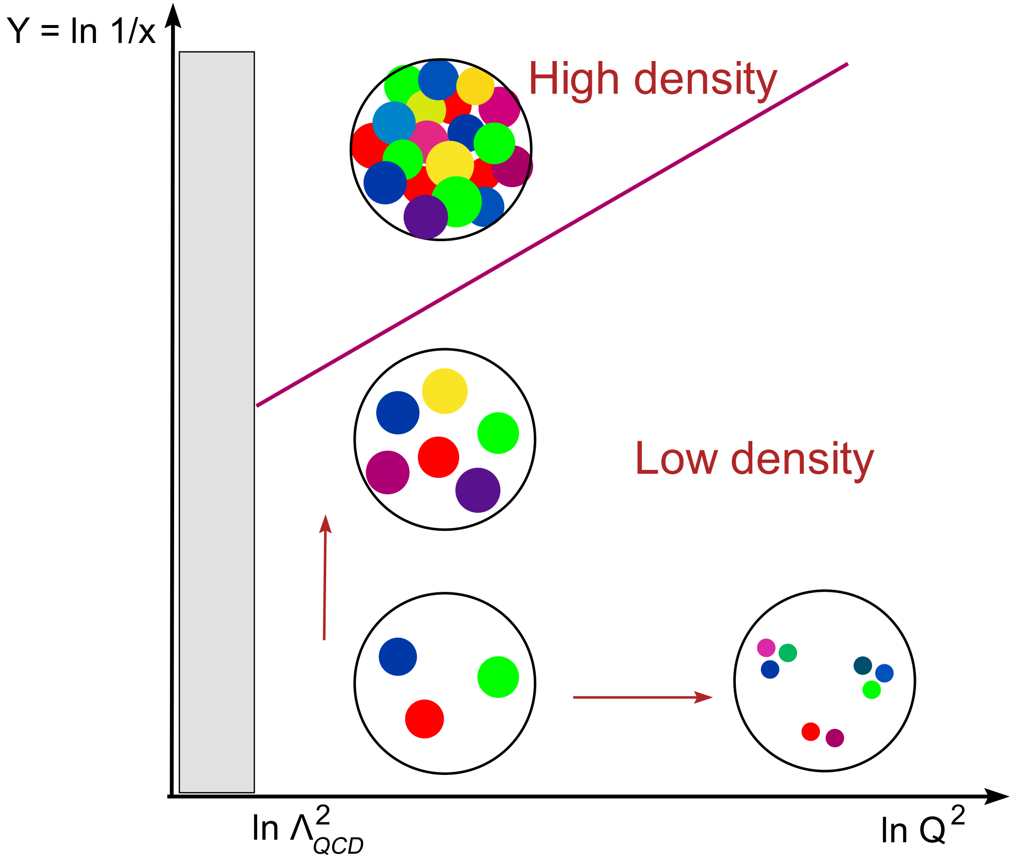
Introduction: What is the CGC ?

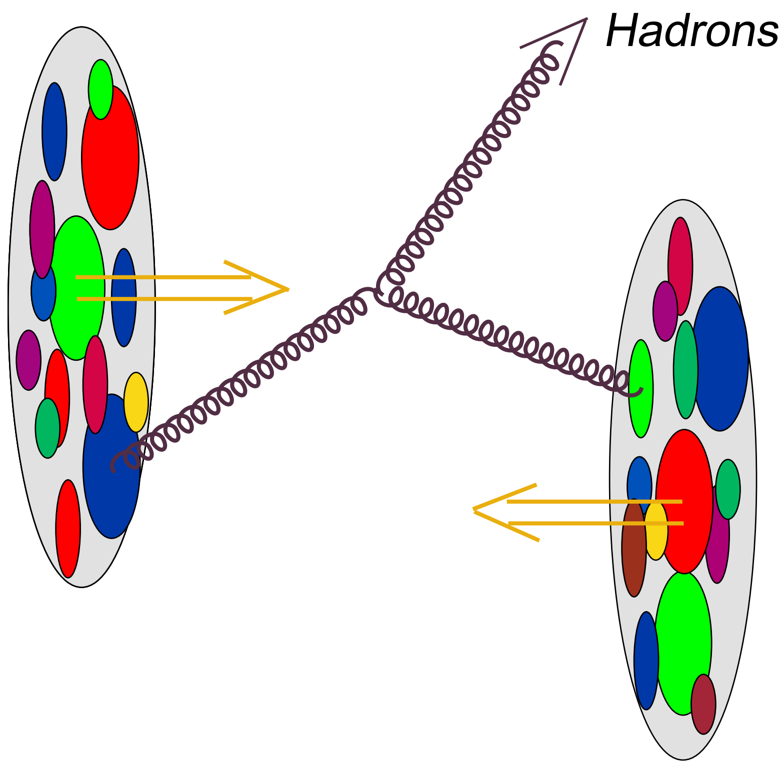
- A physical picture, and a theory (within pQCD), for the 'small–x' part of the wavefunction of an energetic hadron



- Generalization of the parton picture to high gluon density

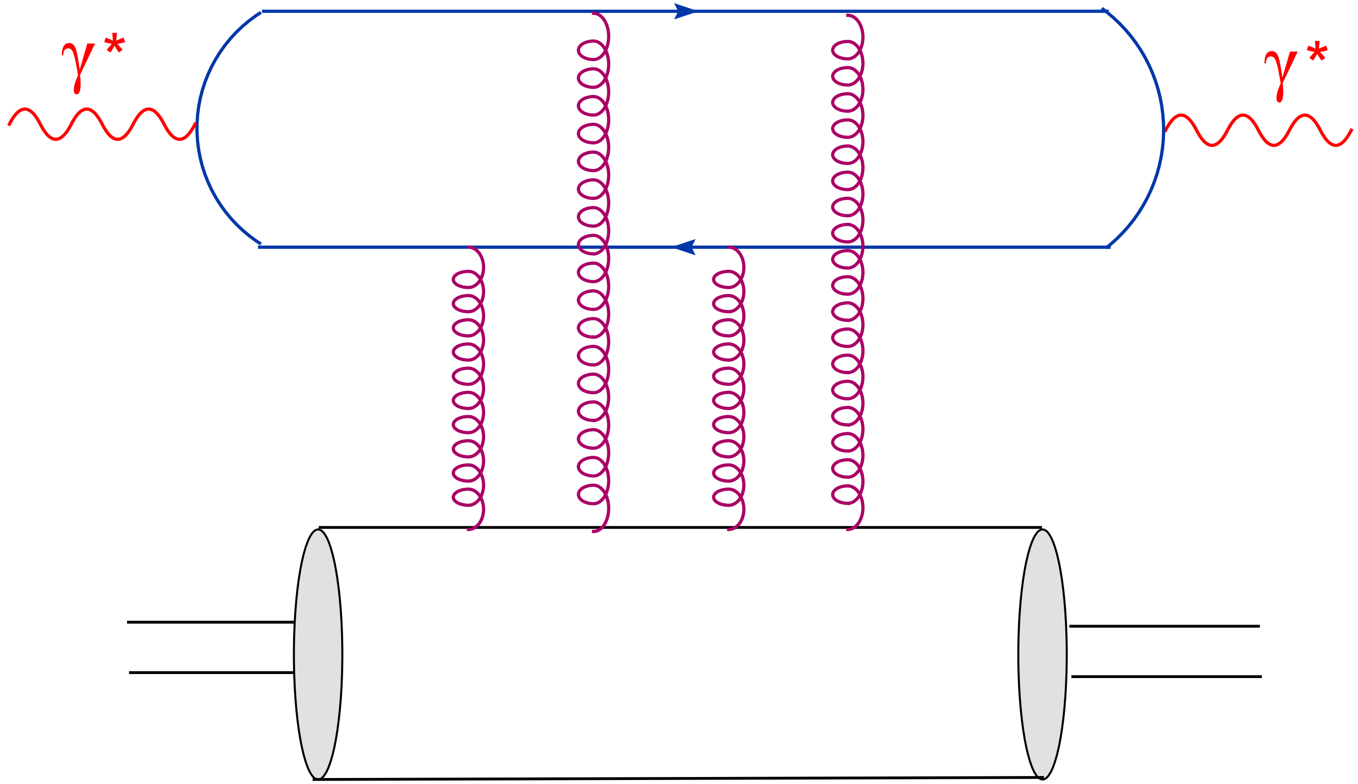
Introduction: What is the CGC ?

- The partonic (generalized) 'distributions' (including correlations) in the transverse plane prior to a collision

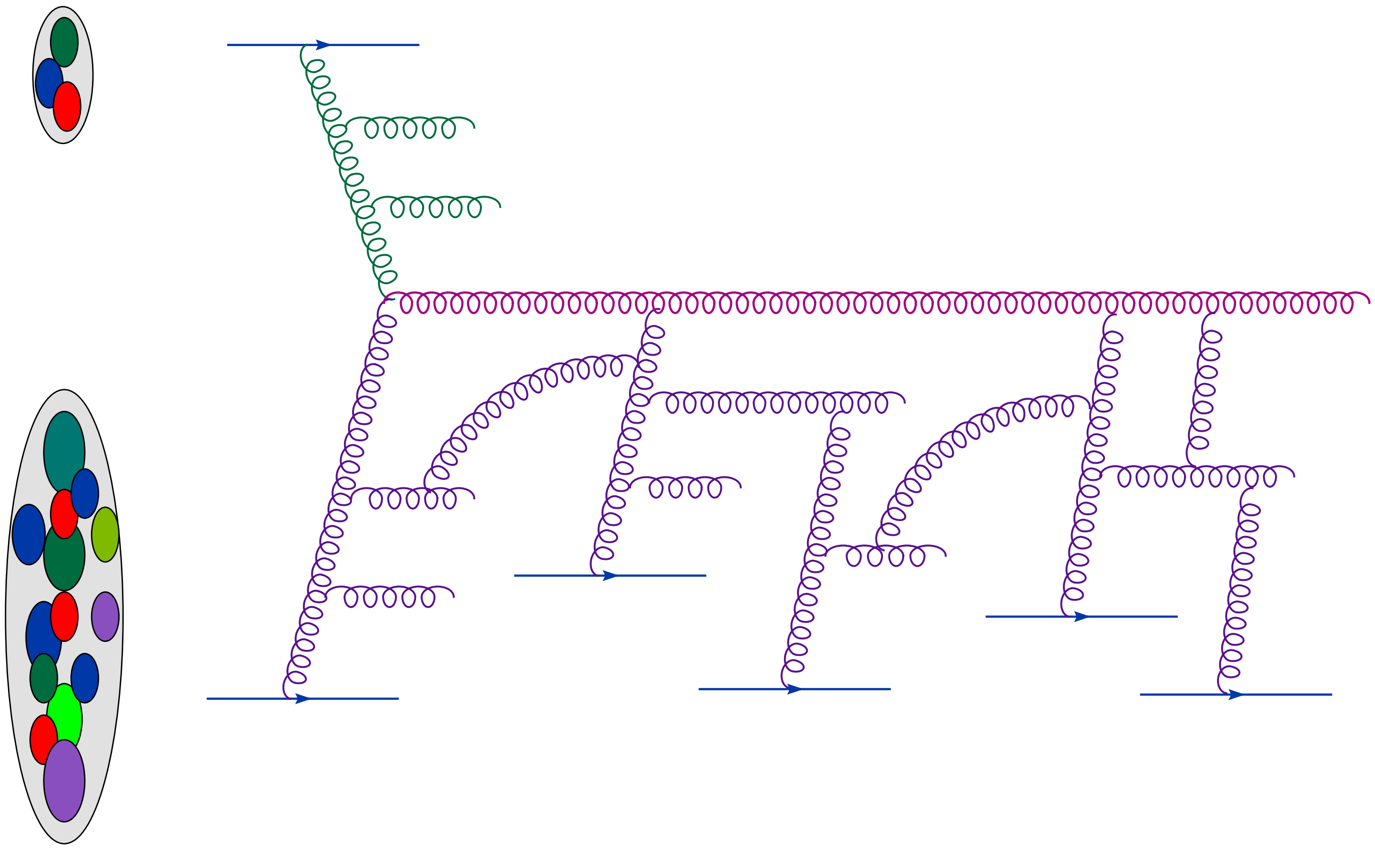


- When supplemented with a factorization prescription, it can be used to compute scattering amplitudes at high energy

Introduction: What is the CGC ?

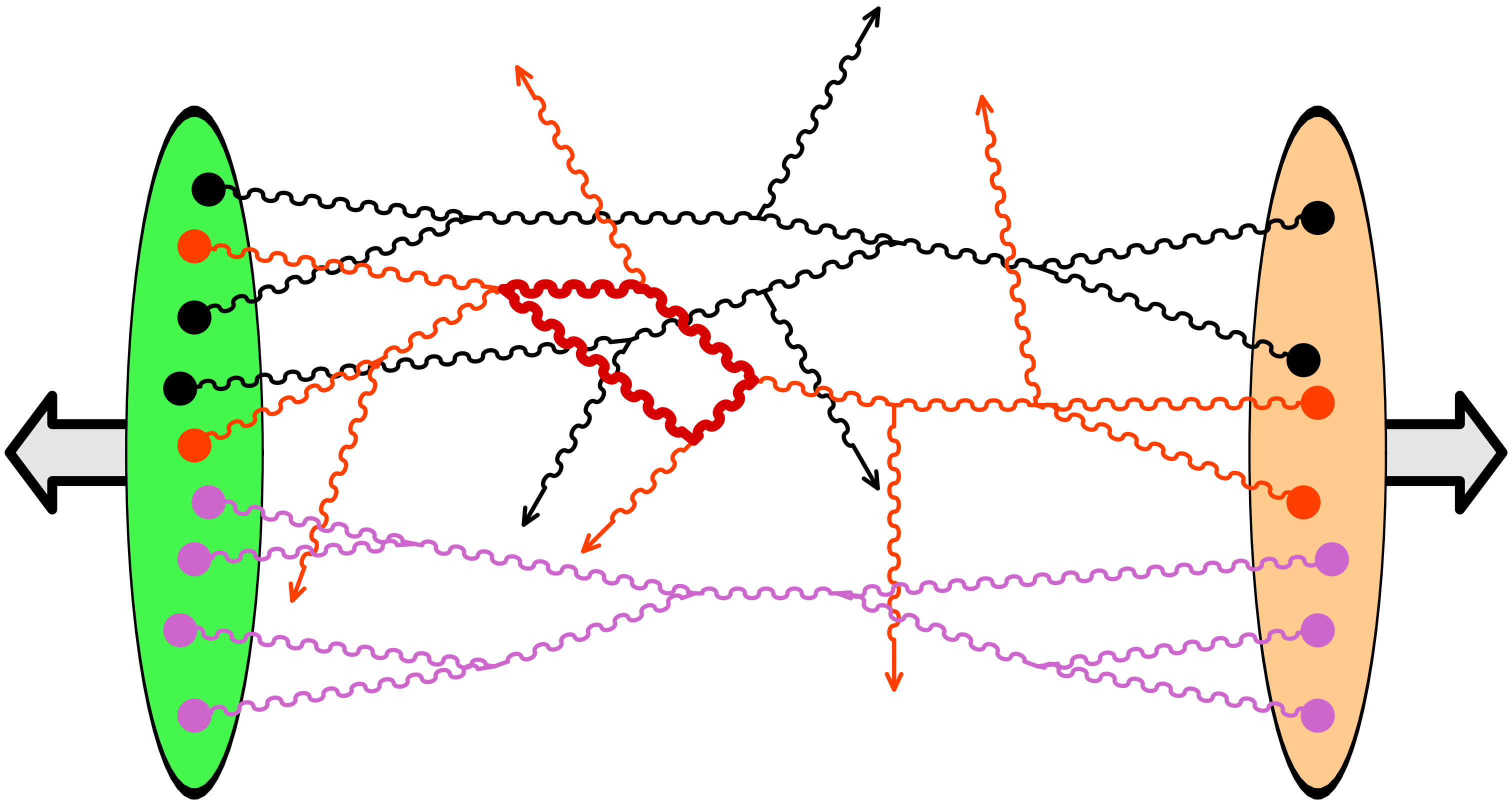


- The factorization is well under control for deep inelastic scattering (DIS) at small–x ...



- ... and also for 'dilute–dense' hadron–hadron collisions (pA or pp at forward rapidities)

Introduction: What is the CGC ?



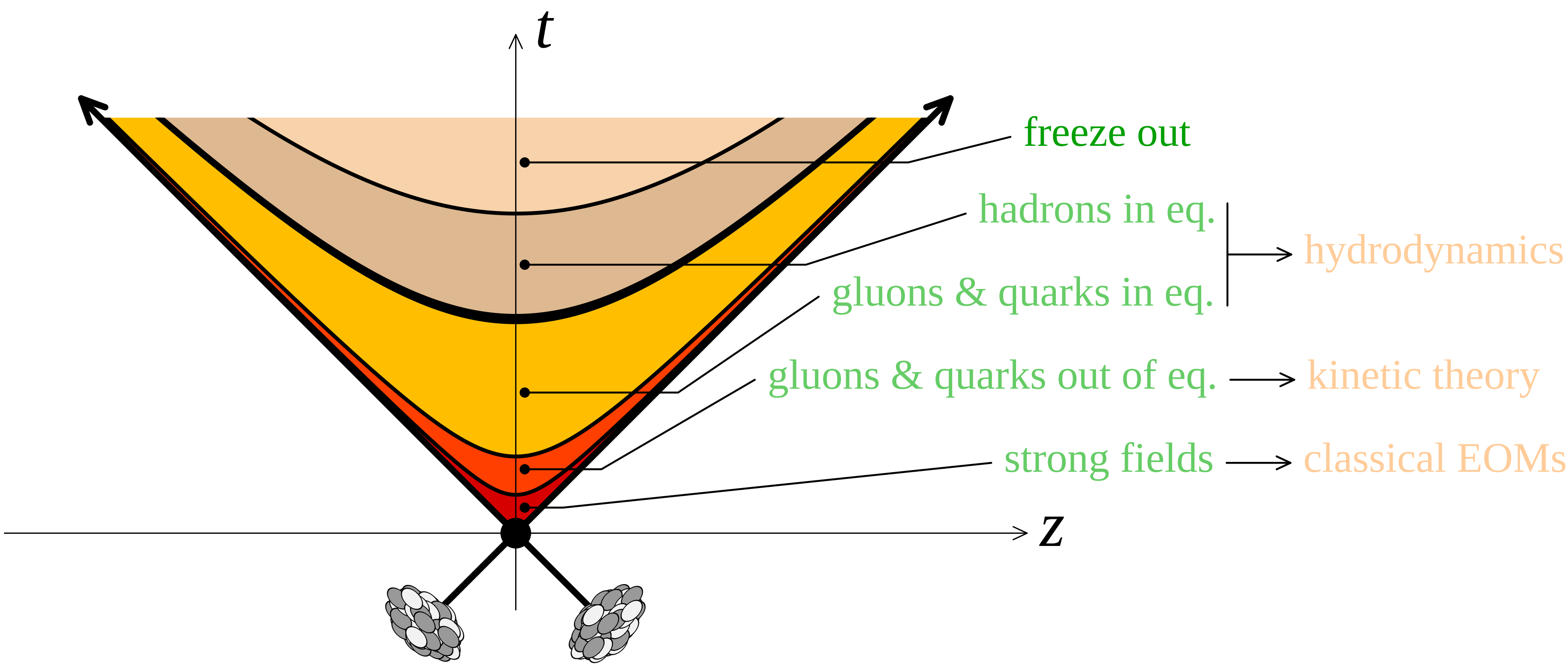
(courtesy of François)

- AA collisions : 'complications' due to final state interactions

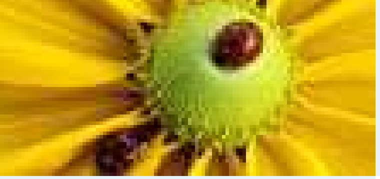
  'complications' = important new physical effects, not encoded in the initial wavefunctions

  (rescattering, thermalization (QGP), hadronization, jet quenching ...)

Introduction: What is the CGC ?

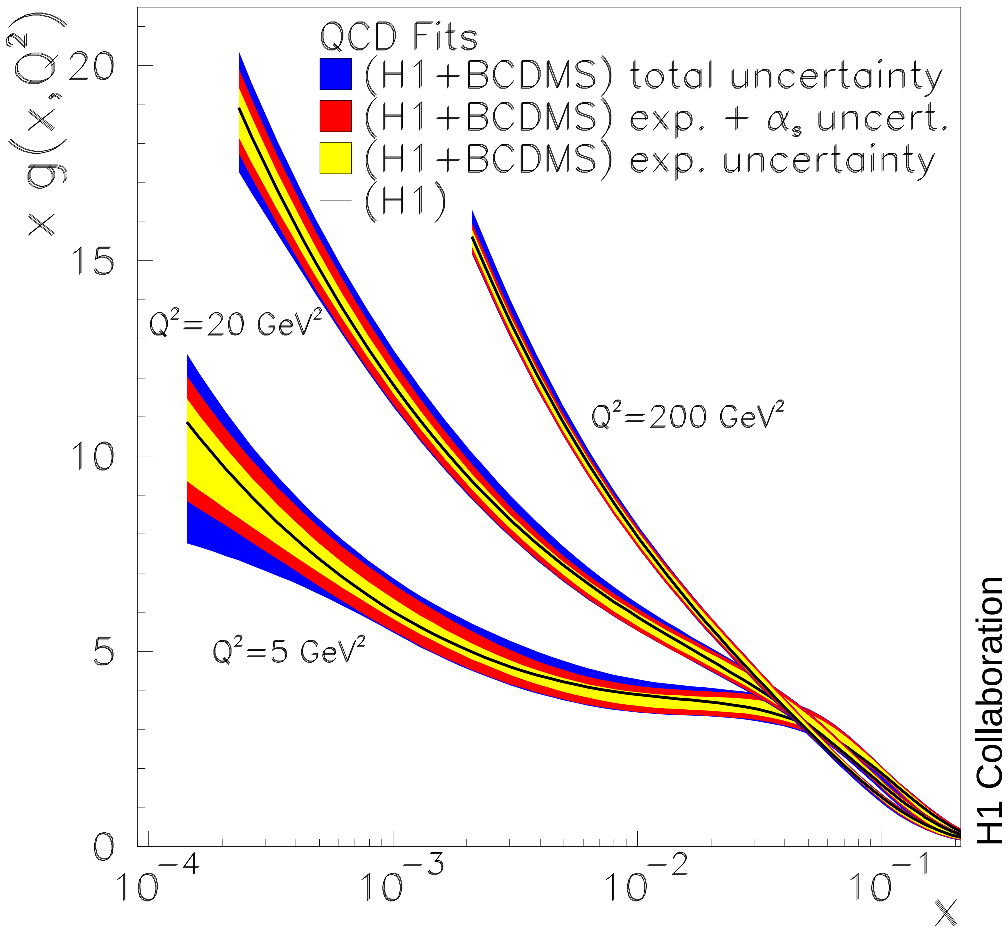


- “CGC gives the initial conditions for the heavy ion collision”
  - calculate the initial production of semi-hard particles
  - prepare the stage for kinetic theory or hydrodynamics



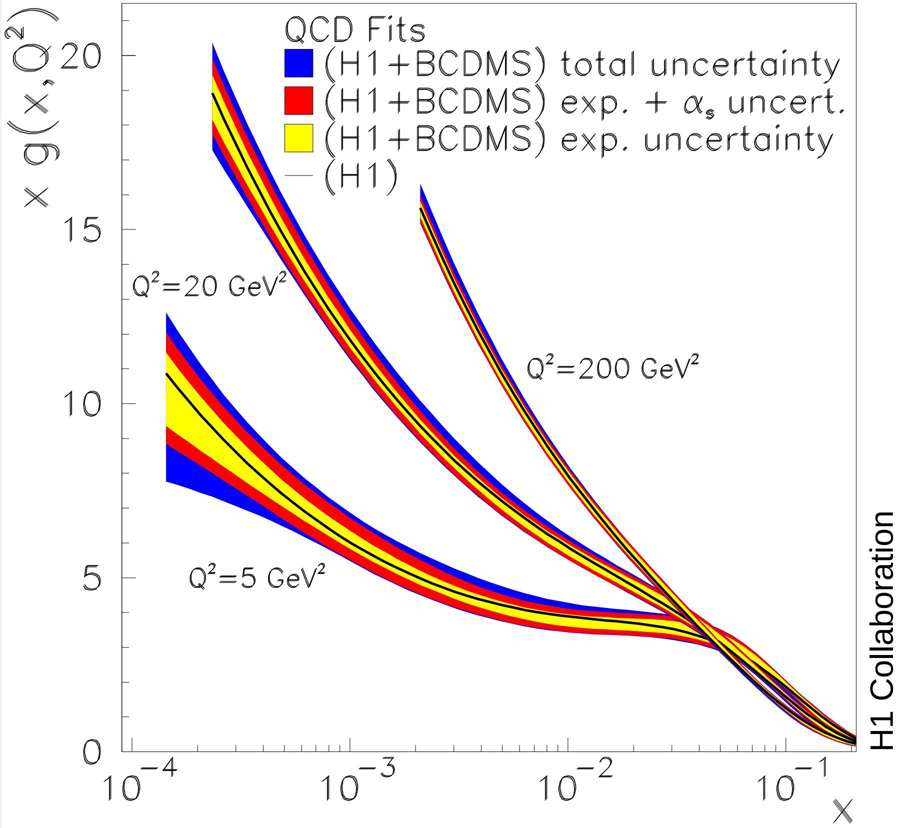
Motivation: Gluons at HERA

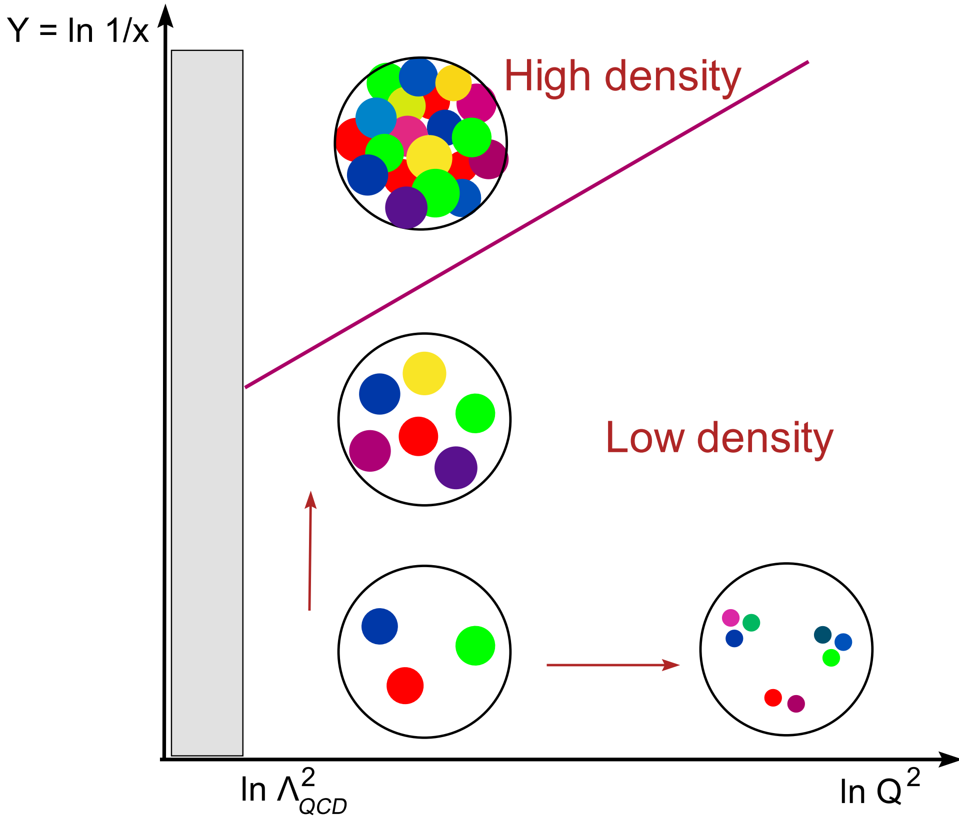
▷ The gluon distribution rises very fast at small x ! $(\sim 1/x^{\lambda})$



$xG(x,Q^2) \approx$ # of gluons with transverse area $\sim 1/Q^2$ and $k_z = xP$

Motivation: Gluons at HERA

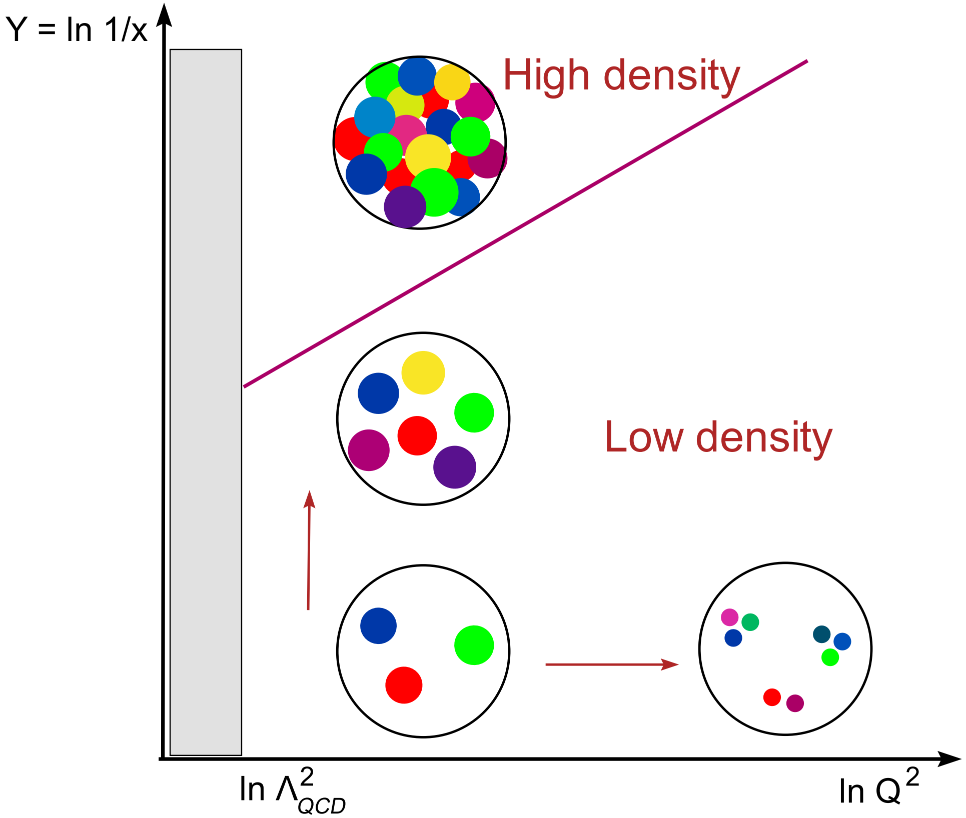




▷ High–Q^2 evolution : The parton density is decreasing

▷ 'Small–x' evolution: An evolution towards increasing density

Gluon occupation number



▷ What matters is not the gluon number, but the occupation number !

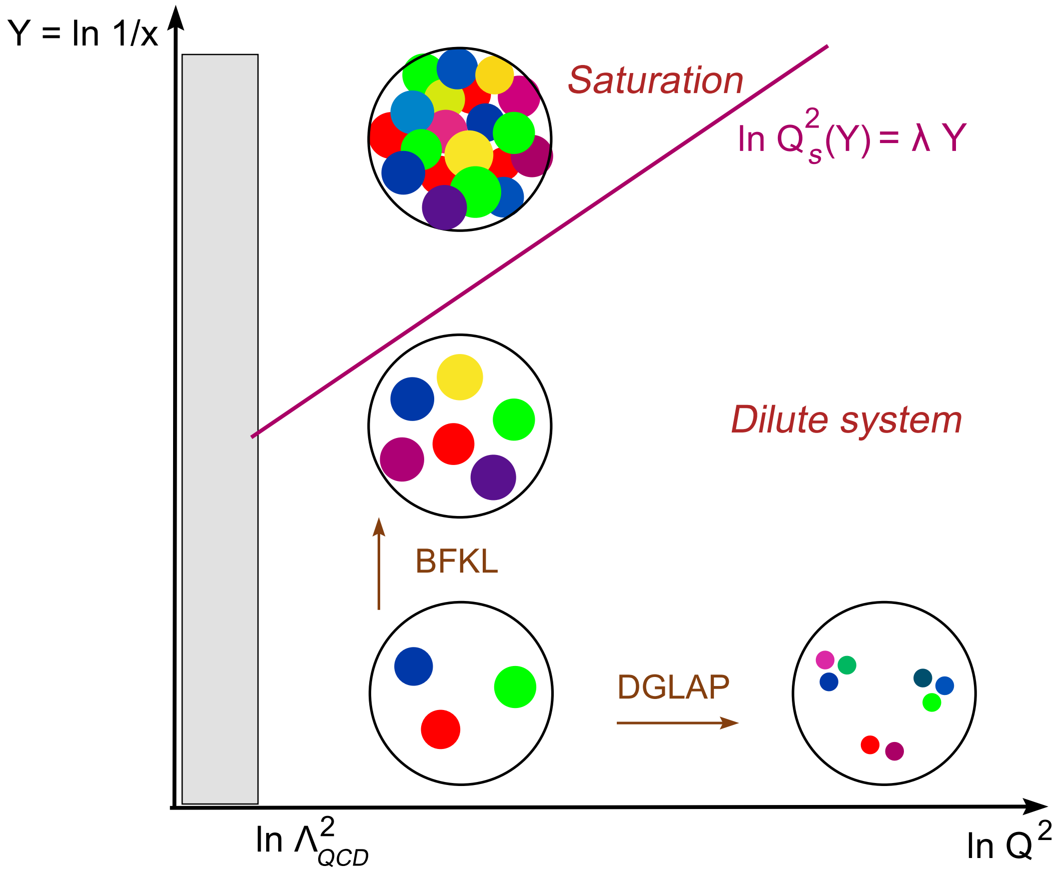
$$\varphi(x, Q^2) \equiv \frac{\pi}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2} \sim \frac{\log Q^2}{Q^2} \frac{1}{x^\lambda}$$

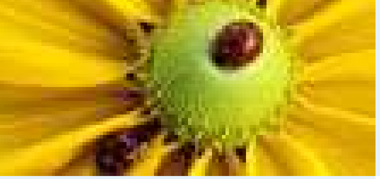
the 'fraction' of the hadron area which is covered by gluons

The Saturation Momentum

- Onset of non–linear physics : $\varphi(x, Q^2) \sim 1/\alpha_s \gg 1$

$$Q_s^2(x) \simeq \frac{\alpha_s}{N_c} \frac{xG(x, Q_s^2)}{\pi R^2} \sim x^{-\lambda}$$





Large nucleus ($A \gg 1$)

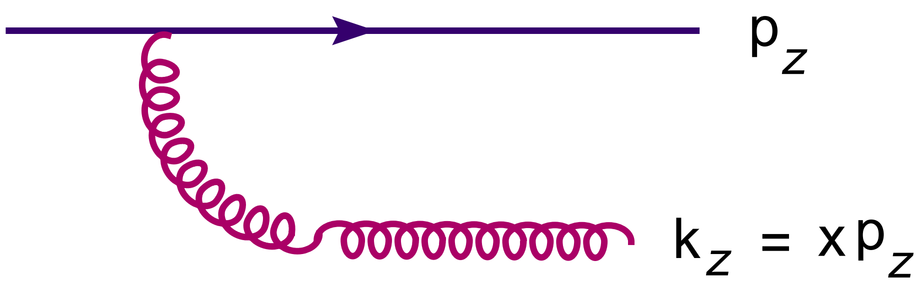
- $xG_A(x,Q^2) \propto A$ and $R_A \propto A^{1/3}$

$$Q_s^2(A,x) \simeq \alpha_s \frac{xG_A(x,Q_s^2)}{\pi R_A^2} \sim x^{-\lambda} A^{1/3}$$

- Non–linear effects already at moderately high energies
- Too optimistic (at very small values x) !
  The A–dependence is reduced by the small–x evolution
- Some estimates: (theory+phenomenology)
  - $Q_s^2 \approx 1.5$ GeV^2 at HERA ($A=1$) for $x \sim 10^{-5}$
    RHIC ($A=208$) for $x \sim 10^{-3}$
  - LHC: at $x \sim 10^{-6}$, $Q_s^2 \approx 3 \div 5$ GeV^2 for protons
    $Q_s^2 \approx 6 \div 12$ GeV^2 for $A = 208$

Gluon evolution at small x

- The 'infrared sensitivity' of bremsstrahlung favors the emission of 'soft' (= small–x) gluons



$$\mathrm{d}\mathcal{P} \propto \alpha_s \frac{\mathrm{d}k_z}{k_z} = \alpha_s \frac{\mathrm{d}x}{x} \equiv \alpha_s\, \mathrm{d}Y$$

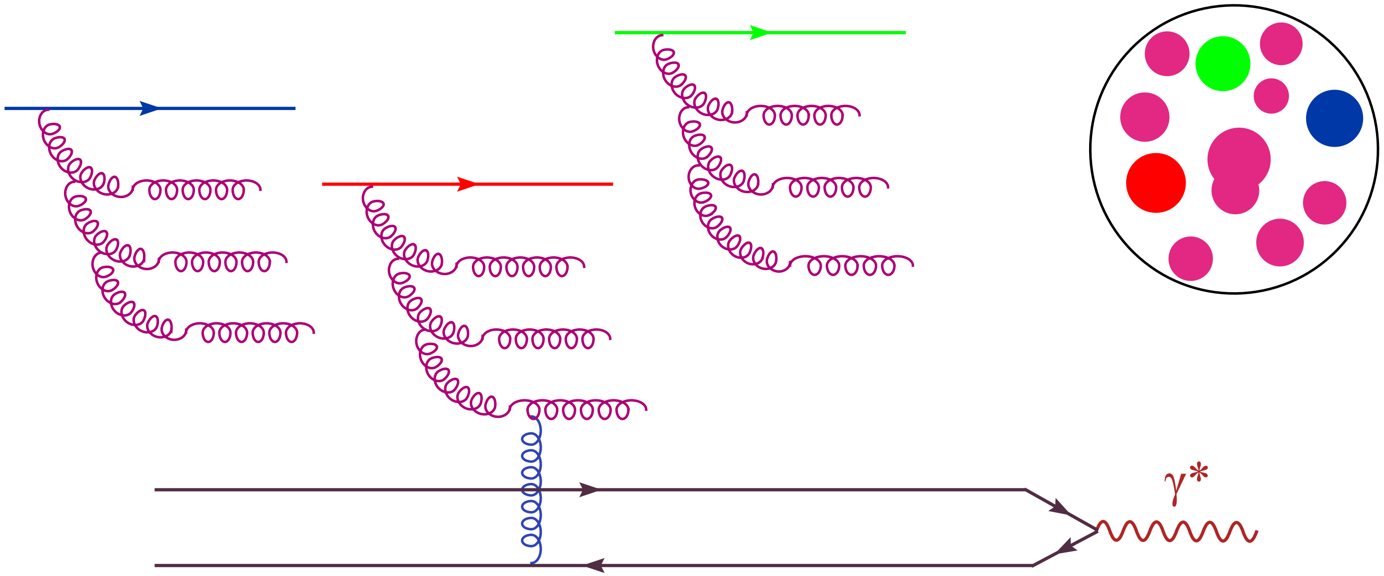
$$Y \equiv \ln\frac{1}{x} \sim \ln s \implies \mathrm{d}Y = \frac{\mathrm{d}x}{x} : \quad \text{“rapidity”}$$

- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity.
- High rapidity $(\alpha_s Y \gg 1) \implies$ Many gluons !

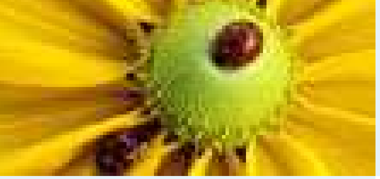
BFKL evolution

- The 'last' gluon at small x can be emitted off any of the 'fast' gluons with $x' > x$ radiated in the previous steps :

$$\frac{\partial \varphi}{\partial Y} \simeq \alpha_s \varphi \qquad \Longrightarrow \qquad \varphi(Y) \propto \mathrm{e}^{\omega \alpha_s Y}$$

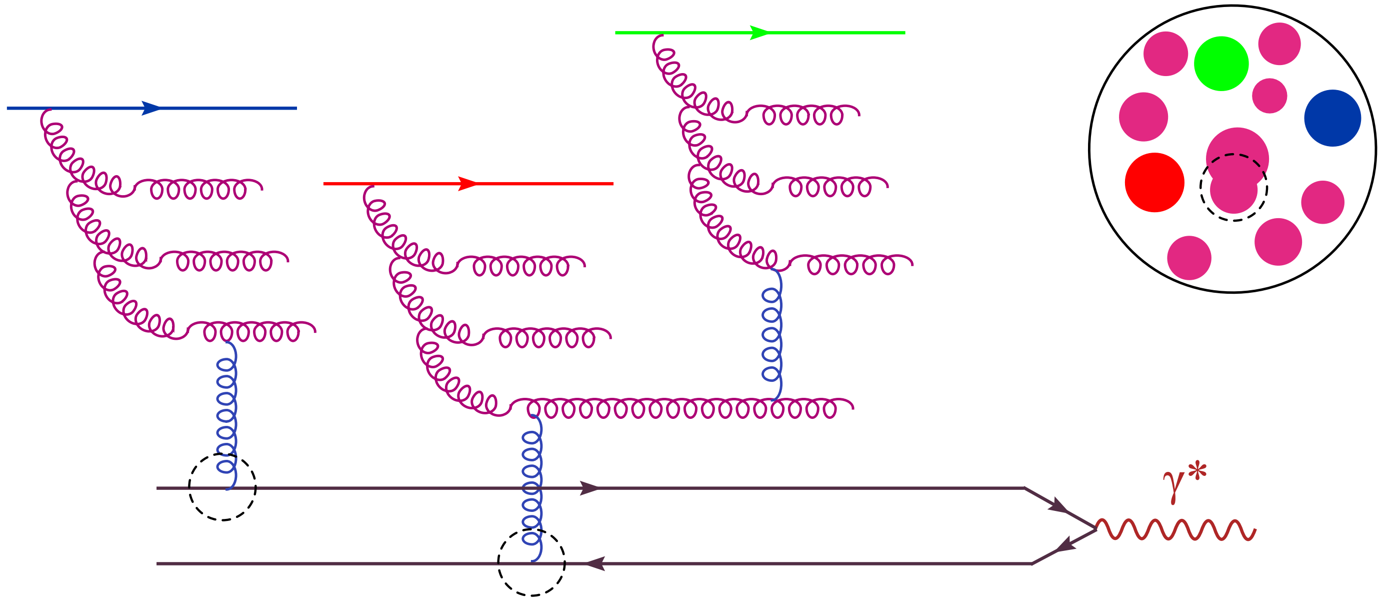


- Cartoon version of BFKL eq. *(Balitsky, Fadin, Kuraev, Lipatov, 78)*
- Valid so long as the density is low enough: $\varphi \ll 1/\alpha_s$



Non–linear evolution

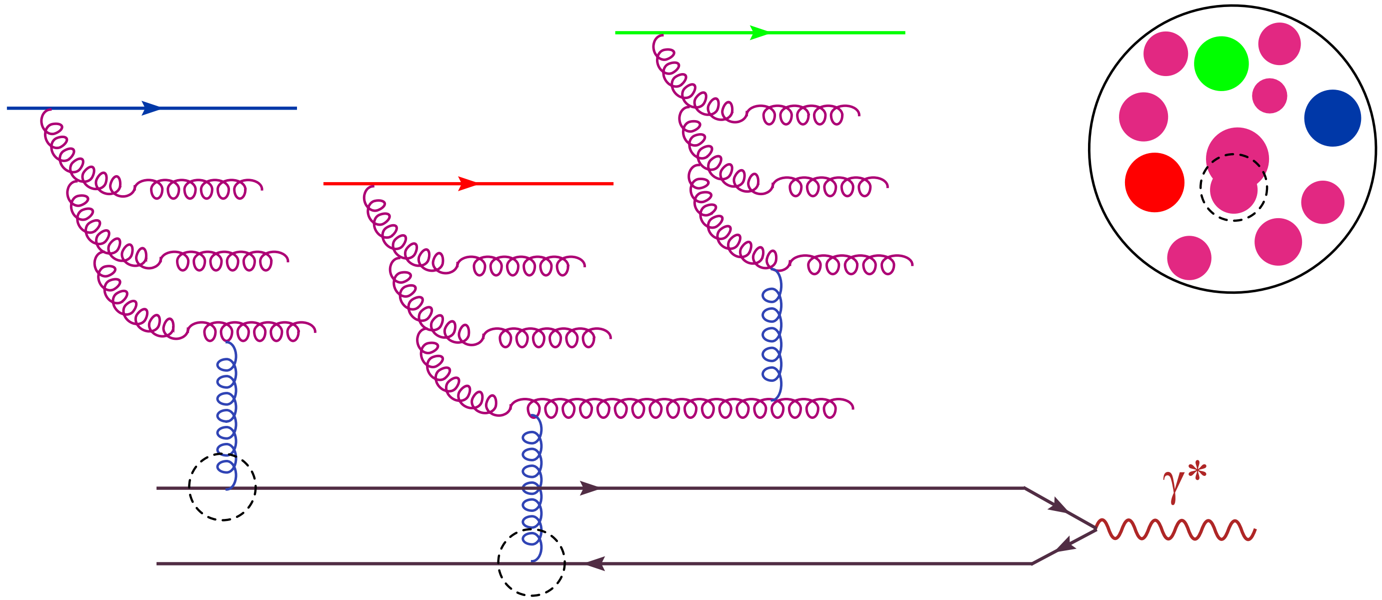
- At high density, non-linear effects become important: saturation, multiple scattering



$$\frac{\partial\varphi}{\partial Y} \simeq \alpha_s\varphi - \alpha_s^2\,\varphi^2 = 0 \quad \text{when} \quad \varphi \sim \frac{1}{\alpha_s} \gg 1$$

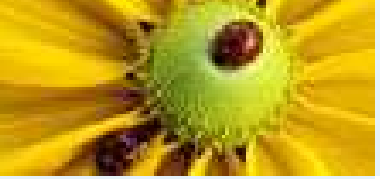
- ‘Fixed point’ at high energy (the evolution stops)
- Cartoon version of the Balitsky–Kovchegov equation (99)

- At high density, non-linear effects become important: saturation, multiple scattering



$$\frac{\partial\varphi}{\partial Y} \simeq \alpha_s\varphi - \alpha_s^2\,\varphi^2 = 0 \quad \text{when} \quad \varphi \sim \frac{1}{\alpha_s} \gg 1$$

- Mean field approximation to the JIMWLK equation (CGC)
  (Jalilian-Marian, E.I., McLerran, Weigert, Leonidov, and Kovner, 97–00)



Next–to–leading order corrections

- So far, the non–linear equations have been fully established only to "leading order" : $\mathcal{O}(\alpha_s^n \ln s^n)$ for any n

$$\frac{\partial \varphi}{\partial Y} \simeq \alpha_s(?)\varphi - \alpha_s^2(?)\,\varphi^2$$

- NLO effects partially known and under active investigation:

  $\mathcal{O}(\alpha_s^{n+1} \ln^n s)$ (e.g. : the running of the coupling)

  Triantafyllopoulos, Munier, Peschanski, Balitsky, Kovchegov, Weigert ...

- The linear (BFKL) equation is known to NLO accuracy

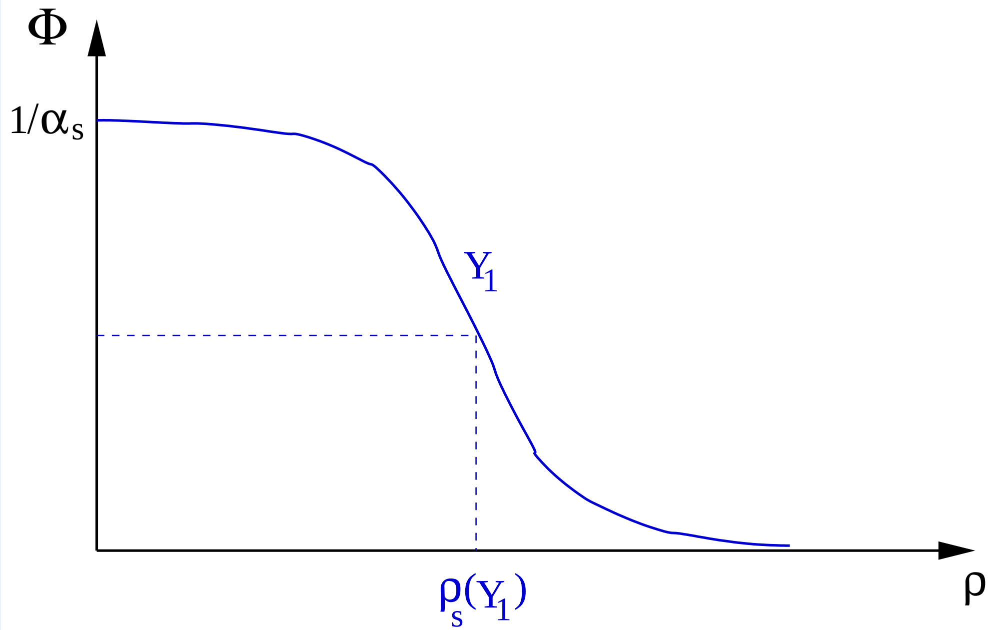
  Fadin, Lipatov, Camici, Ciafaloni, Salam ...

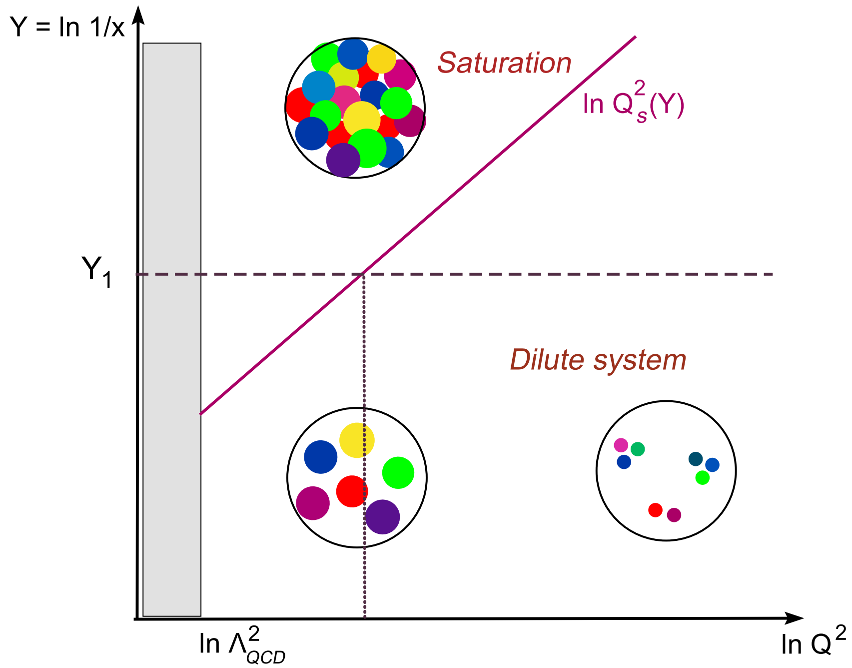
- Quantitatively, and even qualitatively, important !

  Dramatic consequences for the phenomenologie

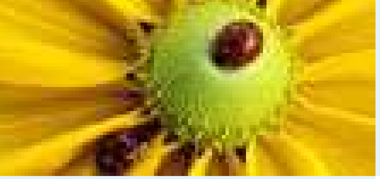
Solution: Saturation front

- Occupation number $\varphi(Y, Q^2)$ as a function of $\rho \equiv \ln Q^2$



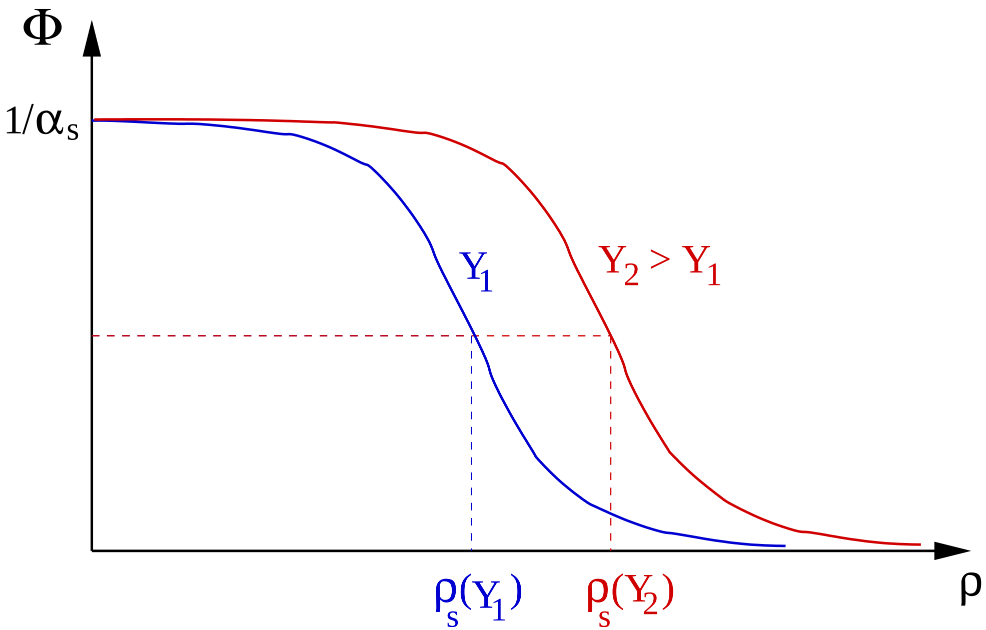


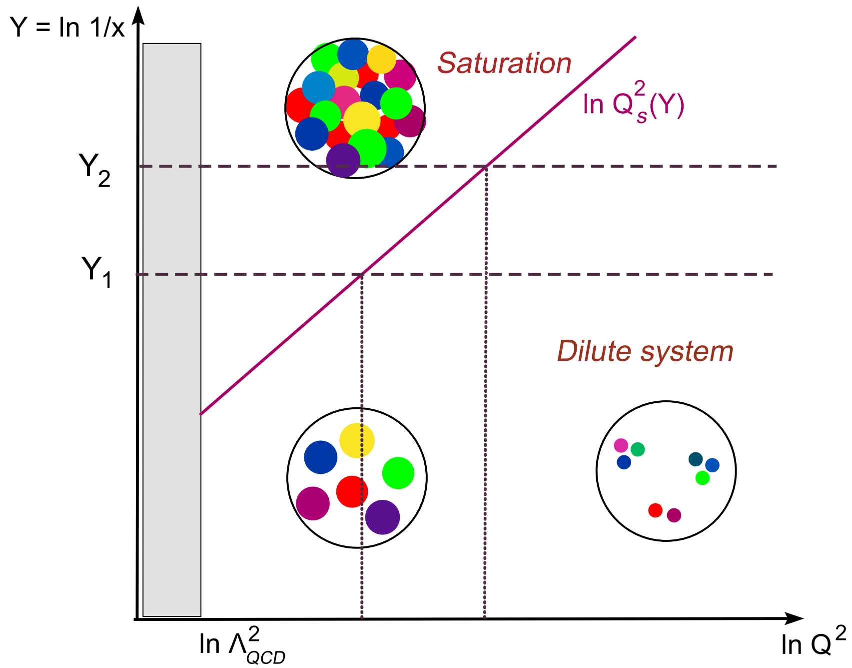
- $\rho_s(Y) \equiv \ln\{Q_s^2(Y)/\Lambda^2_{\rm QCD}\}$: saturation momentum
  - $\rho \gg \rho_s(Y)$: $\varphi \propto \mathrm{e}^{-\rho}\,\mathrm{e}^{\alpha_s Y}$ $\quad (\varphi \propto 1/Q^2)$
  - $\rho \lesssim \rho_s(Y)$: $\varphi \sim 1/\alpha_s$ = const.



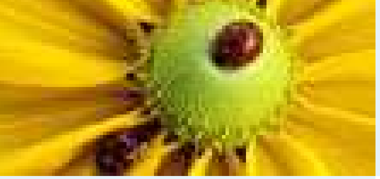
Solution: Saturation front

- $\rho_s(Y)$ increases with Y ('the front propagates')





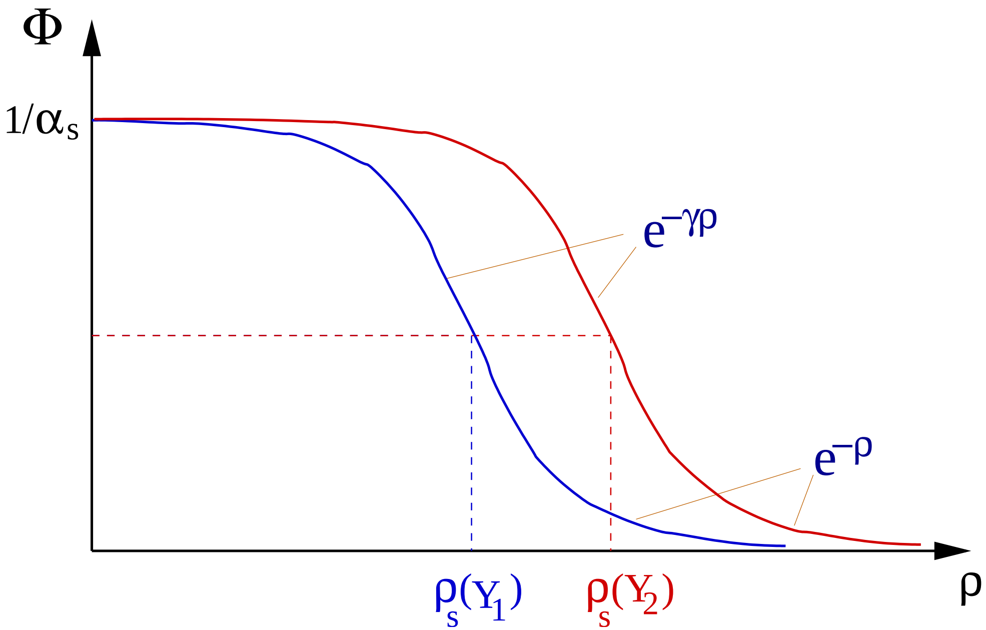
- Fixed coupling : $\rho_s(Y) \simeq \rho_0(A) + \lambda_0 Y$ with $\lambda_0 \sim \mathcal{O}(1)$
- Running coupling : $\rho_s(Y) \simeq \sqrt{\beta\lambda_0 Y + \rho_0^2(A)}$
- Full NLO and Y large enough: $\rho_s(Y) \approx \lambda Y$ with $\lambda \approx 0.3$

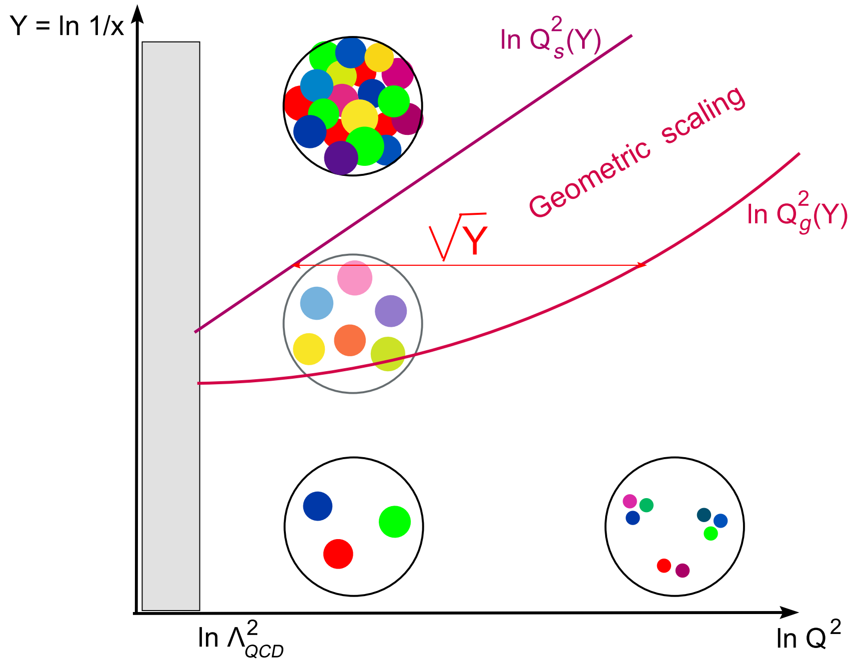


Geometric scaling window

- Shape of the front near the saturation line $(Q^2 > Q_s^2)$

$$\varphi(Y, Q^2) \simeq \mathrm{e}^{-\gamma(\rho-\rho_s(Y))} \equiv \left(\frac{Q_s^2(Y)}{Q^2}\right)^{\gamma} \quad \text{with } \gamma \approx 0.63$$





- Fixed coupling : $\rho_g - \rho_s \propto Y^{1/2}$
- Running coupling : $\rho_g - \rho_s \propto Y^{1/6}$

Geometric Scaling at HERA

(Staśto, Golec-Biernat and Kwieciński, 2000)

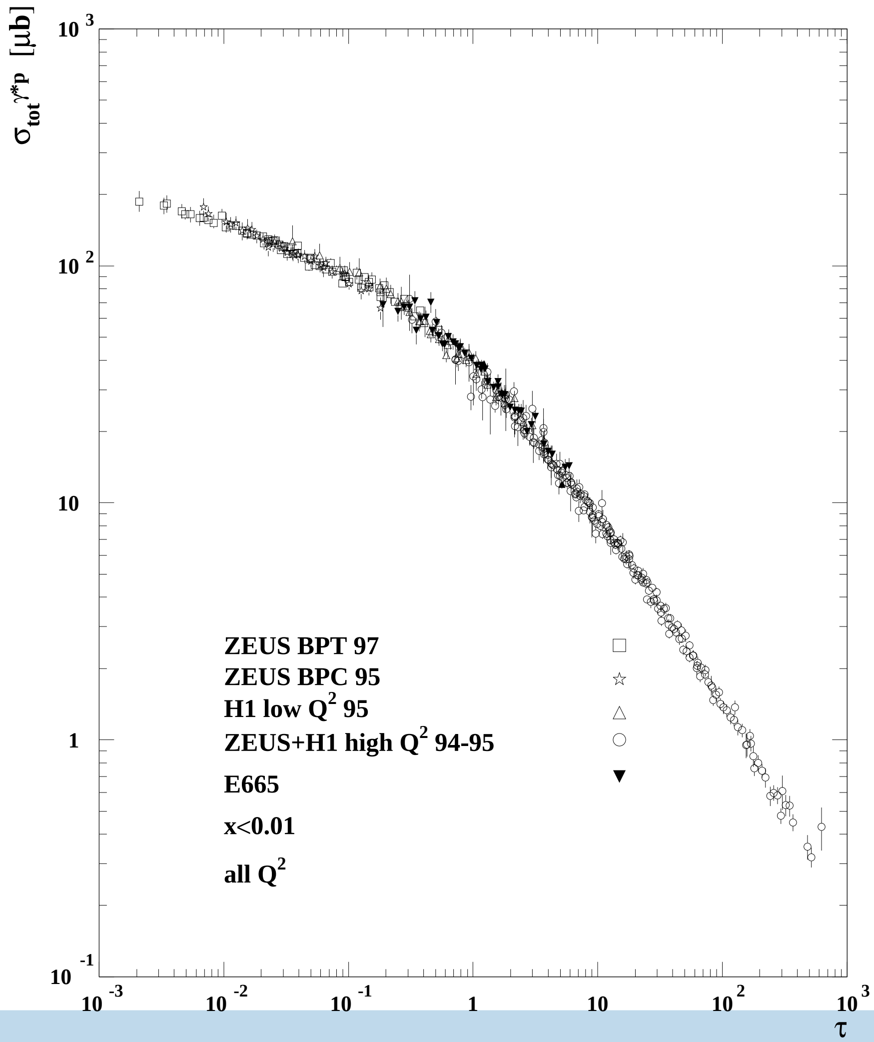
$$\sigma(x, Q^2) \approx \sigma(\tau) \quad \text{with} \quad \tau \equiv Q^2/Q_s^2(x), \quad Q_s^2(x) = (x_0/x)^\lambda \,\text{GeV}^2\,, \quad \lambda \simeq 0.3$$

$x \leq 0.01$

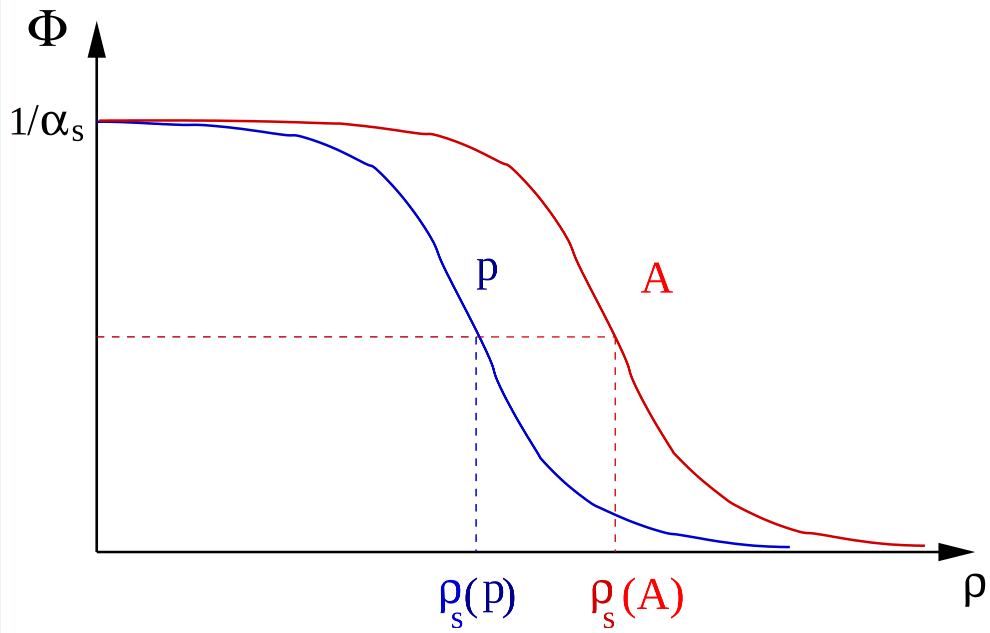
$Q^2 \leq 450$ GeV^2

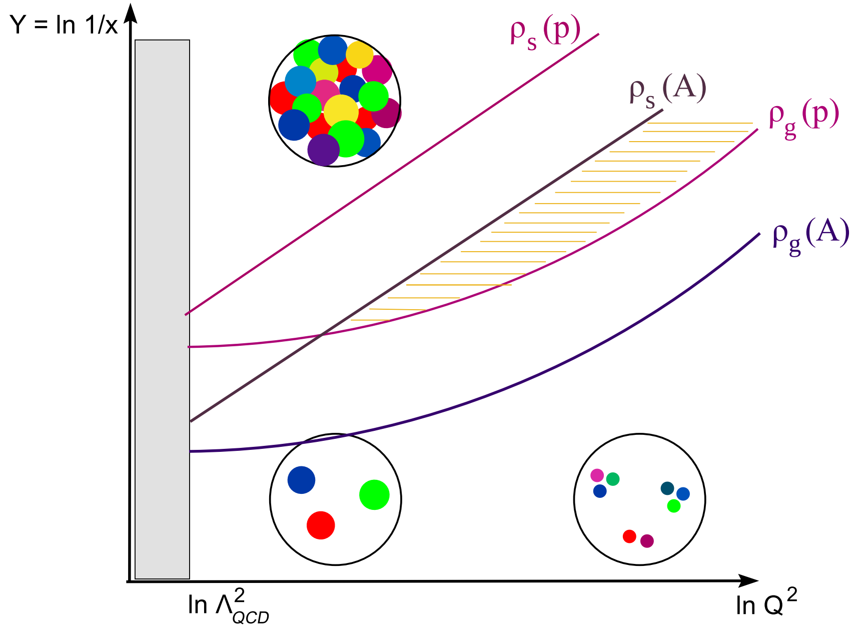
$Q_s^2 \sim 1$ GeV^2

for $x \sim 10^{-4}$



Nuclear effects



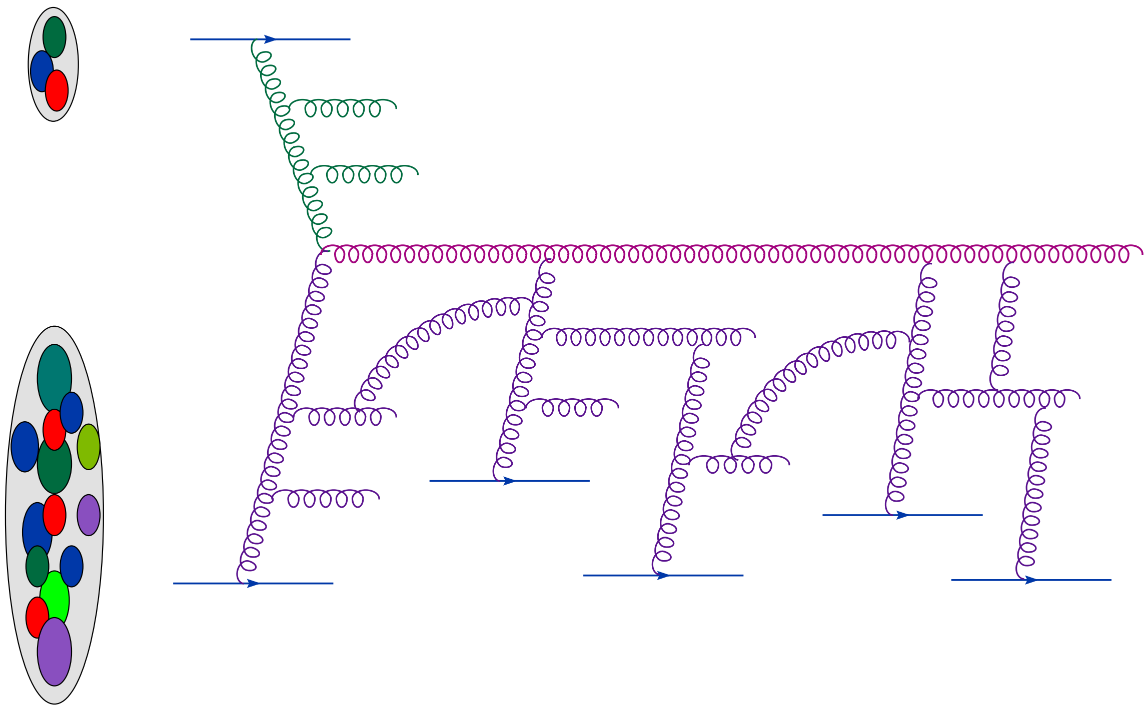


- Assume $Q_s^2(A) \simeq A^{1/3}\, Q_s^2(p)$ at $Y = Y_0$
- Fixed coupling : $\rho_s(A,Y) - \rho_s(p,Y) \simeq \ln A^{1/3}$ (const. !)
- Running coupling : $\rho_s(A,Y) - \rho_s(p,Y) \propto (\ln A^{1/3})^2/\sqrt{Y}$

At very large Y, a nucleus is not denser than a proton !

Gluon production in pp or pA collisions

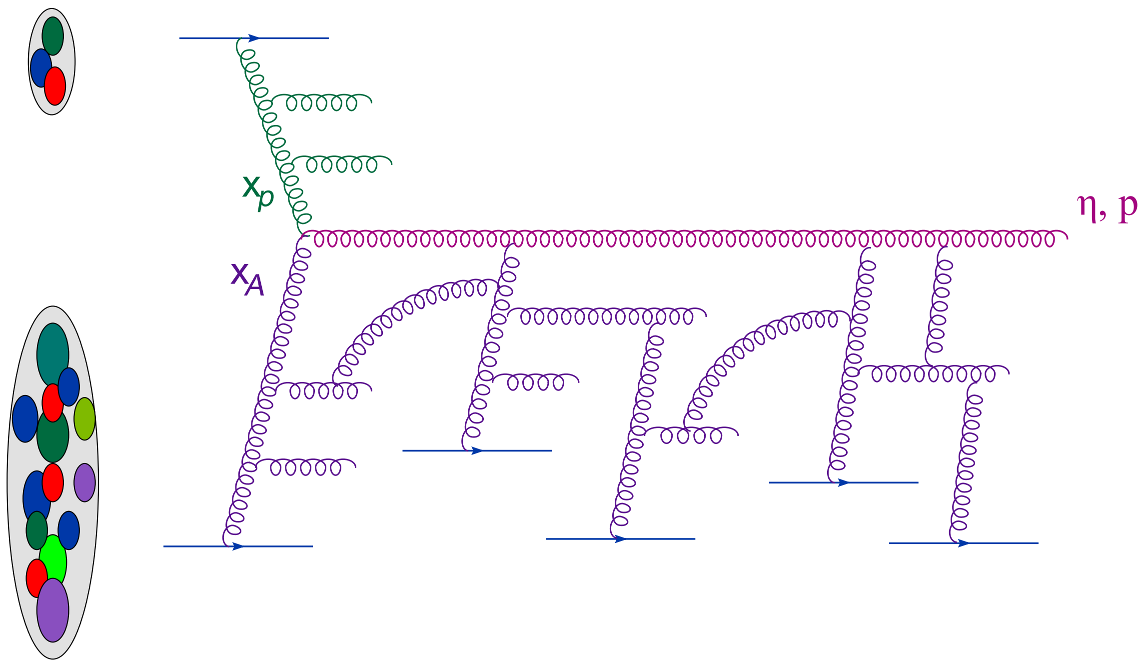
- 'Dense–dilute' scattering
  - pA collisions (RHIC, LHC)
  - pp collisions at 'forward rapidity' (LHC)
- Only one parton from the dilute projectile gets involved



- A probe of the gluon distribution inside the dense target !

Non–linear effects in the target

- Two sources for high-density gluonic matter inside the target:



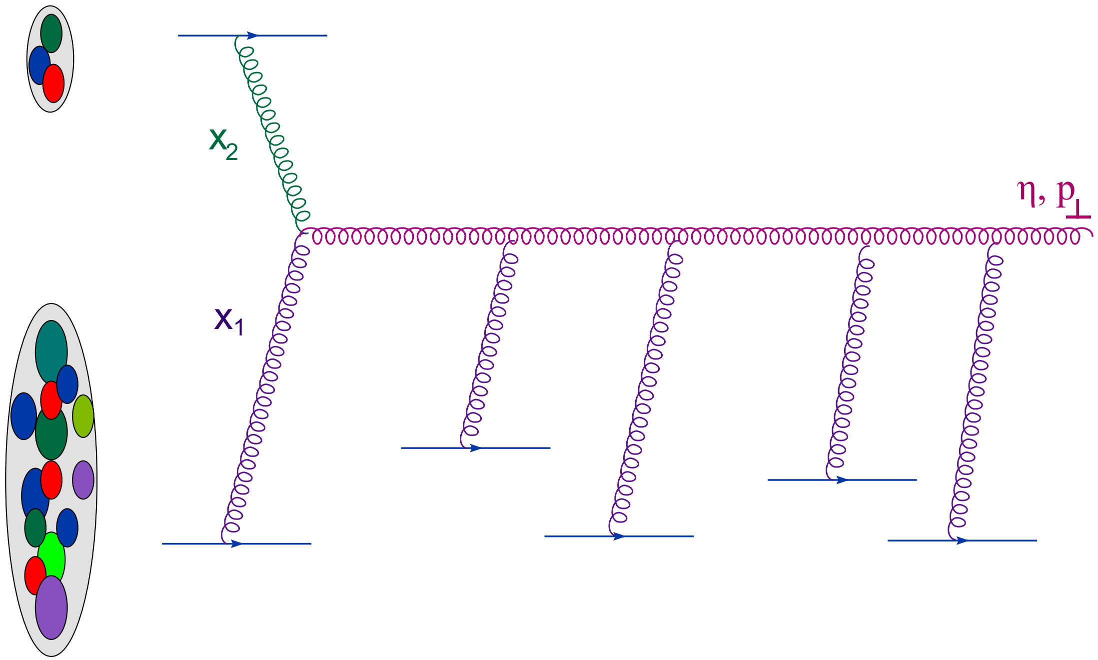
- Small–x target evolution in the 'forward kinematics':

$$x_A = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}\,, \qquad x_p = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

- Increasing $\eta \iff$ Decreasing x_A for the nuclear target

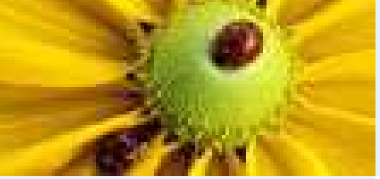
Non–linear effects in the target

- Two sources for high-density gluonic matter inside the target:



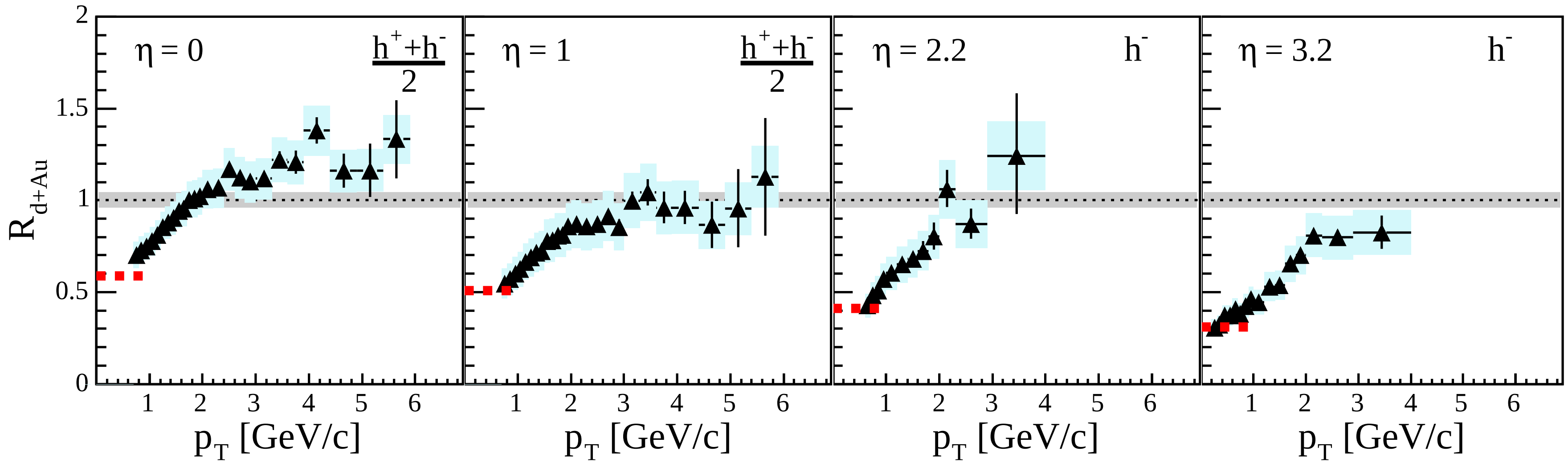
- Large nucleus $A \gg 1$: Many 'color sources' (the $3A$ valence quarks) which emit gluons already at 'tree–level'

- RHIC physics: One can disentangle these two mechanisms by varying the pseudo-rapidity η

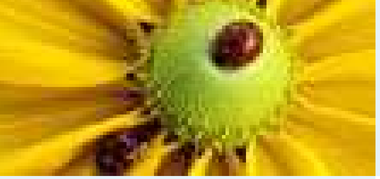


High–$p_\perp$ suppression in d+Au at RHIC

Nuclear modification factor : $R_{\mathrm{d+Au}} \equiv \frac{1}{2A} \frac{dN_{\mathrm{d+Au}}/d^2p_\perp d\eta}{dN_{\mathrm{pp}}/d^2p_\perp d\eta}$



- One finds *(BRAHMS [arXiv:nucl-ex/0403005])* :
  - $\eta = 0$: Cronin peak ($R_{\mathrm{d+Au}} > 1$ for ‘high’ $p_\perp$)
  - $\eta \simeq 3$: Suppression ($R_{\mathrm{d+Au}} < 1$ for all $p_\perp$)
  - very fast evolution with increasing η !
- Qualitatively consistent with CGC (see the talk by Yacine)



Forward physics at LHC

- What should we expect at the LHC ?

Remember: $x_{\rm target} = \frac{p_\perp}{\sqrt{s}}\,{\rm e}^{-\eta}$ where 'target' = A or p

- LHC: Considerably larger values for both s and η

$\Longleftrightarrow$ a much larger phase for the evolution of the target !

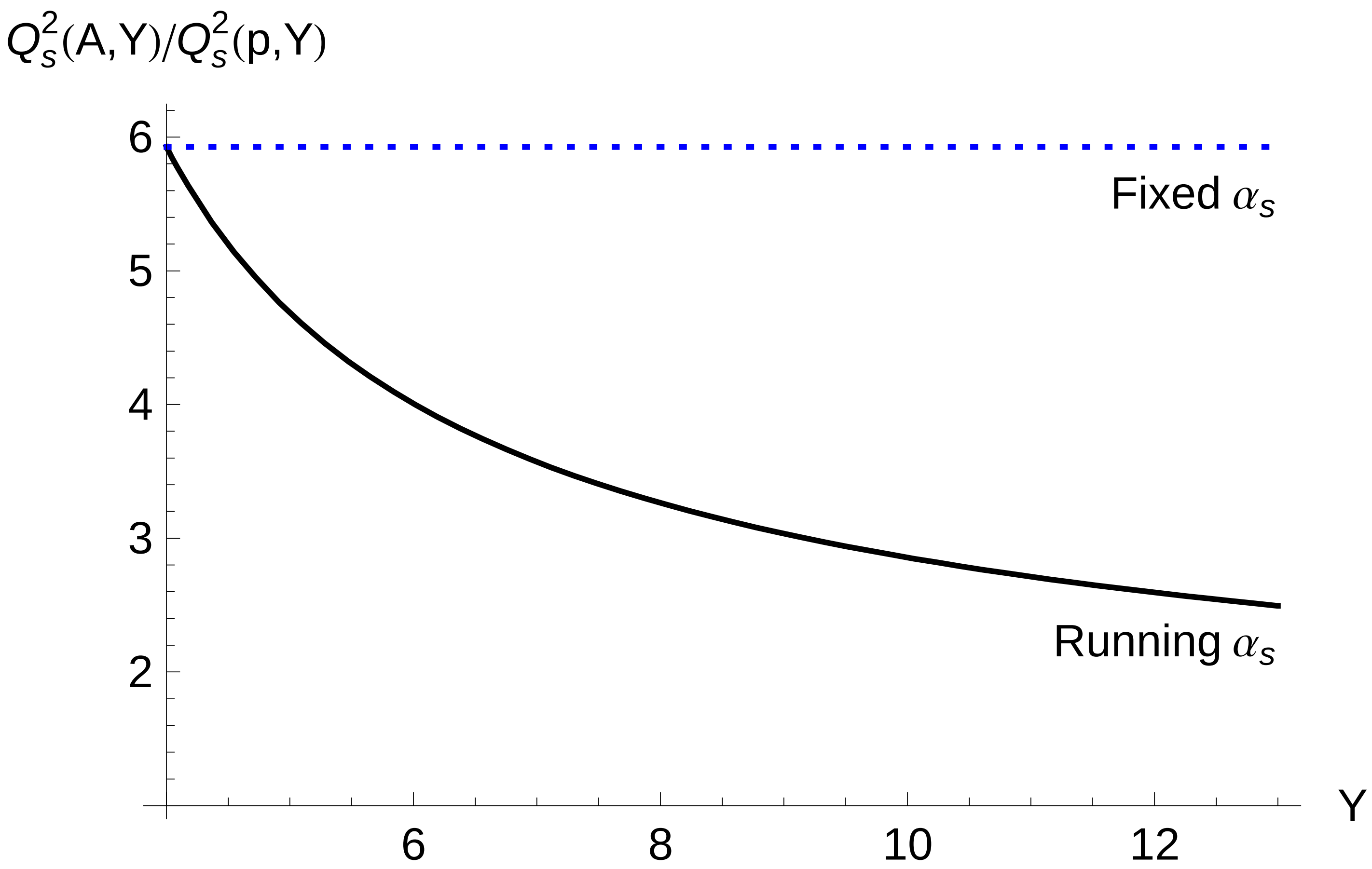
  - RHIC: $\eta \simeq 3$ & $\sqrt{s} = 200$ GeV: $x_1 \sim 10^{-4}$ for $p_\perp = 2$ GeV
  - LHC : $\eta \simeq 6$ & $\sqrt{s} = 8.8$ TeV : $x_1 \sim 10^{-6}$ for $p_\perp = 10$ GeV

- Some 'fine details' of the evolution, like, for instance, the differences between 'fixed' and 'running coupling' scenarios should become manifest.

- Running coupling effects should progressively wash out any difference between a nucleus and a proton target !

Saturation momenta at LHC *(prediction)*

- The ratio of the nuclear ($A = 208$, $A^{1/3} \approx 6$) vs. proton saturation momenta as a function of $Y = \ln(1/x)$



- Decrease by a factor of 2 (from 6 to 3) at $Y = 10$

R_{pA} at high energy : Total shadowing

- A simple analytic estimate for R_{pA} at high $p_\perp$:

$$R_{pA}(\eta, p_\perp) \approx \frac{1}{A^{1/3}} \frac{\varphi_A(x, p_\perp)}{\varphi_p(x, p_\perp)}$$

- RHIC: even at large η, R_{pA} is rather close to one

  $R_{pA}(\eta, p_\perp) \gtrsim 0.8$ for $\eta = 3.2$ and $p_\perp > 2$ GeV

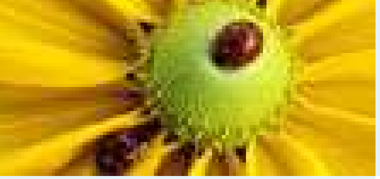
- With increasing energy (η), the nucleus and the proton become closer and closer to each other ...

  ... hence the ratio R_{pA} must decrease towards its minimal possible value :

$$R_{pA}(\eta, p_\perp) \to \frac{1}{A^{1/3}} \approx 0.5 \quad \text{when} \quad \eta \to \infty$$

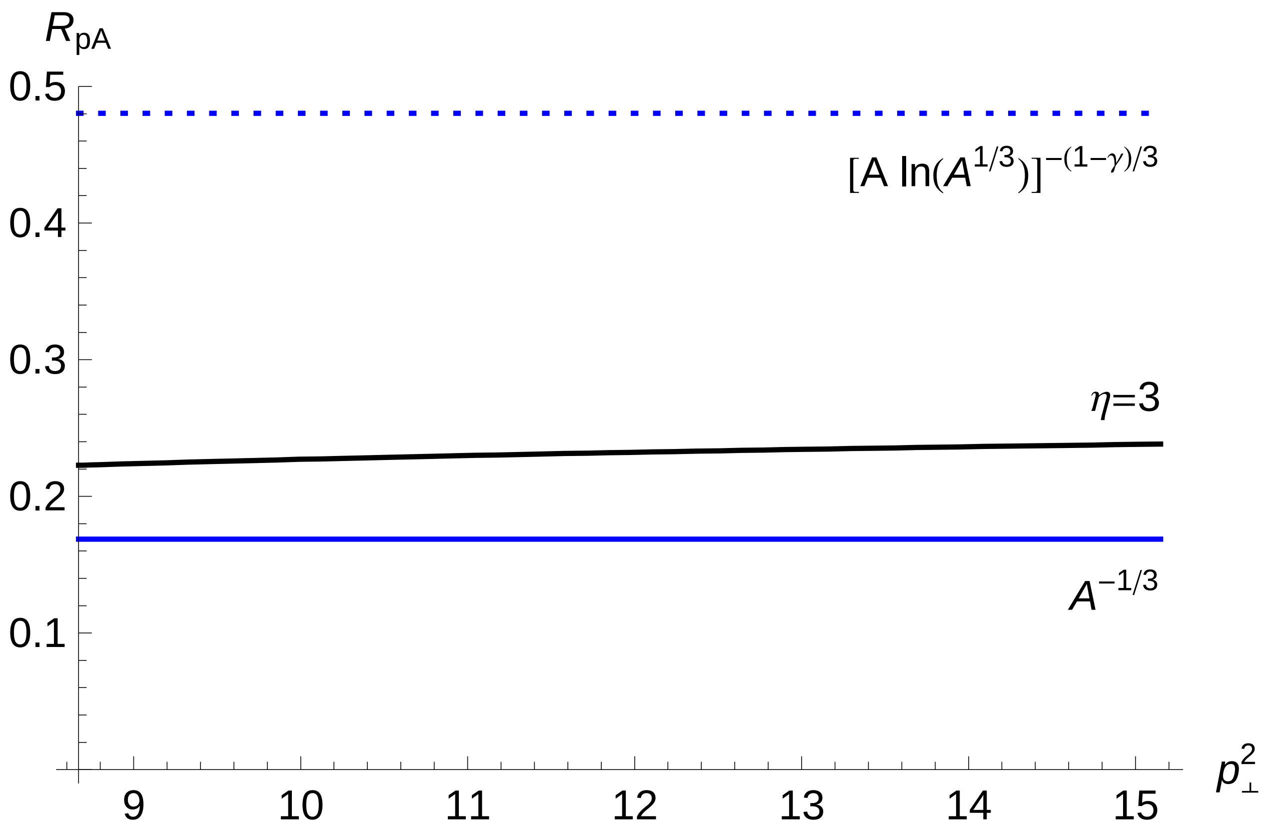
"Total gluon shadowing"

E.I., K. Itakura, D. N. Triantafyllopoulos, hep-ph/0403103



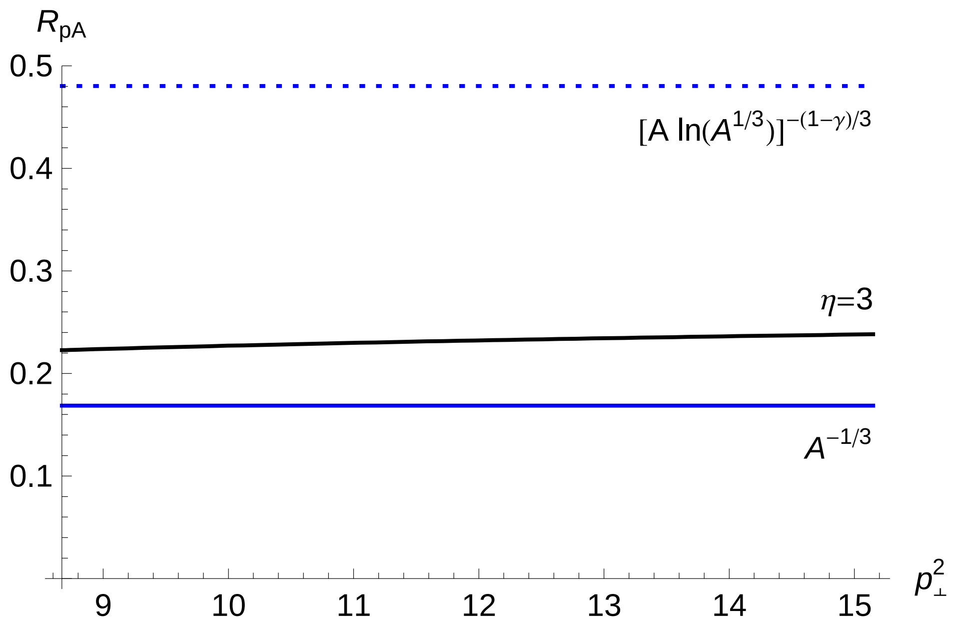
R_{pA} at the LHC *(prediction)*

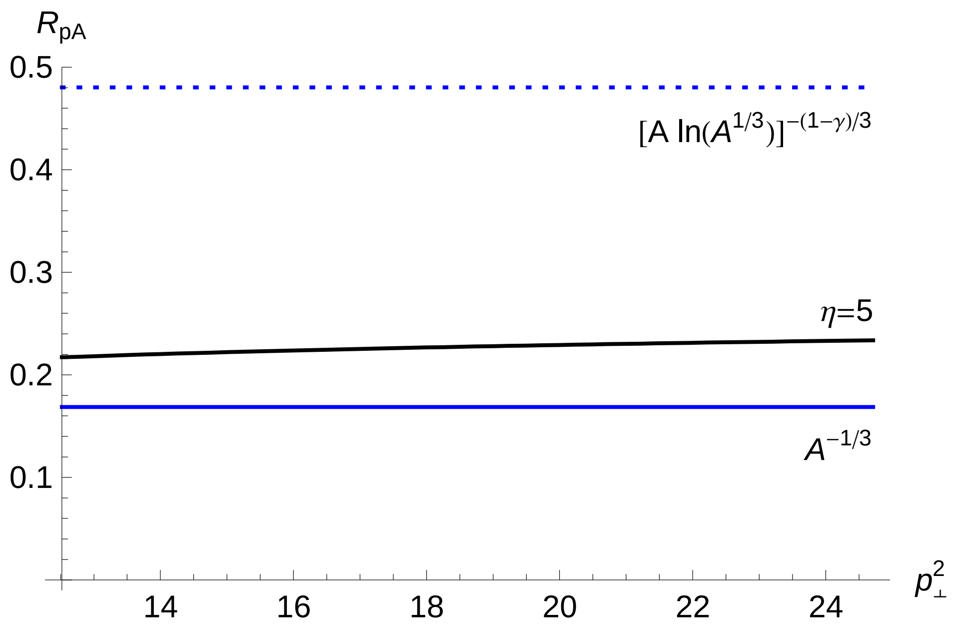
(E.I. and D. Triantafyllopoulos, "Last call for LHC", CERN, May 2007)



- Significant discrepancy from 'fixed coupling' scenario
- Close to total gluon shadowing already for $\eta \sim 3$

R_{pA} at the LHC *(prediction)*



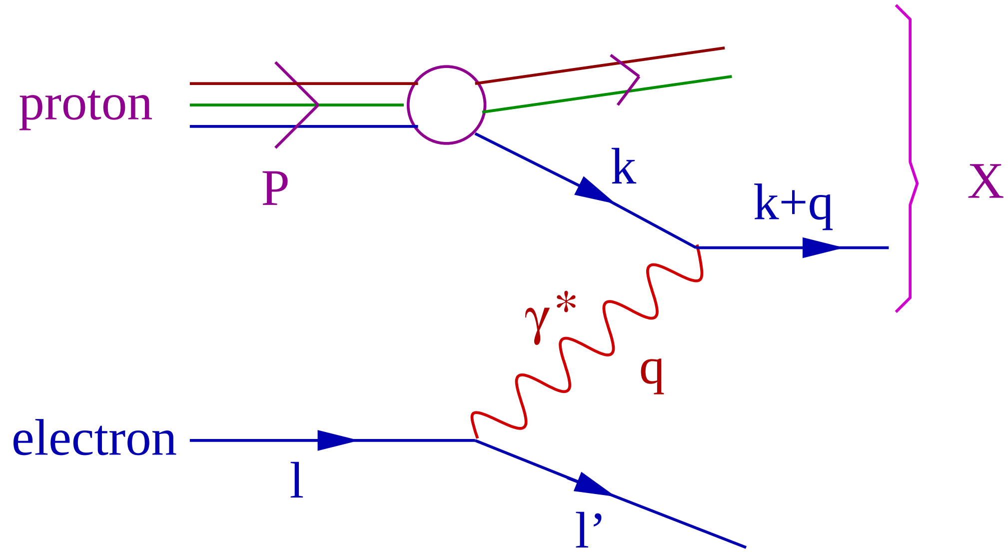


Motivations for the CGC

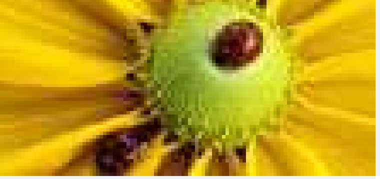
- First indications in favour of a high–density partonic phase at high energy came from theoretical considerations within the framework of perturbative QCD ...
  - BFKL evolution *(Balitsky, Fadin, Kuraev, Lipatov, 75–78)*
  - “Gluon saturation” *(A. Gribov, Levin, Ryskin, 82)*
- ... but they have been largely ignored for quite some time ...
  - the applicability of pQCD at high energy being far from obvious, or widely accepted !
- ... until the advent of the first HERA data (mid 90s) ...
- ... which showed that the gluon distribution rises indeed !

Deep Inelastic Scattering at small–x

electron (l) + proton (P) $\longrightarrow$ electron (l') + X (P_X)



- Two independent kinematical invariants :
  - $Q^2 \equiv -q^\mu q_\mu \geq 0$
  - $x \simeq Q^2/s$ with $s \equiv (P+q)^2 \gg Q^2$
- Virtual photon absorbed by a quark excitation of the proton
  - with transverse size $\Delta x_\perp \sim 1/Q$
  - and longitudinal momentum $k_z = xP$



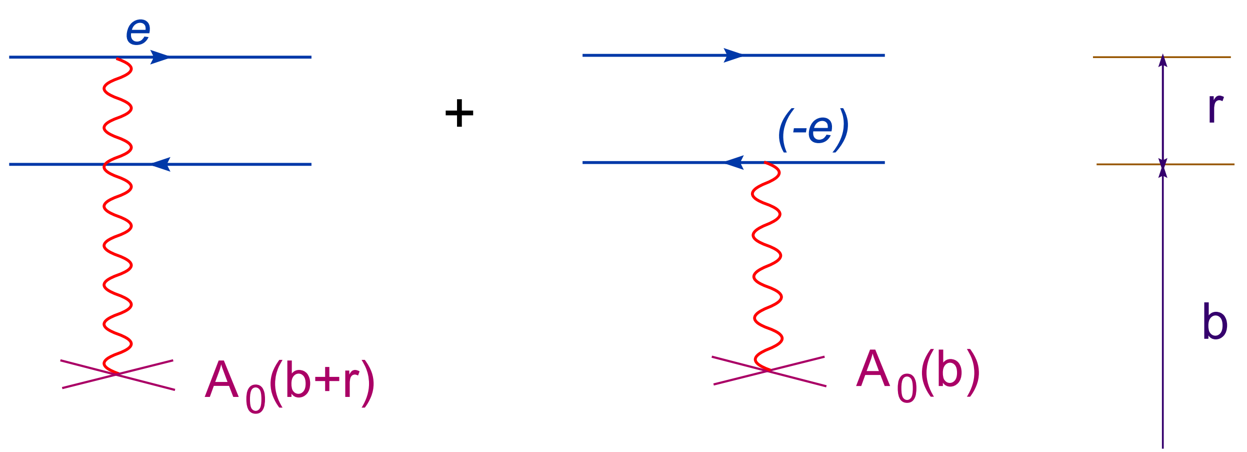
Dipole in a background field

- Reminder (classical electrodynamics) :

  A small dipole 'feels' the electric surrounding field:

$$V(\boldsymbol{r}) = e\left[A_0(\boldsymbol{b}+\boldsymbol{r}) - A_0(\boldsymbol{b})\right] \simeq e\, r^i \partial_i A_0(\boldsymbol{b}) = -e\,\boldsymbol{r}\cdot\boldsymbol{E}(\boldsymbol{b})$$

V(r) =



- QCD : 'Color dipole' = $q\bar{q}$ pair in a color singlet state

$$e\,\boldsymbol{r}\cdot\boldsymbol{E} \;\rightarrow\; g t^a \boldsymbol{r}\cdot\boldsymbol{E}_a$$ + average over color: $\frac{1}{N_c}\mathrm{tr}\{\ldots\}$

The Saturation Momentum

- Parametrization:

$$Q_s^2(A,Y) = \Lambda^2 \exp\sqrt{B(Y-Y_0)+\rho_A^2}$$

with: $\Lambda = 0.2\text{GeV}, \quad B = 2.25, \quad Y_0 = 4, \; Q_s^2(A,Y_0) = 1.5\text{GeV}^2$

- Proton : $\rho_A \; \rightarrow \; \rho_p$ such that $Q_s^2(p,Y_0) = 0.25\text{GeV}^2$

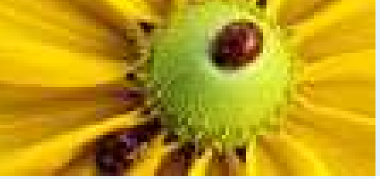
- Consistent with 'geometric scaling' fits to HERA

Gelis, Peschanski, Soyez, Schoeffel, hep-ph/0610435

- Gluon distribution in the geometric scaling window :

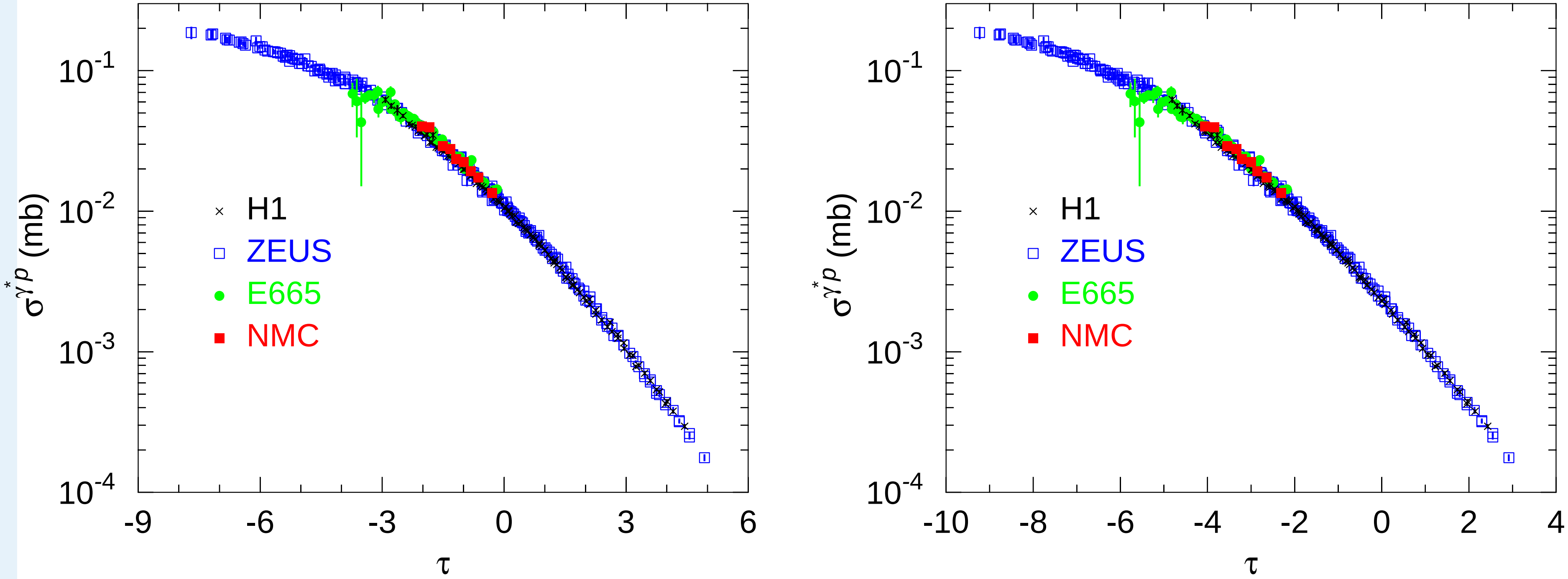
$$\Phi(k_\perp,Y) \;\propto\; \left[\frac{Q_s^2(Y)}{k_\perp^2}\right]^\gamma \left(\ln\frac{k_\perp^2}{Q_s^2(Y)}+c\right)$$

with: $\gamma = 0.63, \quad c = 1/\gamma$



Geometric Scaling in DIS at small x

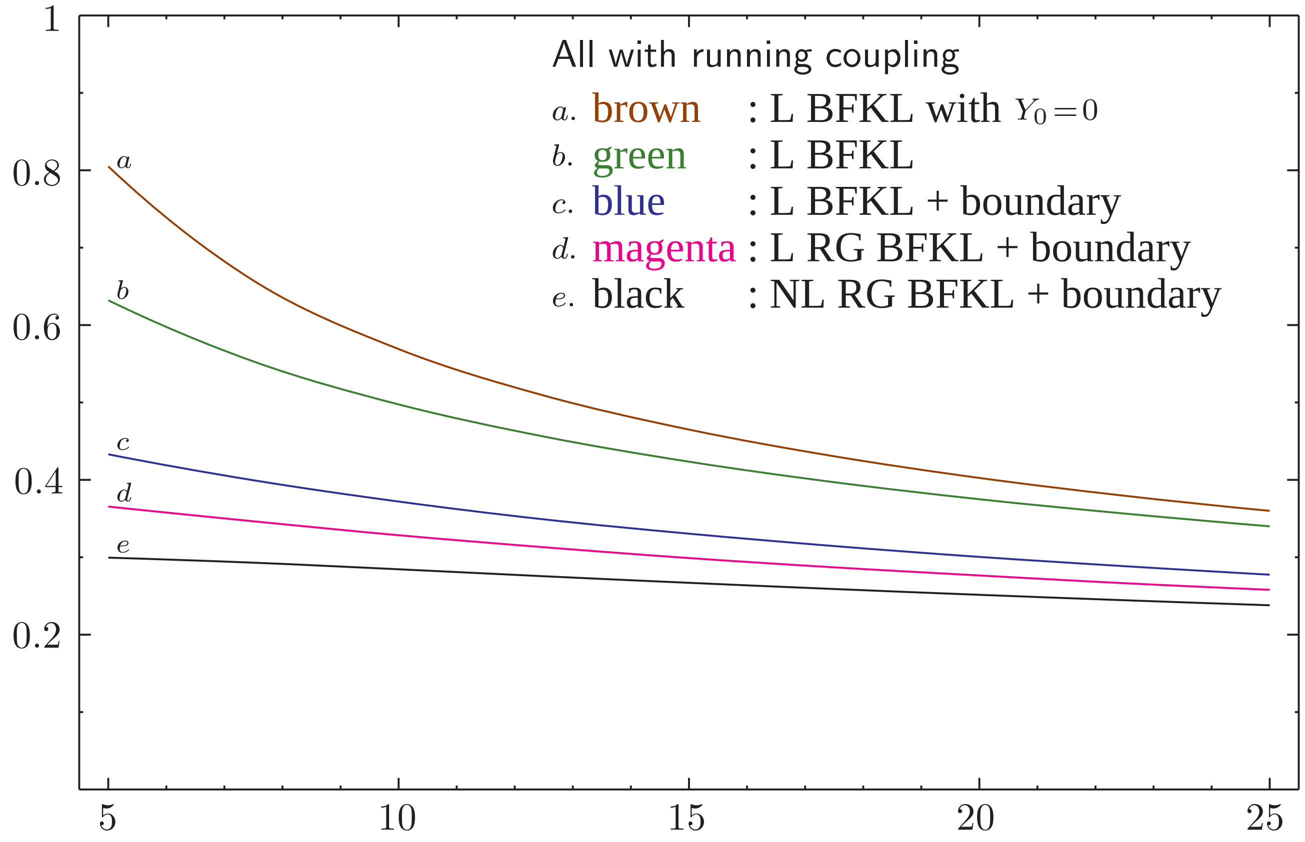
Gelis, Peschanski, Soyez, Schoeffel, hep-ph/0610435



- Left: $\tau \equiv \log Q^2 - \lambda Y$, with $\lambda = 0.32$
- Right: $\tau \equiv \log Q^2 - \lambda\sqrt{Y}$, with $\lambda = 1.62$

The energy dependence of Q_s

D.N. Triantafyllopoulos, 2002

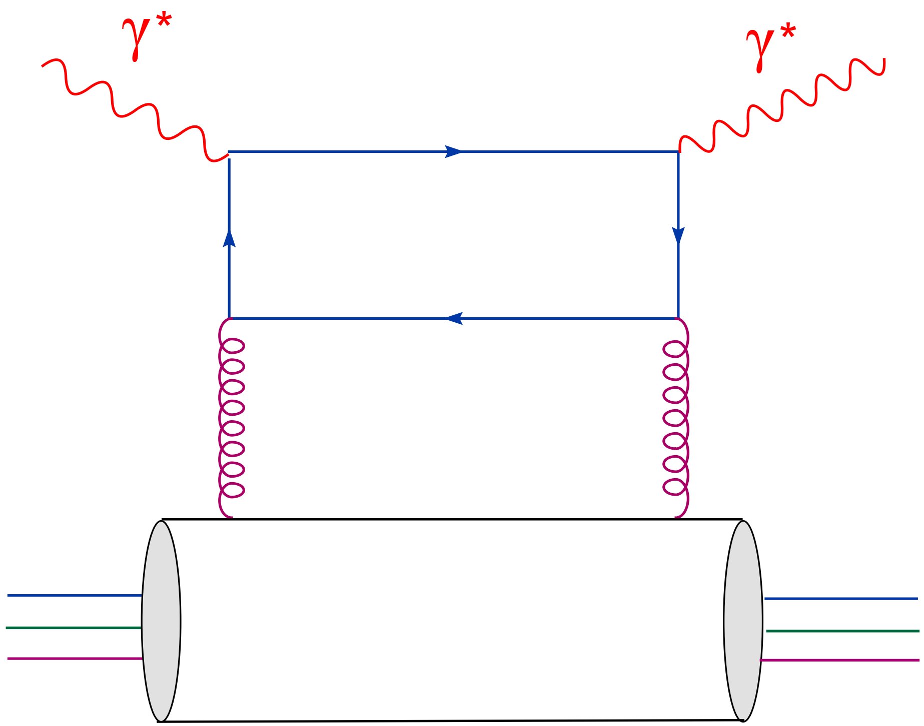


$$\lambda(Y) \equiv \frac{\mathrm{d}\ln Q_s^2(Y)}{\mathrm{d}Y}$$

- NLO corrections dramatically slow down the evolution !

DIS at small x

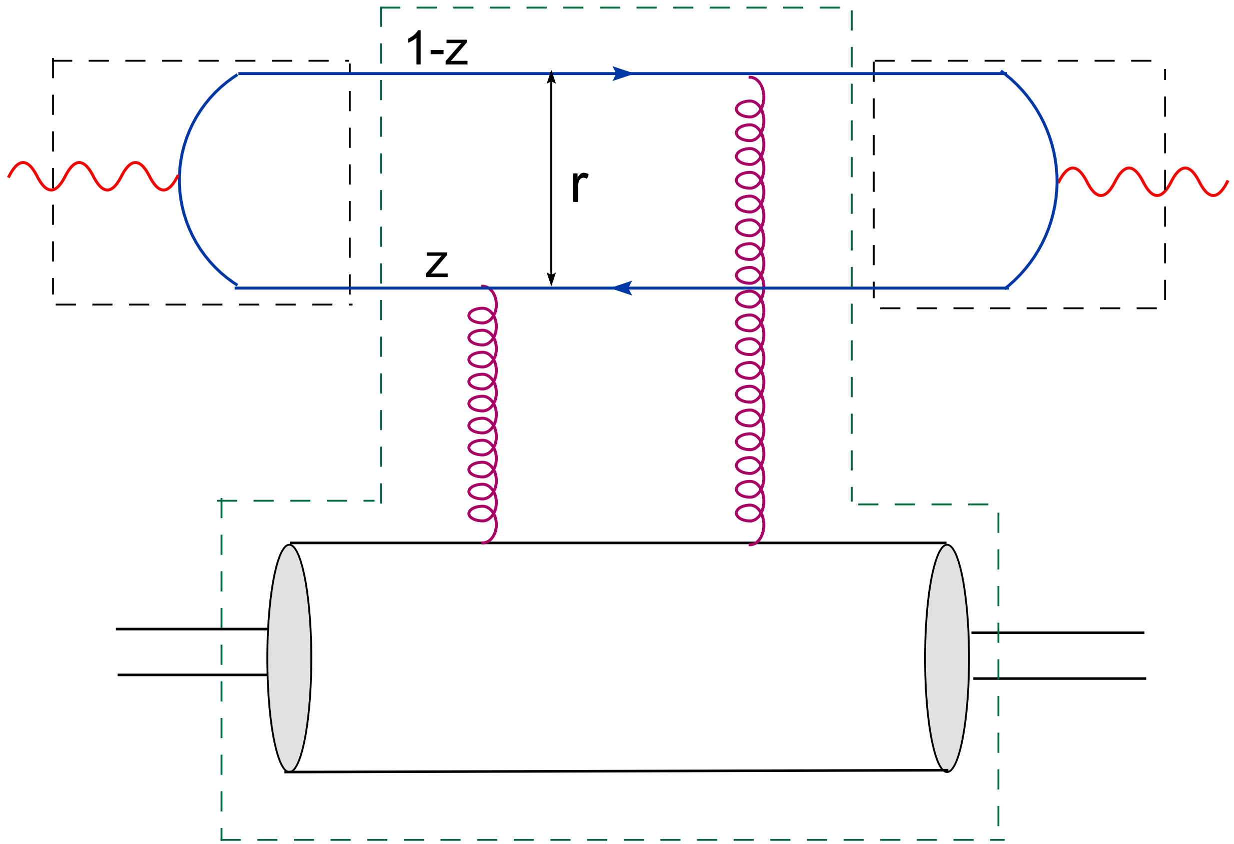
- At small x, the struck quark is typically a sea quark, emitted off the gluon distribution :



- The quark loop can be alternatively associated with the virtual photon wavefunction

Dipole factorization for DIS

$$\frac{d\sigma_{\gamma^* p}}{d^2 b}(x, Q^2) = 2\int_0^1 dz \int d^2\boldsymbol{r}\; |\Psi_\gamma(z, \boldsymbol{r}; Q^2)|^2 \; T_{\text{dipole}}(x, \boldsymbol{r})$$

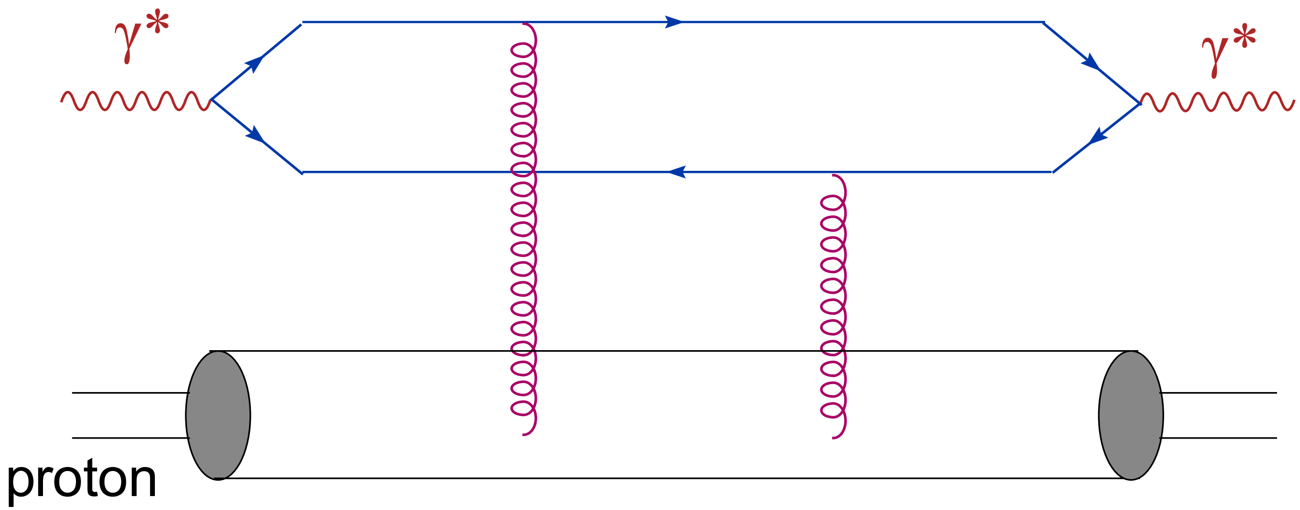


- Unitarity bound on the dipole amplitude: $T(x, \boldsymbol{r}, \boldsymbol{b}) \leq 1$

Dipole scattering

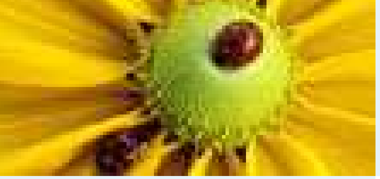
- A small color dipole scatters off the electric field in the target

$$V(\boldsymbol{r}) \simeq g t^a \boldsymbol{r} \cdot \boldsymbol{E}_a \implies T(x,r) \propto \frac{g^2}{N_c} r^2 \langle \boldsymbol{E}_a \cdot \boldsymbol{E}_a \rangle_x$$



$$T(x,r) \simeq \frac{\alpha_s}{N_c} r^2 \frac{xG(x,1/r^2)}{\pi R^2} \propto \alpha_s^2 \frac{r^2}{R^2} \frac{1}{x^\lambda}$$

- When decreasing x and/or increasing r : $T(x,r) \sim \mathcal{O}(1)$



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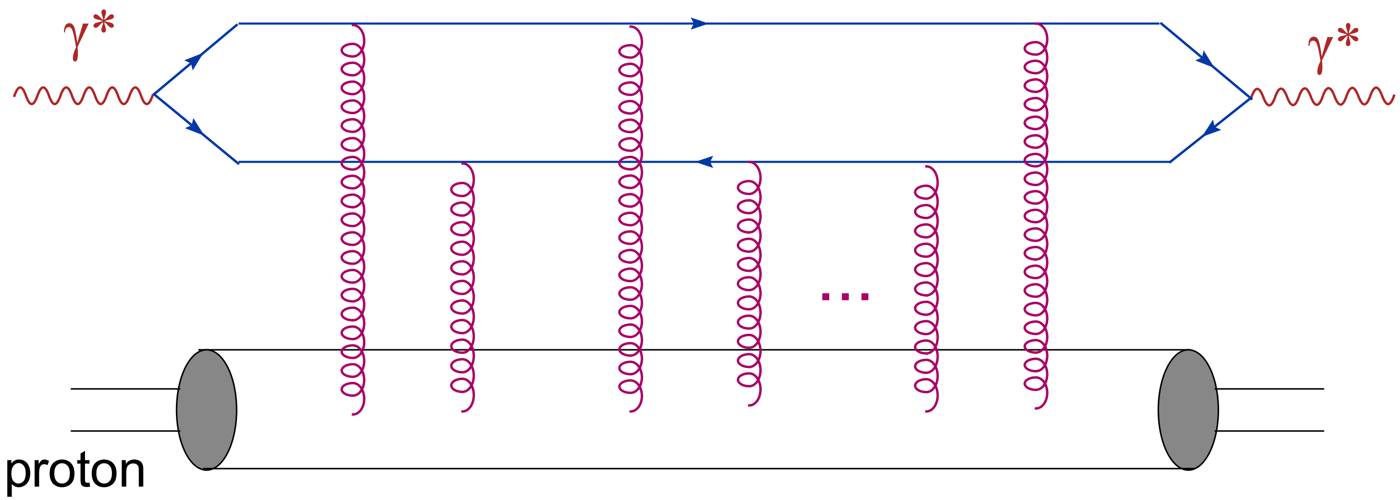
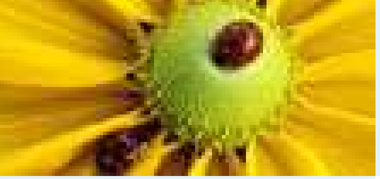
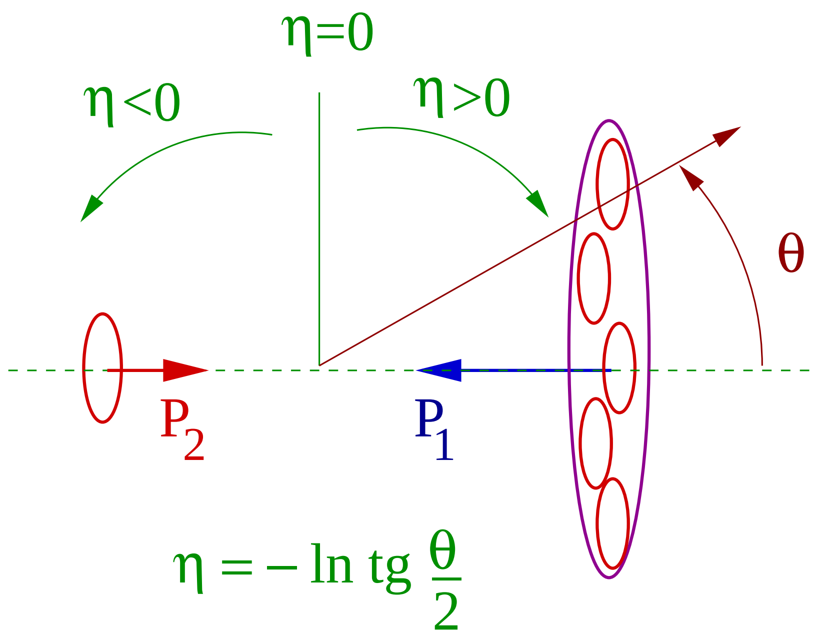
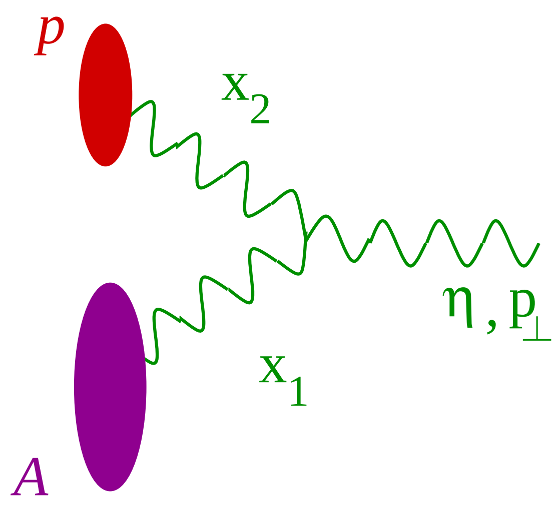


- Multiple scattering becomes important and restores unitarity



Gluon production: Kinematics





$$x_1 = \frac{p_\perp}{\sqrt{s}}\, \mathrm{e}^{-\eta}\,, \qquad x_2 = \frac{p_\perp}{\sqrt{s}}\, \mathrm{e}^{\eta}$$

- Increasing $\eta \iff$ Decreasing x_1 for the nucleus
  - RHIC: $\eta \simeq 3$ & $\sqrt{s} = 200$ GeV: $x_1 \sim 10^{-4}$ for $p_\perp = 2$ GeV
  - LHC : $\eta \simeq 6$ & $\sqrt{s} = 8.8$ TeV : $x_1 \sim 10^{-6}$ for $p_\perp = 10$ GeV

The Color Glass Condensate

- At saturation, gluons have large occupation numbers $n \sim 1/\alpha_s \gg 1$: ‘Bose condensate’ (strong field)
- The small–x gluons are emitted from ‘color sources’ (partons) with larger values of x, which are frozen in some random configuration : ‘Glass’ (frozen disorder)
- Gluons carry color !