QCD collisional energy loss

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SUBATECH, Nantes

– 2$^{\mathrm{e}}$ journées QGP, Etretat, September 2007 –

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'In all affairs it's a healthy thing now and then to hang a question mark on the things you have long taken for granted.' (Bertrand Russell)

RHIC-ognitions

hadron matter at large T
is quark-gluon plasma

$\Longleftarrow$

$R_{AA} \sim$ partonic jet quenching

usually interpreted as radiative energy loss, yet with somewhat *'surprising'* parameters, e.g. $\hat{q}$

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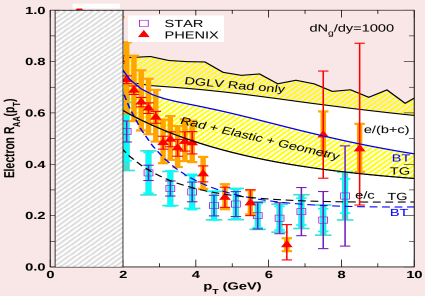

[Wicks et al., 2007]

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$\Updownarrow$

motivation to re-address the 'settled' question of **collisional energy loss**

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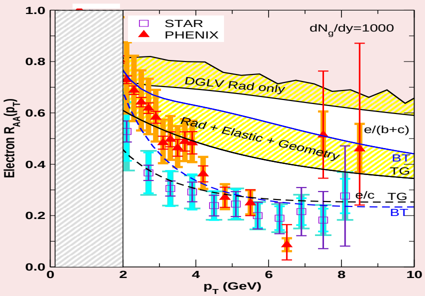
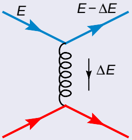


[Wicks et al., 2007]

Two mechanisms

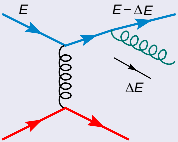
collisional energy-loss



$$\frac{dE_{col}}{dx} \sim \alpha^2 \ln(E)$$

[Bjorken, Gyulassy-Braaten-Thoma]

radiative energy-loss



$$\frac{dE_{rad}}{dx} \sim \alpha^3 E^n, \qquad n = \{1, \tfrac{1}{2}, 0\}$$

[BDMPS(Z), GLV, . . .]

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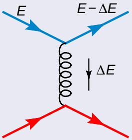

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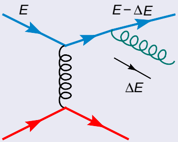

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[BDMPS(Z), GLV, ...]

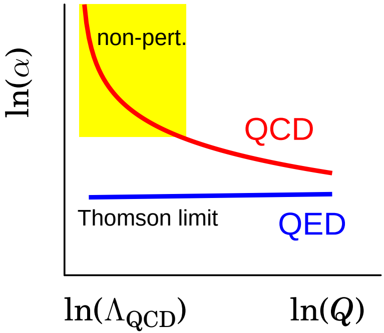
'paradigm': collisional $\ll$ radiative loss

statement clearly depends on value of the coupling α

$$\text{theoretical result} = \alpha^n \times \text{dimension} \times [1 + \text{corrections}]$$

$$\alpha(Q^2) = \frac{4\pi/\beta_0}{\ln(Q^2/\Lambda^2)}$$

	β_0	Λ
QED	$-\frac{4}{3}$	10^{xx} GeV
QCD	$11 - \frac{2}{3} n_f$	~ 0.2 GeV



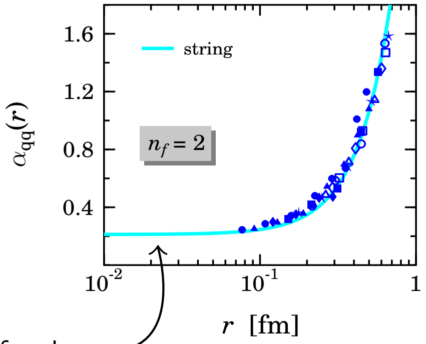
there is **NO** QCD coupling *'per se'* $\Rightarrow$ 'relevant scale' Q is crucial

Strong coupling with perturbative methods?

1. **heavy quark potential** $V_{qq}(r)$ $\longrightarrow$ coupling $\alpha_{qq}(r) = \frac{3}{4}\, r^2 \frac{\partial V_{qq}}{\partial r}$

- lattice QCD [Kaczmarek et al.]
- string parameterization:

$$\begin{aligned} V_{qq}^{\sigma} &= -\tfrac{4}{3} a r^{-1} + \sigma r \\ \rightarrow \alpha^{\sigma} &= a + \tfrac{3}{4}\, \sigma r^2 \end{aligned}$$



NO asympt. freedom

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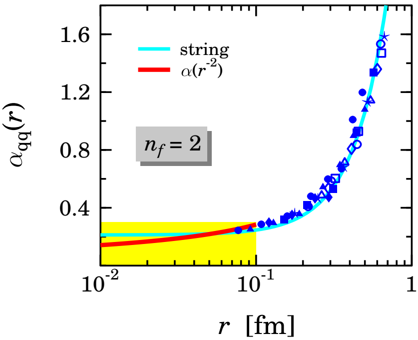

- challenge pQCD

$$\alpha(q^2 = r^{-2})$$

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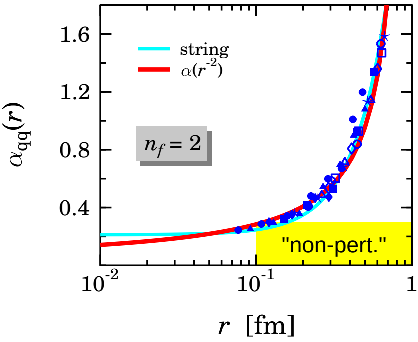
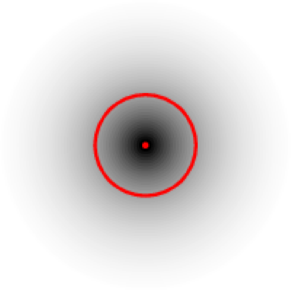


'provocative' observation [AP 2006]

pQCD with $\Lambda = 0.2$GeV can work *quant'ly* even at $\alpha \sim 1$

2. **heavy quark potential at T > 0: Debye screening**

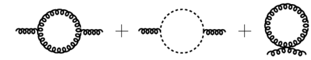


$$V(r) \sim \frac{\exp(-r/l_D)}{r}$$

Debye mass $m_D = l_D^{-1}$ is important *IR regulator*

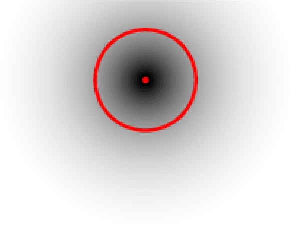
Strong coupling with perturbative methods?

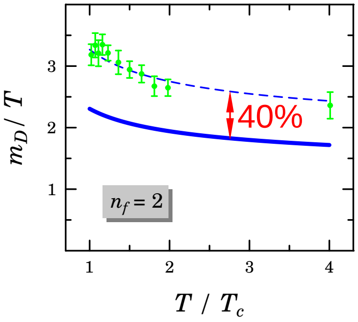
2 heavy quark potential at $T > 0$: Debye screening

QGP: 

$$m_D^2 = \alpha(Q^2)\, 4\pi \left(1 + \tfrac{1}{6} n_f\right) T^2$$

'folklore': $Q \simeq 2\pi T$





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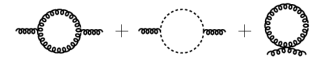
QGP: 

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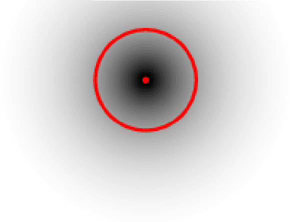
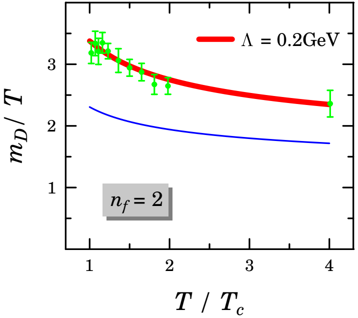
renormalization: $Q = m_D$ [AP 2006]





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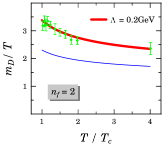
pQCD with $\Lambda = 0.2\text{GeV}$ works near T_c!

Interlude: Why this is more than academic

$\Rightarrow$ *incorrect 'choice' of relevant momentum scale in* $\alpha(Q^2)$ *can lead to false picture of a 'more' strongly coupled QGP!*

	'k-factor'
$\alpha^{1/2}$	~ 1.4



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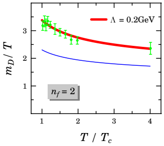
α^1	~ 2



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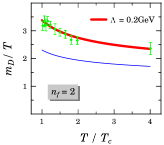
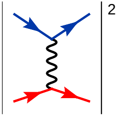
α^2	~ 4





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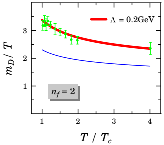

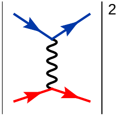

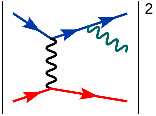

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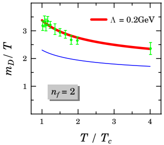

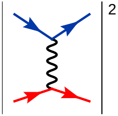

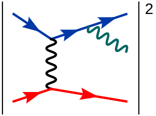

recall: inferred transport coefficient $\hat{q} \sim 10\,\text{GeV}^2/\text{fm}$ seems factor 10 too large

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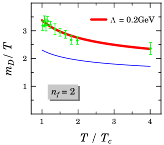

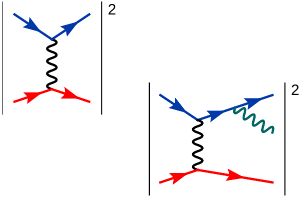

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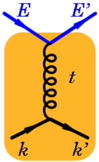
Do we really understand the *BASICS* of QCD energy loss?

QCD collisional energy loss

Critical review

Bjorken '82: $$\frac{dE^{coll}}{dx} \sim \int_{k^3} \frac{n(k)}{2k} \int^{Ek} dt\, t\, \frac{d\sigma}{dt}$$



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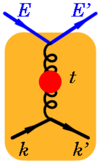

$\frac{d\sigma}{dt} \sim \frac{\alpha^2}{t^2}$: $\mathcal{I}_{Bjorken} \sim \int_{\mu^2}^{Ek} dt\, t\, \frac{\alpha^2}{t^2} = \alpha^2 \ln \frac{Ek}{\mu^2}$

divergence,
IR-cutoff **required**

Gyulassy, Braaten, Thoma calculate $\mu \sim m_D$ with *hard thermal loop* theory

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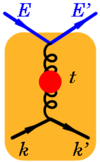

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divergence,
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conceptual inconsistency

thermal fluctuation needed – **vacuum contributions 'forgotten'**

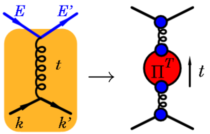
$\Rightarrow$ value of α unspecified in $dE^{coll}_{Bjorken}/dx \sim \alpha^2 T^2 \ln(ET/m_D^2)$

NB: this often-used formula is **NOT PREDICTIVE!**

Revision

calculate loop-corrections and renormalize:

- IR-screening at scale of Debye mass
- bare coupling $\longrightarrow$ running coupling



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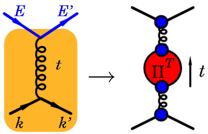

crucial modification of t-integral

$$\mathcal{I}_{Bjorken} \sim \int_{m_D^2}^{Ek} dt\, t\, \frac{\alpha^2}{t^2} = \alpha^2 \ln \frac{Ek}{m_D^2}$$

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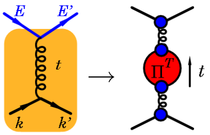

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leading-order (resummed) result

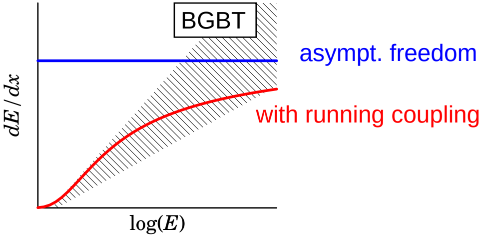
$$\frac{dE^{coll}}{dx} \xrightarrow{E\to\infty} \alpha(m_D^2)\,T^2 \quad \text{vs.} \quad \frac{dE^{coll}_{Bjorken}}{dx} \sim \alpha^2 T^2 \ln \frac{ET}{m_D^2}$$

[AP 2006]

QCD collisional energy loss

$$\frac{dE^{coll}}{dx} \sim \left(\alpha(m_D^2) - \alpha(ET)\right) T^2 \quad \text{vs.} \quad \frac{dE^{coll}_{Bjorken}}{dx} \sim \alpha^2 T^2 \ln(ET/m_D^2)$$

- predictive! (Λ is fixed)
- 1st order in the coupling at soft scale: it can be larger than previously 'assumed'
- for large E: saturation due to asymptotic freedom



Jet quenching

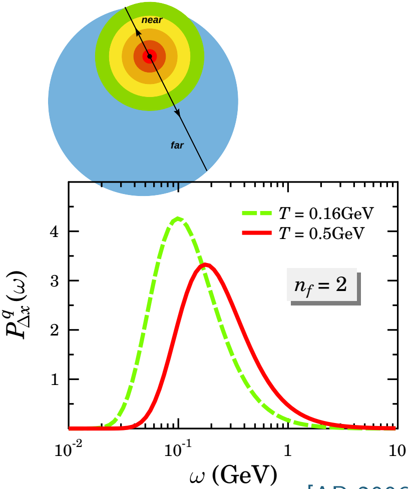
dE/dx insufficient for jet quenching, we need stochastic description [BDMS]

Occam's razor: *only* collisional loss

MC simulation

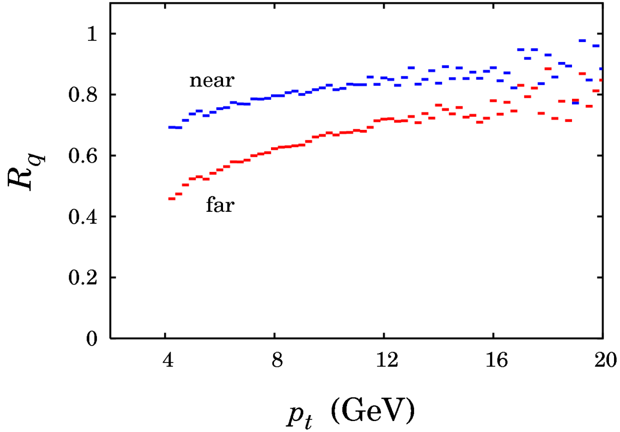
- <u>*local*</u> energy-loss probability $P(\omega)$ in eikonal approx.
- central collisions and mid rapidity
- Bjorken geometry (R) and dynamics

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3}$$



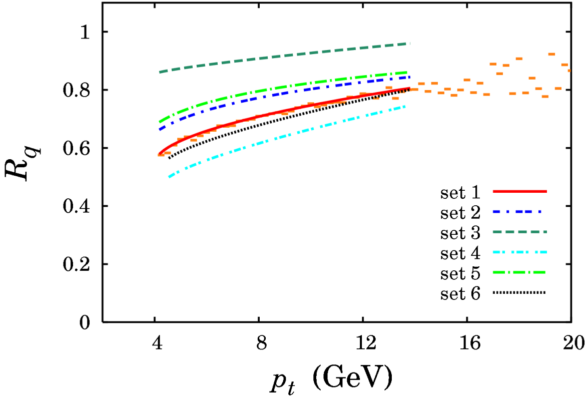
[AP 2006]

partonic suppression ratio $R_q = \frac{dN/dp|_{fin}}{dN/dp|_{ini}}$



$$R = 5\,\mathrm{fm}$$
$$T_0 = 0.5\,\mathrm{GeV}$$
$$\tau_0 = 0.2\,\mathrm{fm}$$

Jet quenching



sizeable collisional contribution to jet quenching

[AP 2006]

	set 1	set 2	set 3	set 4	set 5	set 6
R (fm)	5	3	5	5	5	7
T_0 (GeV)	0.5	0.5	0.3	0.7	0.5	0.5
τ_0 (fm)	0.2	0.2	0.2	0.2	0.1	0.2
τ_c (fm)	6.1	6.1	1.3	16.7	3.1	6.1

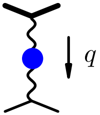
QED collisional energy loss

Critical review

running coupling does not affect QED-‘playground’ muon in $e^{\pm}\gamma$ plasma

n.l.l. calculation [Braaten & Thoma 1991]

- Braaten-Yuan matching method
  - hard contribution: tree-level amplitude
  - soft contribution: *hard thermal loop* propagator
  - matching at intermediate scale $eT \ll q^\star \ll T$



- relevant limit $E \gg M \gg T$, i.e. $v \to 1$

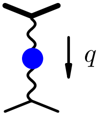
$$-\frac{dE}{dx}\bigg|_{\mathrm{BT}}^{v\to 1} = \frac{e^4 T^2}{48\pi}\left[\ln\frac{2E}{e^2 T} + 2.031\right]$$

Critical review

running coupling does not affect QED-'playground' muon in $e^{\pm}\gamma$ plasma

n.l.l. calculation [Braaten & Thoma 1991]

- Braaten-Yuan matching method
  - hard contribution: tree-level amplitude
  - soft contribution: *hard thermal loop* propagator
  - matching at intermediate scale $eT \ll q^\star \ll T$



- relevant limit $E \gg M \gg T$, i.e. $v \to 1$

$$-\frac{dE}{dx}\bigg|_{\mathrm{BT}}^{v\to 1} = \frac{e^4 T^2}{48\pi}\left[\ln\frac{2E}{e^2 T} + 2.031\right]$$

incorrect assumption: $q \ll E$

$\ln E \sim \int^E \frac{dq}{q}$ $\Rightarrow$ **hard momenta do contribute to n.l.l. order!**

n.l.l. accuracy requires

- thorough re-calculation of hard contribution of $\mu e^{\pm}$-scattering

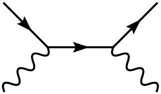
$\Rightarrow$ new result [Peigné & AP, to be publ.]

$$-\frac{dE}{dx}\bigg|^{v\to 1} = \frac{e^4 T^2}{48\pi}\left[\ln\frac{2E}{e^2 T} + \underbrace{\ln 24 - \gamma + \frac{\zeta'(2)}{\zeta(2)}} - \frac{3}{4}\right]$$

Revision

n.l.l. accuracy requires

- thorough re-calculation of hard contribution of $\mu e^{\pm}$-scattering
- calculation of Compton contribution (previously missed by incorrect assumption)



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$$+\frac{e^4T^2}{96\pi}\left[\ln\frac{4TE}{M^2} - \frac{5}{6} - \gamma + \frac{\zeta'(2)}{\zeta(2)}\right]$$

Resumé

principle of cognition

describe one phenomenon $\longrightarrow$ predict others

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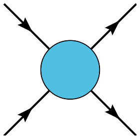
QCD:

- fix $\mathbf{\Lambda}$ *once and for all*
  - heavy quark potential at $T = 0$
- *(re-)calculate*
  - Debye mass
  - collisional energy loss dE/dx
  - jet quenching



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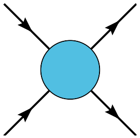

'It's promising for the future. The important matches are coming.'

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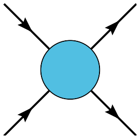

'It's promising for the future. The important matches are coming.'

(Zinedine Zidane)