

Detectors for spaceborne gamma-ray telescopes and polarimeters: state of the art and perspectives

Focus on the angular resolution of e^+e^- pairs telescopes

Denis Bernard,

名 姓

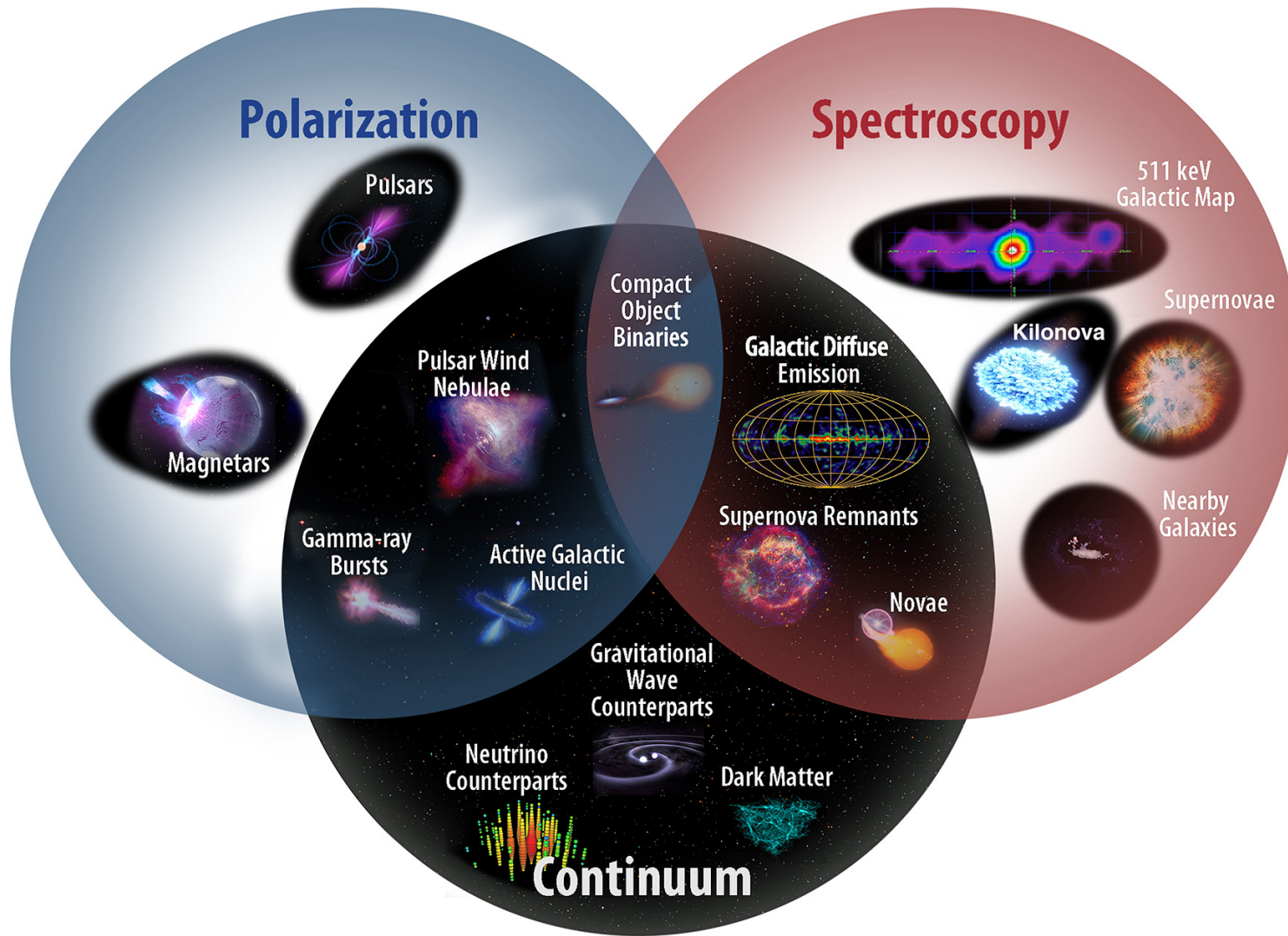
Laboratoire Leprince-Ringuet (LLR), Ecole Polytechnique & CNRS / IN2P3, France

S14, Future spaceborne MeV detectors and related astrophysics

International Conference of Deep Space Sciences, April 2023, Hefei, China



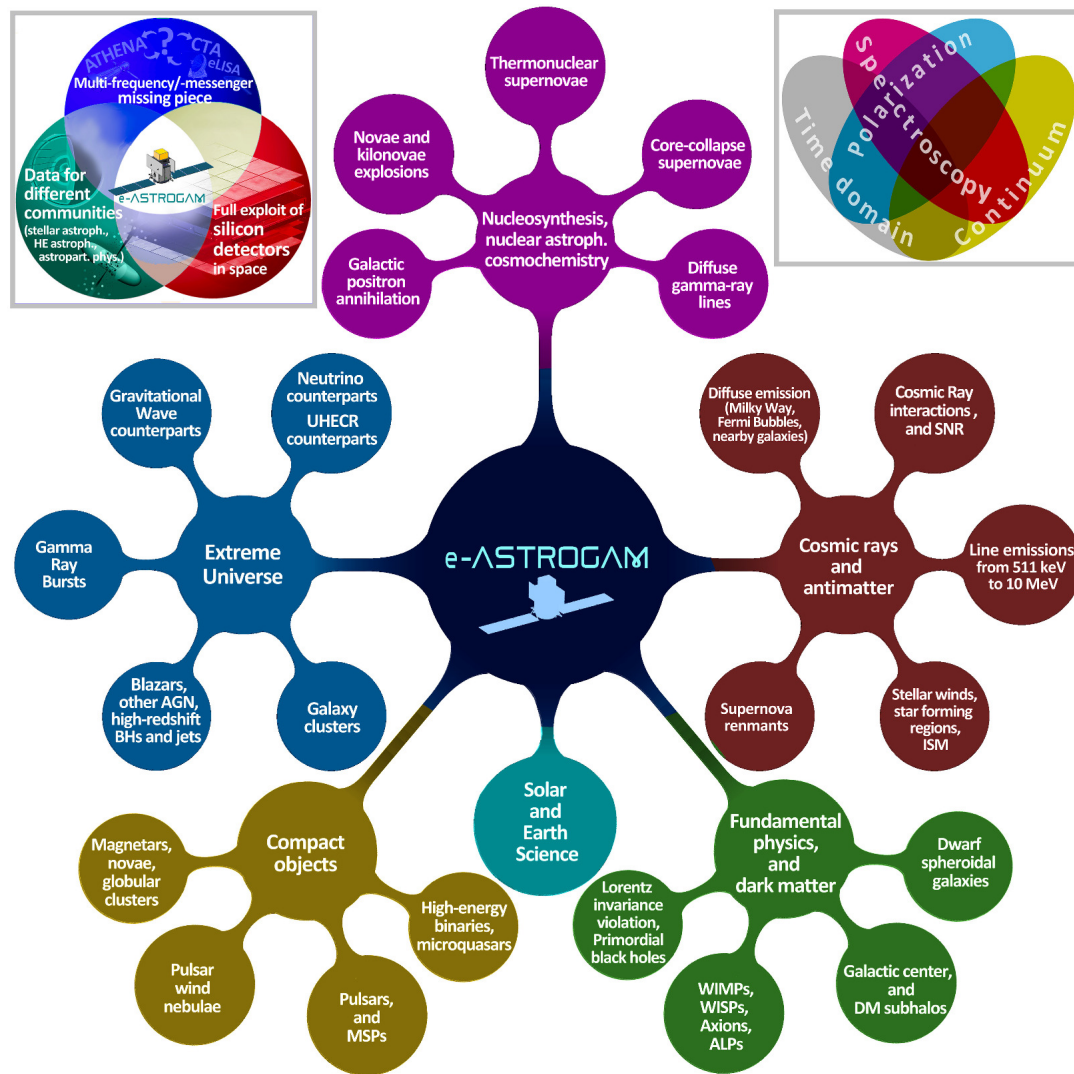
Next MeV gamma-ray space mission: Scientific programme



R. Caputo *et al.*, *J. Astron. Telesc. Instrum. Syst.* **8** (2022) no.4, 044003

AMEGO-X (The All-sky Medium Energy Gamma-ray Observatory eXplorer) is a combined (Compton + pairs) telescope project

Next MeV gamma-ray spece mission: Scientific programme

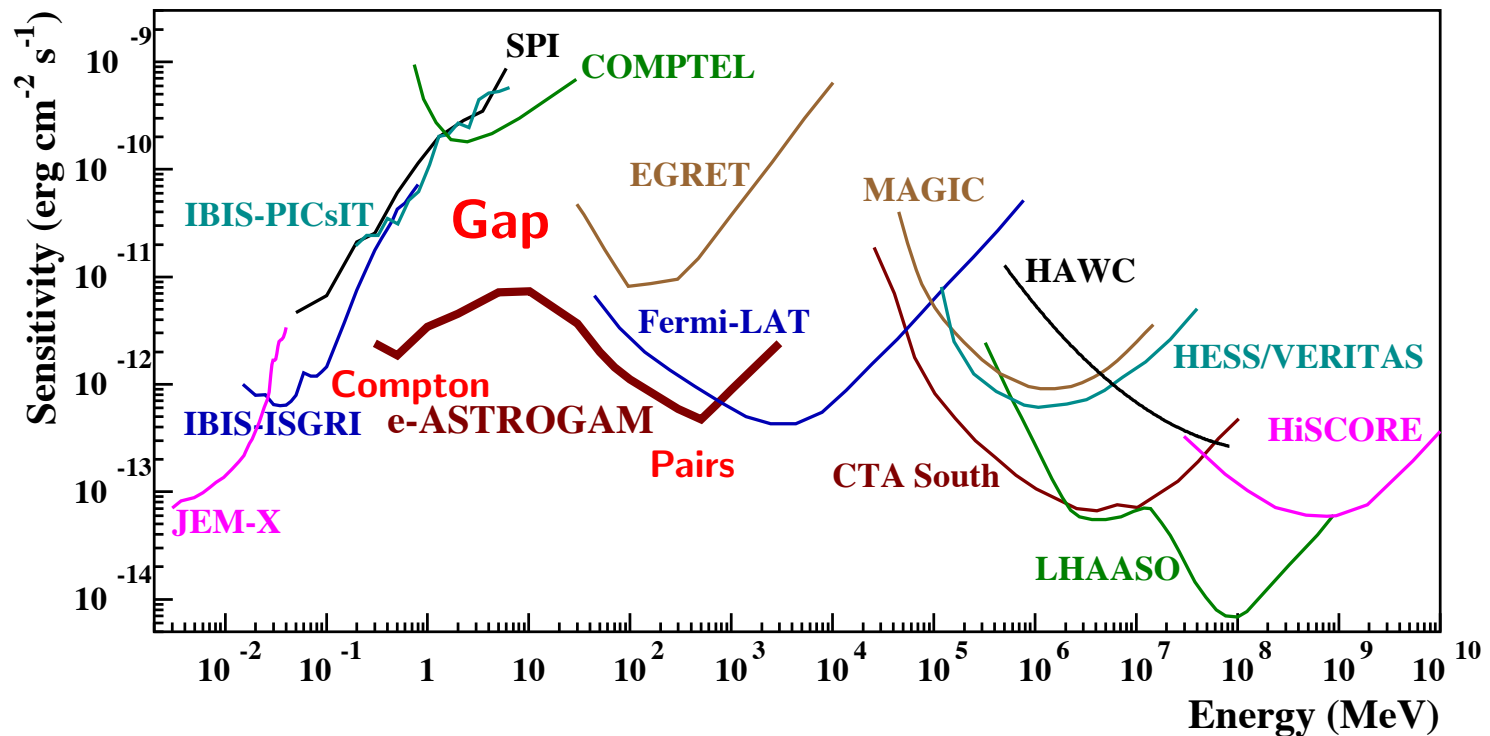


A. De Angelis *et al.* [e-ASTROGAM], JHEAp **19** (2018) 1-106

ASTROGAM is a combined (Compton + pairs) telescope project

Point source continuum differential sensitivity

“**Sensitivity gap**” in the MeV energy range



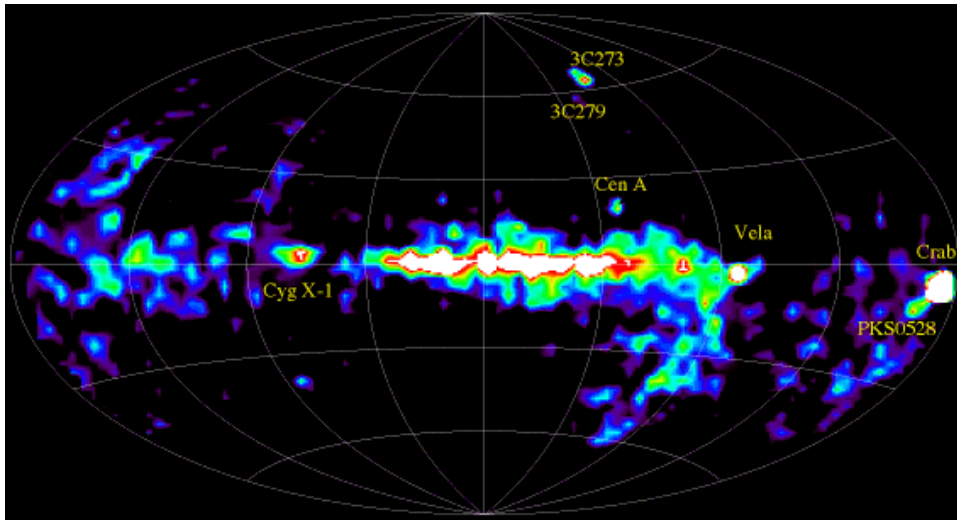
A. De Angelis *et al.* [e-ASTROGAM], JHEAp **19** (2018) 1-106

Differential flux sensitivity: Minimum flux needed to obtain a n -standard-deviation detection from a point-like gamma-ray source, ASTROGAM: $n = 3$, $T = 1$ year.

ASTROGAM is a combined (**Compton** + **pairs**) telescope project

The γ -Ray Sky in the MeV Energy Range

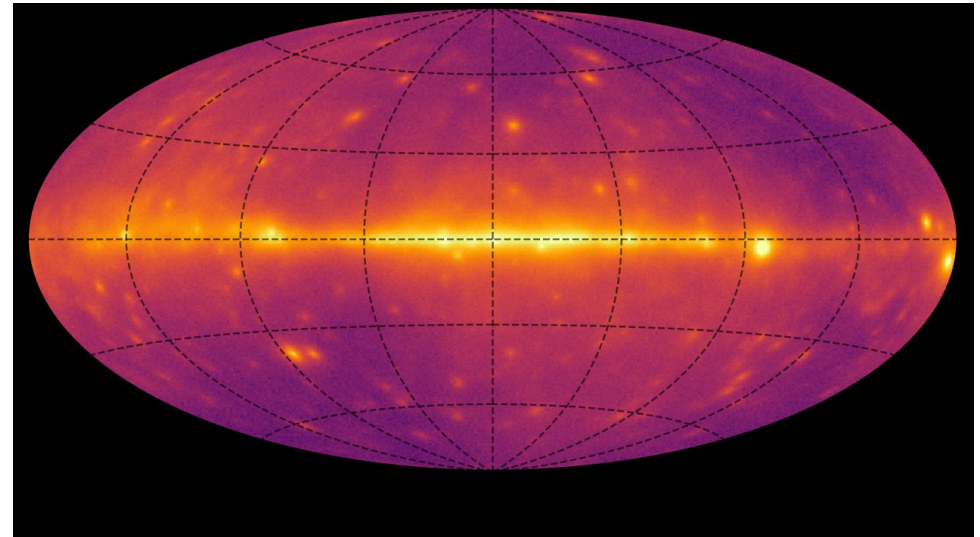
COMPTEL
(Compton)



1 – 30 MeV
9 years

heasarc.gsfc.nasa.gov

Fermi LAT
(Pairs)



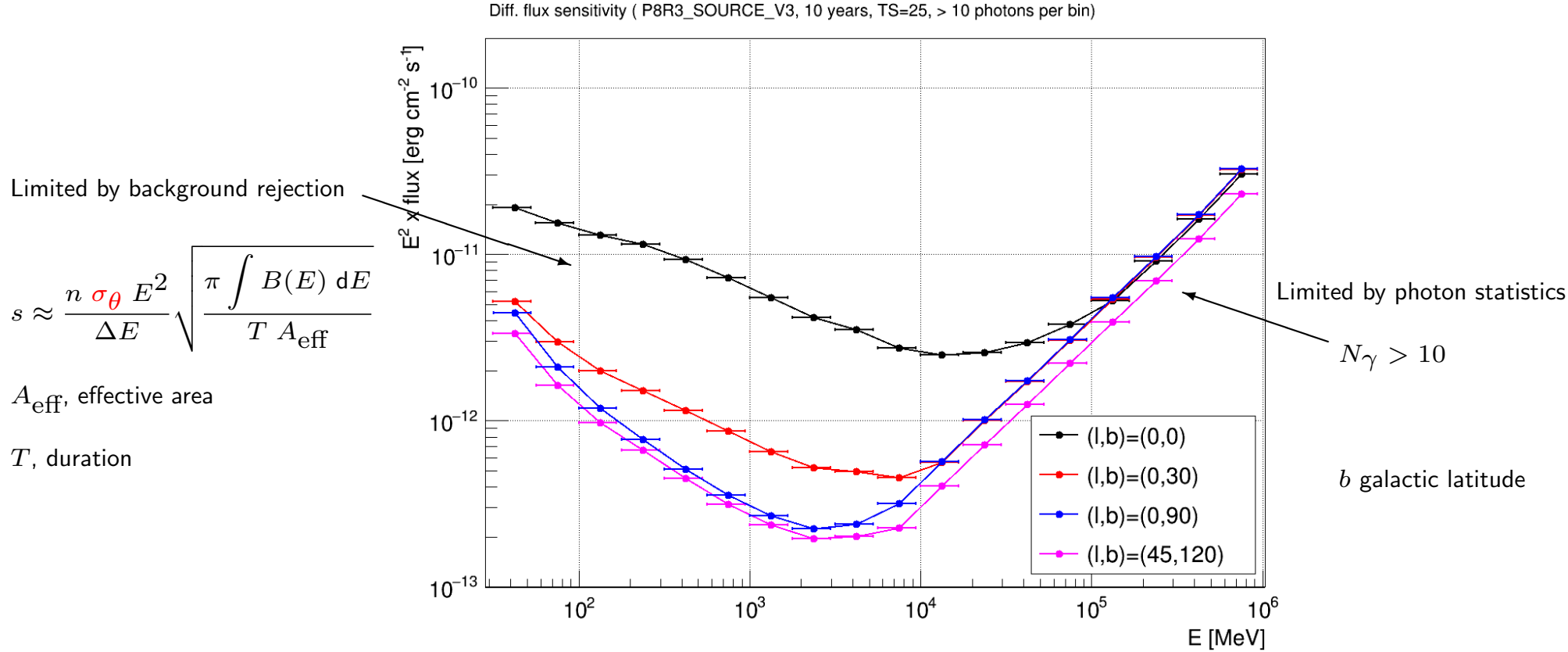
20 – 200 MeV
13 years

10th Fermi Symposium

“We are not facing a sensitivity wall, we are facing an angular resolution wall”

I. Grenier (2016)

Differential flux sensitivity and angular resolution

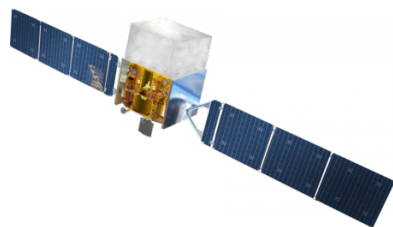


Fermi LAT (Large Area Telescope) [performance.html](https://fermi.gsfc.nasa.gov/science/observing/lat_performance.html)

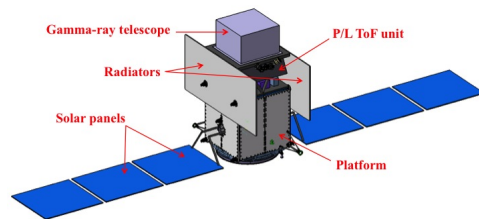
- In this talk I focus on the angular resolution, σ_{θ} .

Differential flux sensitivity: Minimum flux needed to obtain a n -standard-deviation detection from a point-like gamma-ray source, (here for 4 energy bins / decade, $n = 5$)

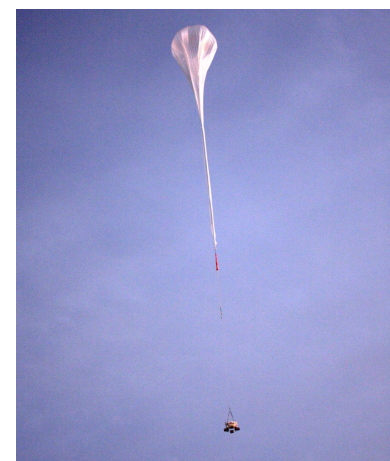
Test Cases



Fermi-LAT



AMEGO-X / ASTROGAM



GRAINE

| | | | |
|---------------------|--|---|---|
| Converter / Tracker | Si + W foils | "Pure" Si | film (emulsion+"base") |
| Carrier | satellite | satellite | balloon |
| Status | In orbit (2008) | projects | project prototype flown |
| | Astrophys. J. 697 (2009) 1071 | J. Astron. Tel. Instr. Syst. 8 (2022) 044003 | Adv. Space Res. 62 (2018) 2945 |

[JHEAp **19** \(2018\) 1-106](#)

Photon angular resolution

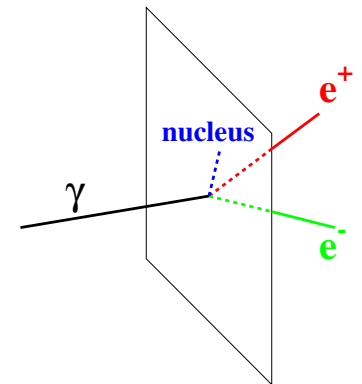
$$\gamma Z \rightarrow e^+ e^- Z$$
$$\vec{k} = \vec{p}_{e^+} + \vec{p}_{e^-} + \vec{p}_r$$

Contributions:

- (a) Un-measured **nucleus recoil momentum**
- (b) **Single-track momentum resolution**
- (c) **Single-track angular resolution,**

Results obtained with the full, 5D, Bethe-Heitler differential cross section

- Includes exact recoil momentum, \vec{p}_r , (angle, magnitude) distributions
- 3D final state (NOT a coplanar conversion)
- Energy-momentum strictly conserved



H. Bethe and W. Heitler, *Proc. R. Soc. Lond. A*, **146** (1934) 83

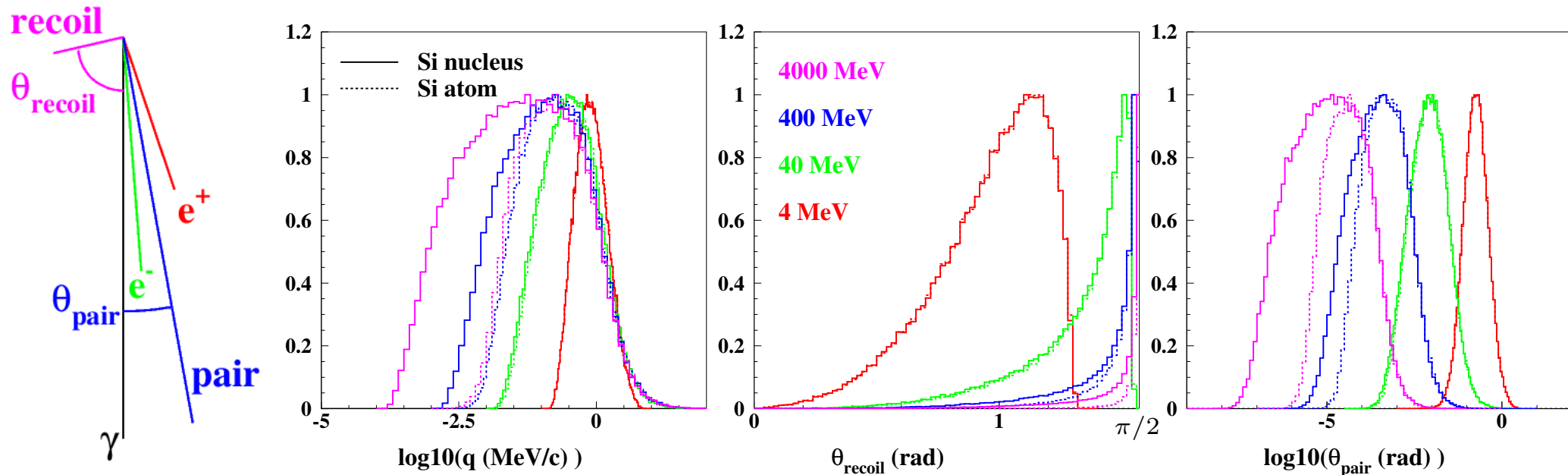
D. Bernard, *Nucl. Instrum. Meth. A* **899** (2018), 85

In Geant4 as *G4BetheHeitler5DModel* ,

default for high-precision EM physics (Opt 4) since [release 10.6 \(Dec. 2019\)](#)

V. Ivanchenko *et al.* [Geant4], *EPJ Web Conf.* **214** (2019), 02046

(a) Unmeasured nucleus recoil momentum

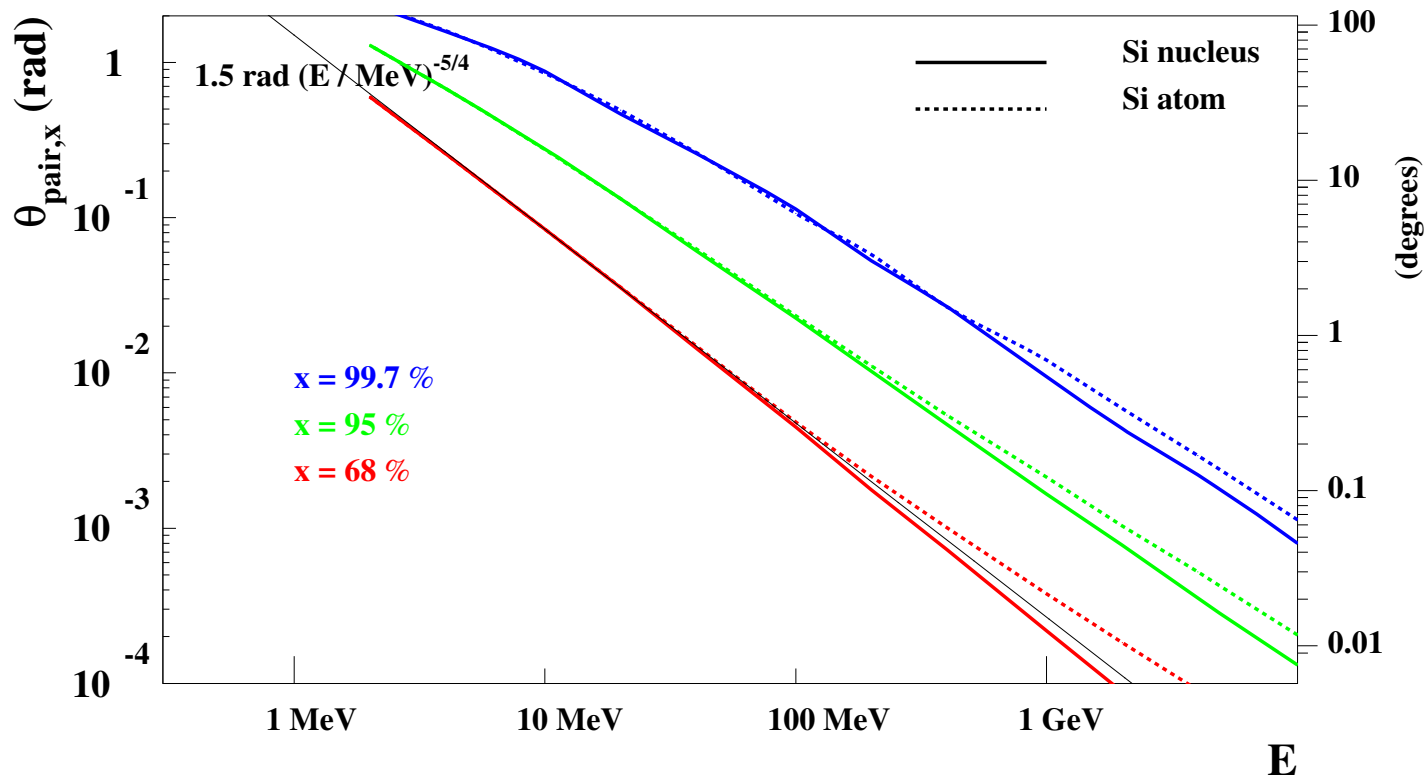


- Conversion on nuclei, recoil momentum distribution extends to $q_{\text{min}} \propto 1/E$
- Conversion on atoms (not on isolated nucleus) & $E > 100 \text{ MeV}$, low- q tail cut by screening
- Recoil direction asymptotically transverse, $\theta_{\text{recoil}} \approx \pi/2$
- Angular resolution, θ_{pair} , scales asymptotically like q/E

D. Bernard, NIM A 729 (2013) 765

P. Gros et al., Astroparticle Physics 88 (2017) 60

Unmeasured nucleus recoil momentum: angular resolution “containment” value



@ 100 MeV, $\theta_{\text{pair},68\%} = 0.27^\circ$, $\theta_{\text{pair},95\%} = 1.35^\circ$ and $\theta_{\text{pair},99.7\%} = 6.56^\circ$

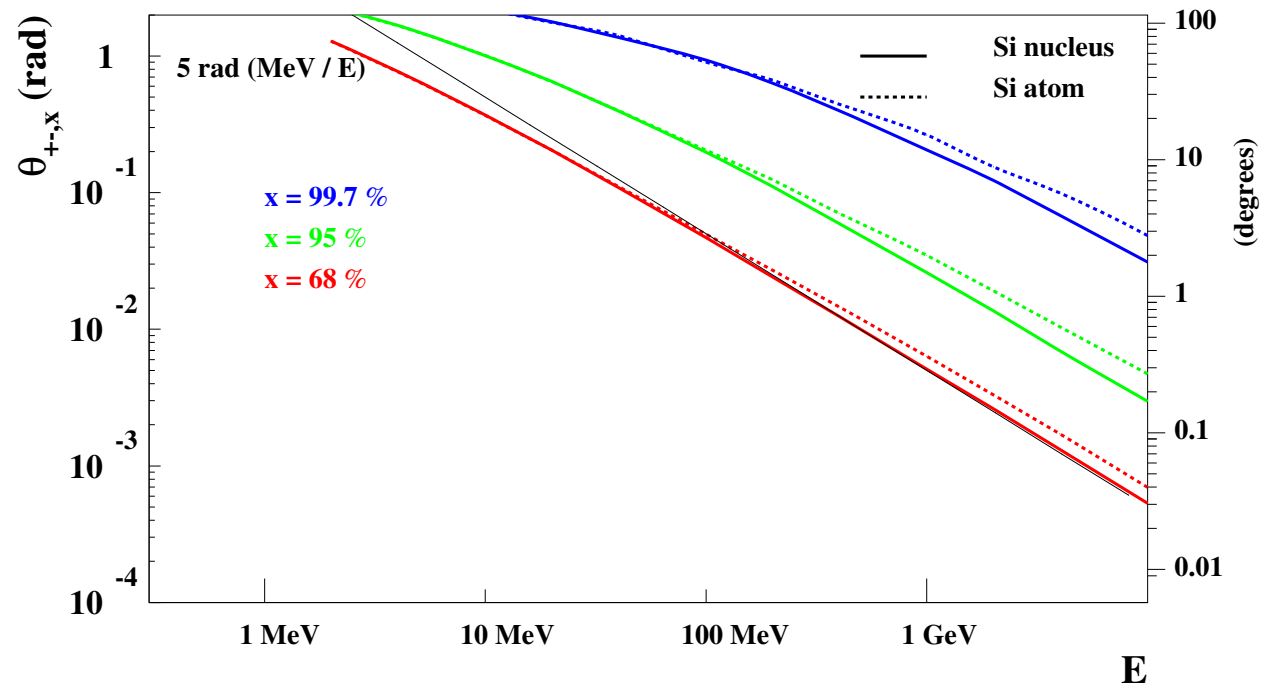
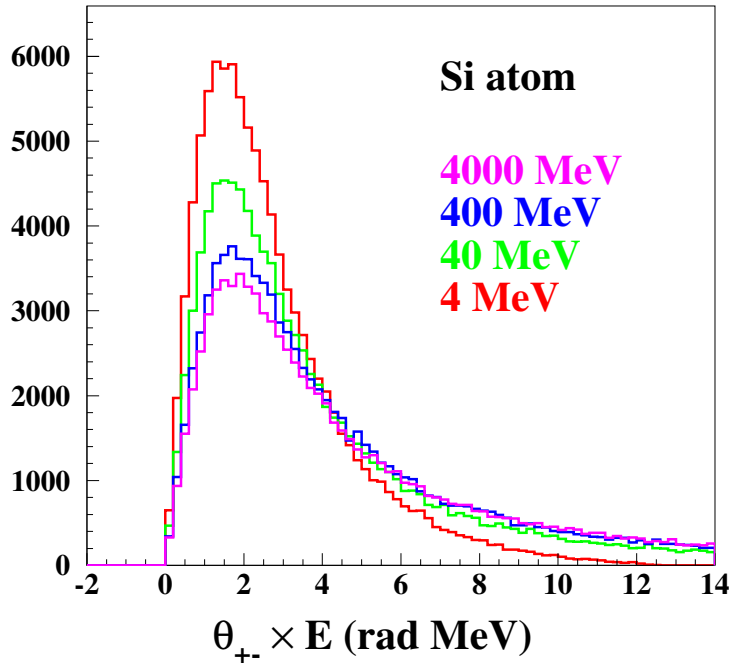
An unavoidable kinematic limit ; Scales like $E^{-5/4}$

Adapted from

D. Bernard, NIM A 729 (2013) 765

P. Gros et al., Astroparticle Physics 88 (2017) 60

(b) Single-track momentum resolution; Pair opening angle, θ_{+-}



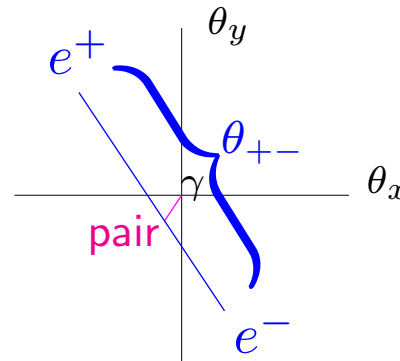
Scales like E^{-1} ; (opening angle) $\theta_{+-,x} \gg \theta_{\text{pair},x}$ (pair polar angle)

High-energy asymptotic expressions

Full analysis

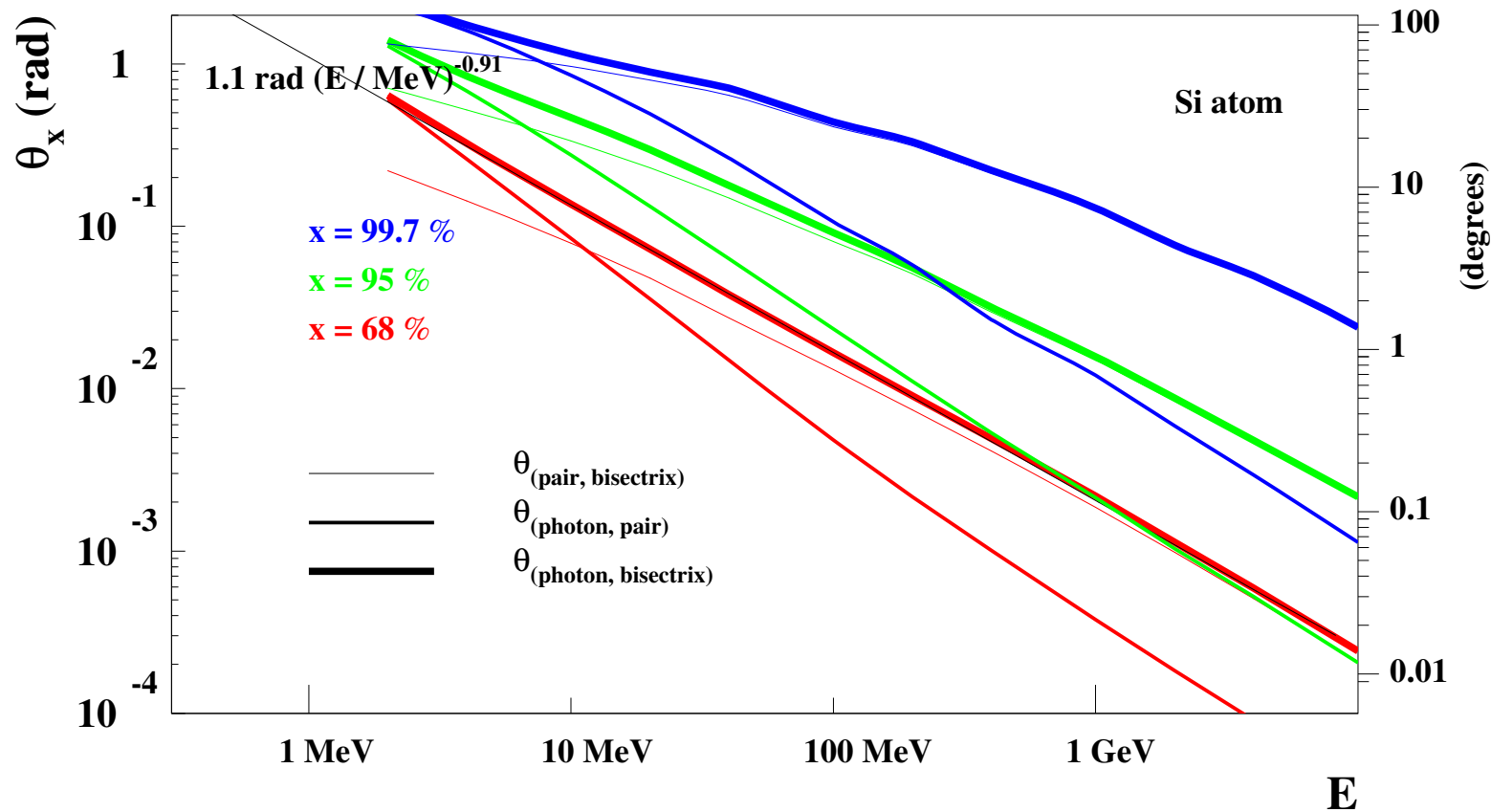
H. Olsen, Phys. Rev. 131 (1963) 406

D. Bernard, NIM A 729 (2013) 765



e^+ / e^- momentum share ?

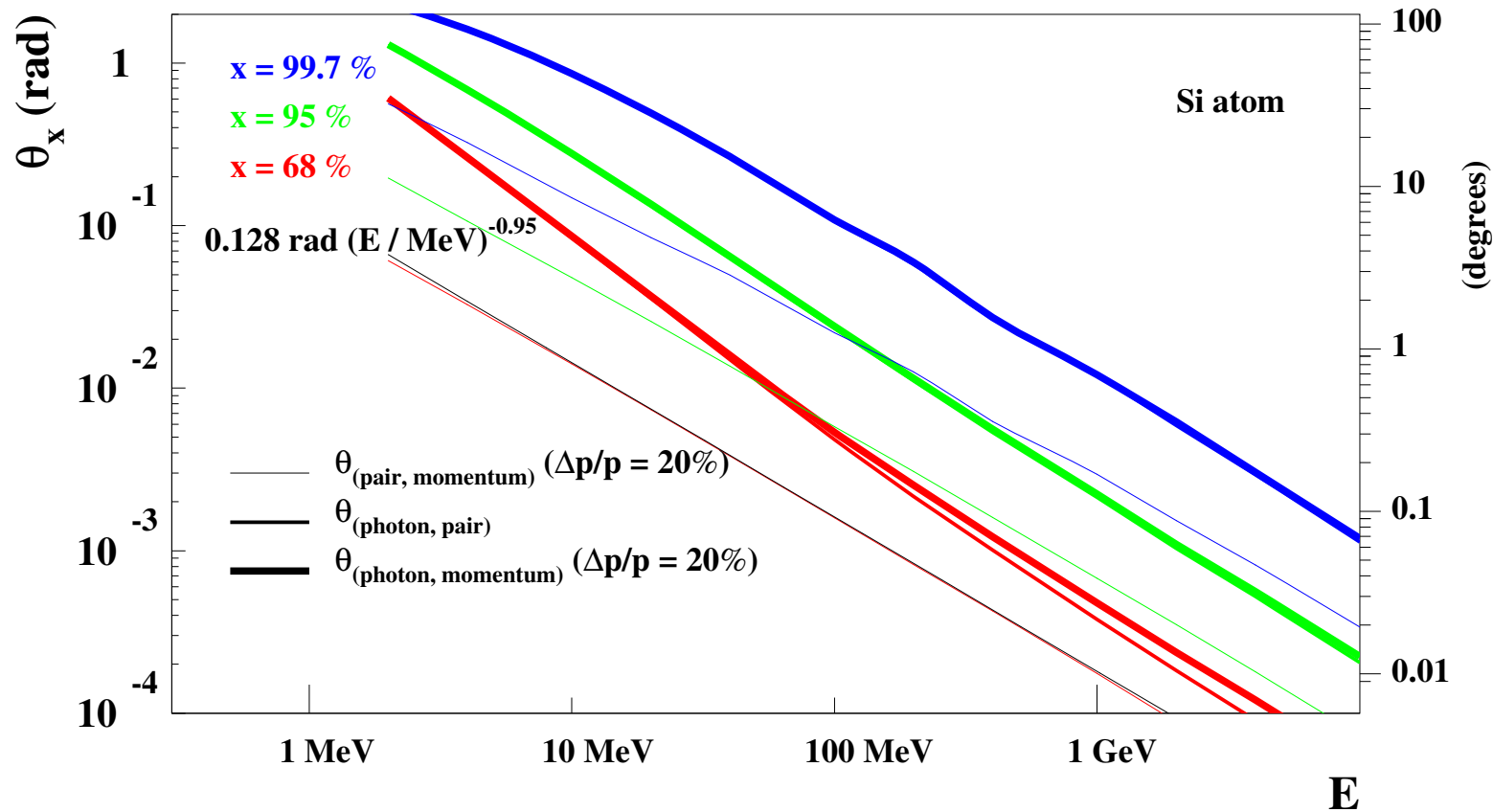
Track momentum measurement ; opening angle ; (1) bisectrix



Assume equipartition: $p_+ = p_- = E/2$. Photon direction “bisectrix” of e^+ , e^-

$\theta_{(\text{pair, bisectrix})}$ dominates over $\theta_{(\text{photon, pair})}$, for $E > 10 \text{ MeV}$

Track momentum measurement ; opening angle ; (2) with momentum



Contribution to the angular resolution induced by a $\sigma_p/p = 20\%$ momentum resolution

Scales like $0.64 \text{ rad } \frac{\sigma_p}{p} \left(\frac{E}{\text{MeV}} \right)^{-0.95}$; negligible compared to recoil

Track Momentum Measurement ; (1) External (to the tracker)

Calorimeter

Fermi LAT

CsI(Tl) (8.6 X_0)

$\Delta E/E = 20\%$ (68%) @ 100 MeV

[Fermi LAT Performance](#)

Magnetic spectrometer

AMS-02

Magnet + Si tracker

$\Delta E/E = 14\%$ @ 200 MeV

[B. Beischer Ph D \(2020\)](#)

A major impact on the mission mass budget

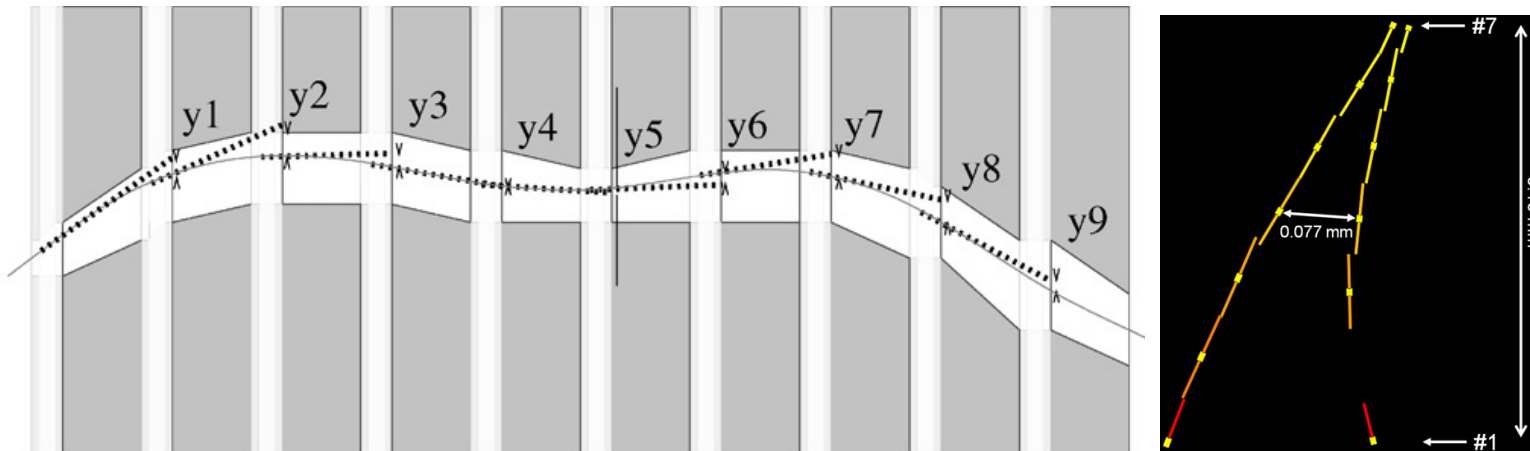
Track Momentum Measurement ; (2) Internal (in the tracker)

Multiple scattering of relativistic $z = 1$ charged particle through thickness Δ

$$\theta_0 \approx \frac{p_0}{p} \sqrt{\frac{\Delta}{X_0}} (1 + \epsilon \log(\Delta/X_0)) \quad , \quad p_0 = 13.6 \text{ MeV}/c$$

R.L. Workman et al. (PDG) Prog. Theor. Exp. Phys. 2022, 083C01 (2022) (pdf)

$$\Rightarrow \sqrt{\langle y^2 \rangle} \propto \frac{p_0}{p}$$



GRAINE: Typical relative precision, $\sigma_p/p = 10 - 20 \%$

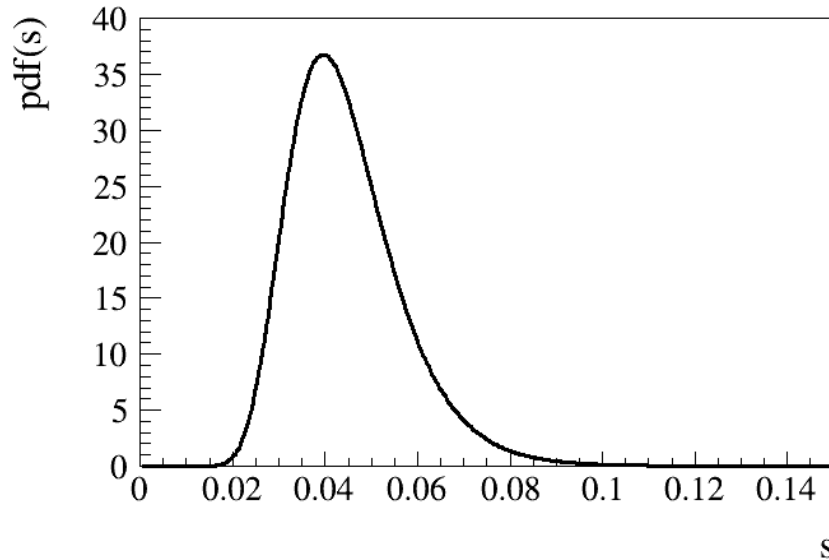
G. Molière Z. Naturforschung A 10 (1955) 177

K. Kodama *et al.* The DONUT (Direct Observation of the Neutrino Tau) Collaboration, Nucl. Instrum. Meth. A **493** (2002) 45

S. Takahashi *et al.*, The GRAINE (Gamma-Ray Astro-Imager with Nuclear Emulsion) Collaboration, PTEP **2015** (2015) no.4, 043H01

Track Momentum Measurement ; (2) Internal: Optimality

- Apply Kalman filter with track momentum hypothesis p



- A Bayesian analysis of the filtering innovations of s -indexed Kalman filters yields an **optimal, unbiased**, estimate of the momentum from multiple measurements of multiple-scattering (and of the other track parameters, BTW).

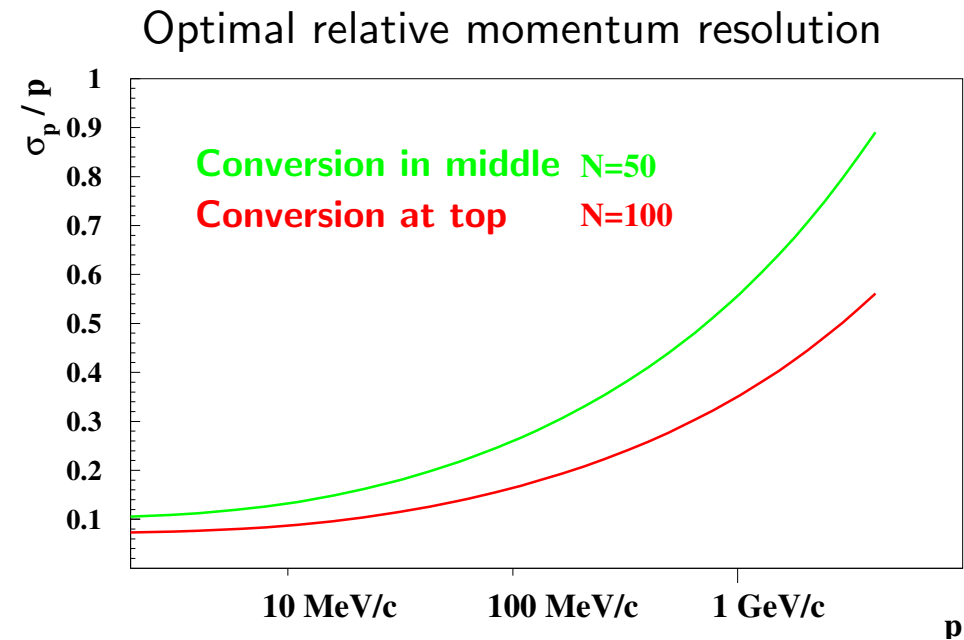
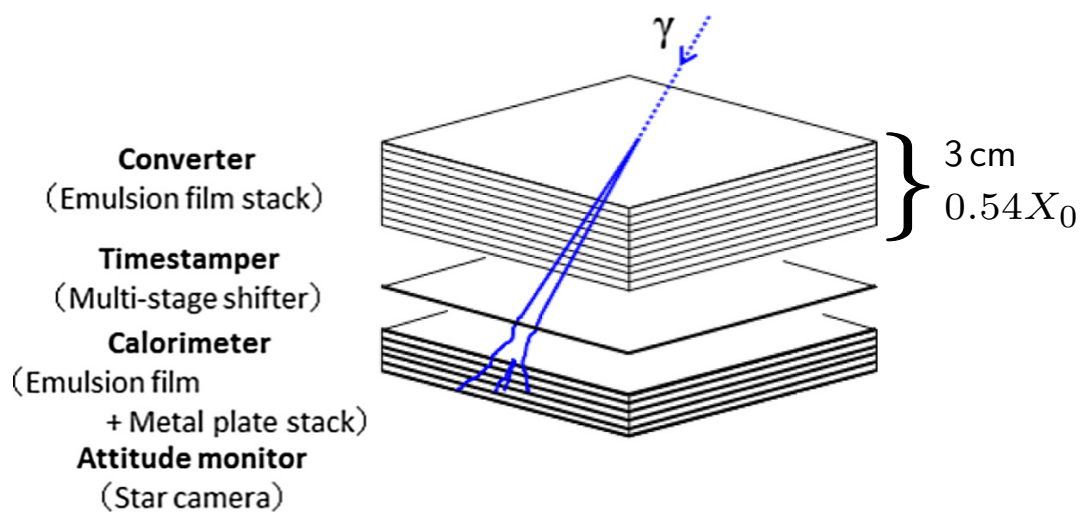
s , average multiple-scattering angle variance per unit track length, $s, \equiv \left(\frac{p_0}{p}\right)^2 \frac{\Delta}{lX_0}$

M. Frosini et al., Nucl. Instrum. Meth. A 867 (2017) 182

(l longitudinal measurement sampling, $l = \Delta$ for homogeneous detectors)

Caveat: no BremsStrahlung nor dE/dx here; just multiple scattering

Track Momentum Measurement ; (2) Internal: Optimality: GRAINE



S. Takahashi *et al.*, *Adv. Space Res.* **62** (2018) 2945

Computed from M. Frosini *et al.*, *Nucl. Instrum. Meth. A* 867 (2017) 182

S. Takahashi *et al.*, *PTEP* **2015** (2015) no.4, 043H01

Converter:

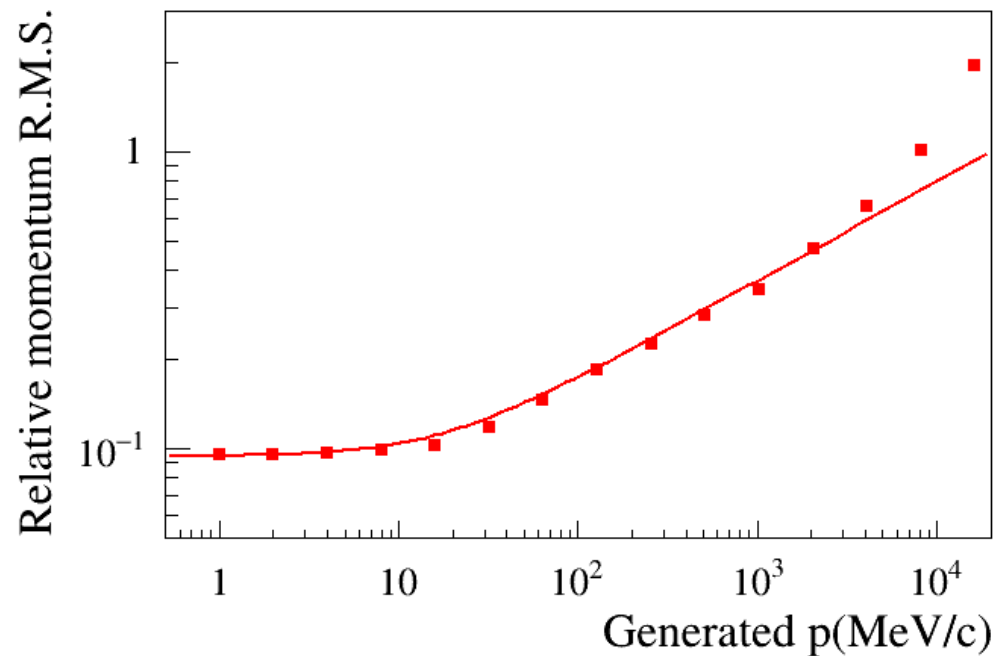
- $N \approx 100$ layers (emulsion film)
- Total thickness $L \approx 3$ cm, $l = L/N = 0.03$ cm
- Total number of radiation lengths $L/X_0 \approx 0.53$,
 Number of radiation lengths per layer $t = l/X_0 \approx 0.0053$,
 Average radiation length $X_0 \approx L/0.53 = 5.66$ cm,
- Single layer spatial resolution, $\sigma = 0.42 \mu\text{m}$!

Track Momentum Measurement ; (2) Internal: Optimality: Si detectors

“Pure” (no W foil), Double-sided silicon strip detector (DSSSD) trackers + Calorimeter

- ASTROGAM, A. De Angelis *et al.* [e-ASTROGAM], *Exper. Astron.* **44** (2017) no.1, 25
- AMEGO-X. R. Caputo *et al.*, *J. Astron. Telesc. Instrum. Syst.* **8** (2022) no.4, 044003

- $X_0 = 9.4$ cm, radiation length
- $N = 56$ layers
- $\Delta = 500$ μm , wafer thickness
- $l = 1$ cm, layer spacing
- $\sigma = 240/\sqrt{12} \approx 70$ μm
single measurement space precision

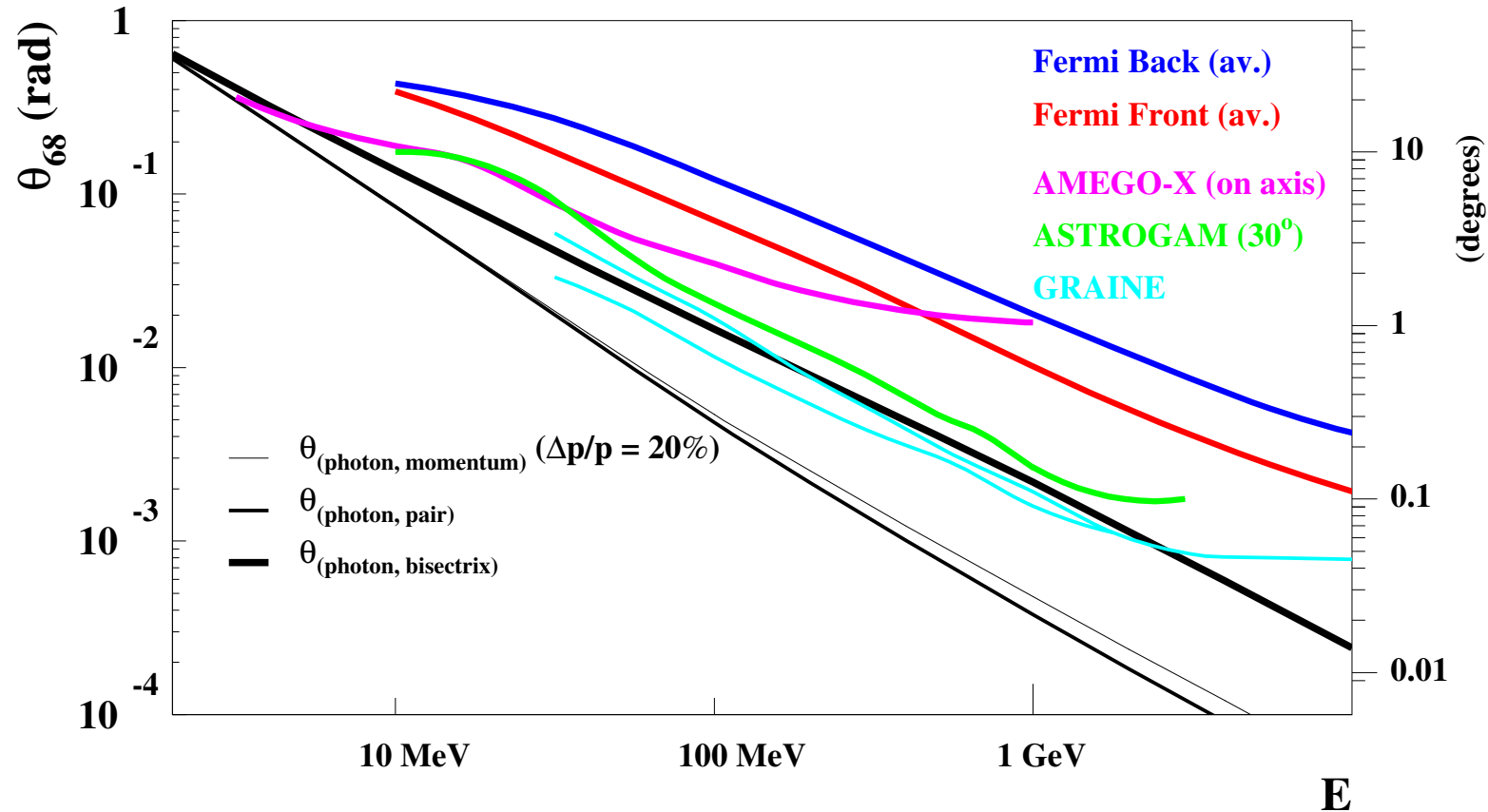


Usable method in the MeV energy range

M. Frosini et al., *Nucl. Instrum. Meth. A* **867** (2017) 182

Calorimetry in the tracker silicon too (≈ 11 MeV loss per track, through 56 wafer pile @ 0°)

Angular resolution: interim wrap-up (a)+(b)



Fermi LAT Performance, Pass 8 Release 3 Version 3

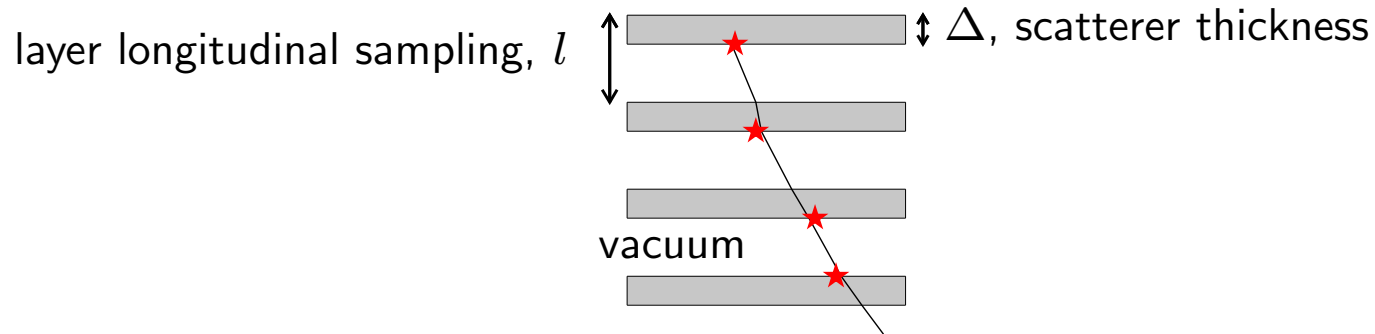
GRAINE, *Adv. Space Res.* **62** (2018) 2945

AMEGO-X, *J. Astron. Telesc. Instrum. Syst.* **8** (2022) no.4, 044003

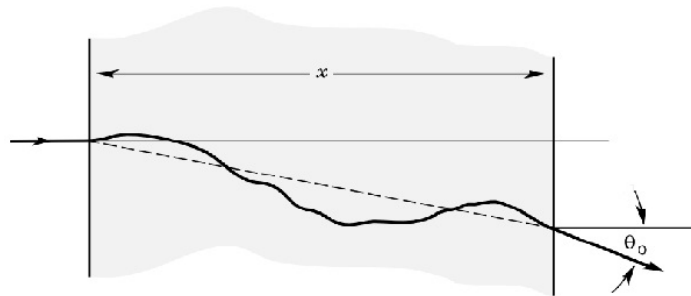
ASTROGAM, *Exper. Astron.* **44** (2017) no.1, 25

Tracking charged particle tracks with multiple scattering

- Typical tracker consisting of N plane “layers”, each with **measurement** precision σ



- Multiple scattering of charged particles in their way through matter



Deflection polar angle, θ_0 , Gaussian distributed,

$$\theta_{\text{RMS},0} \approx \frac{p_0}{p} \sqrt{\frac{\Delta}{X_0}} (1 + \epsilon \log(\Delta/X_0)) \quad , \quad p_0 = 13.6 \text{ MeV}/c$$

(PDG) Prog. Theor. Exp. Phys. 2022, 083C01 (2022) (pdf)

- Tracking: fit track model ((position, direction) at track origin)

Problem of correlation induced by multiple scattering

Tracking with multiple scattering: optimality: Kalman filter

- Assumes Gaussian statistics (detector PSF & multiple scattering deflection)
- Assumes linearity
state (position, direction) at plane i , depends linearly on states at planes j ($j < i$)
- Proceeds by induction:
 - Assuming that we have an optimal **estimate** of the model for planes $i = 1 \cdots n$, (and covariance matrices) and
 - That we know the statistics of the process noise (the RMS of the deflection angle) from plane n to plane $n + 1$, and
 - Given the value of the position **measured** at plane $n + 1$ (and covariance matrix) \Rightarrow the Kalman filter mechanism provides an **optimal estimate** of the model at plane $n + 1$ (a “simple” weighting by the inverse variance)
- Optimality requires two “passes”,
 - 1rst, forward “filtering”
 - 2nd, backward “smoothing”

R. Frühwirth [Nucl. Instrum. Meth. A 262, 444 \(1987\)](#).

If the system is not linear (e.g. tracking with $\vec{B} \neq 0$), linearize it ! \Rightarrow “Extended” Kalman filter

Single-Track (Polar) Angle Resolution

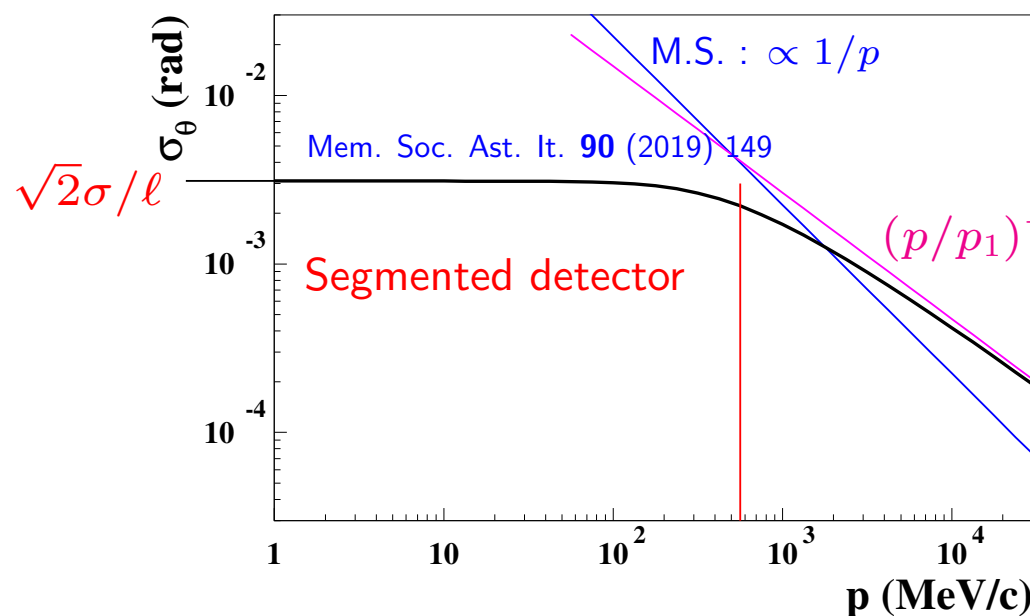
- Optimal tracking (à la Kalman),

M. Frosini et al., *Nucl. Instrum. Meth. A* 867 (2017) 182

Conversion point at bottom of layer

(No dE/dx here)

- Kalman filter tracking useless $<$ few 100s MeV/c



$(p/p_1)^{-3/4}$ homogeneous detector

$$p_1 = 13.6 \text{ MeV}/c \left(\frac{2\sigma}{\ell} \right)^{1/3} \left(\frac{\Delta}{X_{0,W}} \right)^{1/2}$$

D. Bernard, *Nucl. Instrum. Meth. A* 729 (2013) 765

- M.S.: Multiple scattering through (full) conversion layer ; dominates at low energy anyway

Fermi LAT

$l = 3 \text{ cm}$, layer spacing

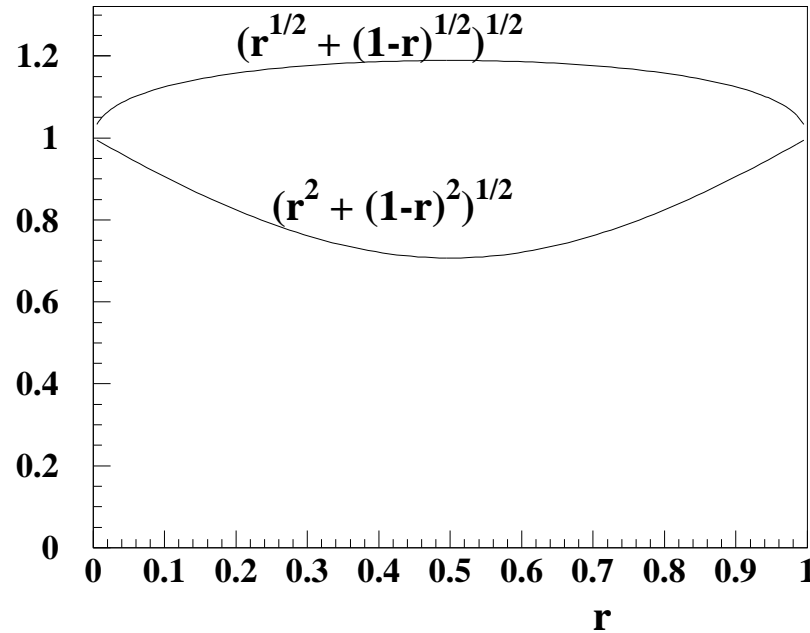
$\sigma = 228 \mu\text{m}/\sqrt{12}$, single measurement space precision

$\Delta = 95 \mu\text{m}$, W foil thickness (front)

$p_1 \approx 0.37 \text{ MeV}/c$,

Polar Angle Resolution: from Single-Track to Single-Photon

$\sigma_{\theta t}(p)$ scaling translate to $\sigma_{\theta\gamma}(E)$ scaling, p , track momentum, E , photon energy.



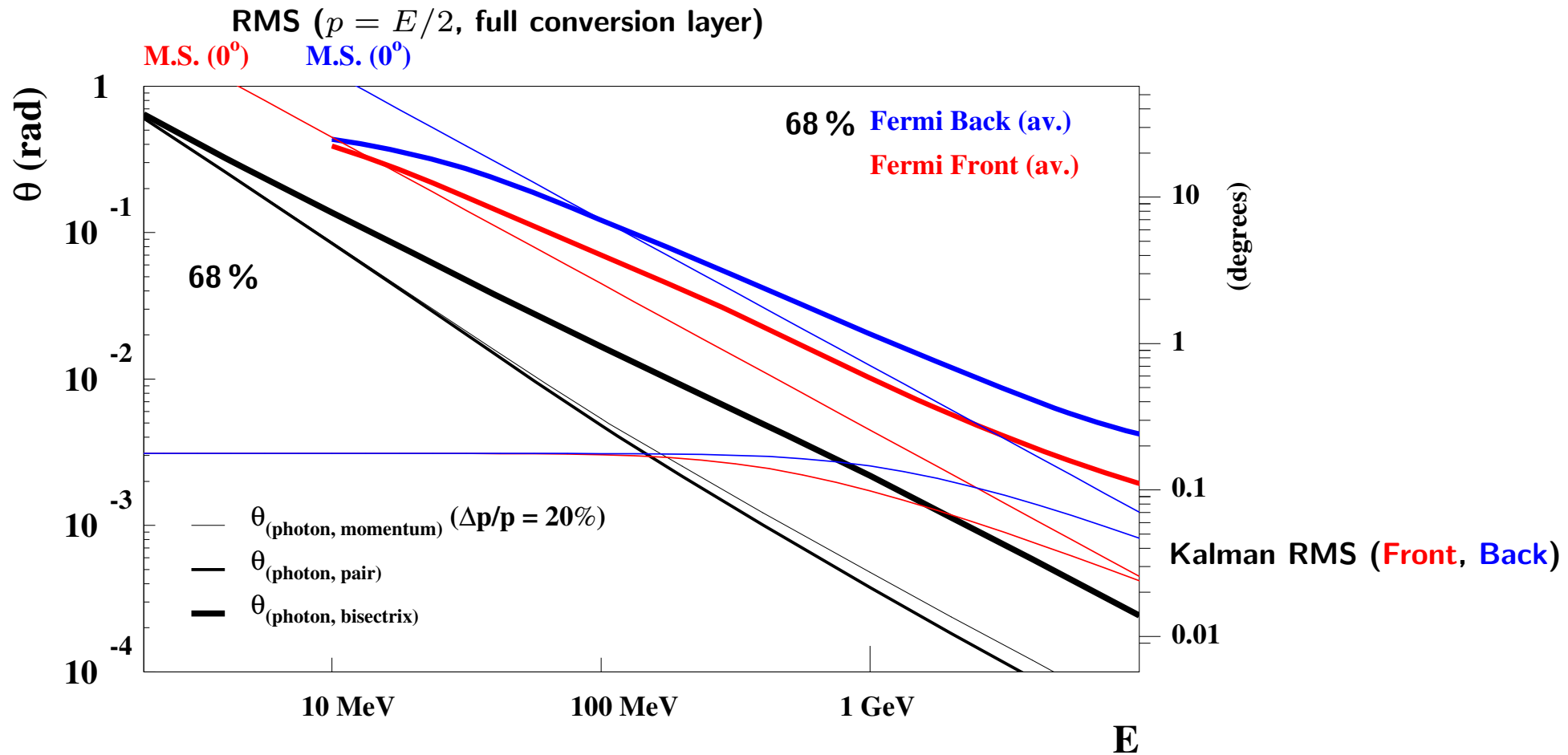
Track-to-photon angular resolution factors as a function of $r = E_+/E_\gamma$

- $\sigma_{\theta\gamma} = \sigma_{\theta t} \sqrt{\sqrt{r} + \sqrt{1-r}}$, multiple scattering dominated regime, $\sigma_{\theta t} \propto (p/p_1)^{-3/4}$
- $\sigma_{\theta\gamma} = \sigma_{\theta t} \sqrt{r^2 + (1-r)^2}$, ultra-high energy regime, $\sigma_{\theta t} = \frac{2\sigma}{L} \sqrt{3/(N+3)}$

D. Bernard , Nucl. Instrum. Meth. A **701** (2013) 225 [err: Nucl. Instrum. Meth. A **713** (2013) 76]

I will neglect this factor

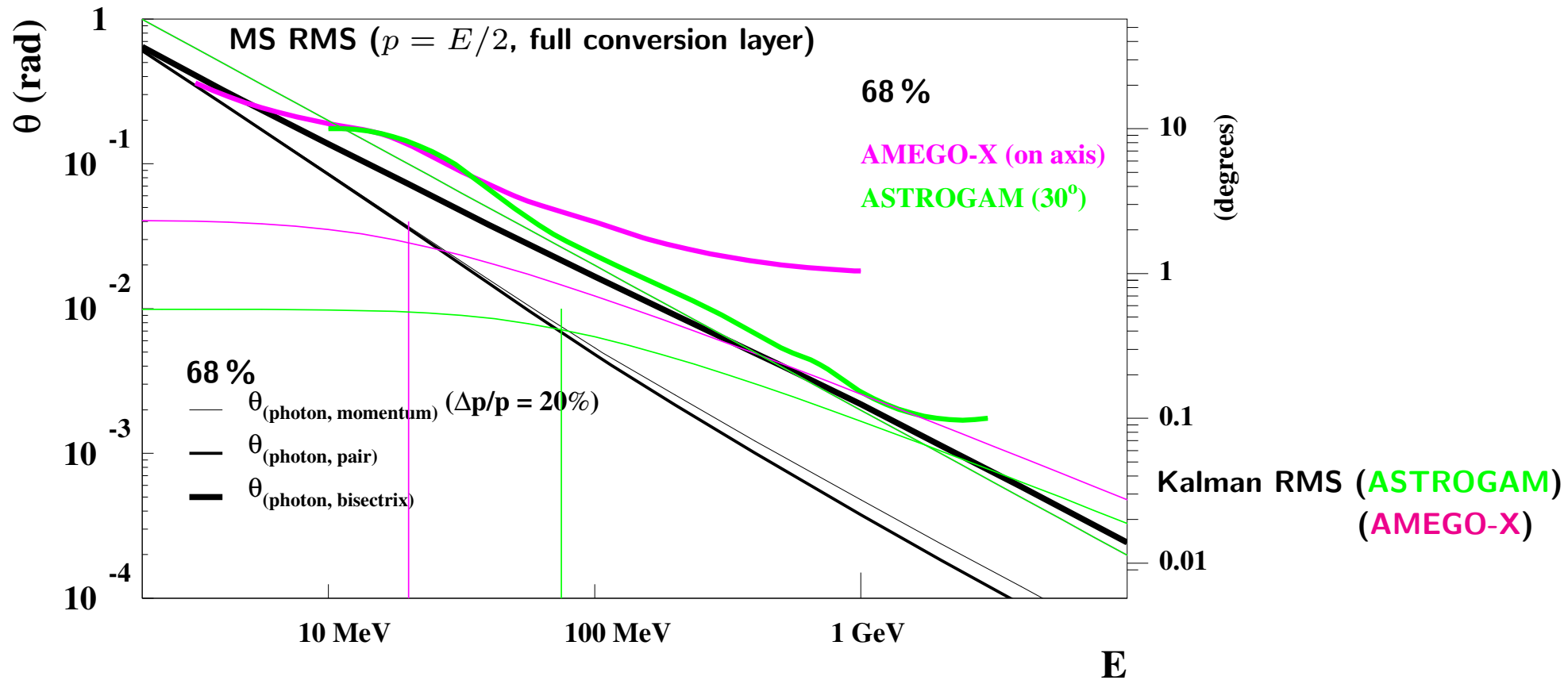
Fermi LAT



- Angular resolution heavily dominated by multiple scattering in conversion layer
- Slope not understood
- Pass8 based on one “main” track though

W. Atwood *et al.* [Fermi-LAT], 2013, eConf C121028, 8,
in Proc. 4th Fermi Symposium, Monterey

AMEGO-X / ASTROGAM



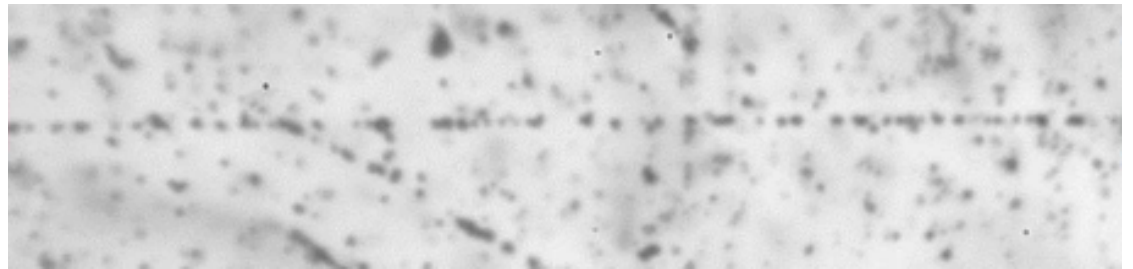
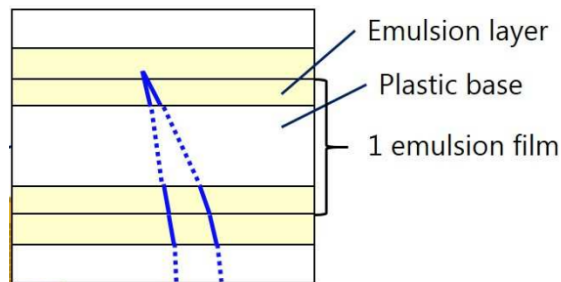
Both dominated by multiple scattering; Bisectrix ?

| | AMEGO-X | ASTROGAM | |
|----------------------------|---------|----------|---------------|
| Segmentation | pixels | strips | |
| N , layer number | 60 | 56 | |
| Δ , Layer thickness | 500 | 500 | μm |
| l , layer spacing | 1 | 1 | cm |
| p , read-out pitch | 240 | 1000 | μm |

GRAINE; Two measurement methods

- **Machine scan** track angle from position difference between top and bottom emulsions of same film

(Absolute position of film in scanning machine (RMS $0.42\ \mu\text{m}$) limiting factor of multi-film analysis)



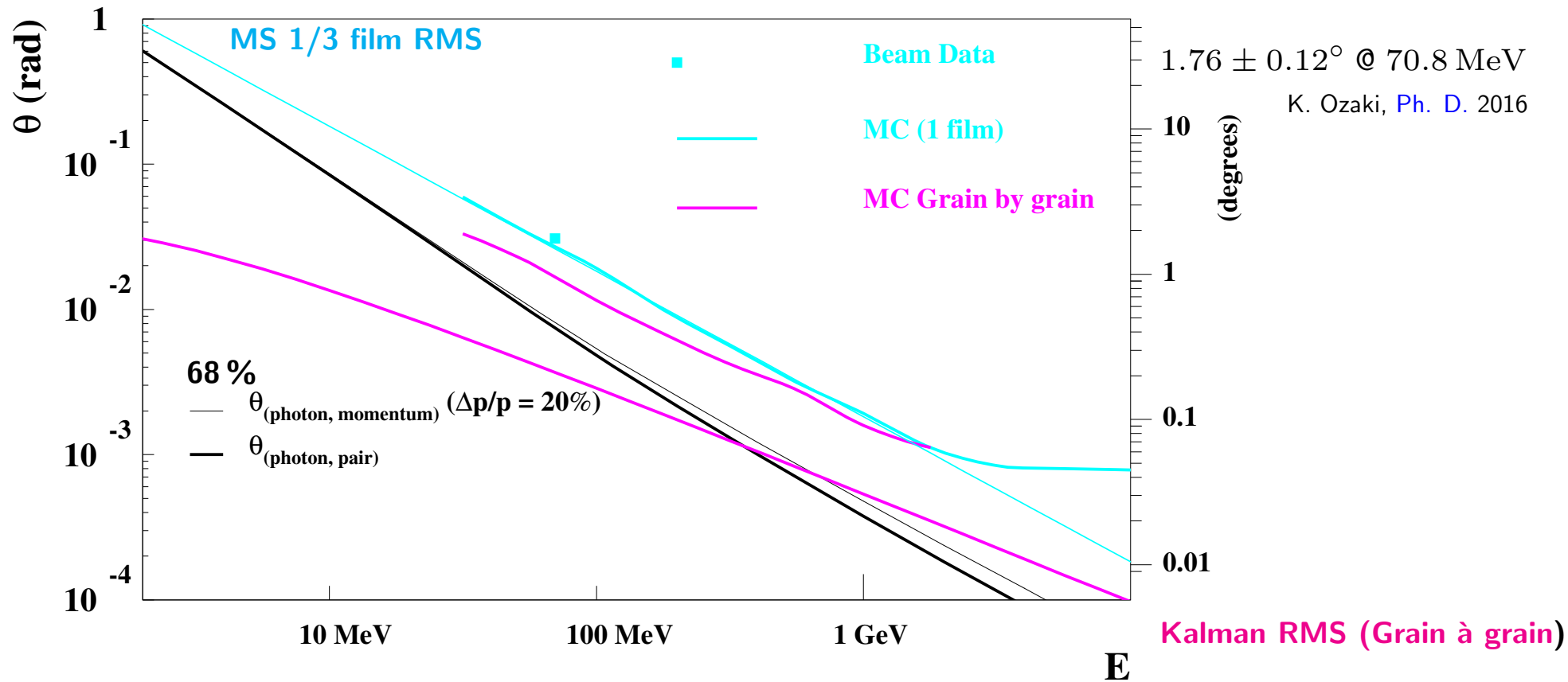
- **Human scan (microscope) grain by grain**

High-density emulsions: Grain density (50 counts / 100 microns)
single grain position measurement 60 nm.

K. Ozaki, [Ph. D.](#) 2016, Kobe University

Momenta measured, see slide 15.

GRAINE



Machine measurement dominated by multiple scattering in film

Grain-by-grain measurement would benefit a Kalman filter fit

- $N = 70 \mu\text{m} / 2 \mu\text{m} = 35$, layer number
- $X_0 = 2.84 \text{ cm}$
- $l = \Delta = 2 \mu\text{m}$, longitudinal sampling
- $\sigma = 0.060 \mu\text{m}$
- $\sigma_p/p = 20 \%$, momentum relative precision

Conclusion; perspectives

- Examined various contributions to the telescope angular resolution

Angular resolution (68 %) scaling

- $E^{-5/4}$ Unmeasured nucleus recoil momentum
 - E^{-1} Multiple scattering in conversion layer
 - $E^{-0.95}$ Track momentum measurement with fixed relative precision
 - $E^{-0.91}$ Bisectrix
 - $E^{-3/4}$ Tracking in homogeneous detector
- Fermi-LAT, AMEGO-X / ASTROGAM, GRAINE (Machine scan): Multiple Scattering in the conversion layer.
 - AMEGO-X / ASTROGAM: bisectrix ?
 - GRAINE (grain by grain): improvement to be expected with Kalman filter tracking ?
- How do better ? Gas TPC (time projection chamber) Talks at this conference:
- SMILE – ETCC: Atsushi Takada
 - HARPO: Shaobo Wang
 - MeGaT: Zhiyong Zhang
- Not in this talk: Pair telescopes poor line sensitivity in the MeV energy range.

Acknowledgements

Fermi LAT Simone Maldera

AMEGO-X Regina Caputo

GRAINE Satoru Takahashi

Errors, misconceptions, outrageous approximations and hazardous assumptions ... mine.