Energy reconstruction of the tagged electron at PS and considerations about the tagging setup

Ph. Bruel
LLR, Ecole Polytechnique,
91128 Palaiseau, France

April 27, 2006

1 Introduction

At PS, the Calibration Unit (CU) will be tested in the so-called tagger mode : the electron beam will be deflected by a magnet and a set of Si chambers (2 upstream and 2 downstream the magnet) will measure the trajectory of the deflected electron, and consequently its energy. The difference of the beam energy and the energy of the electron gives the energy of the Bremsstrahlung gamma emitted by the electron upstream the magnet. The energy resolution of the emitted gamma σ_{PS} depends on the error on the beam energy (typically between 1 and 2 %) and how accurately we are able to reconstruct the energy of the electron.

One of the measurements we want to perform at PS is to check the energy reconstruction with the CU over the wider possible phase space (energy and incidence angle). However, this measurement is only possible when σ_{PS} is small enough compared to σ_{CU}, the energy resolution of the CU .

Figure 1 shows the CU energy resolution for gammas entering the center of a complete tower at various incidence angles as function of the logarithm of the incoming gamma energy.

Figure 2 shows how the energy reconstruction distributions are modified when taking into account that we do not know perfectly the energy of the incoming gamma : the width of these distributions increases as soon as σ_{PS} gets larger and the mean of these distributions shifts as well. Requiring that $\sigma_{\mathrm{PS}} < 0.5\ \sigma_{\mathrm{CU}}$ ensures that the width of the distributions in figure 2 do not increase by more than 12 %.

This note presents some results about the measurement of the energy of the deflected electron and some considerations about the setup that we should use at PS in order to have the wider possible phase space where $\sigma_{\mathrm{PS}} < 0.5\ \sigma_{\mathrm{CU}}$ is true.

2 The measurement of the energy of the deflected electron

A set of 4 Si chambers are available at PS : Si_0 and Si_1 upstream the magnet, and Si_2 and Si_3 downstream, as can be seen in figure 3. These 4 Si chambers provide the measurement of 4 positions ($P_{0,1,2,3}$) along the trajectory of the electron.

Figure 4 shows the trajectories of electrons downstream the magnet when the beam energy is 1 GeV and the magnetic field is 1 T. One can see that, with the actual size of the two downstream chambers, the maximum reachable gamma energy is ~ 450 MeV.

The knowledge of the 4 positions $P_{0,1,2,3}$ is redundant and therefore various methods are available to reconstruct the energy of the electron. This section will be devoted to the method that only uses the angular information.

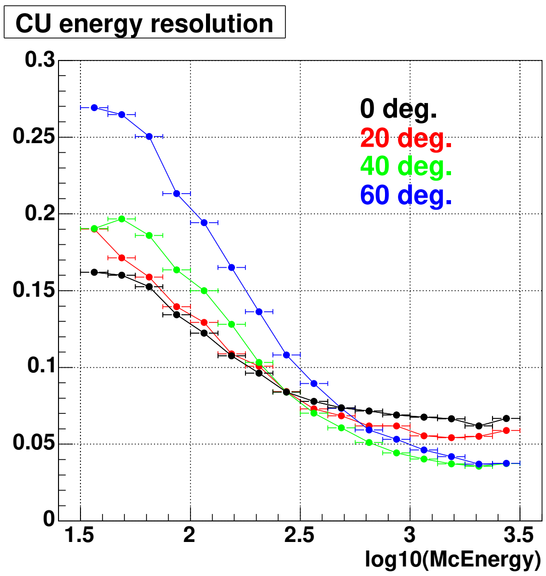

Figure 1: The CU energy resolution for gammas entering the center of a complete tower at various incidence angles as function of the logarithm of the incoming gamma energy.

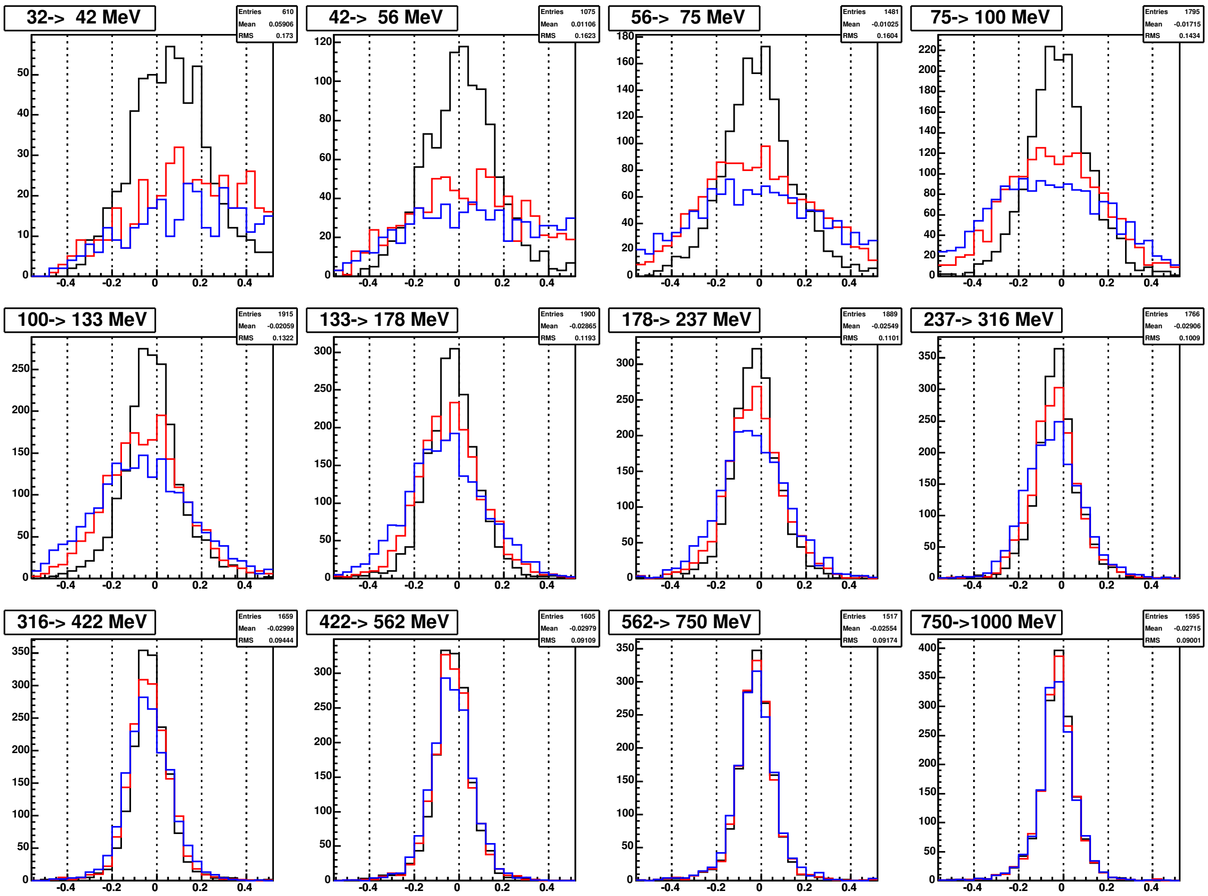

Figure 2: For various bins in $\log \text{McEnergy}$, the distribution of $\text{EvtEnergyCorr}/\text{McEnergy} - 1$ for on-axis gammas entering the center of a complete tower. In black : no smearing is applied, i.e in the ideal case when we know perfectly the energy of the incoming gamma. In red : a smearing is applied simulating a 1 % error on the beam energy and a 1.3 % error on the electron energy. In blue : a smearing is applied simulating a 2 % error on the beam energy and a 1.3 % error on the electron energy.

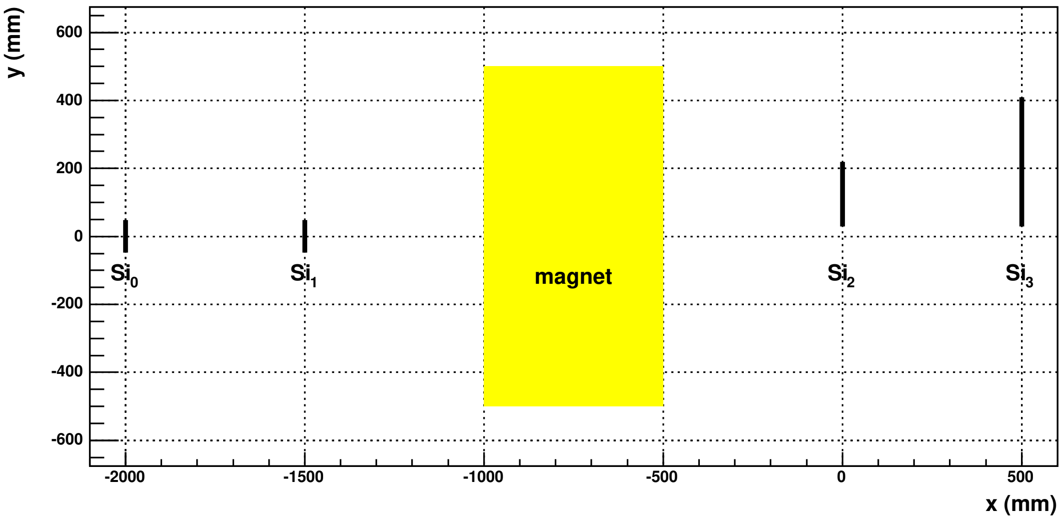

Figure 3: Setup at PS as used in beamtest06 v4r1p1.

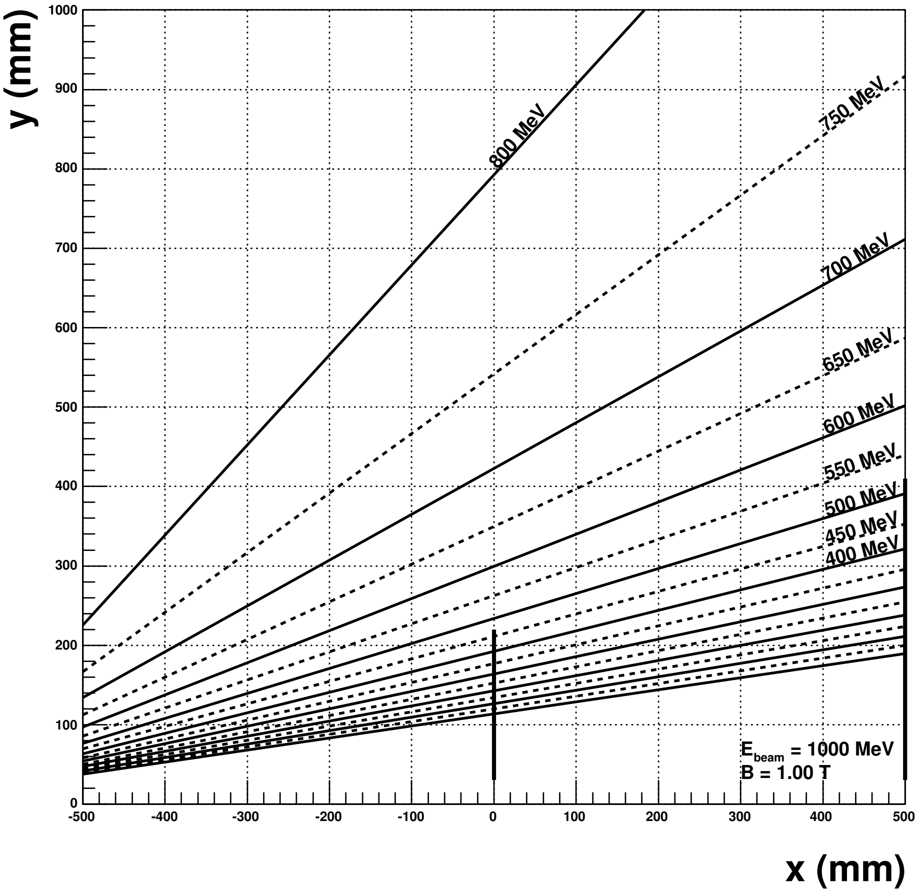

Figure 4: Trajectories of electrons downstream the magnet (the energy of the corresponding radiated gamma is indicated). The beam energy is 1 GeV and the magnetic field is 1 T. The two wide vertical lines represent the 2 downstream Si chambers. Before entering the magnet, the electron direction is $(1, 0, 0)$.

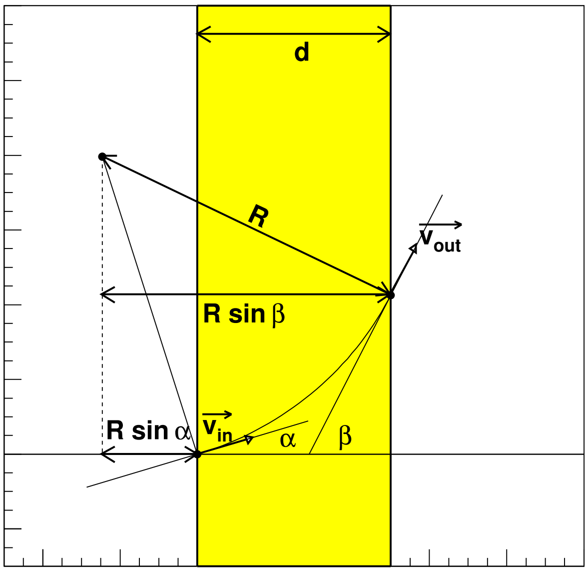

Figure 5: Trajectory of an electron entering a magnet with an angle α and going out with an angle β. R is the radius of curvature of the electron inside the magnet.

2.1 Reconstructing the electron energy using the angular information

Let the electron momenta upstream and downstream the magnet be :

$$\overrightarrow{p_{in}} = \begin{pmatrix} \cos\theta\cos\alpha \\ \cos\theta\sin\alpha \\ \sin\theta \end{pmatrix} \qquad \text{and} \qquad \overrightarrow{p_{out}} = \begin{pmatrix} \cos\theta\cos\beta \\ \cos\theta\sin\beta \\ \sin\theta \end{pmatrix}$$

Figure 5 shows the trajectory of the electron in the $z = 0$ plane. From this figure, one can derive the basic formula that relates α, β and the radius of curvature R :

$$R\sin\beta = d + R\sin\alpha \Rightarrow R = \frac{d}{\sin\beta - \sin\alpha}$$

The momentum of the electron p is therefore :

$$p = \frac{0.3Bd}{(\sin\beta - \sin\alpha)\cos\theta}$$

The 4 positions $P_{0,1,2,3}$ are used to derive the angles α, β and θ :

$$\begin{aligned} \alpha &= \arctan(\overrightarrow{P_0P_1}.\vec{y}/\overrightarrow{P_0P_1}.\vec{x}) \\ \beta &= \arctan(\overrightarrow{P_2P_3}.\vec{y}/\overrightarrow{P_2P_3}.\vec{x}) \\ \theta &= \left(\arcsin(\overrightarrow{P_0P_1}.\vec{z}/|P_0P_1|) + \arcsin(\overrightarrow{P_2P_3}.\vec{z}/|P_2P_3|)\right)/2 \end{aligned}$$

One can see that θ is the average of the angles of the momentum with respect with the $z = 0$ plane upstream and downstream the magnet. This is done in order to reduce the effect of multiple scattering in Si_1 and Si_2.

A simulation of the PS setup has been used to test the performance of this reconstruction. The setup used for this simuation has been modified with respect to the setup shown in figure 3 : the y dimension

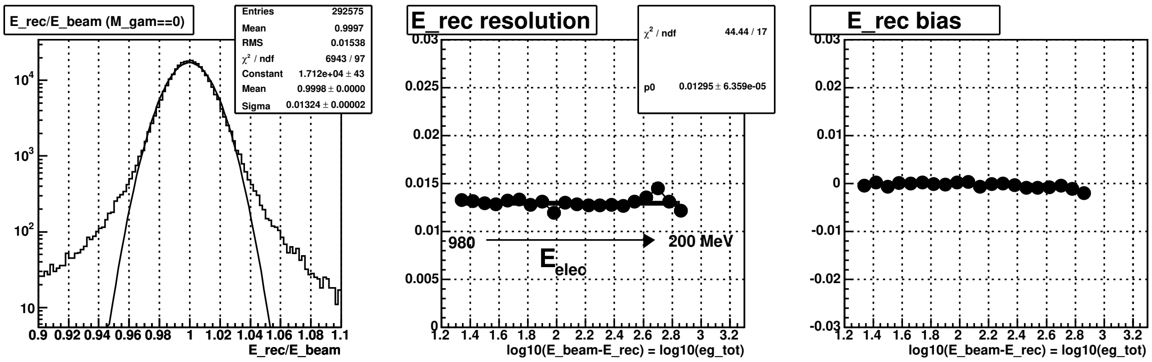

Figure 6: Left : the distribution of $E_{rec}/E_{\rm beam}$ for electrons which do not have radiated. Center and right : the electron energy resolution and bias as function of $\log(E_{\rm beam} - E_{e^-}^{MC}) = \log(E_\gamma^{MC})$.

of the two downstream chambers has been set to 1000 mm in order to be able to detect electrons which have radiated a ~ 800 MeV gamma.

Figure 6 shows the distribution of $E_{rec}/E_{\rm beam}$ for electrons which do not have radiated, and the electron energy resolution and bias as function of $\log(E_{\rm beam} - E_{e^-}^{MC}) = \log(E_\gamma^{MC})$.

2.2 Multiple scattering considerations

From the resolution graph, it can be seen that the electron energy resolution above 200 MeV is constant, about 1.3 %. One can be surprised by this result since the main uncertainty comes from multiple scattering, which is proportional to $1/p$.

Multiple scattering can be modelled by 3 contributions :

- $\delta\alpha$ corresponding to the projection on to the $z = 0$ plane of multiple scattering occuring in Si_1;
- $\delta\beta$ corresponding to the projection on to the $z = 0$ plane of multiple scattering occuring in Si_2;
- $\delta\theta$ corresponding to the projection on to the z-axis of multiple scattering occuring in in Si_1 and Si_2.

Let us derive the electron energy resolution :

$$
\begin{aligned}
p &= \frac{0.3Bd}{(\sin\beta - \sin\alpha)\cos\theta} \\
\Rightarrow \delta p &= \frac{0.3Bd\cos\alpha\ \delta\alpha}{(\sin\beta - \sin\alpha)^2\cos\theta} \oplus \frac{0.3Bd\cos\beta\ \delta\beta}{(\sin\beta - \sin\alpha)^2\cos\theta} \oplus \frac{\sin\theta\ \delta\theta}{(\sin\beta - \sin\alpha)\cos^2\theta} \\
\Rightarrow \frac{\delta p}{p} &= \frac{p}{0.3Bd}\cos\theta\cos\alpha\ \delta\alpha \oplus \frac{p}{0.3Bd}\cos\theta\cos\beta\ \delta\beta \oplus \frac{1}{0.3Bd}\tan\theta\ \delta\theta
\end{aligned}
$$

The angular scale for multiple scattering is :

$$\theta_{MS} = \mu/p \ (\mathrm{GeV}) \qquad \text{where } \mu = 0.0136 \times \sqrt{x/X_0} \times (1 + 0.0038\ln(x/X_0))$$

One Si chamber is 0.08 cm wide and for Si $X_0 = 9.36$ cm, leading to $x/X_0 \sim 0.09$ and $\mu \sim 10^{-3}$.

Since the main uncertainty comes from multiple scattering, we have $\delta\alpha = \delta\beta = \delta\theta = \theta_{MS}$. And for $p > 200$ MeV we can assume $\alpha \sim \theta \sim \theta_{MS} \ll 1$. The energy resolution becomes :

$$\frac{\delta p}{p} \sim \frac{\mu}{0.3Bd}\left(1 \oplus \cos\beta \oplus \frac{\mu}{p^2}\right)$$

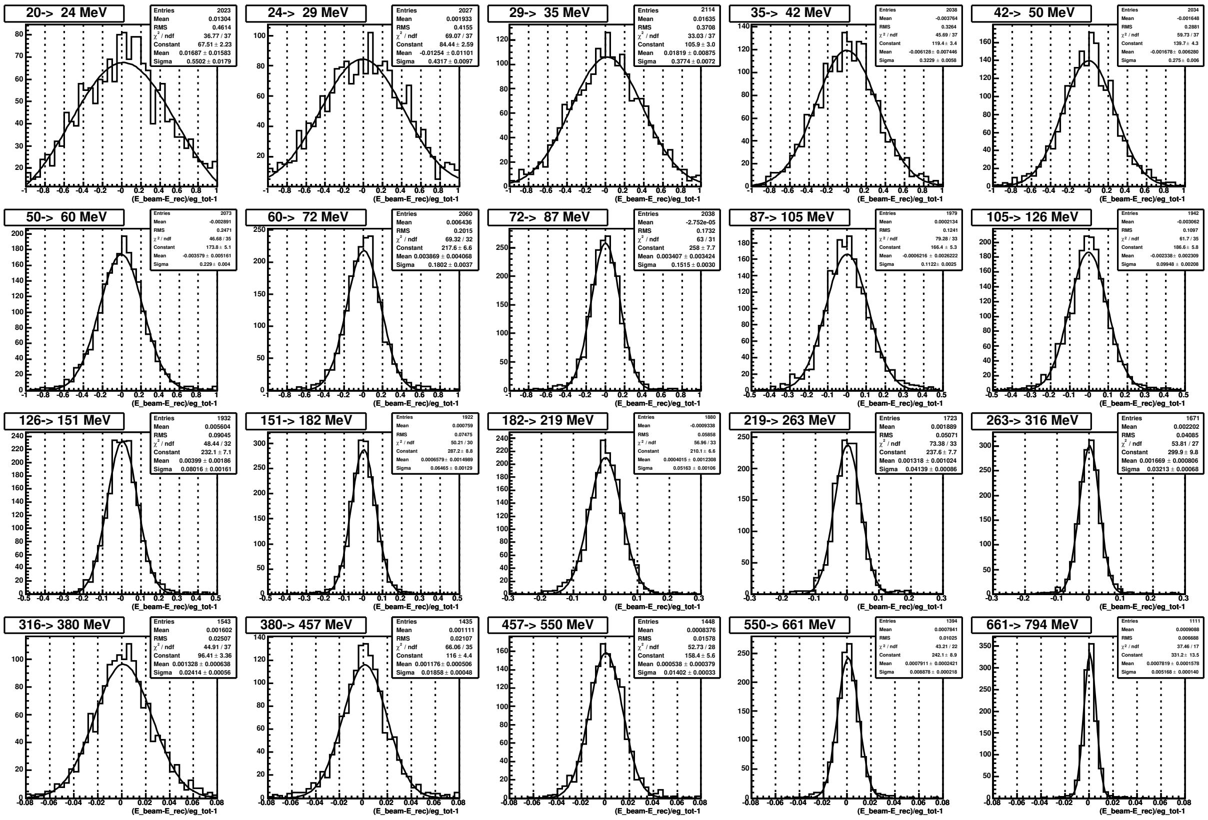

Figure 7: The distribution of $(E_\gamma^{rec}/E_\gamma^{MC} - 1)$ for different bins in $\log(E_\gamma^{MC})$.

So we can see that there is no $1/p$ dependence coming from $\delta\alpha$: when p decreases, the increase in multiple scattering is compensated by the increase in deflection (the radius of curvature is proportional to p). For the same reason, the contribution coming from $\delta\beta$ does not introduce any $1/p$ dependence and it is even reduced by a factor $\cos\beta$ because the multiple scattering occurs after deflection. To the contrary, $\delta\theta$ introduces a $1/p$ dependence because there is no deflection along the z-axis, but it is tempered by the factor μ. This implies that, for $p > 200$ MeV, the contribution of $\delta\theta$ to the energy resolution tends to be negligible : $\mu/p^2 \ll 1 \oplus \cos\beta$.

Estimating the energy resolution leads to $\delta p/p \sim 1$ %, which is not too far from 1.3 % obtained with the simulation.

2.3 The energy resolution of the radiated gamma

Figure 7 shows the distribution of $(E_\gamma^{rec}/E_\gamma^{MC} - 1)$ for different bins in $\log(E_\gamma^{MC})$. Figure 8 shows the gamma energy resolution and bias as function of $\log(E_\gamma^{MC})$. Since the electron energy resolution is constant, the gamma energy resolution can be fit by the following function :

$$\sigma_{\mathrm{PS}}(E_\gamma) = \sigma_\gamma(E_\gamma) = \sigma_{\mathrm{beam}} \frac{E_{\mathrm{beam}}}{E_\gamma} \;\oplus\; \sigma_{e^-} \left(\frac{E_{\mathrm{beam}}}{E_\gamma} - 1 \right)$$

where σ_{beam} is the error on the beam energy and σ_{e^-} is the electron energy resolution.

Figure 8 corresponds to a simulation done with $\sigma_{\mathrm{beam}} = 0$, so the fit of the gamma energy resolution function gives $\sigma_{e^-} \sim 1.3$ %. Figures 9 and 10 correspond respectively to $\sigma_{\mathrm{beam}} = 1$ % and 2 % and, as expected, the fit still gives $\sigma_{e^-} \sim 1.3$ %.

Now we can compare σ_{PS} to σ_{CU}, the energy resolution of the CU.

Figure 11(left) compares σ_{CU} for on-axis gamma and σ_{PS} with $\sigma_{\mathrm{beam}} = 1$ %. The requirement $\sigma_{\mathrm{PS}} < 0.5\ \sigma_{\mathrm{CU}}$ implies $E_\gamma \gtrsim 300$ MeV. Figure 11(right) do the same comparison but with $\sigma_{\mathrm{beam}} = 2$ %. In that

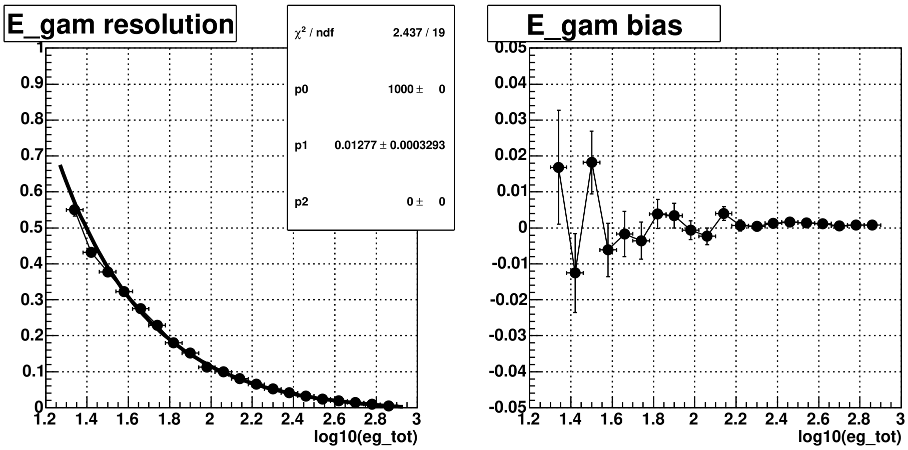

Figure 8: The gamma energy resolution and bias as function of $\log(E_\gamma^{MC})$ for $\sigma_{\text{beam}} = 0$ %.

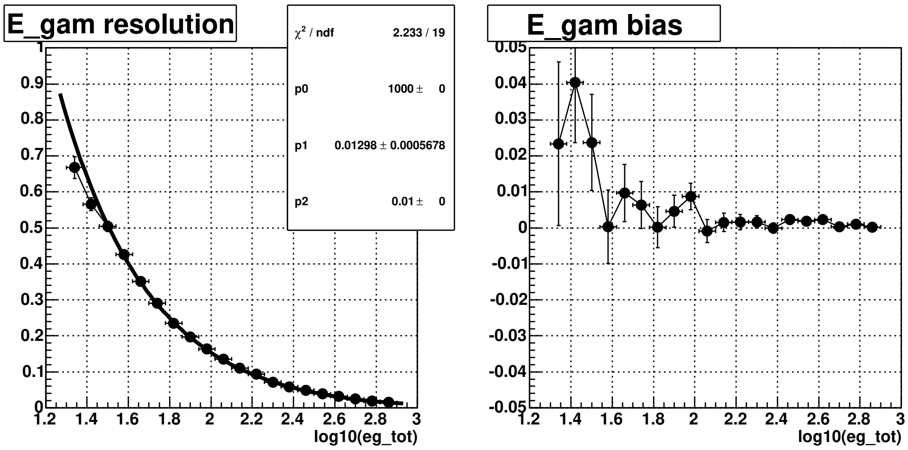

Figure 9: The gamma energy resolution and bias as function of $\log(E_\gamma^{MC})$ for $\sigma_{\text{beam}} = 1$ %.

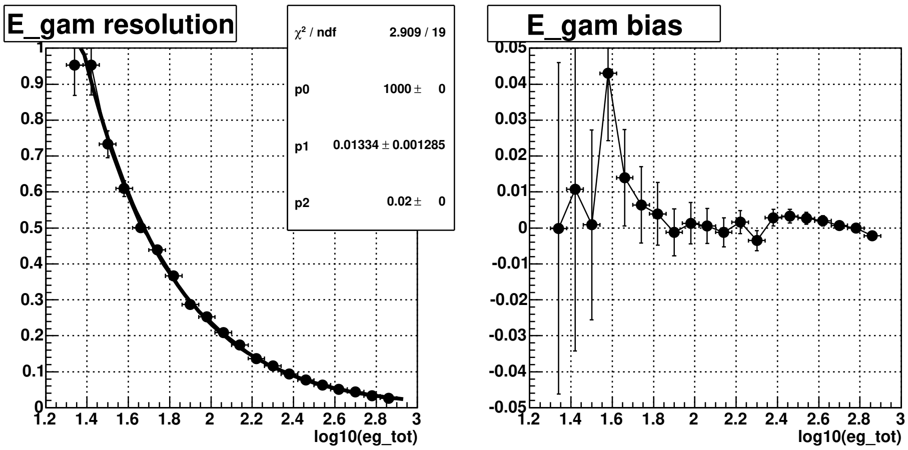

Figure 10: The gamma energy resolution and bias as function of $\log(E_\gamma^{MC})$ for $\sigma_{\text{beam}} = 2$ %.

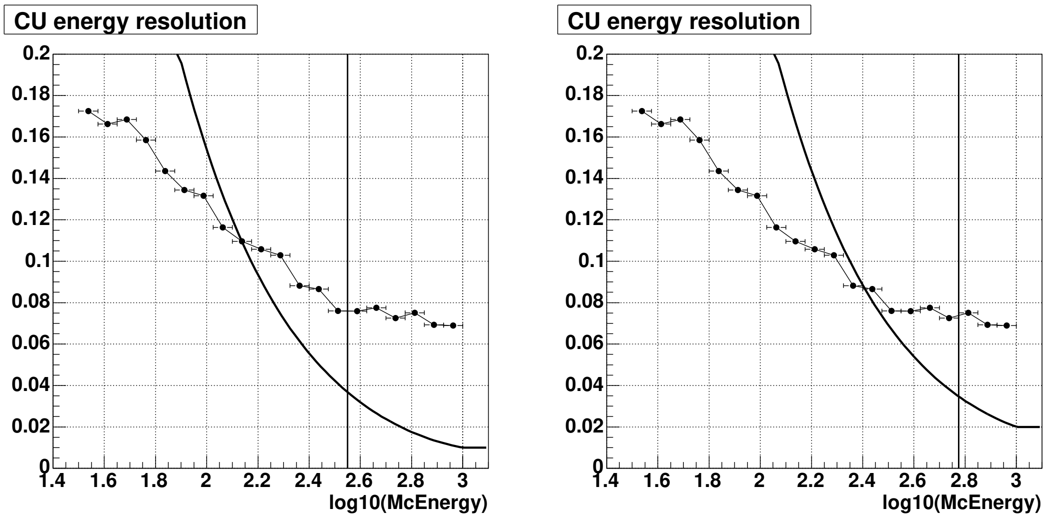

Figure 11: The energy resolution of the CU for on-axis gammas (graph) compared to the resolution with which we measure the energy of the radiated gamma at PS (function) for $\sigma_{\text{beam}} = 1$ % (left) and $\sigma_{\text{beam}} = 2$ % (Right).

case, the same requirement implies $E_\gamma \gtrsim 500$ MeV, which is impossible with the actual setup since the maximum reachable energy is lower than 500 MeV.

3 Alternative methods

Another way to reconstruct the electron energy is to minimize a χ^2. One has to find which χ^2 definition gives the best electron energy resolution.

Here are some quantities, calculated during the fit, that can be used in the definition of χ^2 :

- $\overrightarrow{T_{fit}}$: the trajectory of the electron downstream the magnet (it's a straight line)
- δ_{in} : the angle between $\overrightarrow{p_{in}}$ and $\overrightarrow{p_{in}}(fit)$
- δ_{out} : the angle between $\overrightarrow{p_{out}}$ and $\overrightarrow{p_{out}}(fit)$

The obvious free parameter is the electron momentum p. But one can try to add two additional parameters : $\delta y, \delta z$, the displacements in Si_0 along the y and z-axis, which are used to take into account multiple scattering occuring in Si_1.

For each χ^2 definition, the electron energy resolution is determined. The following table summarizes the results :

χ^2	free parameters	σ_{e^-}
$\text{dist}^2(\overrightarrow{T_{fit}}, P_2)$	p	1.59 %
$\text{dist}^2(\overrightarrow{T_{fit}}, P_3)$	p	1.37 %
$\text{dist}^2(\overrightarrow{T_{fit}}, P_2) + \text{dist}^2(\overrightarrow{T_{fit}}, P_3)$	p	1.42 %
$(\delta_{out}/\theta_{MS})^2$	p	1.28 %
$\text{dist}^2(\overrightarrow{T_{fit}}, P_2) + (\delta_{out}/\theta_{MS})^2$	p	1.34 %
$\text{dist}^2(\overrightarrow{T_{fit}}, P_3) + (\delta_{out}/\theta_{MS})^2$	p	1.32 %
$\text{dist}^2(\overrightarrow{T_{fit}}, P_2) + \text{dist}^2(\overrightarrow{T_{fit}}, P_3) + (\delta_{out}/\theta_{MS})^2$	p	1.36 %
$(\delta_{in}/\theta_{MS})^2 + (\delta_{out}/\theta_{MS})^2$	$p, \delta y, \delta z$	1.28 %
$\text{dist}^2(\overrightarrow{T_{fit}}, P_2) + \text{dist}^2(\overrightarrow{T_{fit}}, P_3) + (\delta_{in}/\theta_{MS})^2 + (\delta_{out}/\theta_{MS})^2$	$p, \delta y, \delta z$	1.36 %

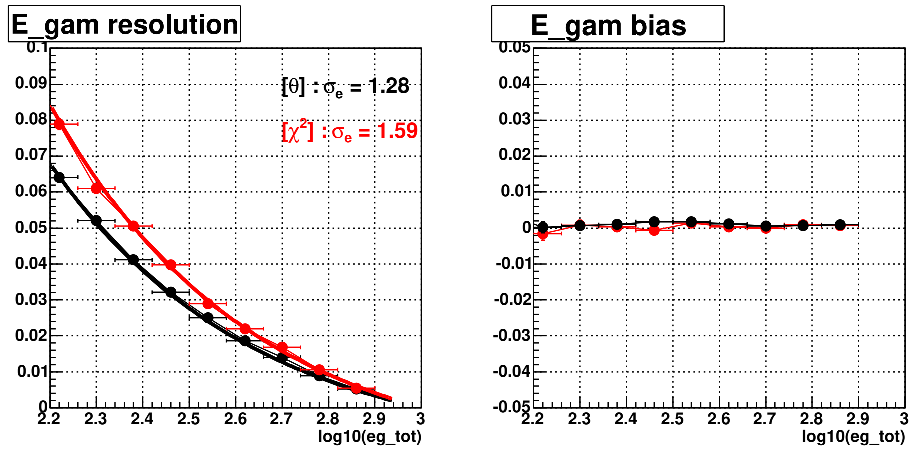

Figure 12: The gamma energy resolution and bias when using the angular information (black) and when minimizing $\chi^2 = \text{dist}^2(T_{fit}, P_3)$(red).

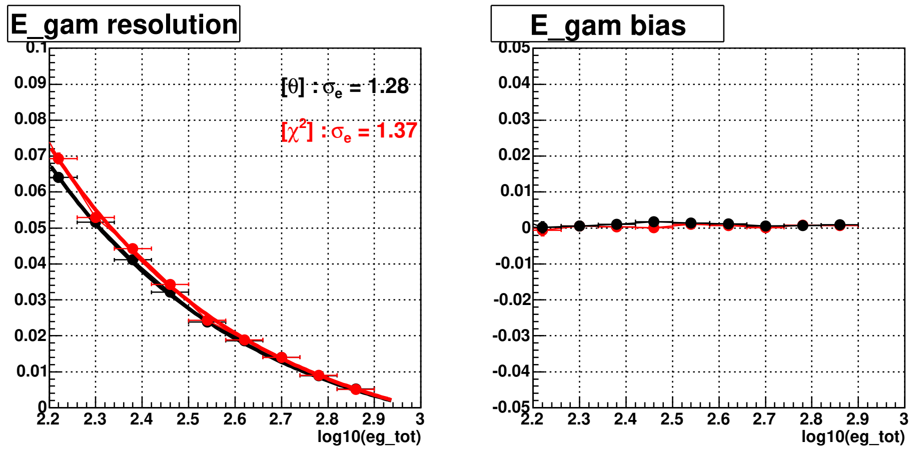

Figure 13: The gamma energy resolution and bias when using the angular information (black) and when minimizing $\chi^2 = \text{dist}^2(T_{fit}, P_3)$ (red).

Since the electron energy resolution achieved with the angular information is 1.28 %, the first conclusion is that none of the χ^2 definitions gives a better resolution. The other conclusion is that, even when using only one point, either P_2 or P_3, the resolution is not much larger than 1.28 %. That means that the corresponding gamma energy resolution will be almost the same at high gamma energy (i.e above 500 MeV), as can be seen in figures 12 and 13, showing the gamma energy resolution obtained when minimizing $\chi^2 = \text{dist}^2(\overrightarrow{T_{fit}}, P_2)$ and $\chi^2 = \text{dist}^2(\overrightarrow{T_{fit}}, P_3)$ respectively.

4 Modifying the setup

From the preceeding sections we know that :

- the actual setup do not provide any tagged gamma above $E_{max} \sim 450$ MeV;
- the best gamma energy resolution (i.e corresponding to the best electron energy resolution) is obtained using the angular information, that is to say with 2 Si chambers downstream the magnet;
- the best electron energy resolution is proportional to $\mu/(0.3Bd)$;
- above ~ 500 MeV, measuring only one position downstream the magnet gives a resolution as good as the best one obtained when measuring 2 positions;

- the PS gamma energy resolution is good enough to check the CU energy resolution only for gamma with energy greater than $E_{min} \sim 300$ MeV (for $\sigma_{\text{beam}} = 1$ %).

We will now see how it is possible to widen the energy region $[E_{min}, E_{max}]$.

4.1 A lower E_{min}

If the minimal beam energy is 1 GeV, one must lower σ_{beam} or σ_{e^-} in order to lower E_{min}.

We have seen that when $\sigma_{\text{beam}} = 2$ %, we have $E_{min} > 500$ MeV. It is clear that this configuration will not allow us any good verification of the CU energy resolution. As a consequence, we have to run with $\sigma_{\text{beam}} = 1$ %.

Lowering σ_{e^-} implies lowering $\mu/(0.3Bd)$. Since $B = 1$ T is already the maximum magnetic field, we have to increase d by rotating the magnet. By rotating the magnet by 45°, we can hope to lower σ_{e^-} down to 0.9 %. But we only get $E_{min} \sim 250$ MeV.

So it seems that the only efficient way to lower E_{min} down to ~ 100 MeV is to run with a beam energy lower than 1 GeV.

4.2 A higher E_{max}

To reach a higher E_{max}, we have to increase the setup coverage. Figure 14 shows one possibility where Si_3 is placed such that :

- below ~ 500 MeV : the electron passes through Si_2 and Si_3 and its energy is determined using the angular information;
- from ~ 500 MeV upto 700 MeV : the electron passes only through Si_3 and its energy is determined minimizing $\chi^2 = \text{dist}^2(\overrightarrow{T_{fit}}, P_3)$

Figure 15 shows that with this setup the electron energy resolution is 1.3%, as with the actual setup, but the maximum reachable gamma energy is ~ 700 MeV instead of ~ 450 MeV.

5 Conclusion

The results presented in this note show that the actual tagging setup at PS does not allow us to perform a good verification of the CU energy resolution over a wide energy range. In order to widen this energy range, one has to modify the setup. As shown in this note, a simple modification (moving the last Si chamber) would already make the situation better.

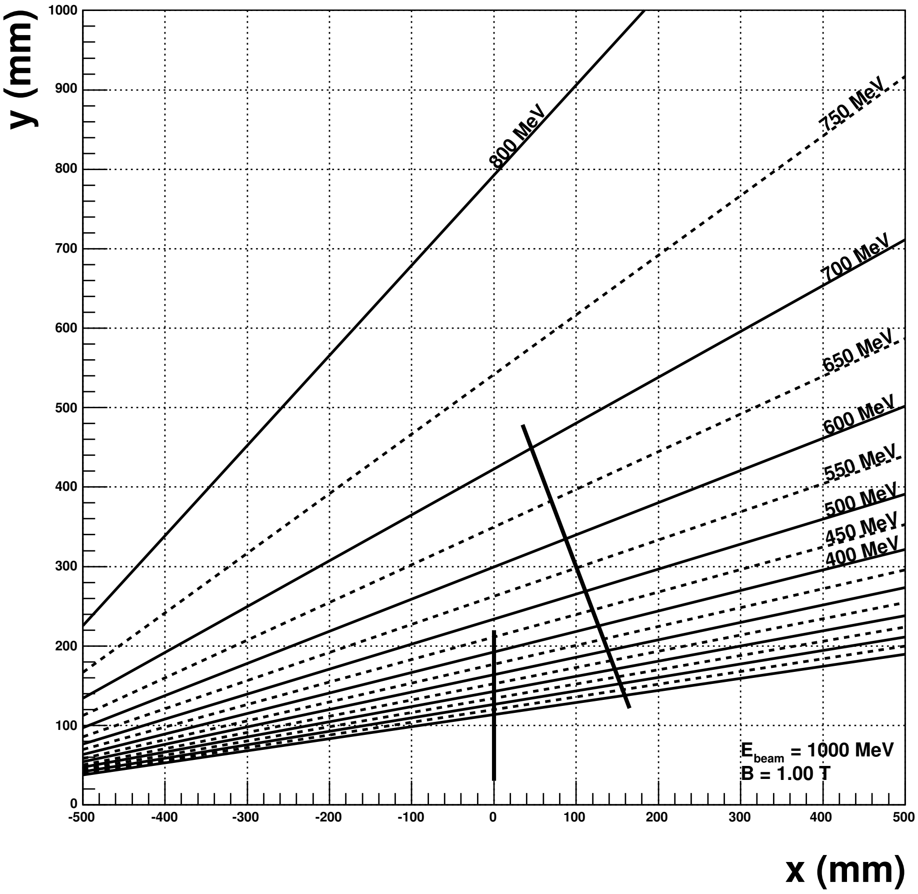

Figure 14: Trajectories of electrons downstream the magnet (the energy of the corresponding radiated gamma is indicated). The beam energy is 1 GeV and the magnetic field is 1 T. The two wide lines represent the 2 downstream Si chambers. The last one has been moved in order to tag gammas with up to $\sim$ 700 GeV.

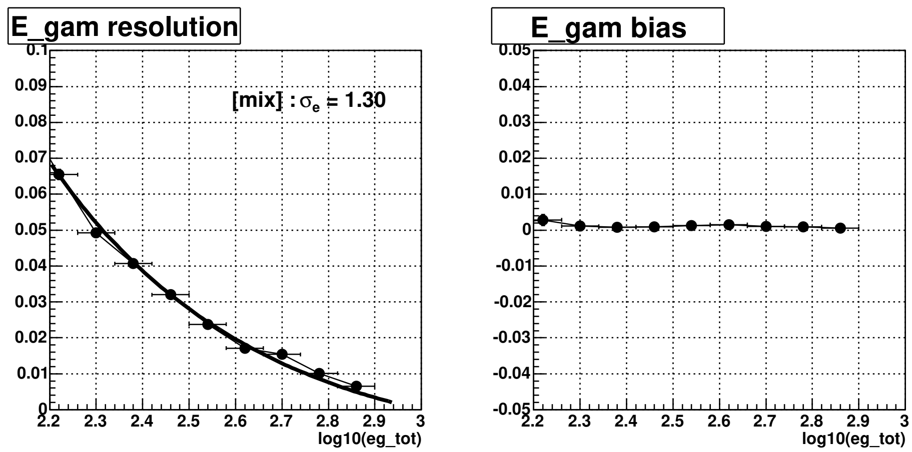

Figure 15: The gamma energy resolution and bias with the modified setup.