

Large anisotropies in the Little Bang

Rencontres QGP-France 2014
Etretat, September 15th, 2014



Jean-Yves Ollitrault,
Institut de physique théorique, Saclay (France)
<http://ipht.cea.fr>

Li Yan, JYO, PRL 112 (2014) 082301

Li Yan, JYO, Art Poskanzer, 1405.6595 & 1408.0921

Anisotropic flow

- Particles are emitted with a *probability distribution* that is not isotropic in azimuthal angle

$$P(\phi) = 1 + 2 \sum_{n>0} v_n \cos(n(\phi - \Psi_n))$$

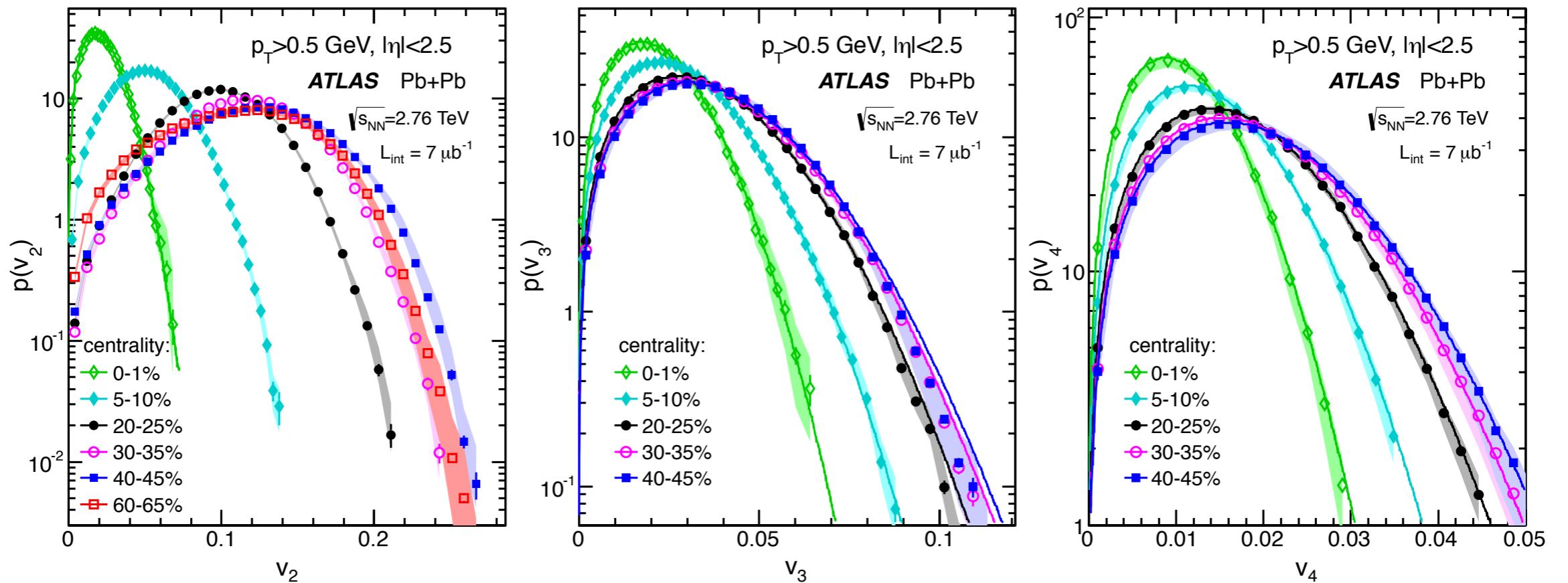
- v_n **≡ anisotropic flow**
 v_2 **≡ elliptic flow**
 v_3 **≡ triangular flow...**
- Finite number of particles → trivial anisotropies from statistical fluctuations.
- v_n can be measured only after statistical fluctuations are subtracted (“unfolded”)

Flow fluctuations

- v_n fluctuates event to event
(PHOBOS, 2005)
- v_n itself has a *probability distribution* for a given system and centrality.

New data in Pb-Pb

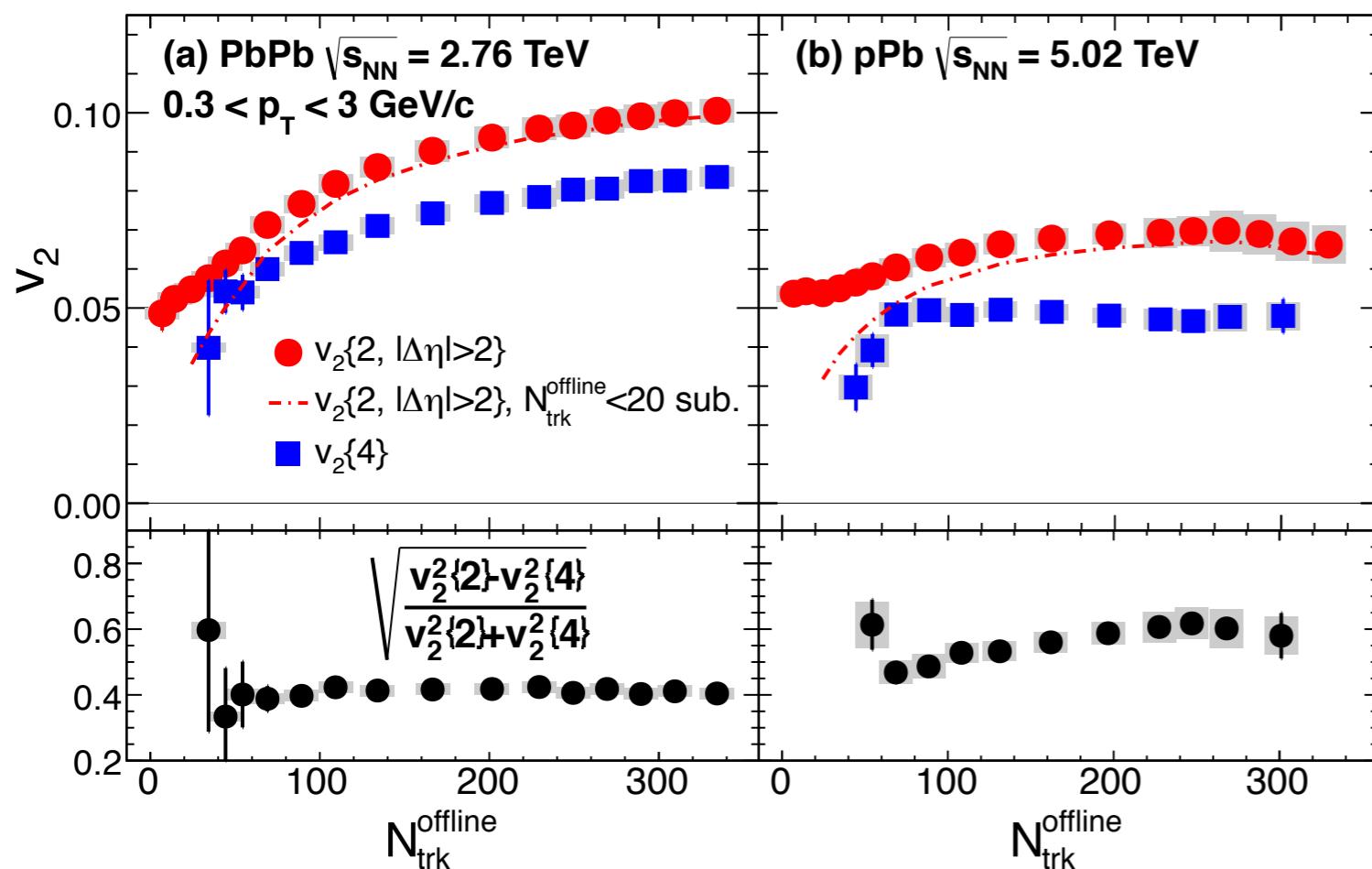
The probability distribution of v_2, v_3, v_4
for various centralities



ATLAS 1305.2942

New data in p-Pb

First 2 cumulants of the distribution of v_2
(less detailed than the full distribution)



$$v_2\{2\} = (\langle v_2^2 \rangle)^{1/2}$$

$$v_2\{4\} = (2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle)^{1/4}$$

If v_2 doesn't fluctuate,

$$v_2\{2\} = v_2\{4\} = v_2$$

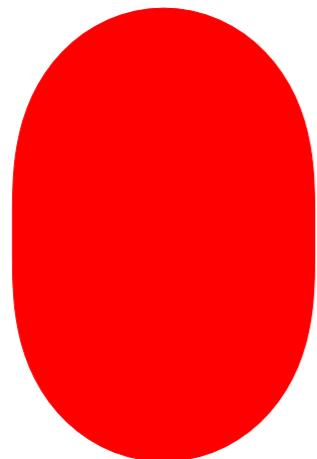
In general $v_2\{4\} < v_2\{2\}$

CMS 1305.0609

- Do we understand these new data?
- What can we learn from them?

The origin of anisotropic flow

*Initial transverse
density profile*

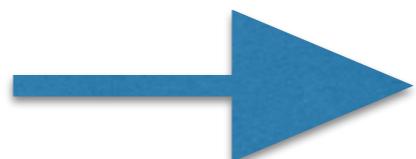
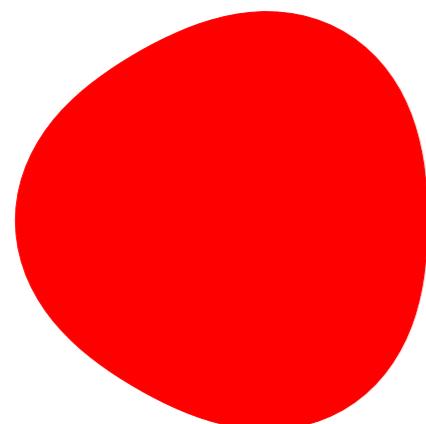


Expansion



Final distribution

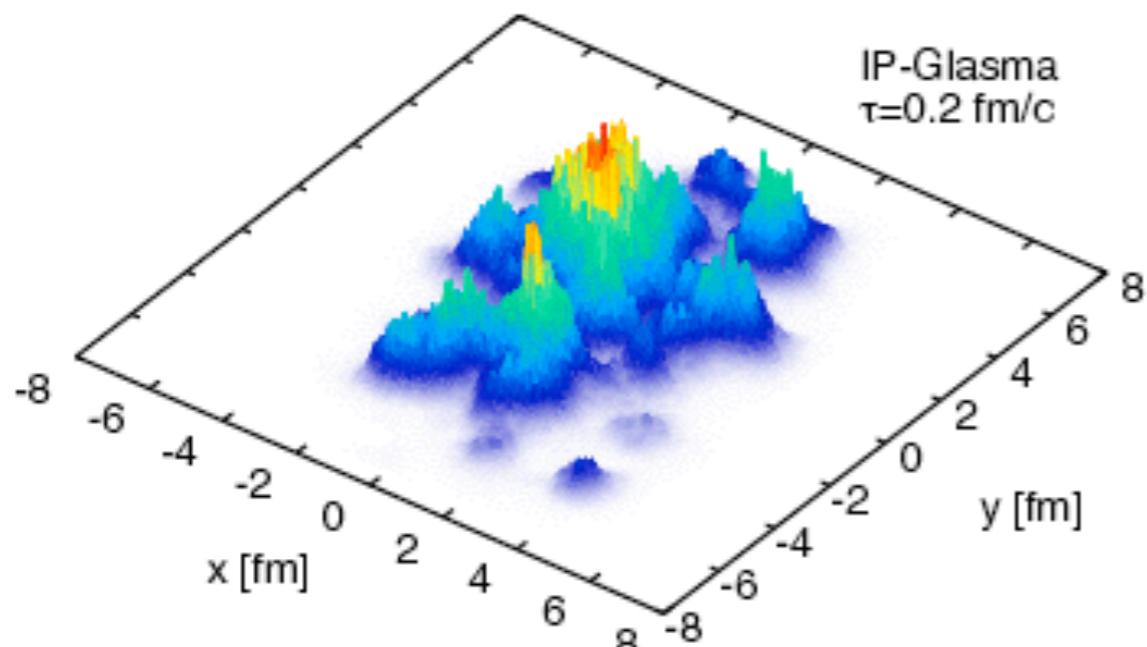
Elliptic flow v_2



Triangular flow v_3

Initial anisotropies

= Fourier decomposition of the initial density profile $\rho(x,y)$



$$\varepsilon_n \equiv \frac{\left| \int r^n e^{in\phi} \rho(r, \phi) r dr d\phi \right|}{\int r^n \rho(r, \phi) r dr d\phi}$$

$\varepsilon_2 \equiv$ initial eccentricity

$\varepsilon_3 \equiv$ initial triangularity

Gale Jeon Schenke 1301.5893

$|\varepsilon_n| < 1$ by definition

Anisotropic flow \approx initial anisotropy

$$v_2 \approx K_2 \epsilon_2$$

$$v_3 \approx K_3 \epsilon_3$$

response coefficients

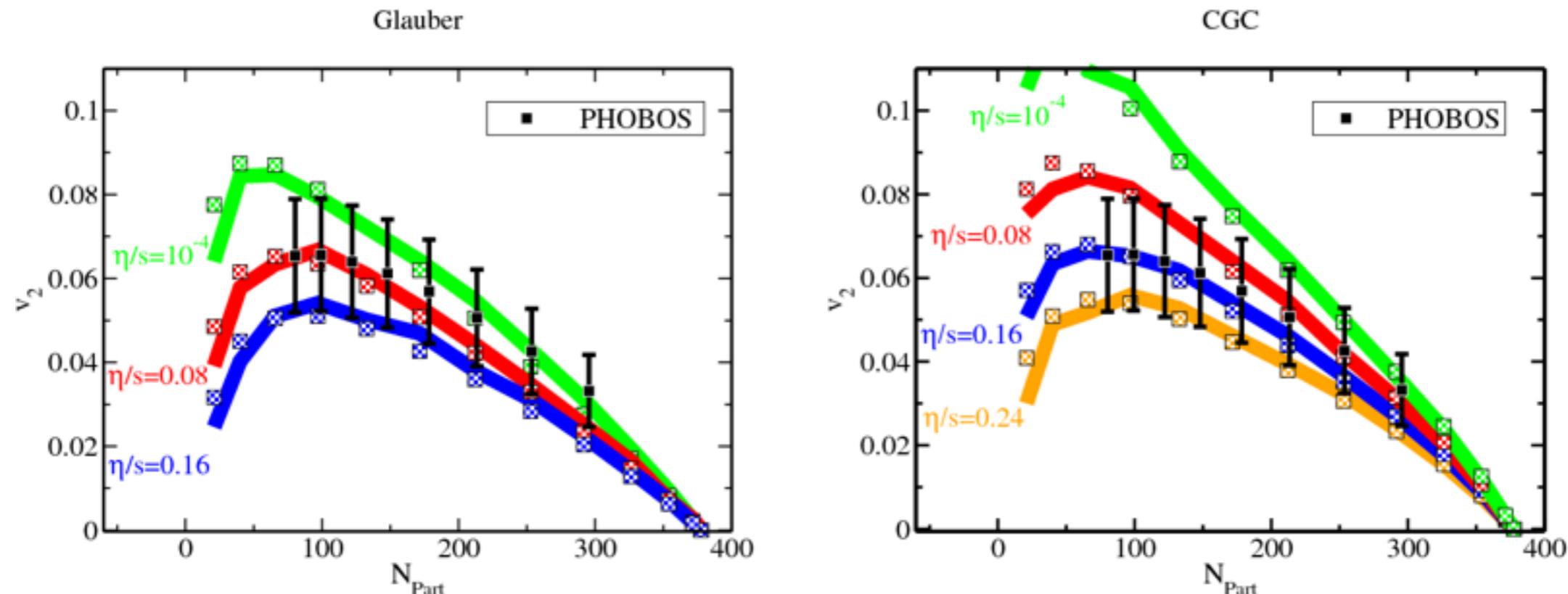
depend on system and centrality

in hydro, depend on viscosity

fluctuate event to event.

v_n fluctuations are due to ϵ_n fluctuations

Problem: can we disentangle the initial anisotropy from the response?



Luzum Romatschke 0804.4015

A long-standing problem in heavy-ion physics:
for any model of initial conditions (Glauber and CGC), i.e.,
for any ϵ_n , one can tune the viscosity — the response K_n —
to match the observed v_n

Is there a general law that describes anisotropy fluctuations?

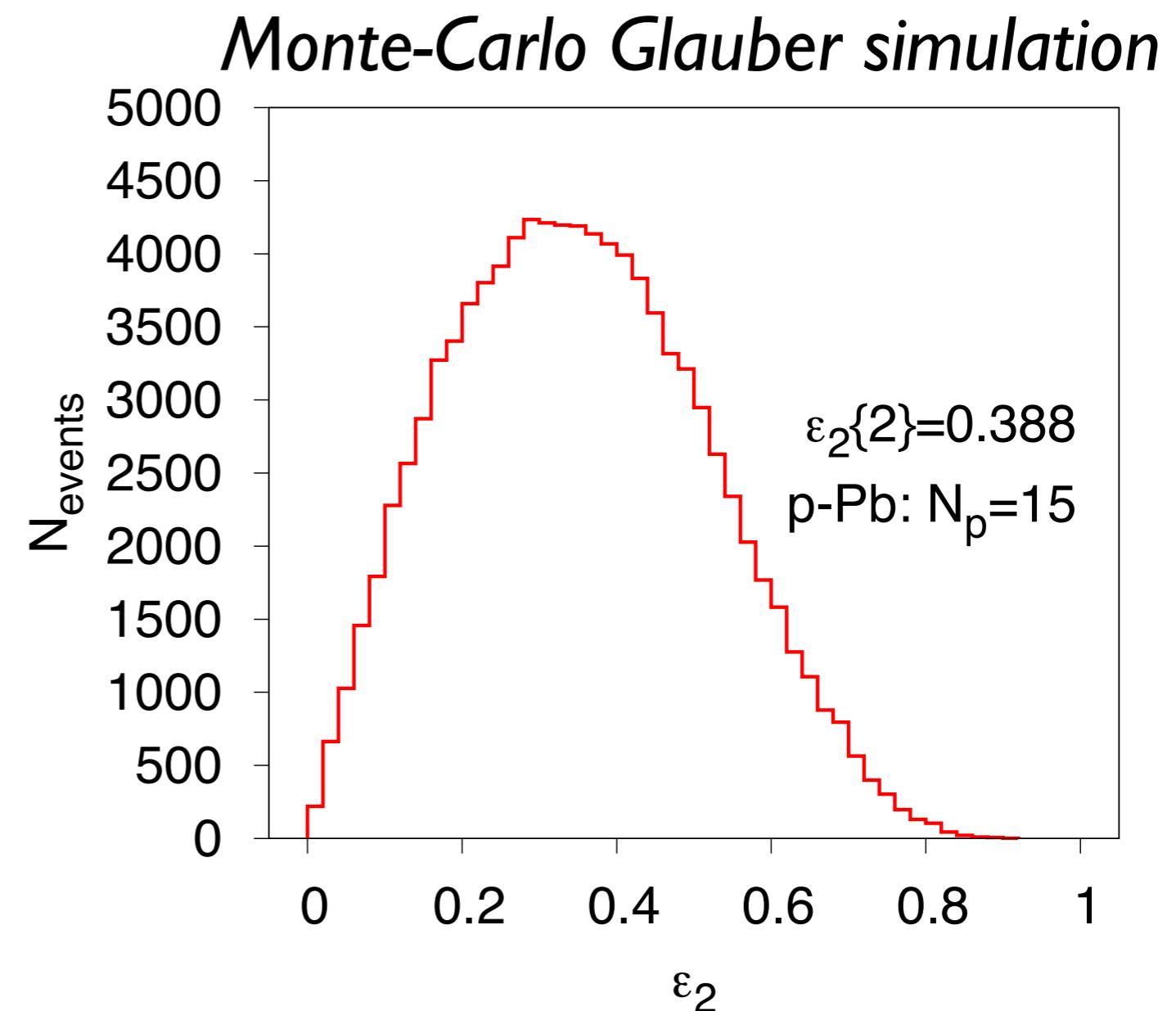
- If we know the statistics of the initial ε_n , then the distribution of observed v_n is the distribution of ε_n , rescaled by the response K_n
- State of the art (as of 2013): Gaussian fluctuations
 $P(\varepsilon_n) \propto \varepsilon_n \exp(-\varepsilon_n^2/\sigma^2)$ *Voloshin et al 0708.0800*
- Then the distribution of v_n is also a Gaussian, of width $K_n \times \sigma$: we are still unable to disentangle the initial state from the response.

The statistics of initial fluctuations

$$\varepsilon_2 = \frac{|\int r^2 e^{2i\phi} \rho(r, \phi) r dr d\phi|}{\int r^2 \rho(r, \phi) r dr d\phi}$$

central p+Pb collision:
initial density $\rho(r, \phi)$ =
independent of ϕ up to
fluctuations

small system: *large*
fluctuations & *anisotropies*



Is there a simple law that describes this distribution?

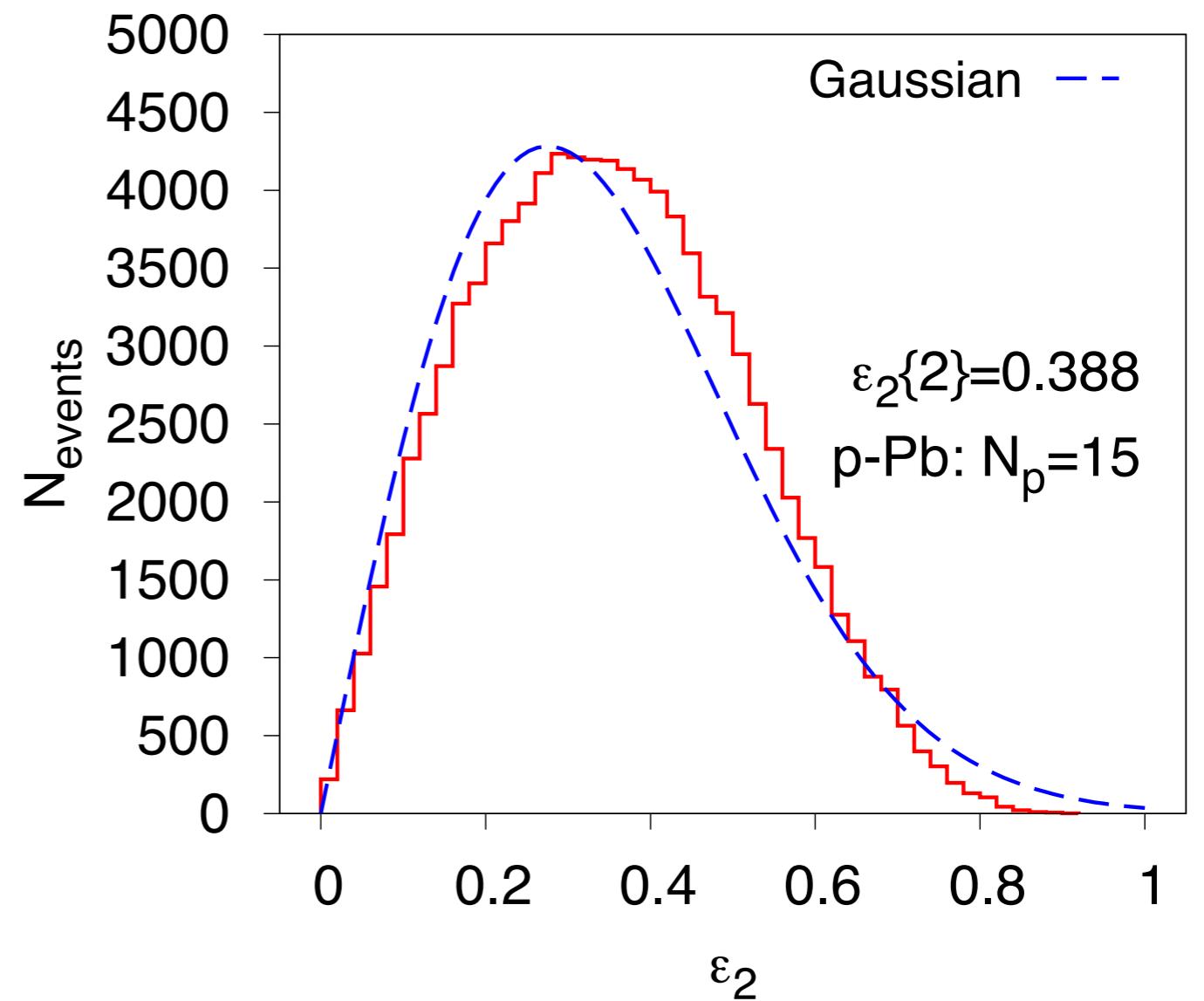
Gaussian?

Central limit theorem

$$P(\varepsilon_2) = 2(\varepsilon_2/\sigma^2) \exp(-\varepsilon_2^2/\sigma^2)$$

Not a good fit.

Does not implement
the condition $\varepsilon_2 < 1$



New “Power” distribution

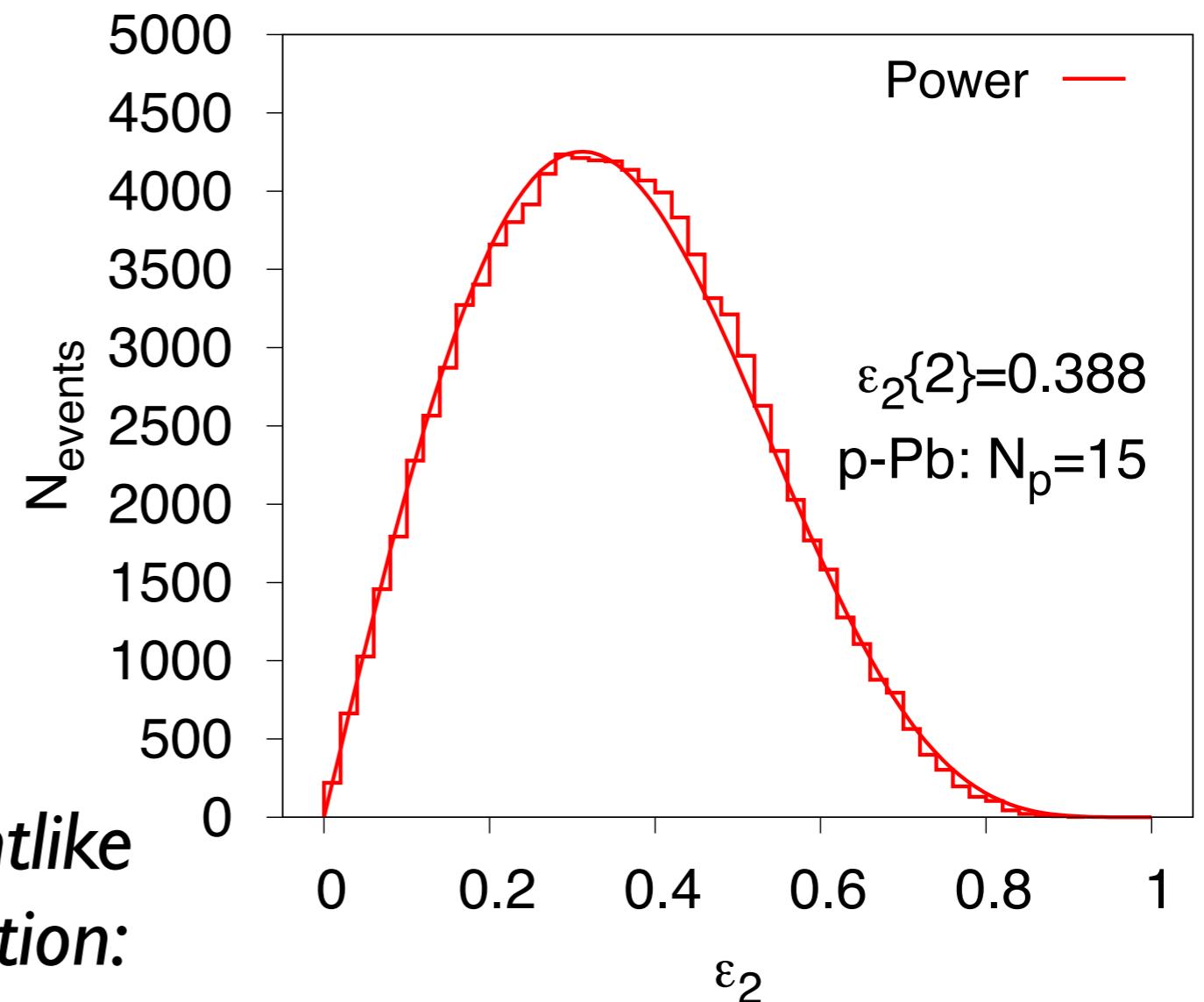
$$P(\varepsilon_2) = 2\alpha\varepsilon_2(1-\varepsilon_2^2)^{\alpha-1}$$

Equivalent to Gaussian for
 $\alpha \gg 1$

Naturally implements the
condition $\varepsilon_2 < 1$.

*Exact result for $N=2\alpha+1$ pointlike
sources with Gaussian distribution:*

JYO, PRD 46 (1992) 229



Much better fit to Monte-Carlo results!

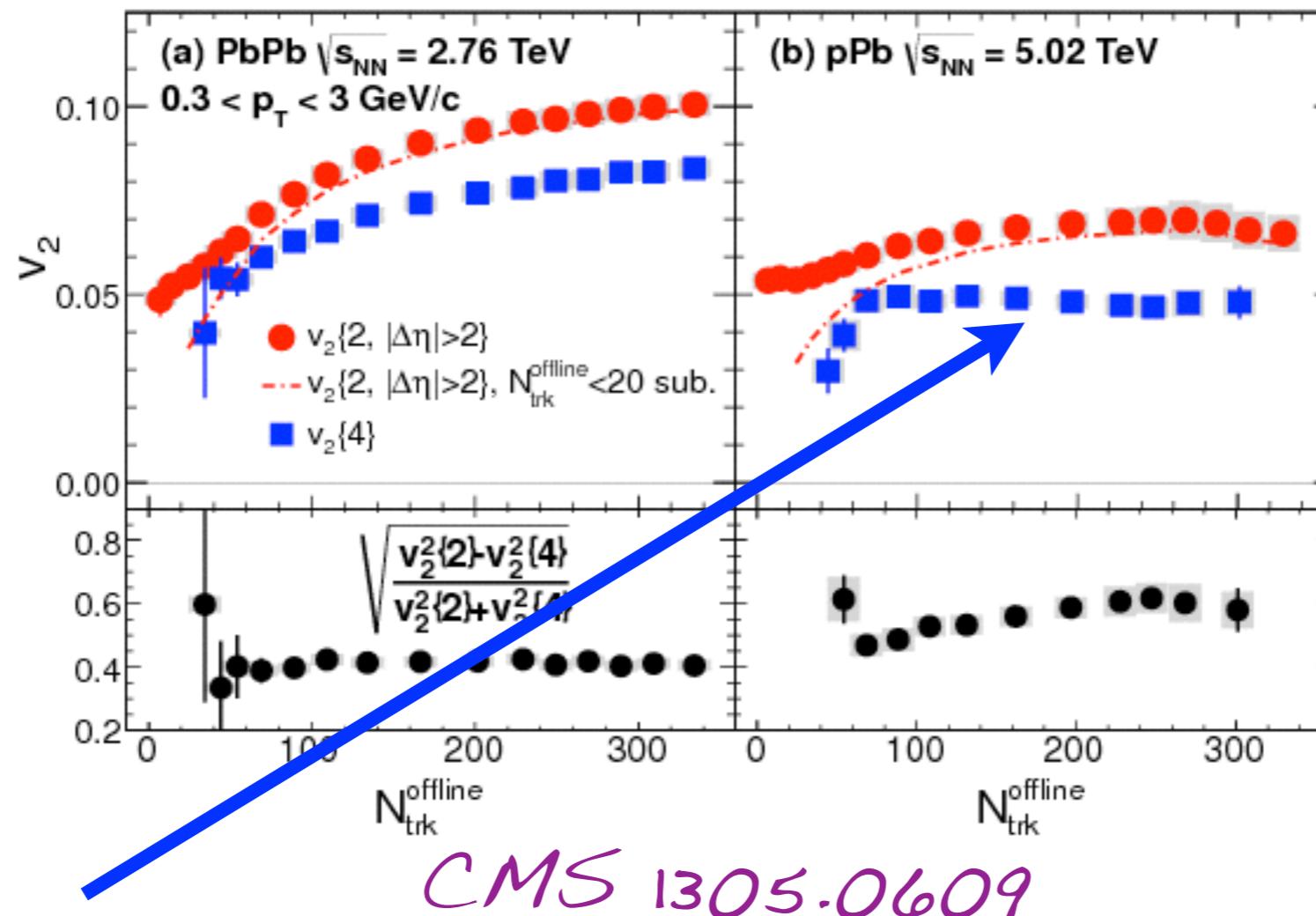
Universality of initial anisotropy fluctuations

- The *Power* distribution fits several models of initial conditions (MC Glauber, MC KLN, IP-Glasma, DIPSY) when the anisotropy is solely created by fluctuations: ε_2 in p-p collisions, ε_2 and ε_3 in p-Pb collisions, ε_3 in Pb-Pb or Au-Au collisions.

Li Yan, JYI, PRL 112 (2014) 082301

- We postulate that it is universal, to a good approximation.

Natural explanation for $v_2\{4\}$ in pPb

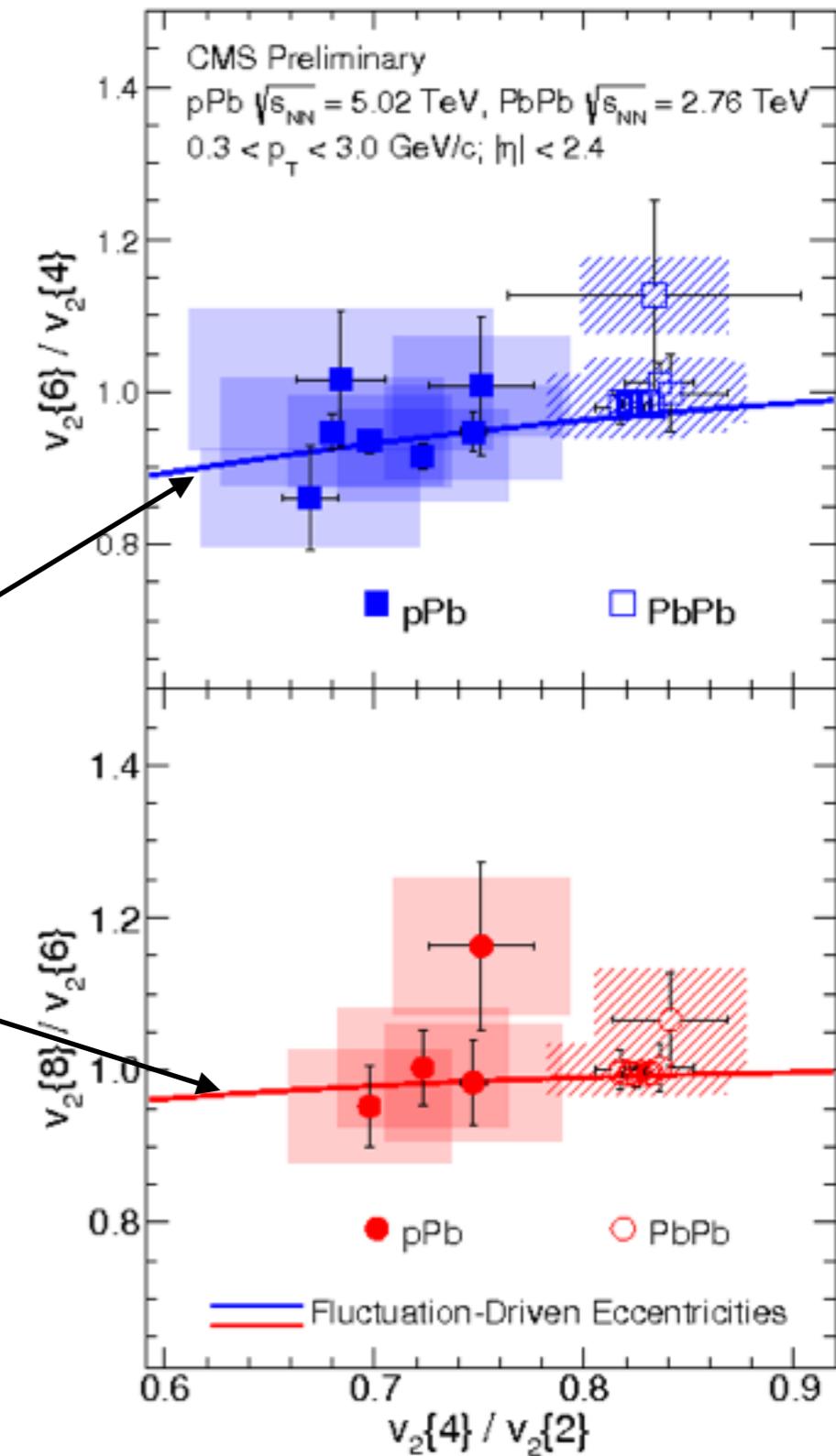


Gaussian fluctuations give $v_2\{4\}=0$.

Our new Power distribution naturally predicts a large $v_2\{4\}$ in p-Pb.

Predictions: higher-order cumulants

- Using as input the experimentally measured ratio $v_2\{4\}/v_2\{2\}$
- Quantitative **prediction** for higher-order cumulants $v_2\{6\}$ and $v_2\{8\}$
- New CMS data (QM2014) in good agreement with our prediction



Generalization to ε_2 in Pb-Pb

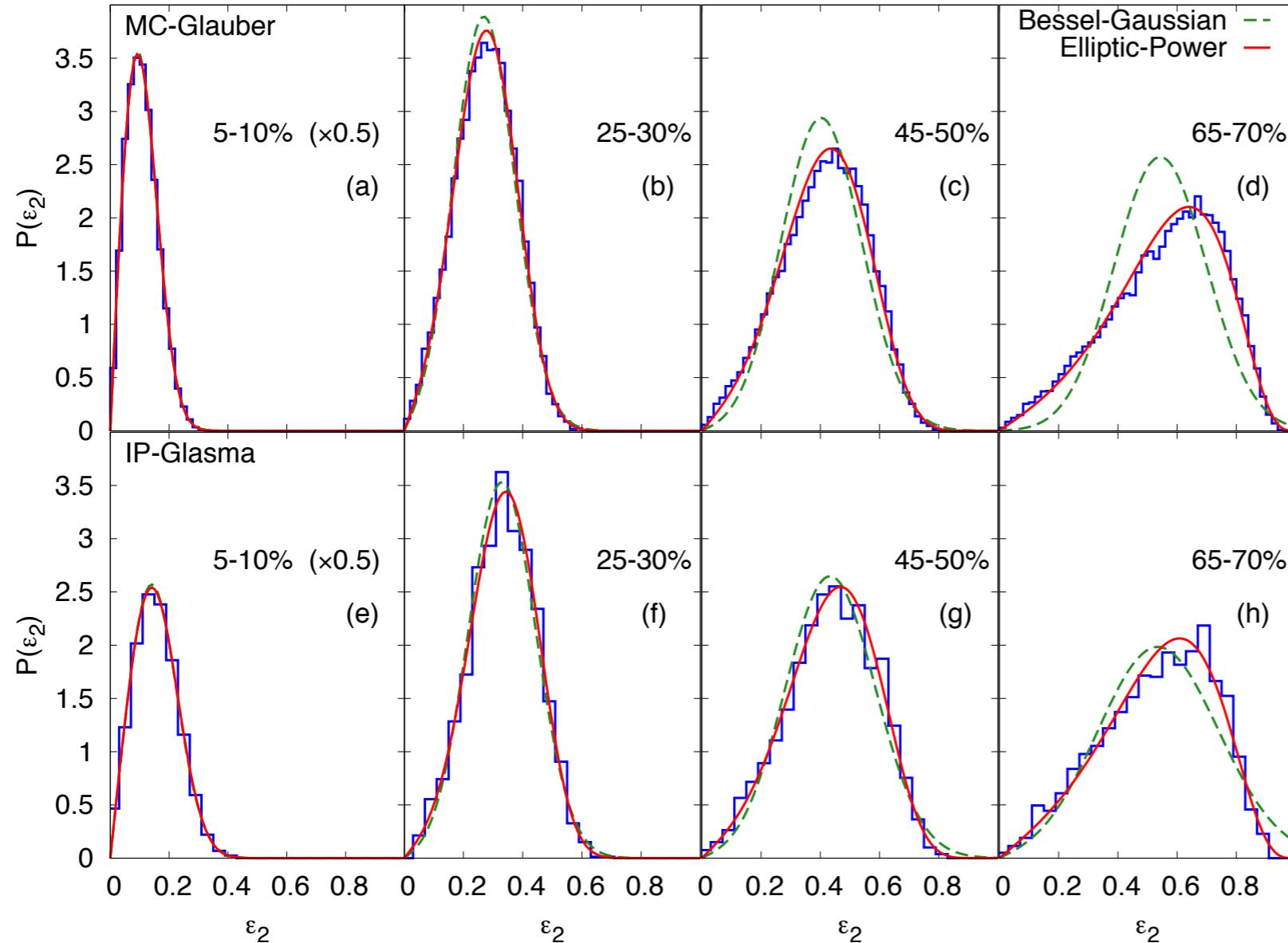
- For ε_2 in non-central Pb-Pb or Au-Au collisions, there is a **mean anisotropy in the reaction plane** in addition to **fluctuations**: requires a generalized distribution with 1 extra parameter: the *Elliptic Power* distribution

$$\frac{dn}{d\varepsilon} = \frac{2}{\pi} \varepsilon \alpha (1 - \varepsilon^2)^{(\alpha-1)} (1 - \varepsilon_0^2)^{(\alpha+1/2)} \int_0^\pi (1 - \varepsilon_0 \varepsilon \cos \phi)^{-(1+2\alpha)} d\phi$$

Reduces to the Power distribution for $\varepsilon_0 = 0$

Li Yan, JYO, Art Poskanzer, 1405.6595

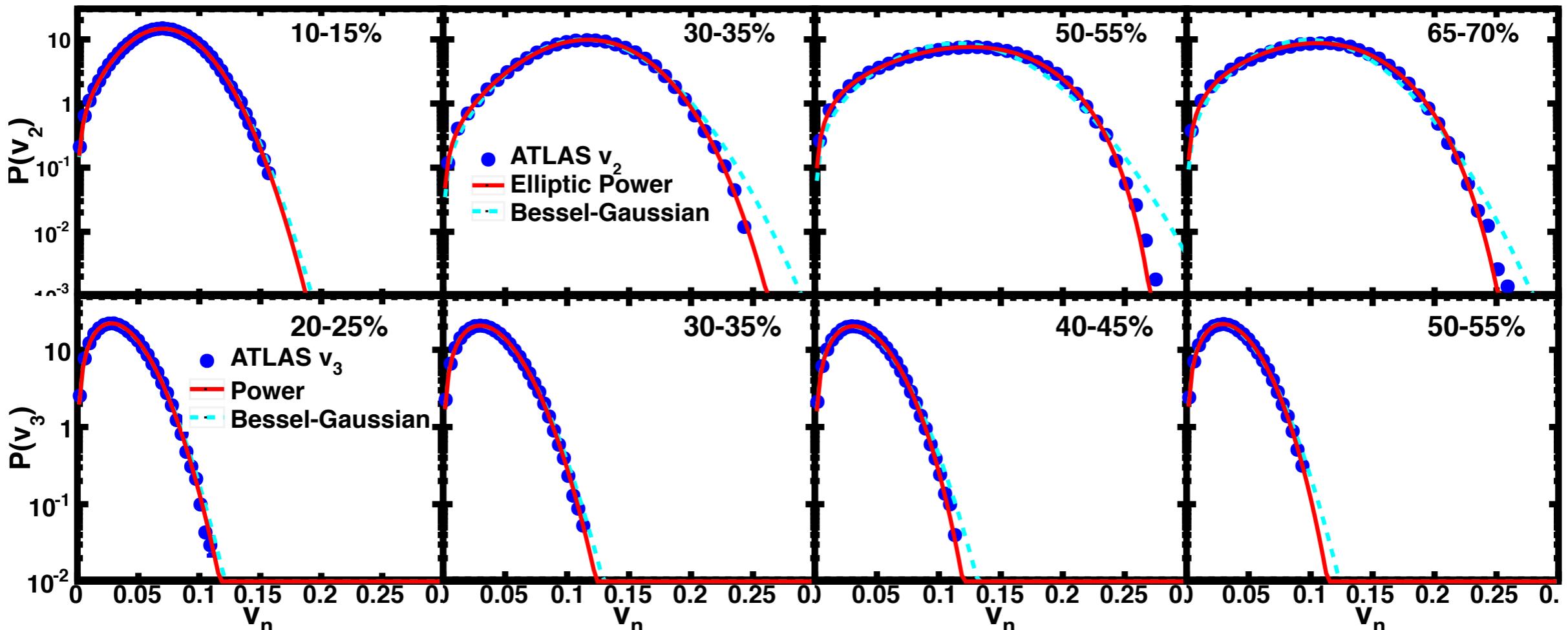
Testing the *Elliptic Power* distribution for ϵ_2



2 models of the initial density:
- Monte-Carlo Glauber
- IP Glasma
Good fits for both models, all centralities

Li Yan, JYO, Art Poskanzer, 1405.6595

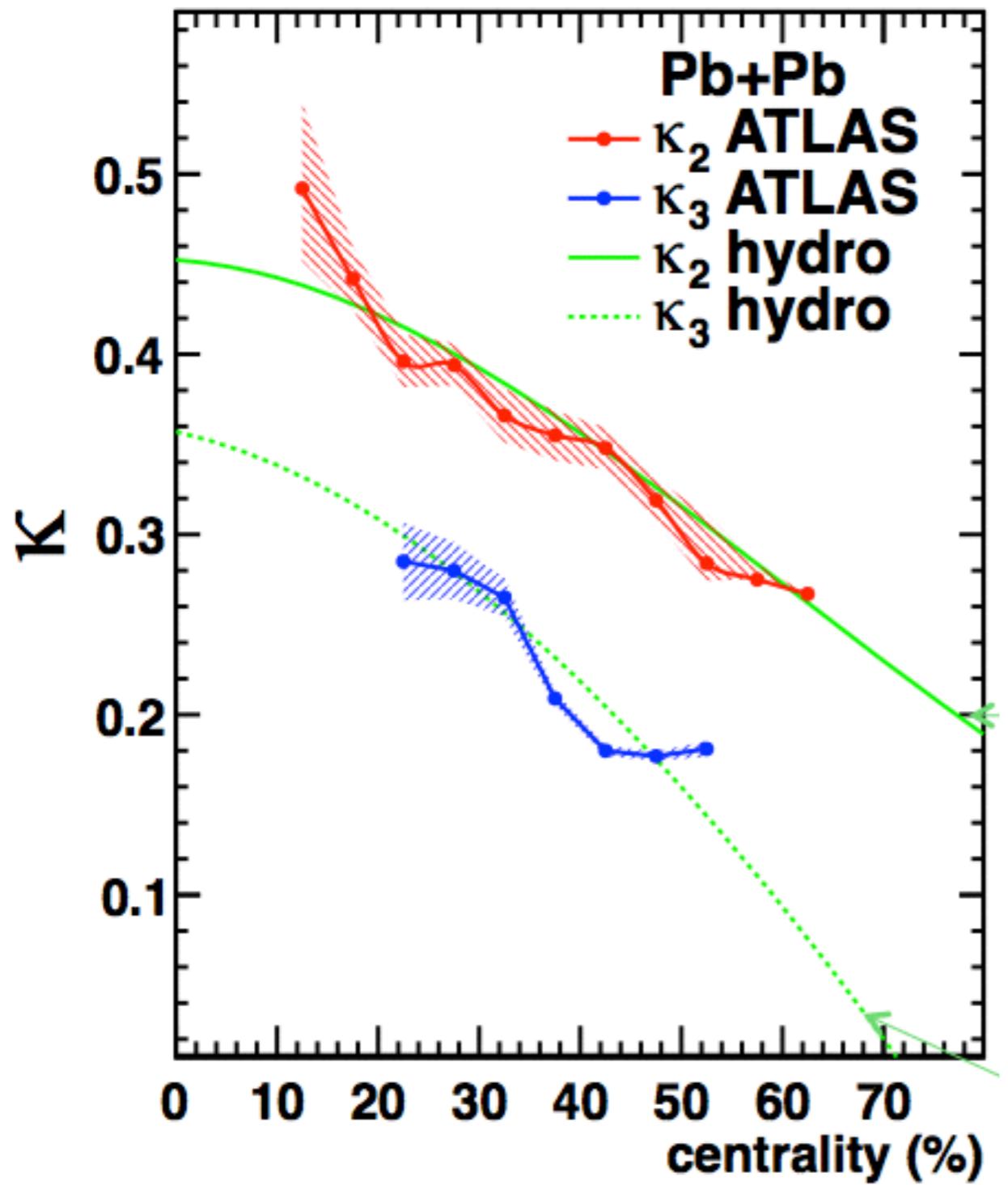
Fitting ATLAS v_3 and v_2 distributions with rescaled *Power* and *Elliptic-Power*



Li Yan, JYO, Art Poskanzer, 1408.0921

Good fits to data for v_2 and v_3 , all centralities

Extracting the hydro response from ATLAS data

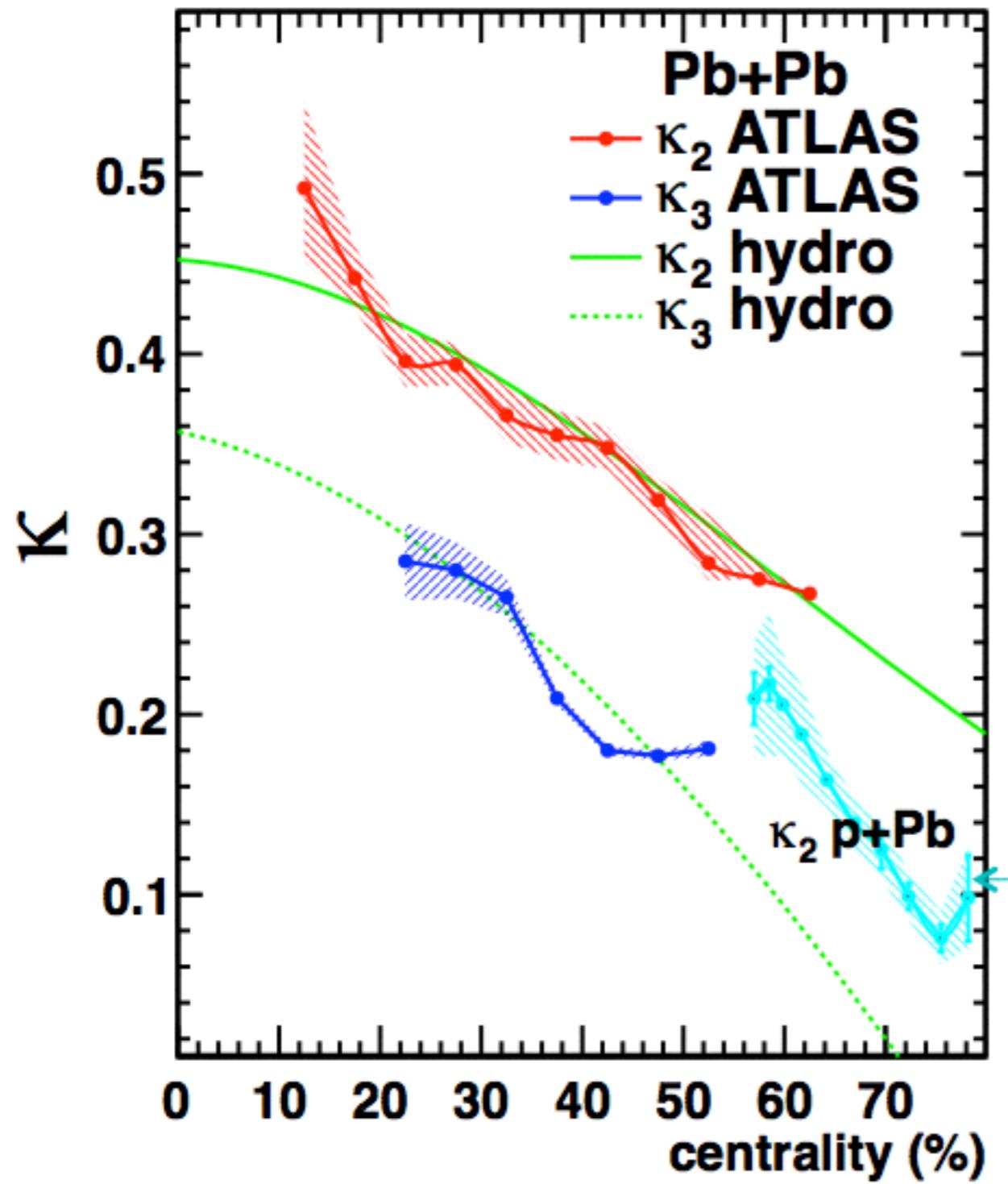


Both κ_3 and κ_2 decrease for peripheral collisions because the smaller size leads to larger viscous damping

Viscous hydro fit: $\eta/s=0.14-0.18$
But global rescaling required

κ_3 : large syst. errors not shown.

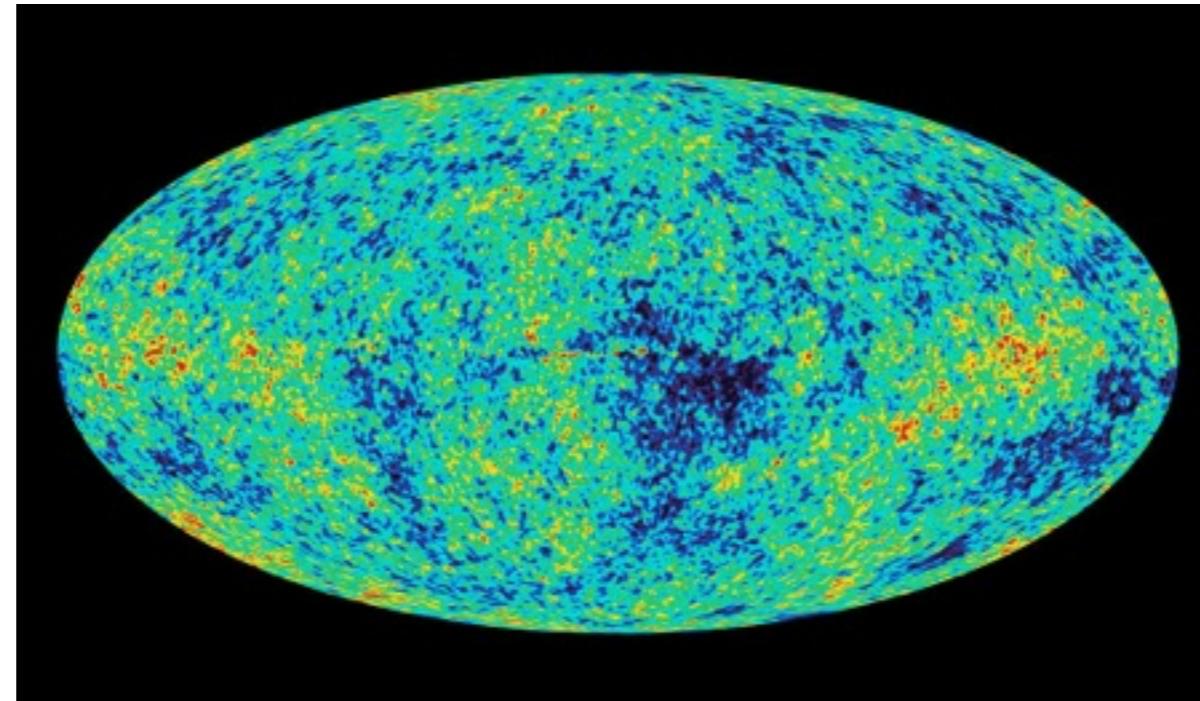
Hydro response in p+Pb



v_2 distributions not available:
we use CMS data on $v_2\{4\}$
and $v_2\{2\}$

The response is somewhat
smaller than in Pb+Pb at the
same equivalent centrality.

Big Bang versus Little Bang



WMAP

Small anisotropies observed in the cosmic microwave background are thought to originate from quantum fluctuations in the early Universe.

Anisotropic flow at RHIC and LHC is a similar phenomenon, occurring within a tiny system with large fluctuations.

The non-Gaussianity of these fluctuations, and the fact that they are universal, allows us to disentangle initial fluctuations from the response.

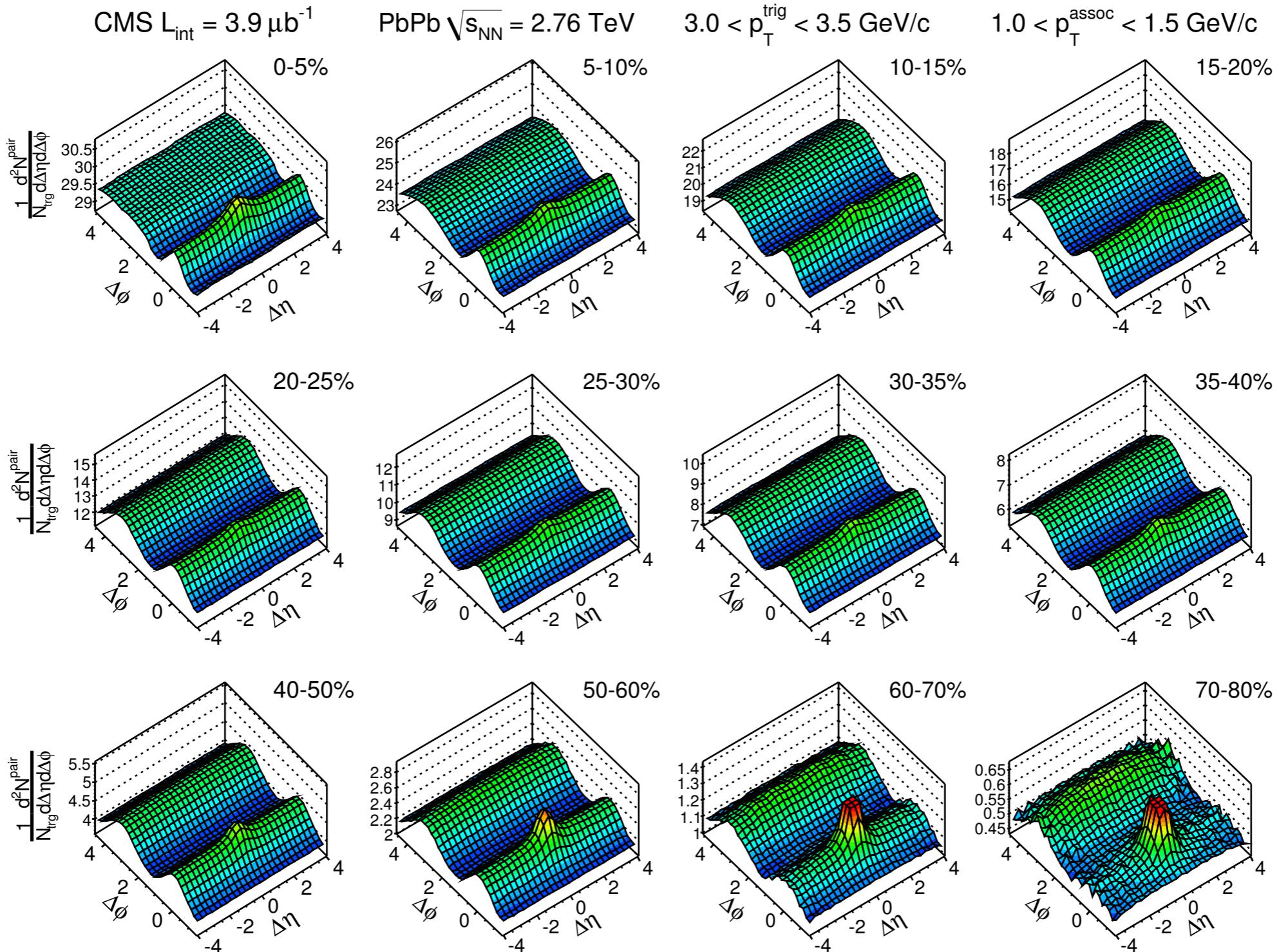
Conclusions, perspectives

- Direct evidence from experimental data that anisotropic flow in p-Pb and Pb-Pb collisions is driven by **large anisotropies** in the initial state: the statistics of ε_n *hits the boundary* $\varepsilon_n < 1$
- The statistics of large fluctuations is not described by the central limit theorem but nevertheless **universal** to a good approximation
- We can extract both the initial anisotropy and the “hydrodynamic” response K_n from experimental data without any prior assumption about the initial state, but with approximations: errors may be large.

Backup

Anisotropic flow

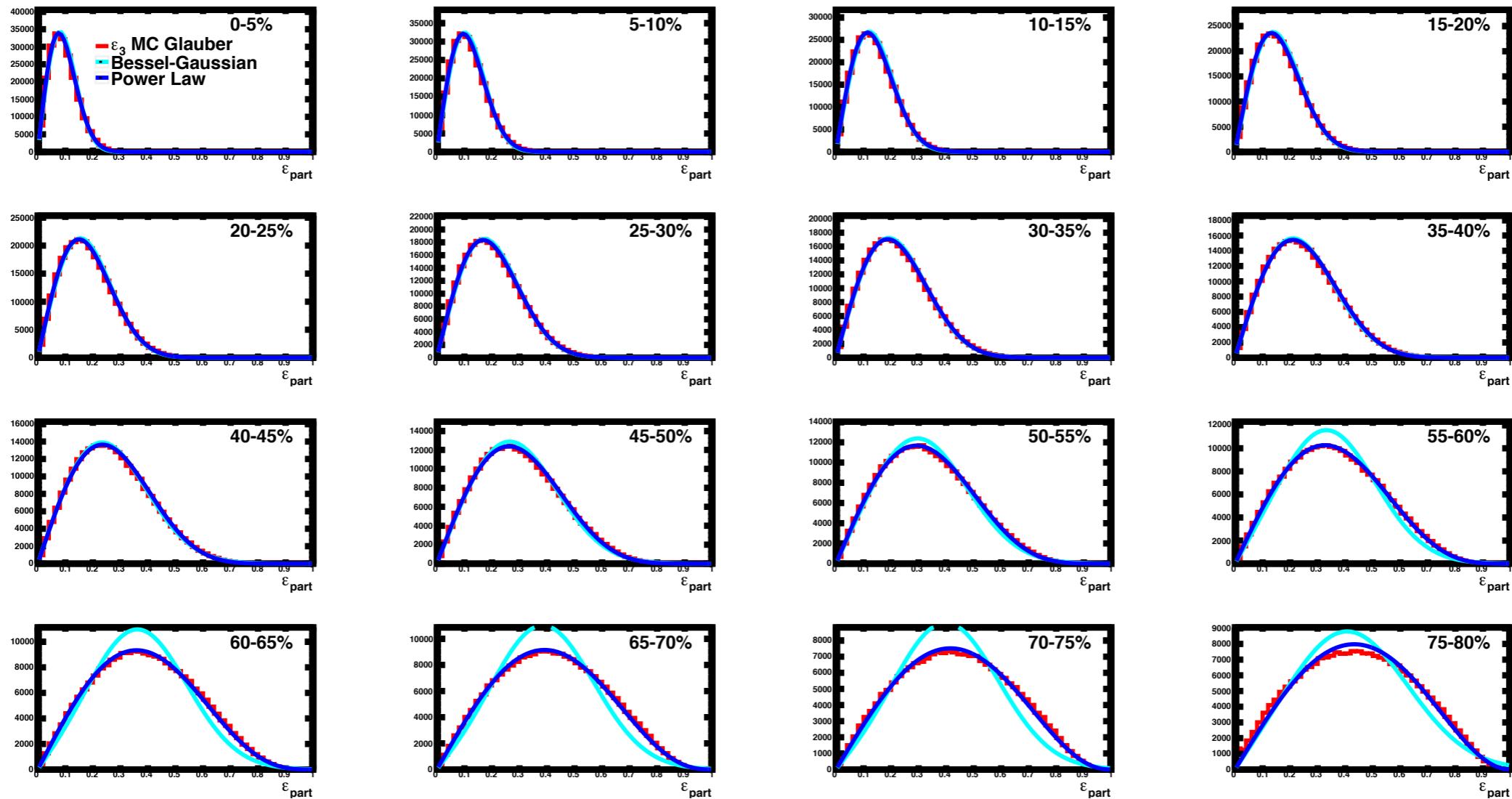
○
central



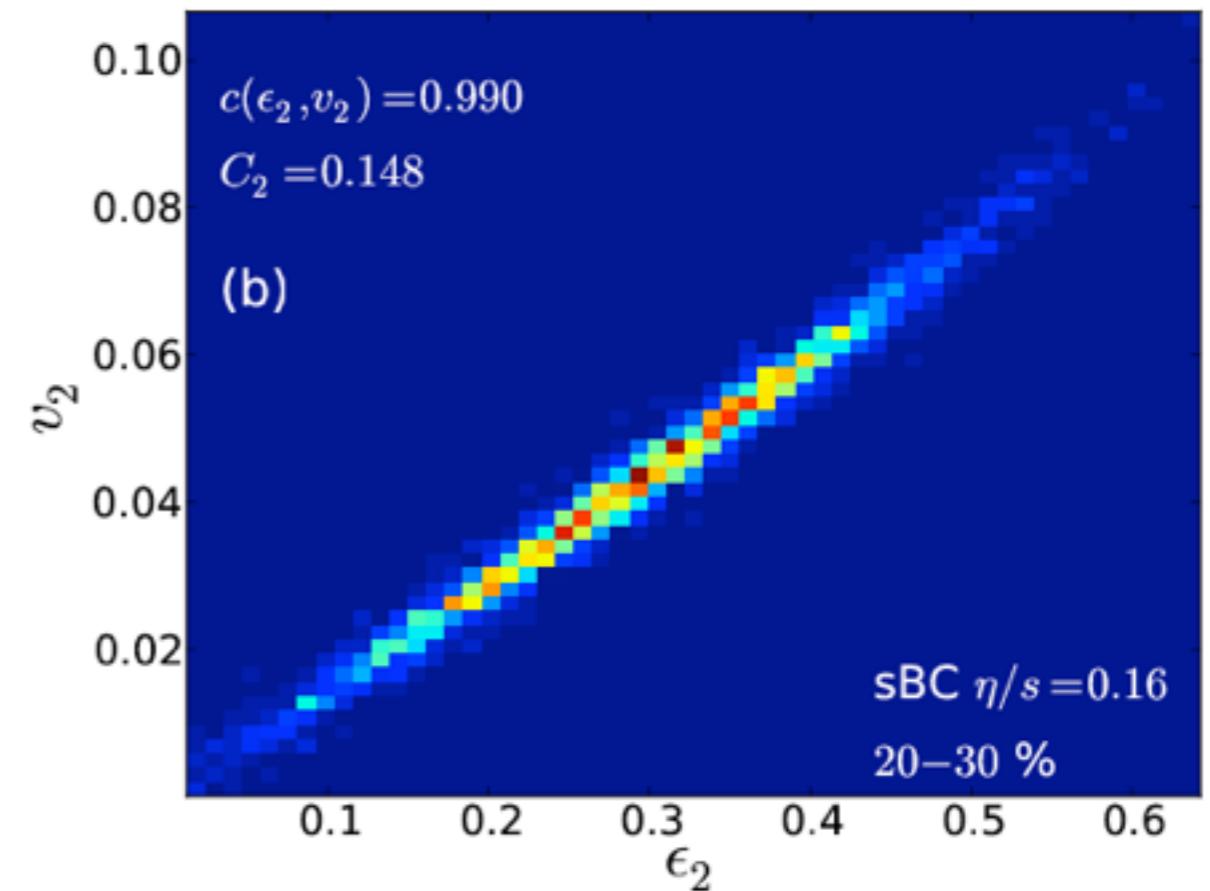
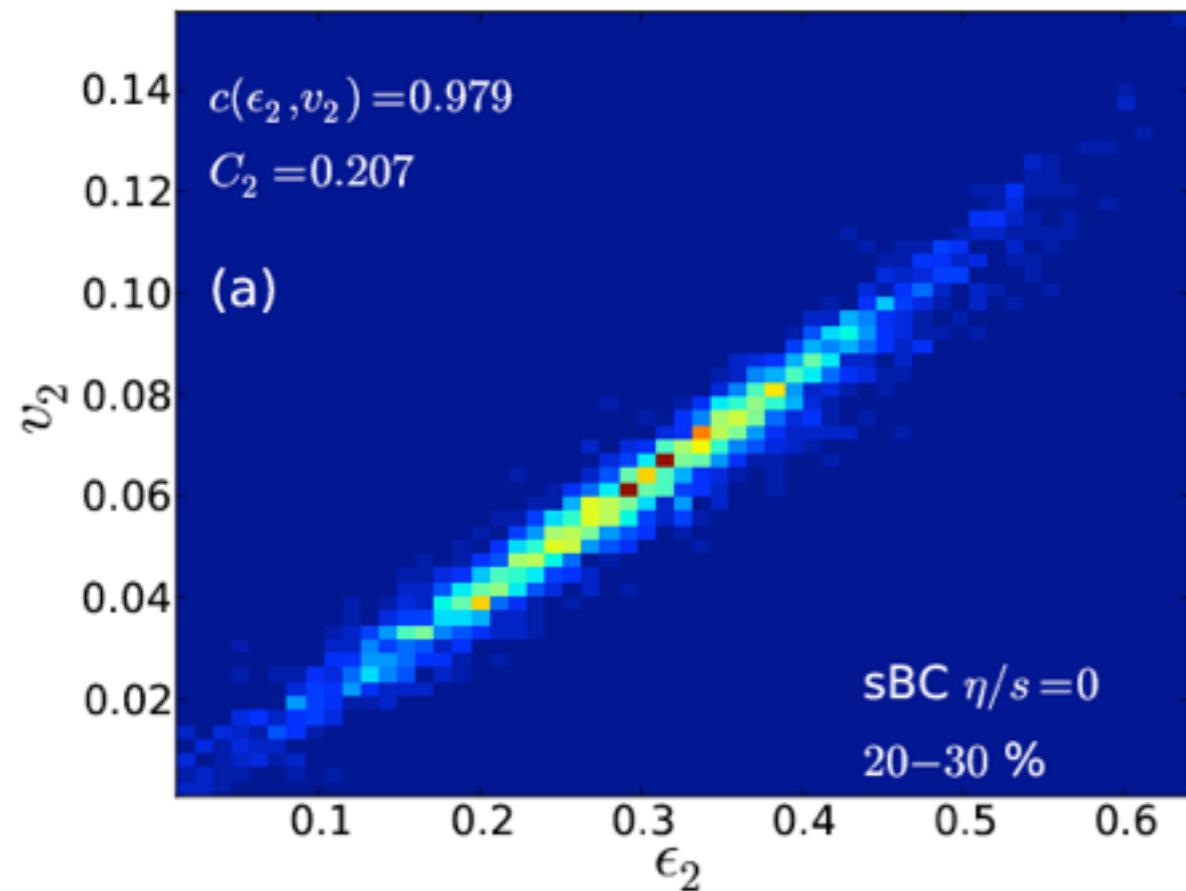
○○
peripheral

CMS 1201.3158

Testing the Power distribution for ε_3 in Au-Au collisions



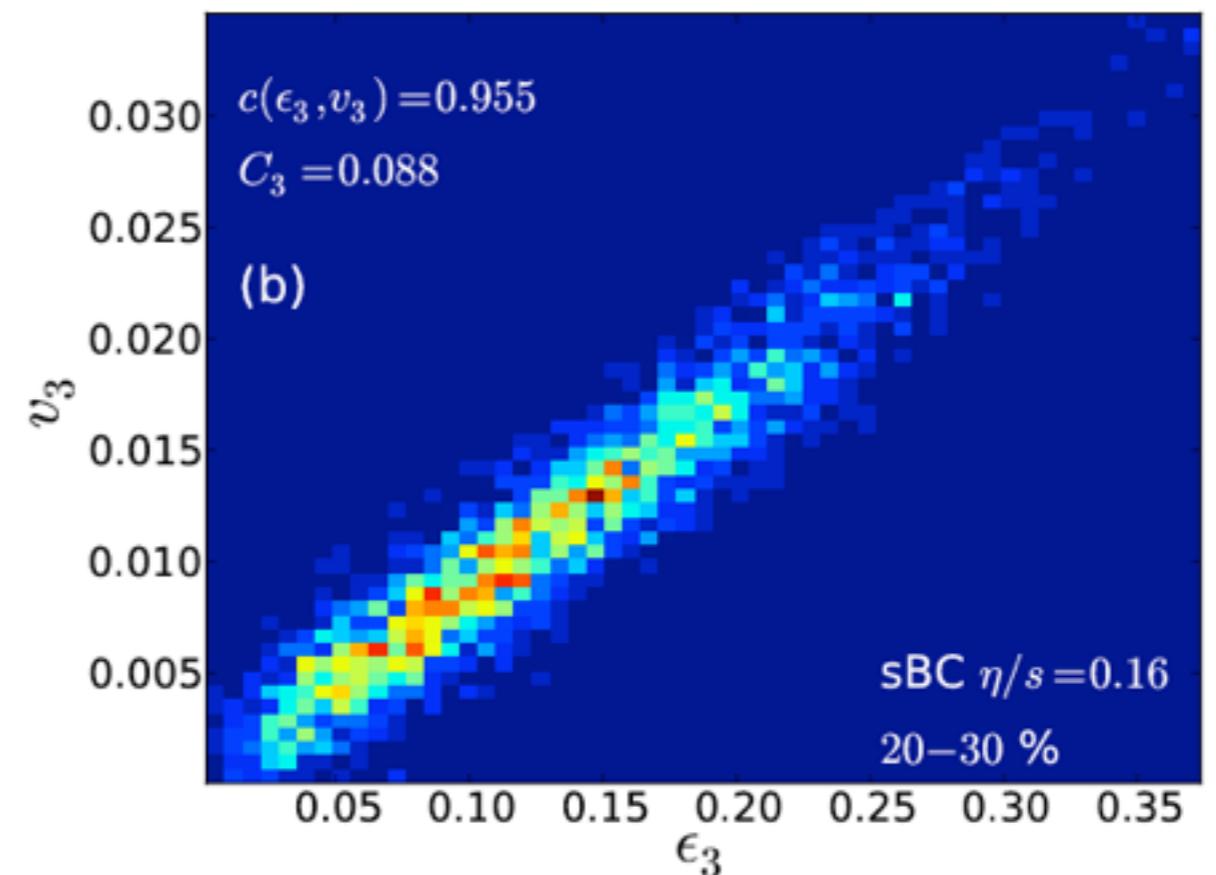
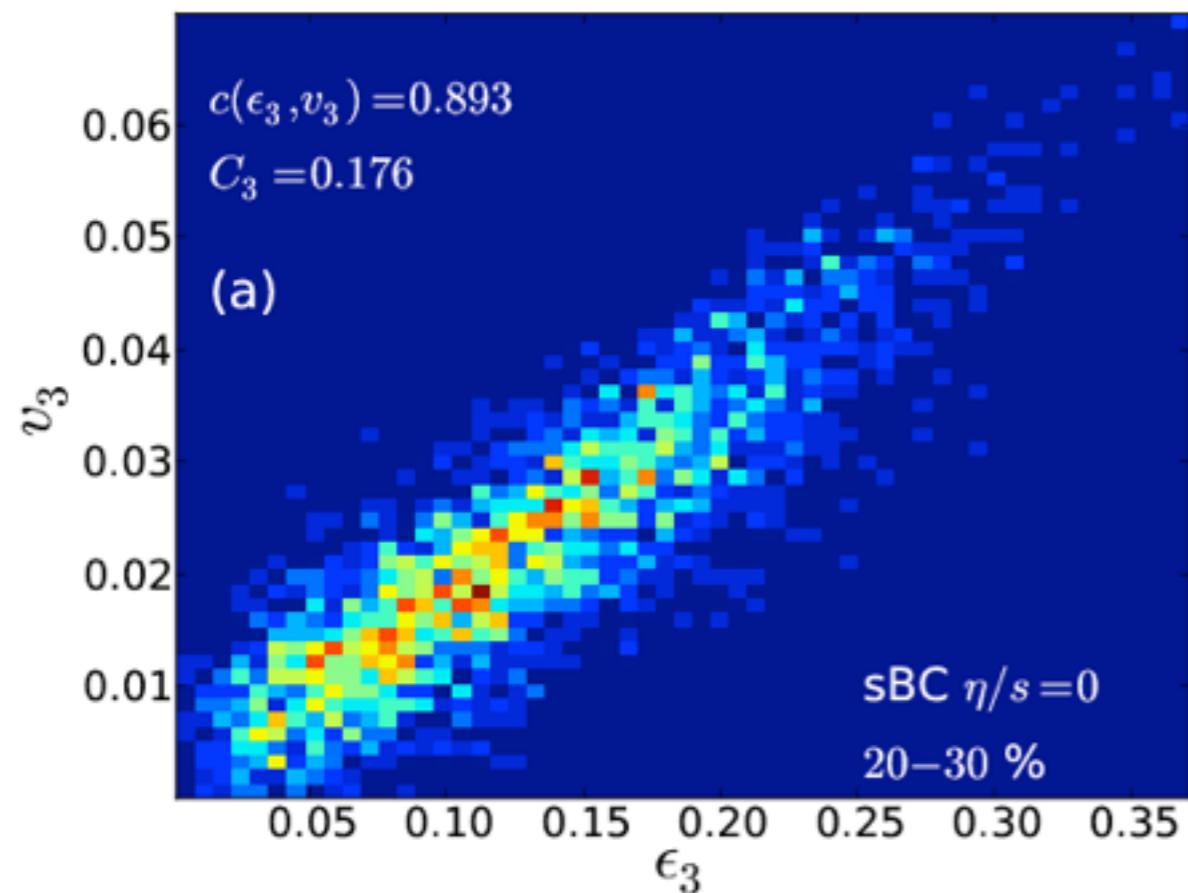
Elliptic flow v_2 versus initial eccentricity ϵ_2



Niemi Denicol Holopainen Huovinen 1212.1008

Each point=different initial density profile.
 v_2 is almost perfectly linear in ϵ_2

Triangular flow v_3 versus initial triangularity ϵ_3



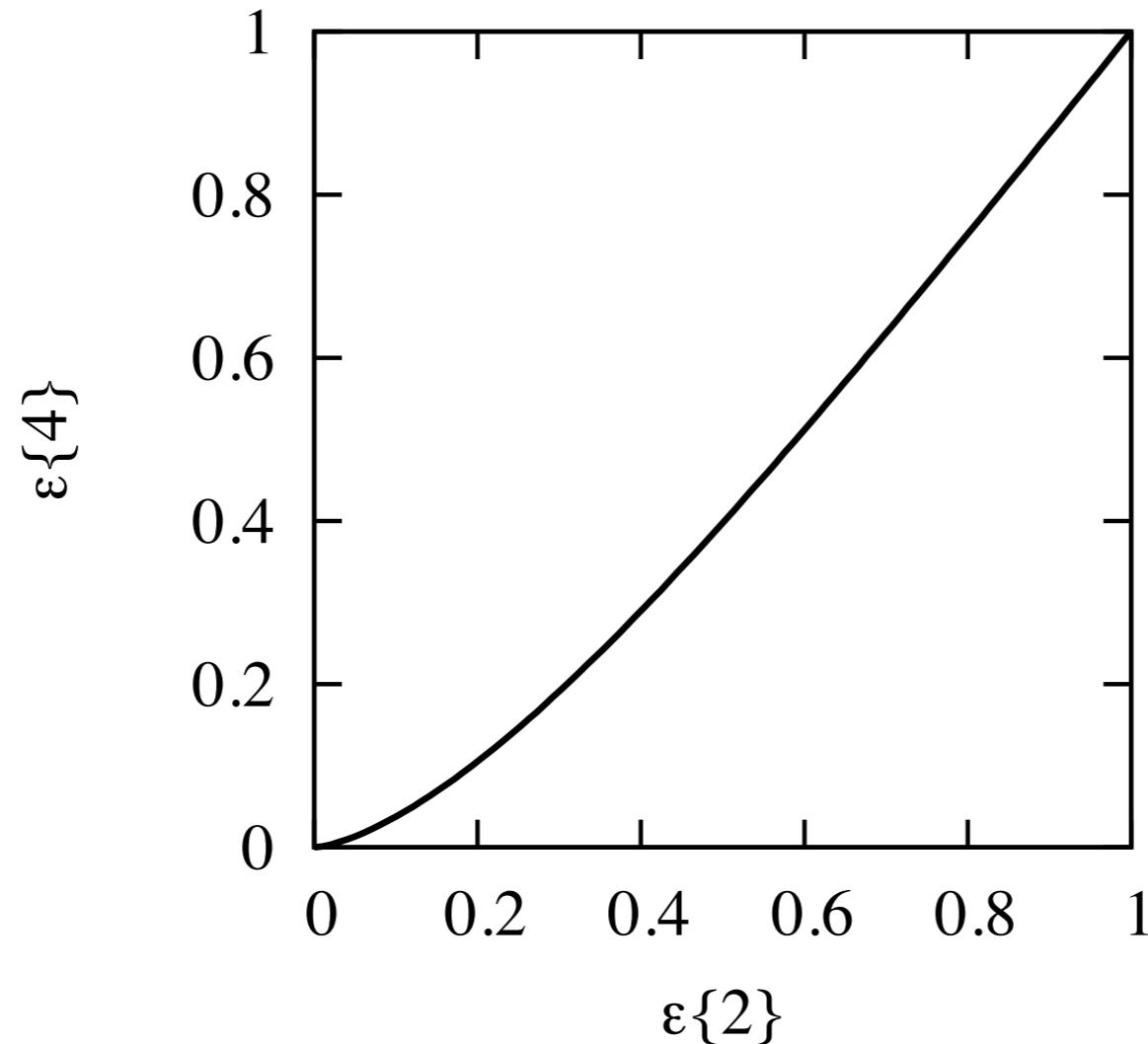
Niemi Denicol Holopainen Huovinen 1212.1008

v_3 is also strongly correlated with ϵ_3

Cumulants

- 2-dimensional Gaussian: Wick's theorem
 $\langle \varepsilon^4 \rangle = 2 \langle \varepsilon^2 \rangle^2$ where $\langle \dots \rangle \equiv$ average over events
- Define $\varepsilon\{2\} \equiv \langle \varepsilon^2 \rangle^{1/2}$ (rms anisotropy)
$$\varepsilon\{4\} \equiv (2 \langle \varepsilon^2 \rangle^2 - \langle \varepsilon^4 \rangle)^{1/4}$$
- $\varepsilon\{4\} = 0$ for Gaussian.
- The power distribution predicts a universal, relation between $\varepsilon\{4\}$ and $\varepsilon\{2\}$

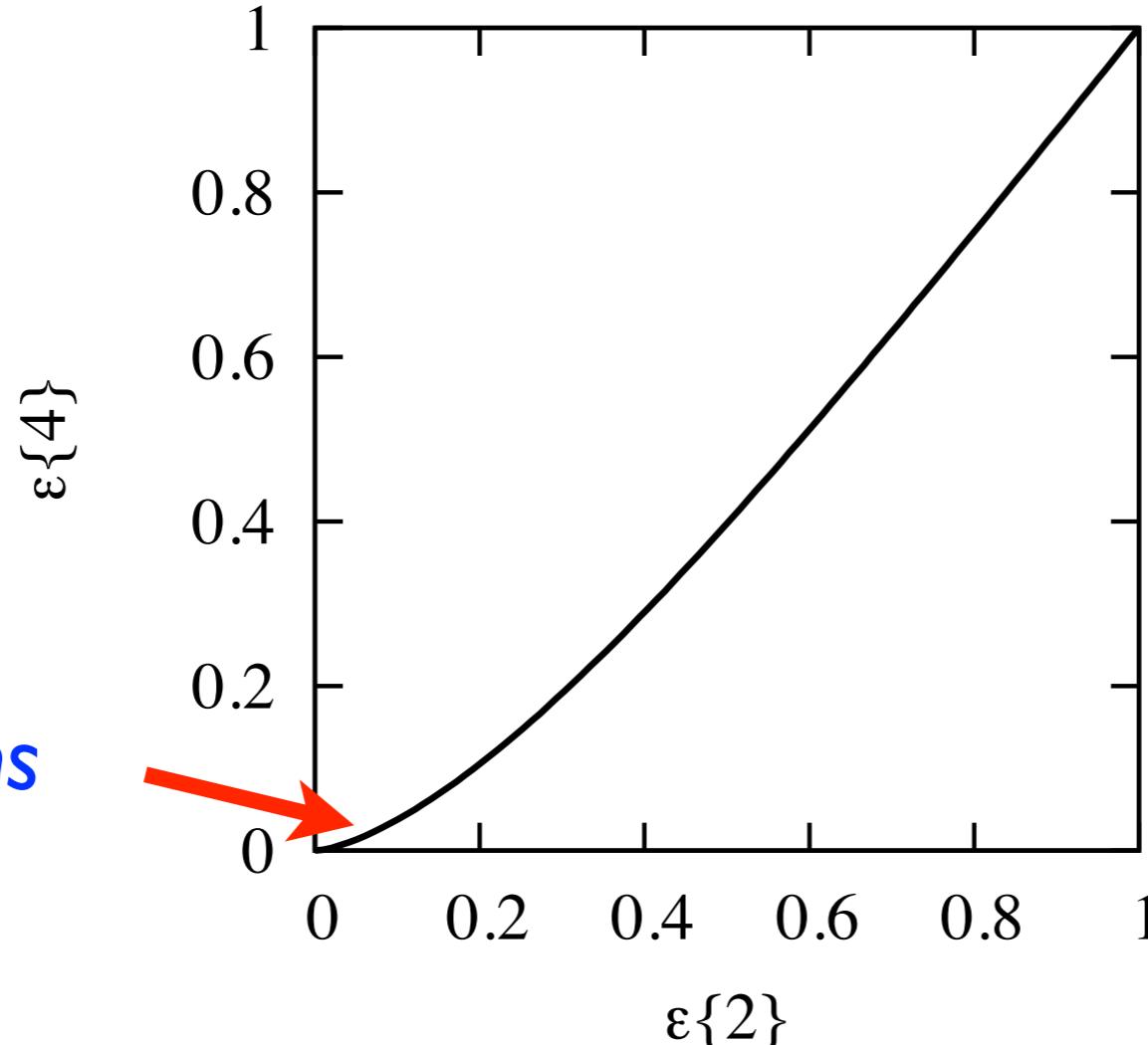
Testing universality with cumulants



Prediction of the power distribution

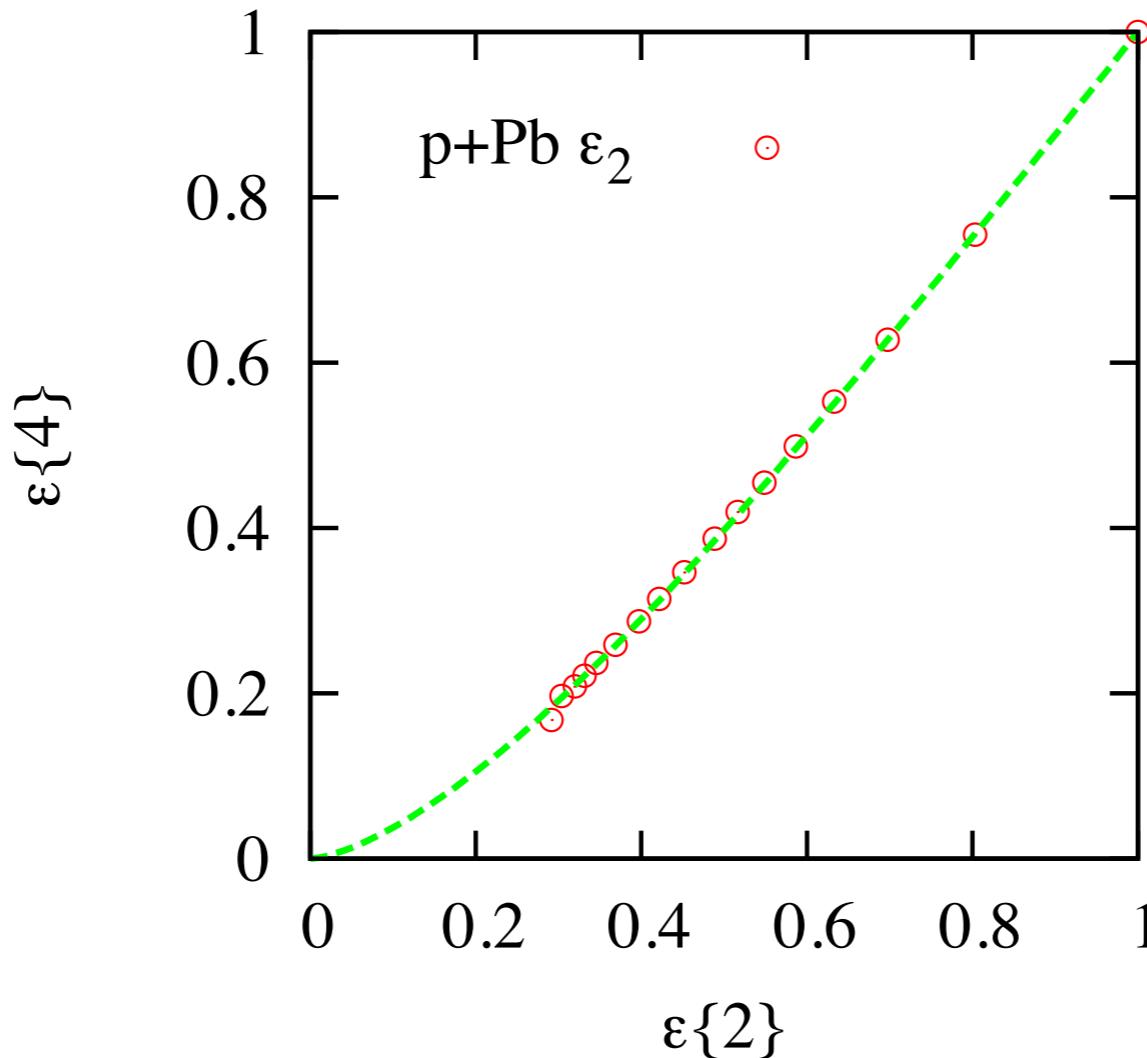
Testing universality with cumulants

Central limit:
large system,
small fluctuations
 $\varepsilon\{2\} \ll 1$ and
 $\varepsilon\{4\} \ll \varepsilon\{2\}$



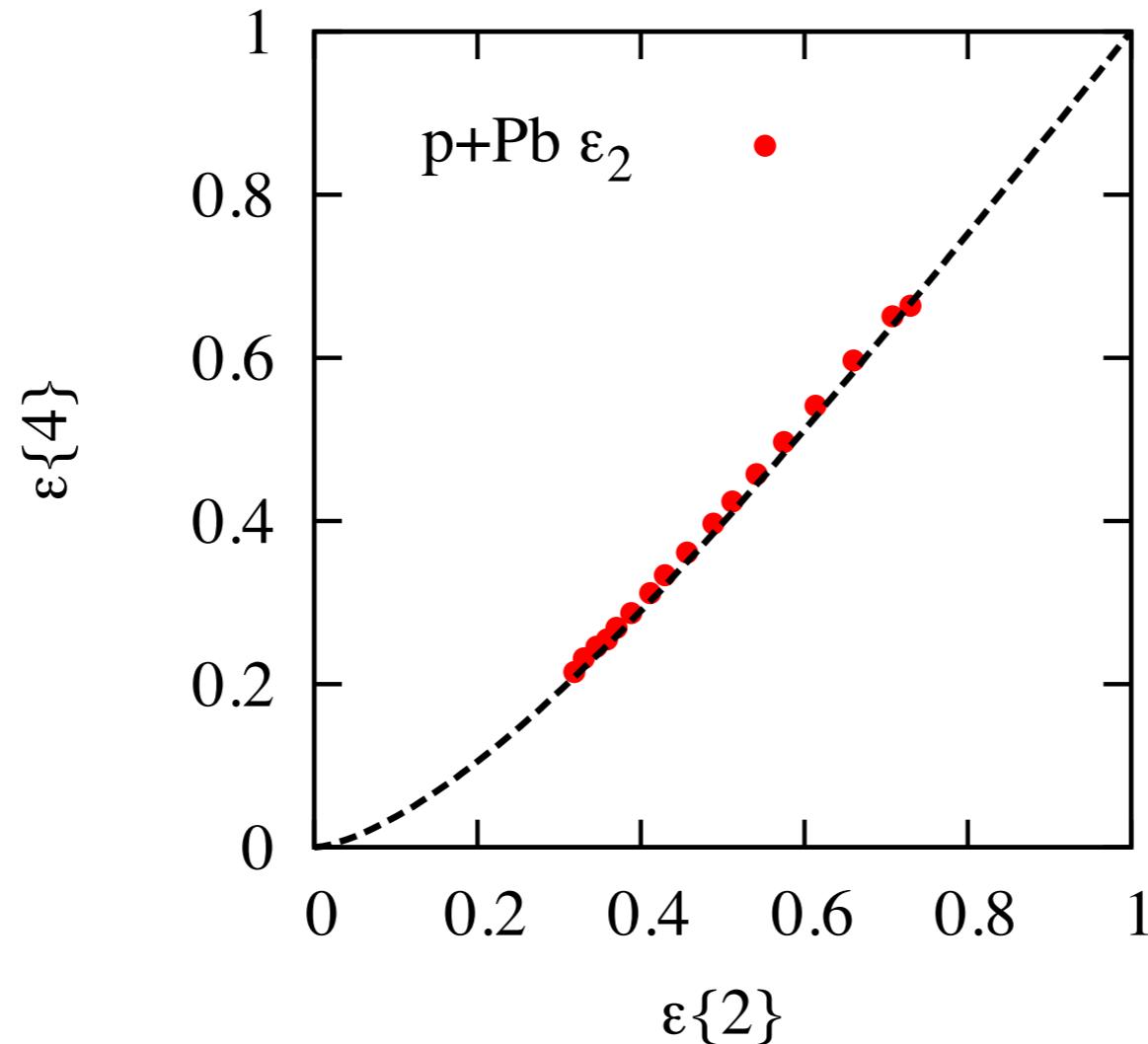
Prediction of the power distribution

Testing universality with cumulants



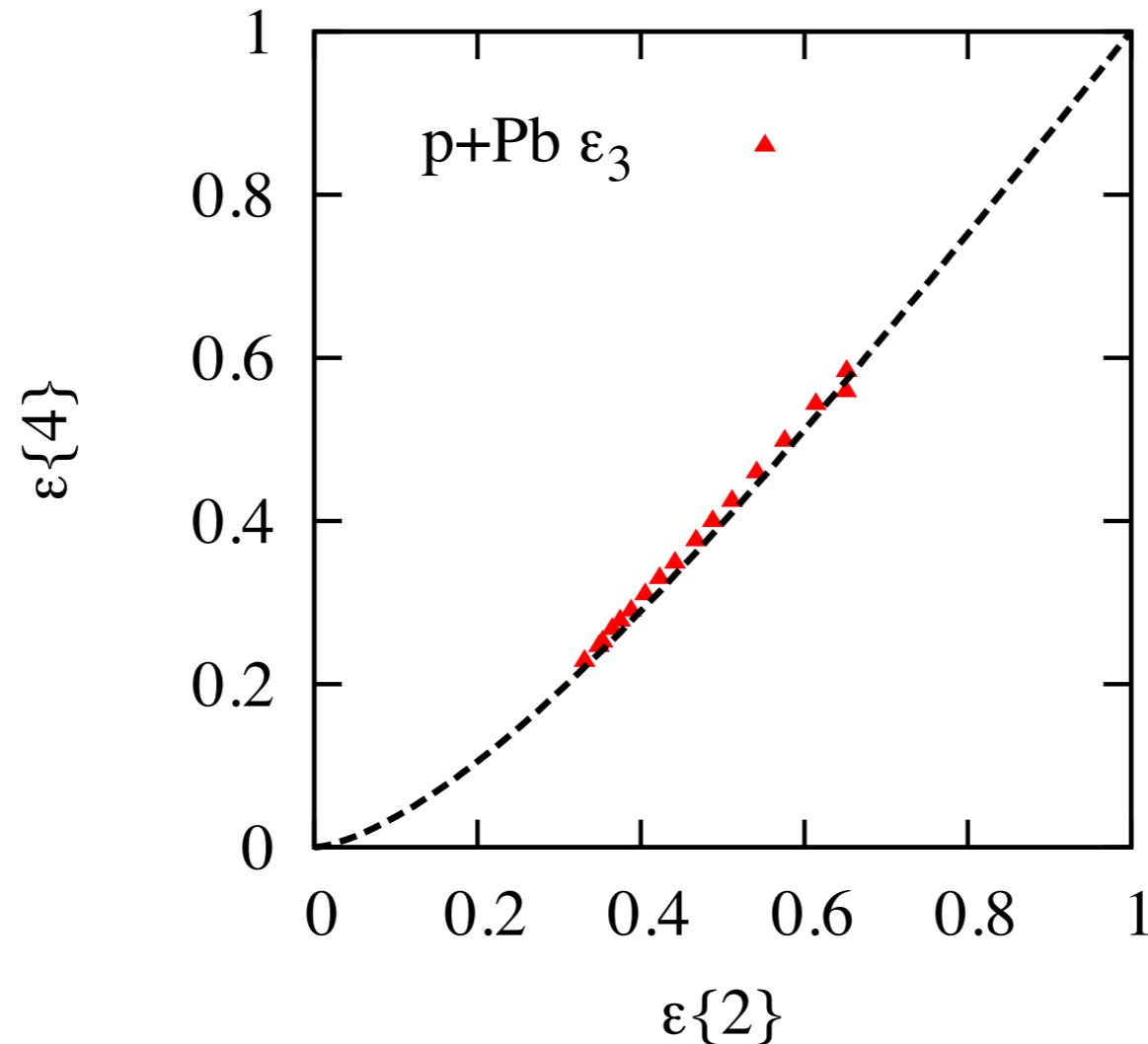
Pointlike sources with Gaussian distribution:
power distribution=exact=test of Monte-Carlo

Testing universality with cumulants



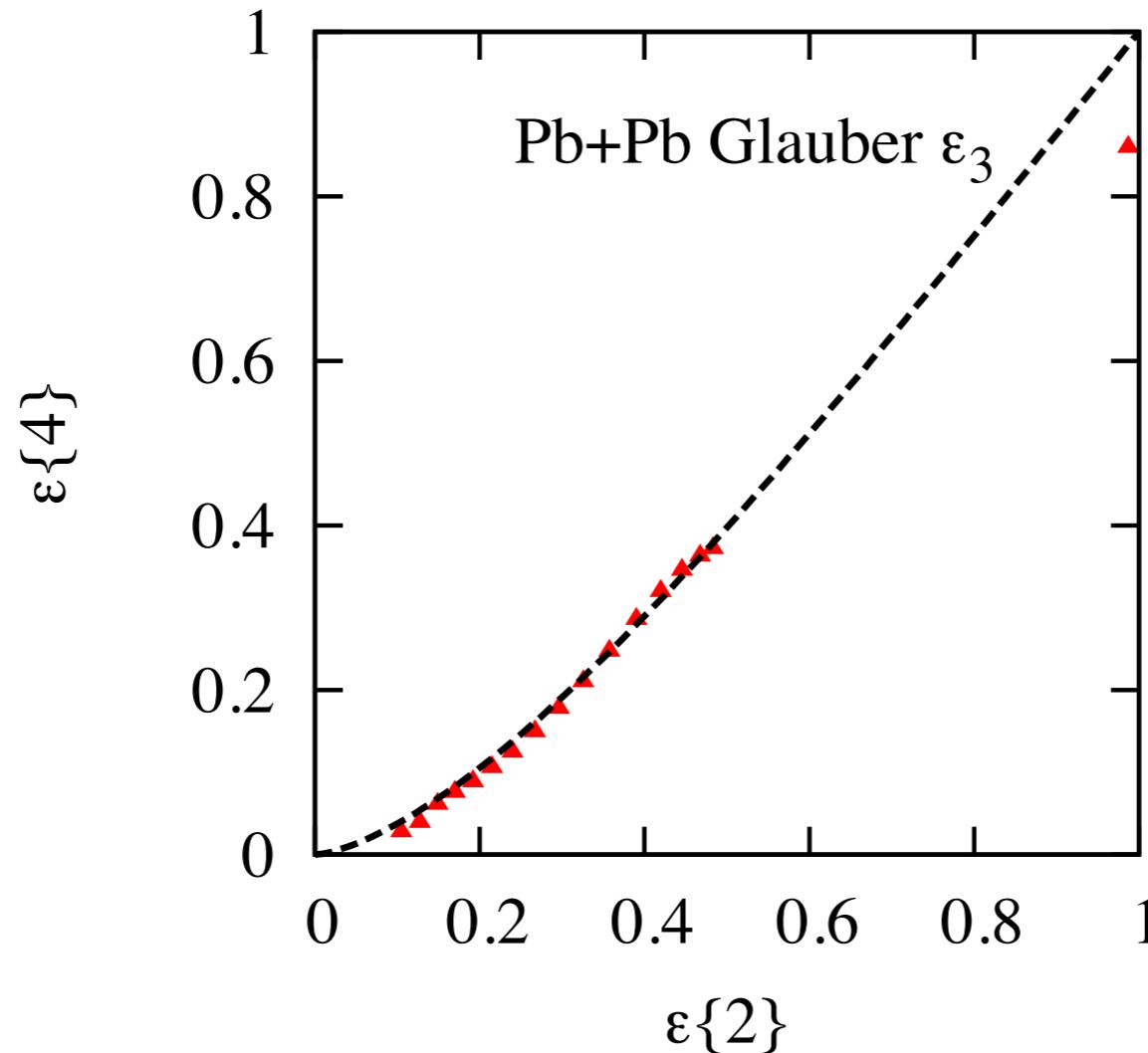
Each point: different number of hit nucleons in target

Testing universality with cumulants



Each point: different number of hit nucleons in target

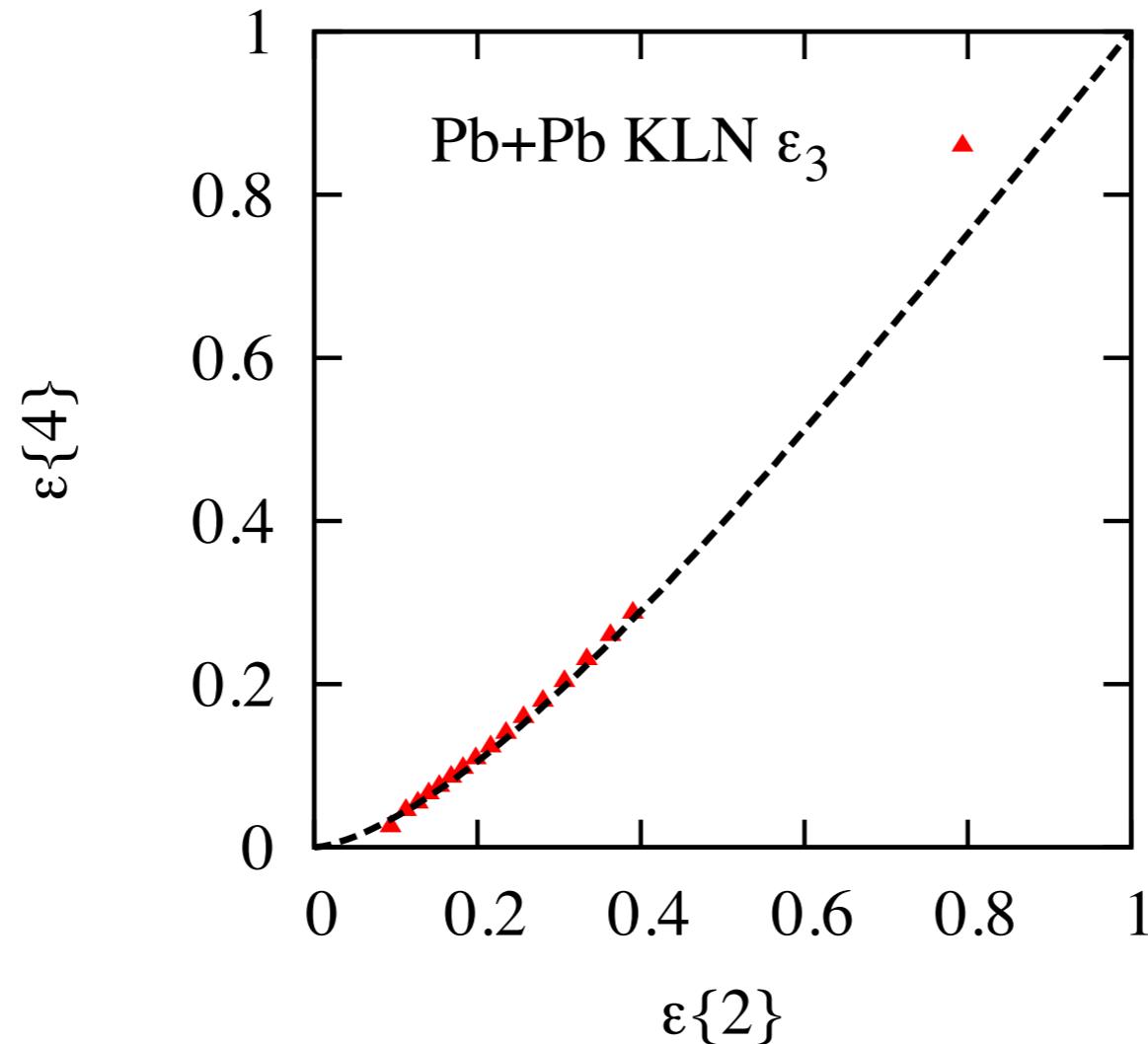
Testing universality with cumulants



Each point: different centrality

Pb-Pb: Larger system: smaller anisotropies

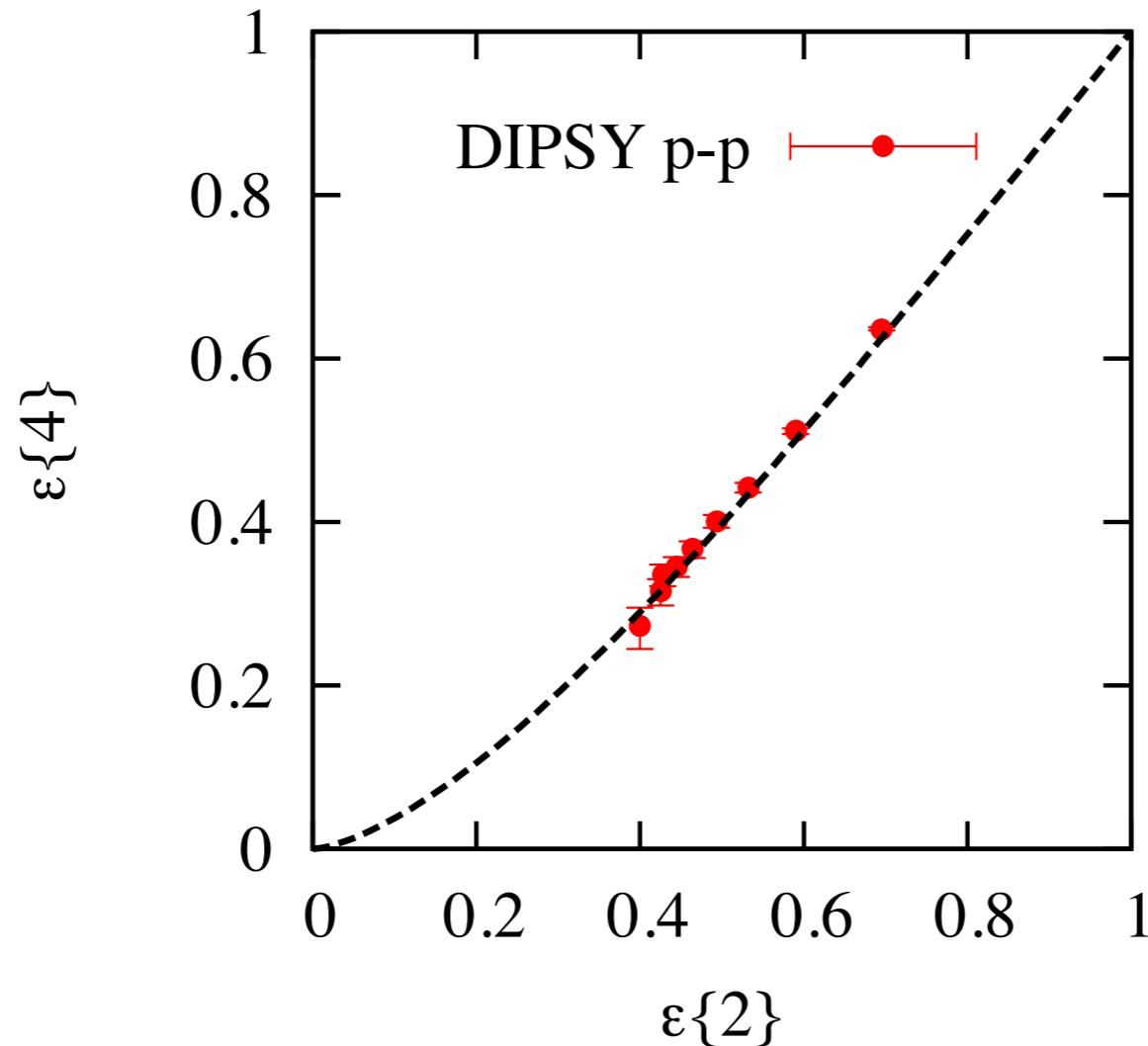
Testing universality with cumulants



Each point: different centrality

Pb-Pb: Larger system: smaller anisotropies

Testing universality with cumulants



data from Avsar Flensburg Hatta JYO Ueda 1009.5643

Each point: different parton multiplicity

Applying the power distribution to experimental data

If $v_n = K_n \varepsilon_n$, with constant K_n

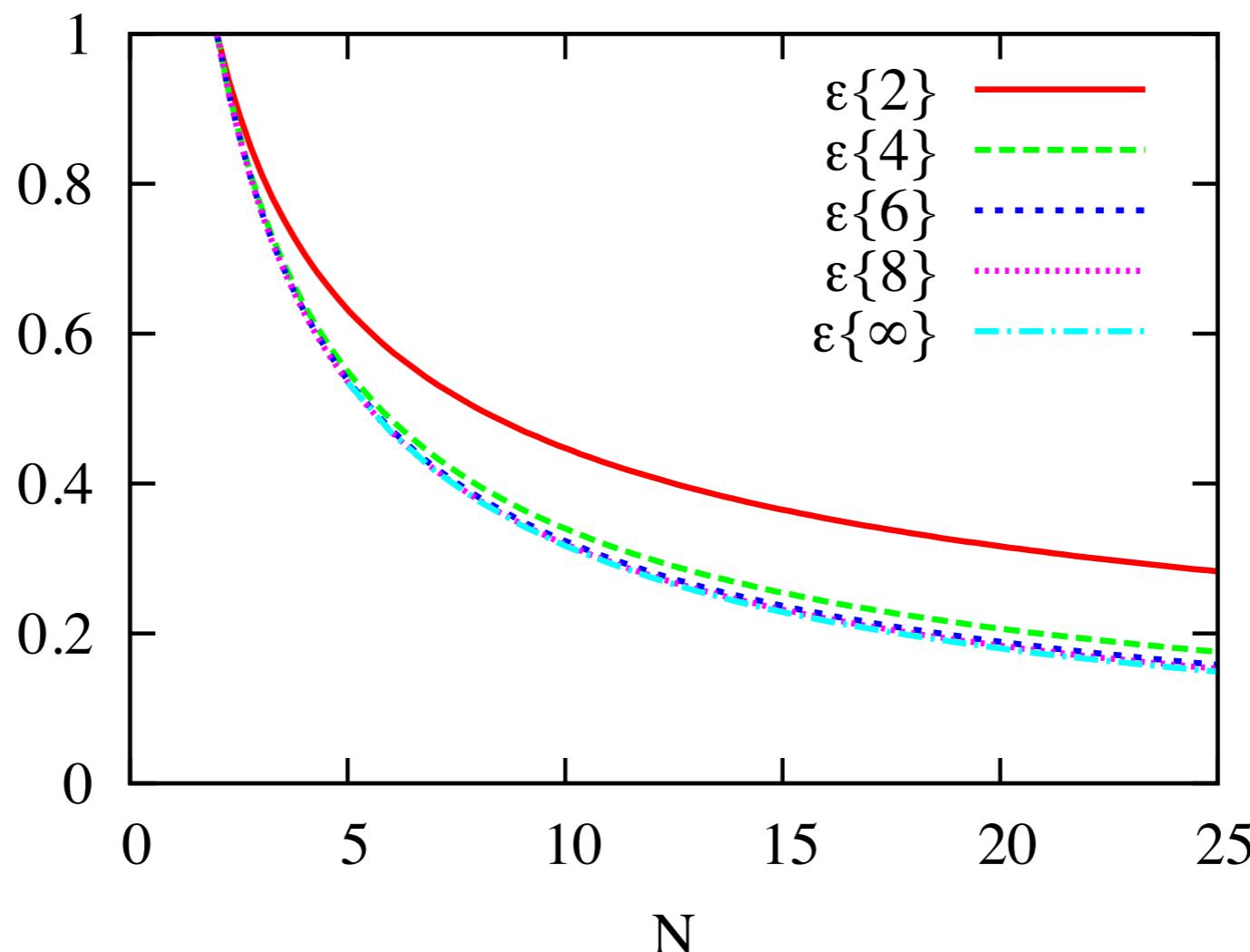
then $v_n\{4\}/v_n\{2\} = \varepsilon_n\{4\}/\varepsilon_n\{2\}$

we can read off the parameter α from the experimentally-measured ratio $v_n\{4\}/v_n\{2\}$

Simple predictions from eccentricity scaling

- Experimentally, one can measure moments (or cumulants) of the distribution of v_n .
- Eccentricity scaling implies that, e.g.
$$\langle v_n^4 \rangle / \langle v_n^2 \rangle^2 = \langle \varepsilon_n^4 \rangle / \langle \varepsilon_n^2 \rangle^2$$
- Thus one can check if a particular model of the initial state is compatible with data.

Higher-order cumulants (predicted by the power distribution)



$\varepsilon\{n\}$ quickly converges as order n increases

κ from Cumulants of Power Distribution

When $\varepsilon_0 = 0$

assuming $v_n = \kappa_n \varepsilon_n$ (linear)

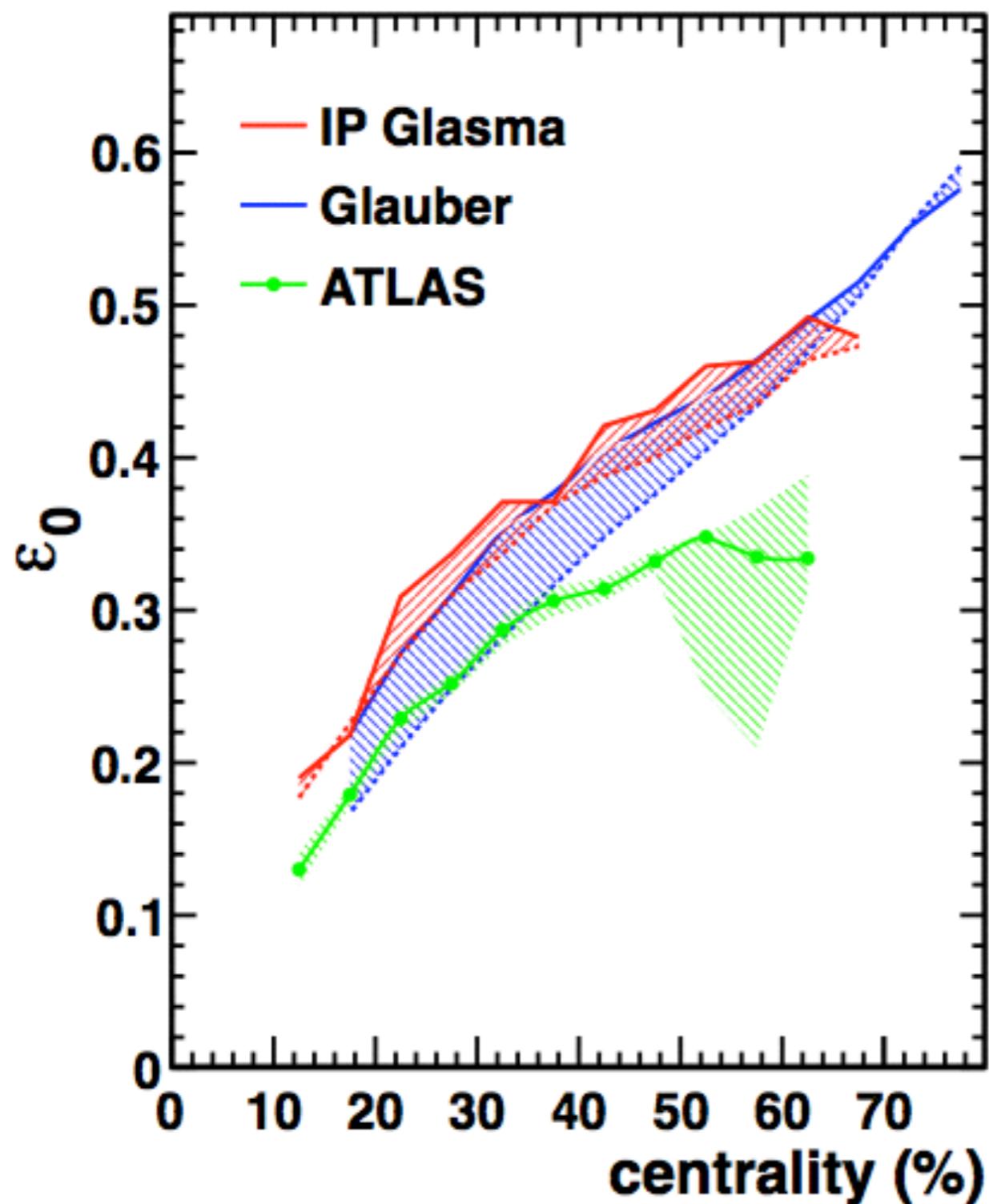
$$\kappa_n = v_n\{2\} \sqrt{2 \left(\frac{v_n\{2\}}{v_n\{4\}} \right)^4 - 1}$$

Independent of the ε distribution!

The cumulant ratio gives the non-Gaussian shape

So why not cumulants of the Elliptic Power?

Reaction-plane eccentricity



Error bar on each model results from the fact that the fit using the Elliptic Power distribution is not perfect.

Eccentricity from data seems too low — compensates the hydro response which is too large.

Higher-order cumulants

- Fit ε_2 distribution from a model (Glauber) with Elliptic Power.
- Calculate cumulants from the model (lines) or from the fit (symbols)

