Hadrons in the NJL model

Benjamin Sintes

Subatech

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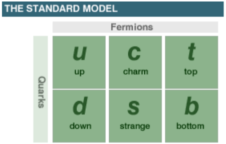
Outline

Quantum Chromo Dynamics

Confinement

- Quarks are confined in hadronic matter, baryons or mesons, and are never observed separately.



Quantum Chromo Dynamics

Perturbative QCD

Can be used for high energy physics.

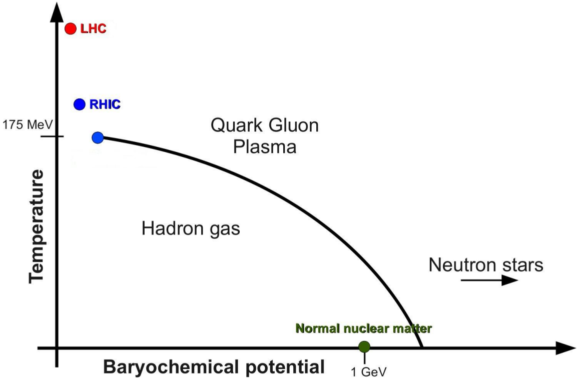
Lattice QCD

Is used to solve numerically the QCD Lagrangian on a lattice of points in space and time.

Low energy models

Such as bag model or NJL model

Phase diagramm of QCD



- Nature of the phase transition
- Realization of chiral symmetry

The Nambu and Jona-Lasinio model

- Originaly a theory of nucleons similar to the BCS theory of superconductivity.
- We only use quarks as degrees of freedom because we assume gluon degrees of freedom are frozen in the low energy limit.
- Constructed to have the same symmetries as QCD.

QCD symmetries

- $L_{QCD} = \bar{\psi}(i\gamma_\mu D^\mu - m_o)\psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$

$D^\mu = \partial_\mu - ieA_\mu$

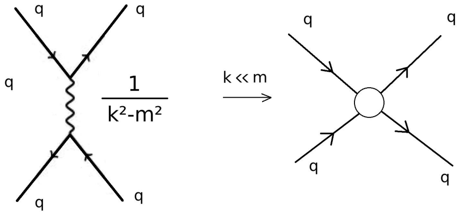
- A symmetry in the Lagrangian implies a conserved current.

Symmetry	Name	Current
$U_v(1)$	Baryonic	$\bar{\psi}\gamma_\mu\psi$
$U_A(1)$	Axial	$\bar{\psi}\gamma_\mu\gamma_5\psi$
$SU_V(3)$	Vector	$\bar{\psi}\gamma_\mu\lambda_a\psi$
$SU_A(3)$	Chiral	$\bar{\psi}\gamma_\mu\gamma_5\lambda_a\psi$

- We assume $m^0_u = m^0_d$

Lagrangian NJL

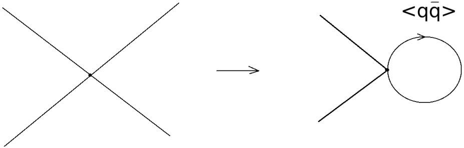
- Lagrangian : $L_{NJL} = \bar{\psi}(i\partial - m_o)\psi + G\sum_a[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2] - K[det\bar{\psi}(1+\gamma_5)\psi + det\bar{\psi}(1-\gamma_5)\psi]$
- Static approximation : Interaction between two quark currents by the exchange of a pointlike gluon.



- Non-renormalizable theory, we need to apply a cut-off.
- Theory does not include confinement

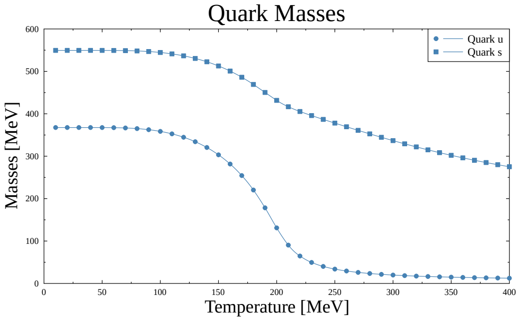
Gap equation

- The Hartree approximation reduces the mutual two body interaction to an interaction with a mean field.
- $(\bar{\psi}\lambda_a\psi)^2 \rightarrow 2\bar{\psi}\lambda_a\psi. < \bar{\psi}\lambda_a\psi >$
- The linearization of the interaction in the mean field approximation is like closing the quark loop.



- This defines a dynamical fermion mass :
  $m_i = mo - 2G < q_i\bar{q}_i > -2K < q_j\bar{q}_j >< q_k\bar{q}_k >$
- Breaking of the chiral symmetry

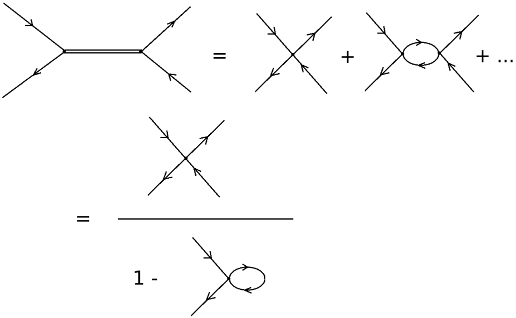
Quark masses



- Quark condensates are the order parameter of the transition

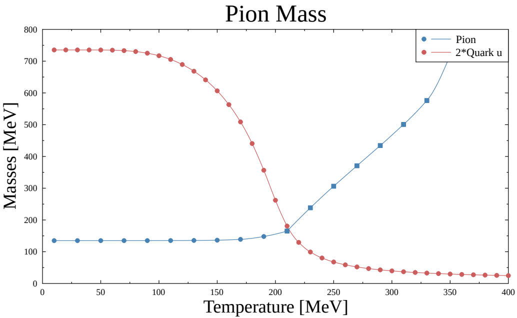
Bethe Salpether

- In the Random Phase Approximation :

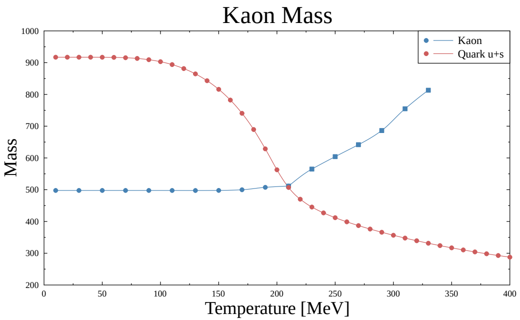


- $T(q^2) = G + G\Pi(q^2)G + G\Pi(q^2)G\Pi(q^2)G + ... = \frac{G}{1-G\Pi(q^2)}$
- $T(q^2) = K_1 . \frac{i.g_{\pi q \bar{q}}}{q^2 - m^2} . K_2$
- The mass of the pion mode is determined by the pole.

Pion


Pion Mass
Pion
2*Quark u
Masses [MeV]
Temperature [MeV]
0
100
200
300
400
500
600
700
800
0
50
100
150
200
250
300
350
400

Kaon


Kaon Mass
Kaon
Quark u+s
Mass
Temperature [MeV]
1000
900
800
700
600
500
400
300
200
0
50
100
150
200
250
300
350
400

Parameters

Parameters (MeV)	Costa[1]	P1
Mo_u	5.5	4.75
Mo_s	140.7	147
Λ	602.3	708
$G.\Lambda^2$	1.835	1.922
$K.\Lambda^5$	12.36	10

[1] : Pseudoscalar Mesons in Hot, Dense Matter, P.Costa, M.C.Ruivo, C.A.de Sousa arXiv0304025v3

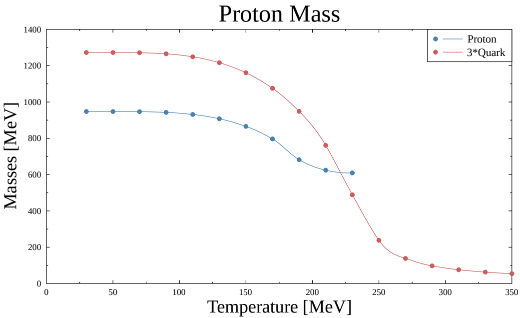
Baryons

Baryons

- Quark-Diquark exchange

Baryons as Relativistic Bound States of Quark and Diquark, M.Oettel ArXiv 12067v1

Proton


Proton Mass
Proton
3*Quark
Masses [MeV]
Temperature [MeV]
0
200
400
600
800
1000
1200
1400
0
50
100
150
200
250
300
350

Results

Masses (MeV)	Costa	P1	Experimental
u	367,6	424,2	
s	549,5	626,5	
Pion	135,0	135,9	135
Kaon	497,7	548,5	498
Diquark [ud]	525,6	599,1	623[2]
Diquark [us]	700,9	794,8	-
Proton	926,1	947,5	938
Λ	1106,1	1196,1	1116
Ξ	1246,8	1220,2	1315
Σ	1232,2	1320,1	1189

[2] Diquark masses from lattice QCD, M.Hess, F.Karsch, E.Laermanm, I. Wetzorke Phys.Rev.D58:111502

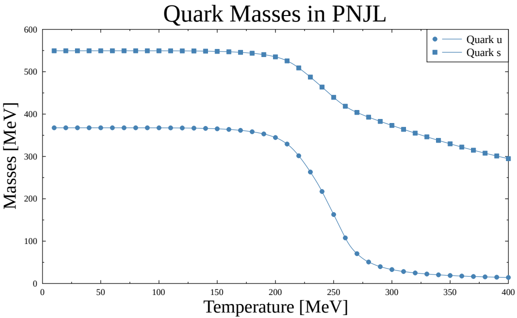
PNJL

- The Polyakov loop serves as an order parameter for the confinement in a pure gauge theory
- Parameters are from pure-gauge lattice data and some thermodynamic quantities
- The expectation value of the Polyakov loop is related to the change of free energy
- We add a potential to our lagrangian

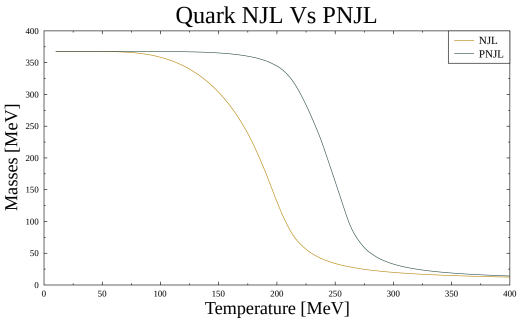
PNJL Lagrangian

- $L_{NJL} = \bar{\psi}(i\partial - m_o)\psi + G\sum_a[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2] - K[det\bar{\psi}(1+\gamma_5)\psi + det\bar{\psi}(1-\gamma_5)\psi] - U(\phi, \bar{\phi}, T)$

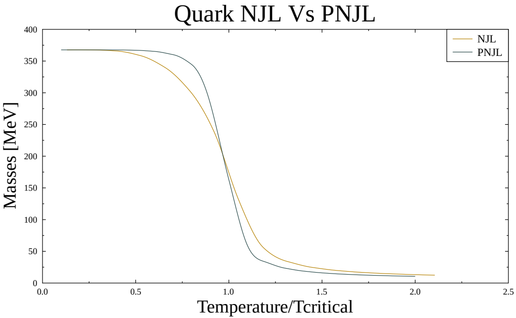
Quarks


Quark Masses in PNJL
Quark u
Quark s
Masses [MeV]
Temperature [MeV]
0
100
200
300
400
500
600
0
50
100
150
200
250
300
350
400

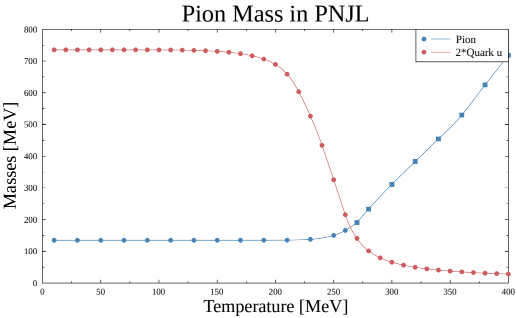
Quarks


Quark NJL Vs PNJL
NJL
PNJL
Masses [MeV]
Temperature [MeV]
0
50
100
150
200
250
300
350
400
0
50
100
150
200
250
300
350
400

Quarks


Quark NJL Vs PNJL
Masses [MeV]
Temperature/Tcritical
NJL
PNJL
0
50
100
150
200
250
300
350
400
0.0
0.5
1.0
1.5
2.0
2.5

Hadrons


Pion Mass in PNJL
Pion
2*Quark u
Masses [MeV]
Temperature [MeV]
0
100
200
300
400
500
600
700
800
0
50
100
150
200
250
300
350
400

Summary

- NJL model is usefull to understand the role of the chiral symmetry
- We can reproduce mesons, diquarks and baryons masses to study the transition phase

- Outlook
    - Cross sections need to be added
    - All implemented in a transport theory