

Joint seminar  CPHT / 

The proton spin structure

Cédric Lorcé

IPN Orsay - IFPA Liège

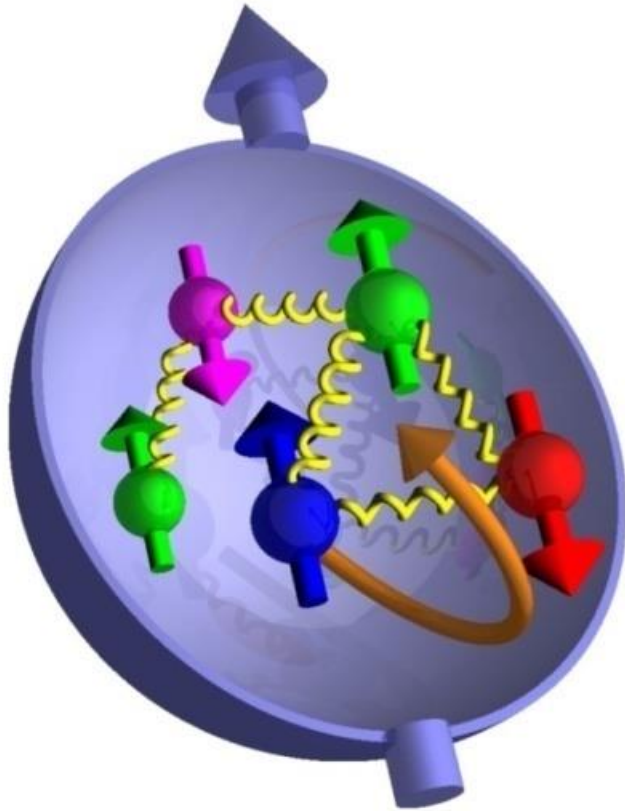


Comprendre le monde,
construire l'avenir®



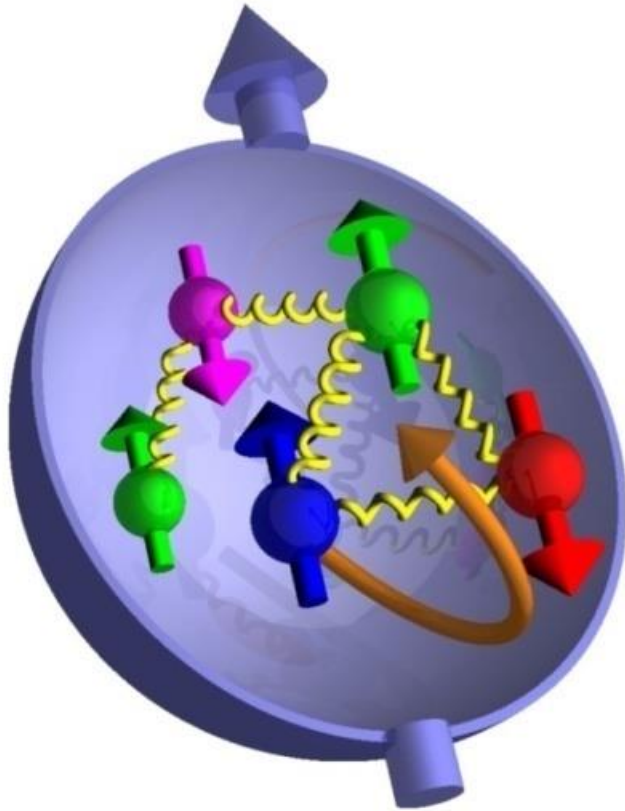
March 21st 2014, Ecole Polytechnique, Palaiseau, France

The outline



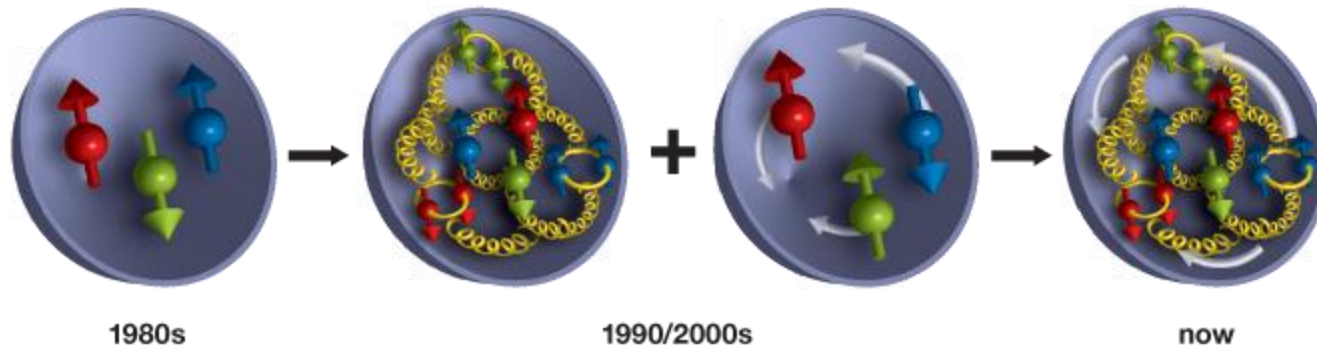
- Why spin and OAM ?
- Proton spin decompositions
- Phase-space distributions and OAM
- Spin-orbit correlation
- Conclusions

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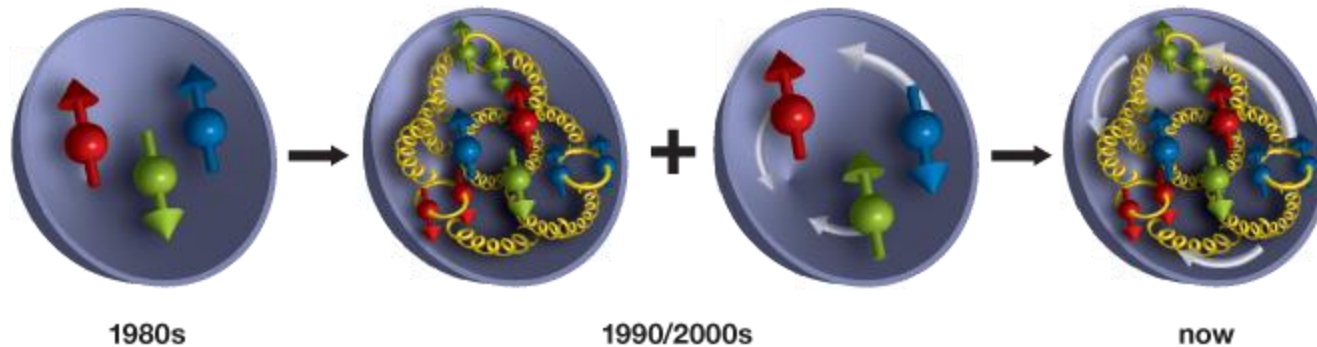
Our picture of the proton evolves !



But many questions remain unanswered ...

- **What is the proton size ?**
- **Why are quarks and gluons confined ?**
- **How are constituent quarks related to QCD ?**
- **How are quarks and gluons distributed inside the nucleon ?**
- **Where does the proton spin come from ?**
- ...

Our picture of the proton evolves !



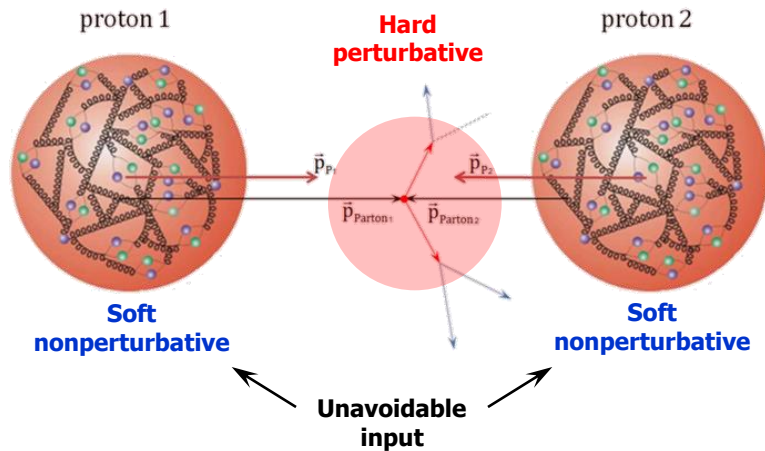
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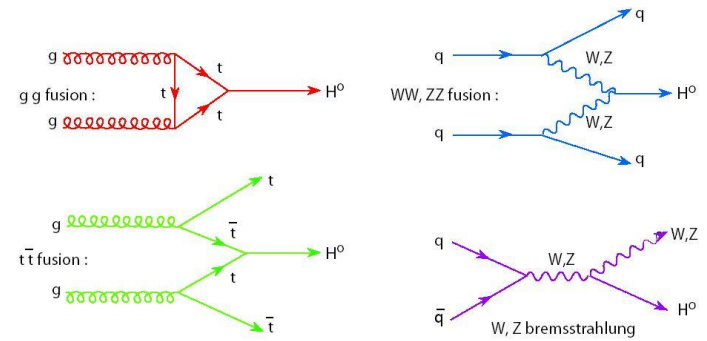


Why are parton distributions interesting ?

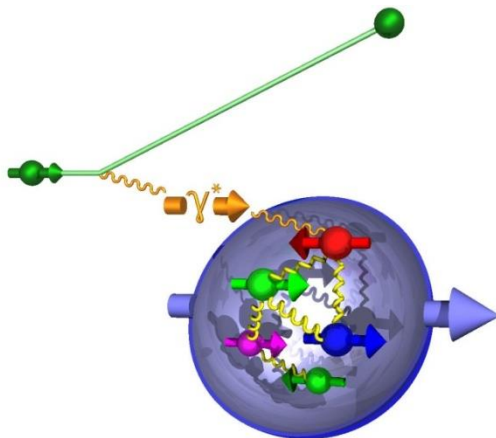
At the energy frontier (LHC, Tevatron, ...)



Standard Model



At the intensity frontier (JLab, COMPASS, MAMI, ...)



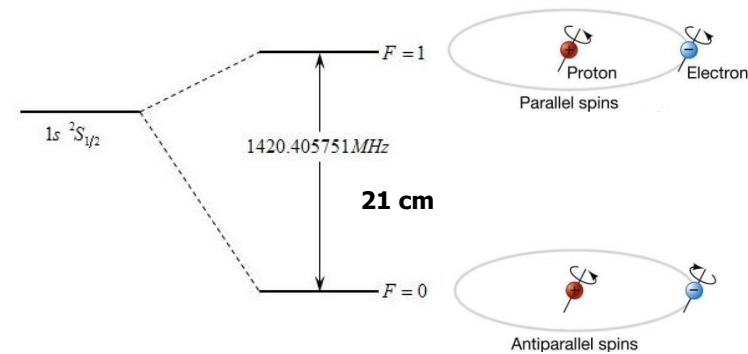
Encode:

- nonperturbative QCD
- nucleon shape
- nucleon spin structure
- spatial/momentum distribution of quarks and gluons

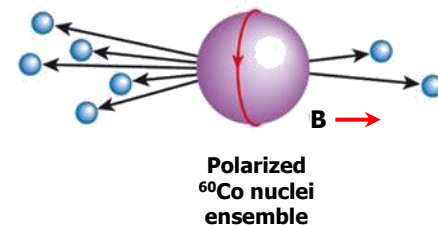


Why is spin interesting ?

Fundamental, quantum, discrete degree of freedom

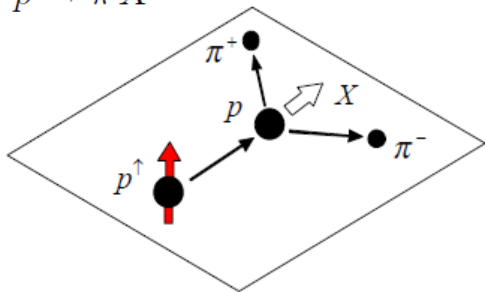


Weak interactions are sensitive to spin (V-A coupling)

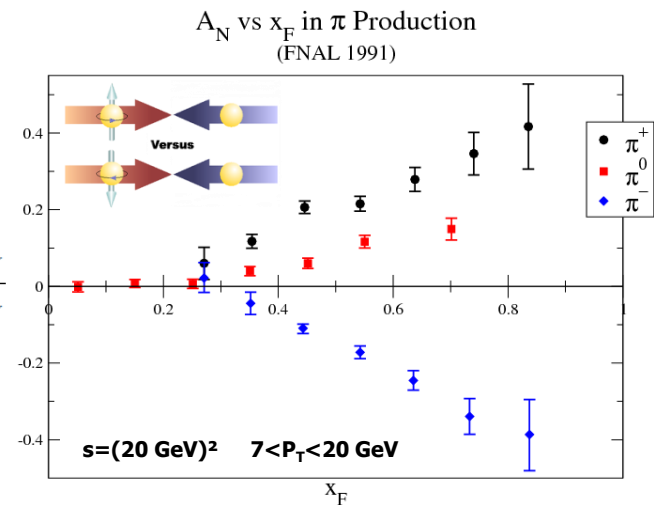


Large spin asymmetries in QCD

$$p^\uparrow p \rightarrow \pi X$$

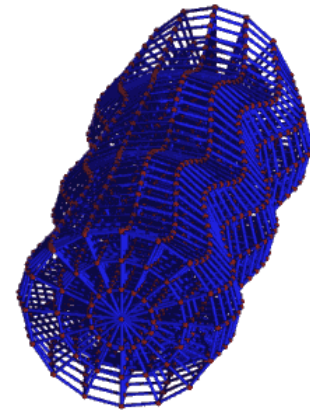
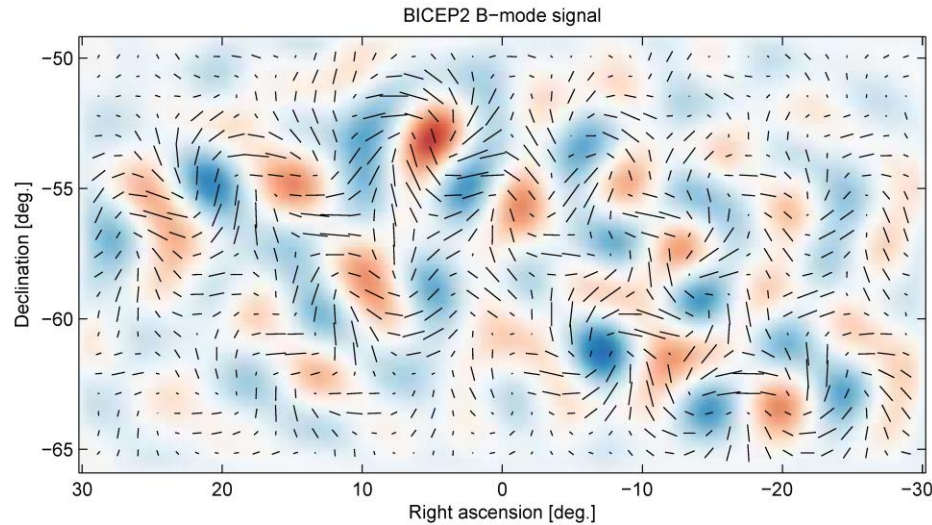


$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$



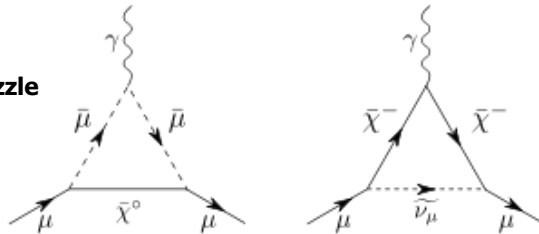
Why is spin interesting ?

Light polarization has fundamental impact on cosmology



New physics (BSM, Quantum gravity, ...) also likely sensitive to spin

Muon g-2 puzzle

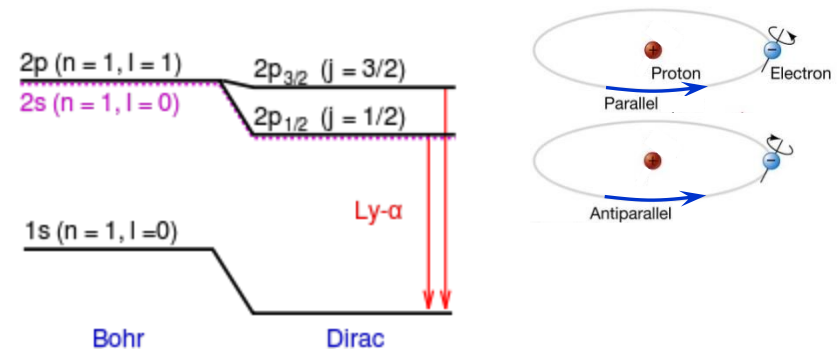


Space-time with torsion

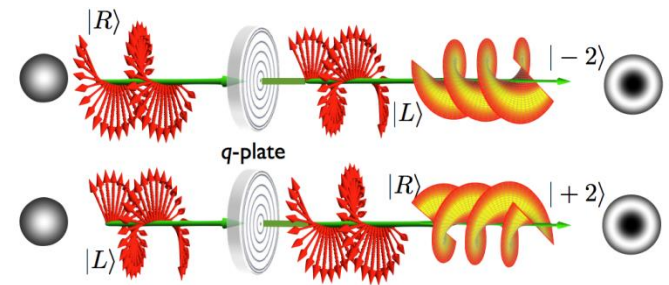
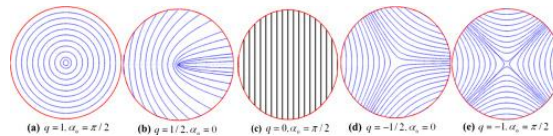


Why is OAM interesting ?

Spin and OAM are coupled

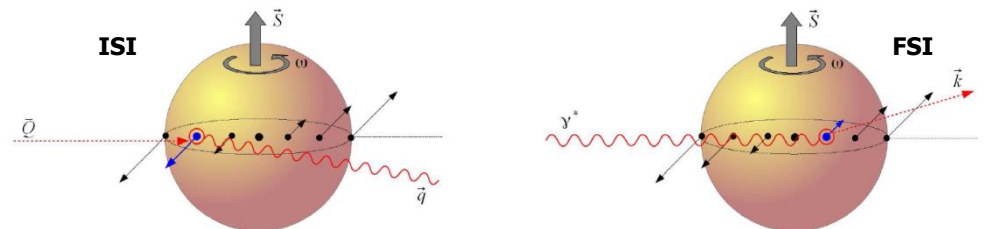


Twisted photons

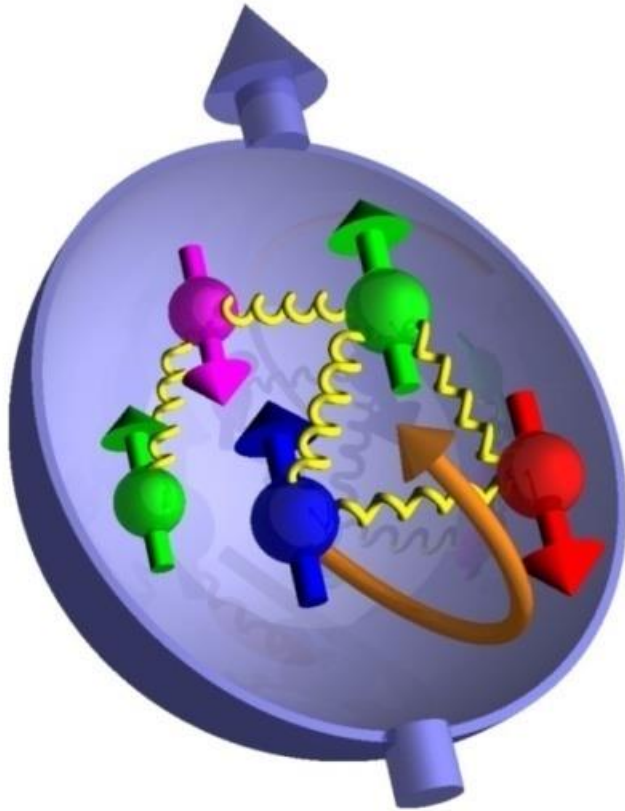


Naive T-odd asymmetries (nucleon or nucleus)

$$\sim (\vec{S} \times \vec{k}) \cdot \vec{P}$$

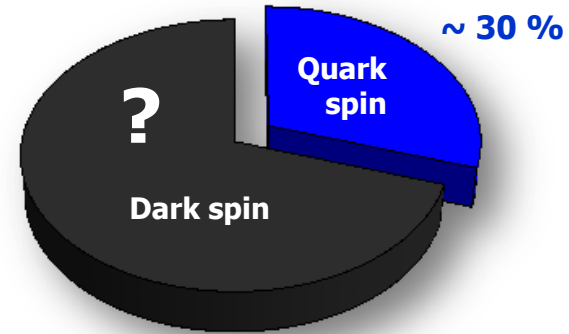


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- **Proton spin decompositions**
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Proton spin puzzle



$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

< 20 %

Many questions/issues :

- Spin *vs* OAM
- Quarks *vs* gluons
- Frame dependence ?
- Gauge invariance ?
- Density ?
- Uniqueness ?
- Measurability ?
- ...

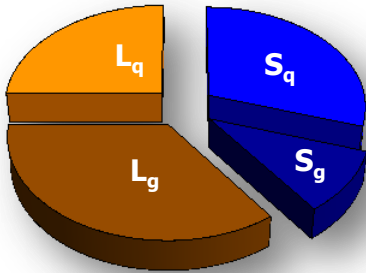
Review: [Leader, C.L. (2014)]

The decompositions in a nutshell

Canonical

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

[Jaffe-Manohar (1990)]



$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{\mathcal{L}}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{\nabla}) \psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}^a$$

$$\vec{\mathcal{L}}_g = \int d^3r E^{ai} \vec{r} \times \vec{\nabla} A^{ai}$$

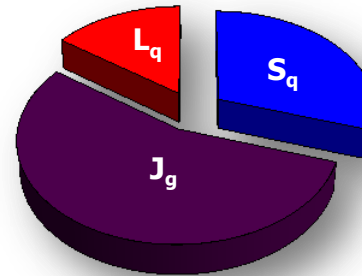
Gauge non-invariant!

Kinetic

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

[Ji (1997)]

$$\vec{D} = \vec{\nabla} + ig\vec{A}$$



$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

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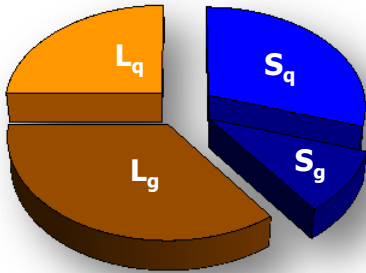
$$\vec{J}_g = \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

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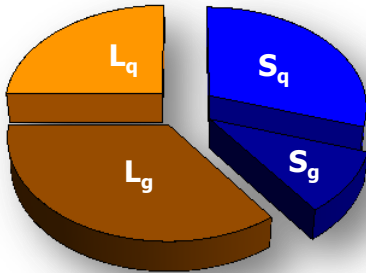


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Gauge non-invariant!

[Chen *et al.* (2008)]

$$A = A_{\text{pure}} + A_{\text{phys}}$$



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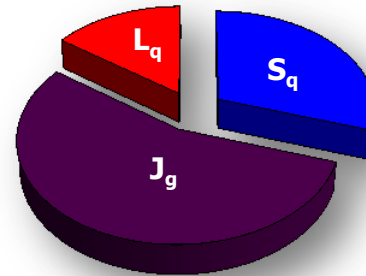
Gauge-invariant extension (GIE)

Kinetic

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

[Ji (1997)]

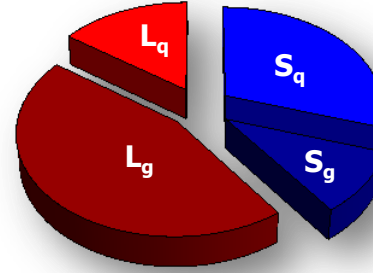
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[Wakamatsu (2010)]

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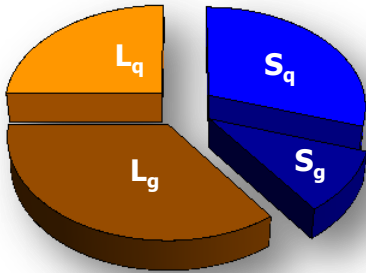
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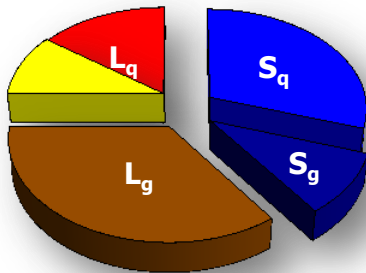
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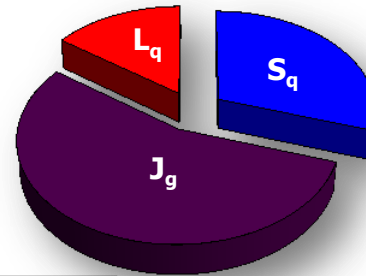
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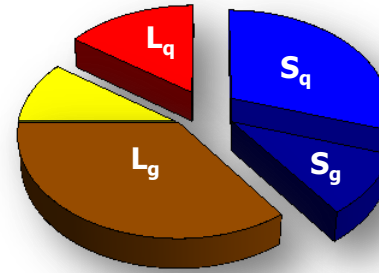
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[tsu (2010)]

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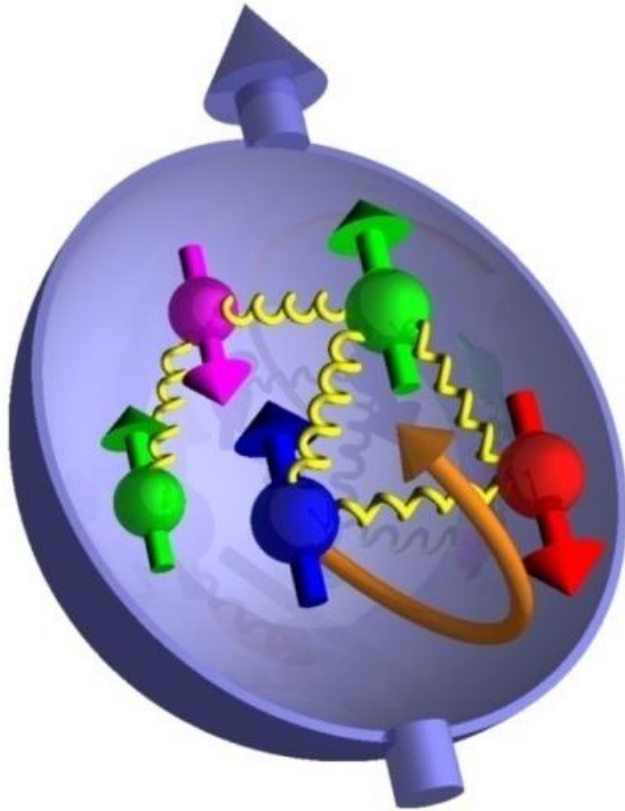
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$$\int d^3r \vec{r} \times [(\vec{A}_{\text{phys}}^a \times \vec{D}_{\text{pure}}) \times \vec{E}^a] \leftarrow$$

[C.L. (2013)]

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Physical processes



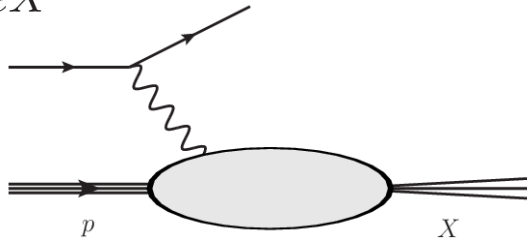
Proton
« tomography »

(Semi-)inclusive

Exclusive

Deep Inelastic Scattering

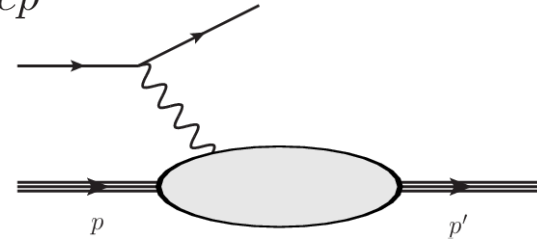
$$ep \rightarrow eX$$



1D picture

Elastic Scattering

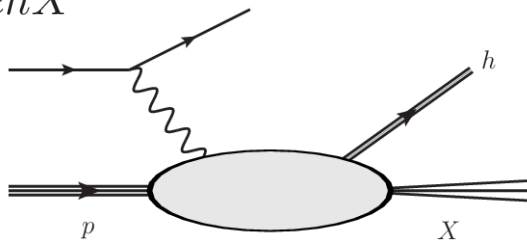
$$ep \rightarrow ep$$



2D picture

Semi-Inclusive DIS

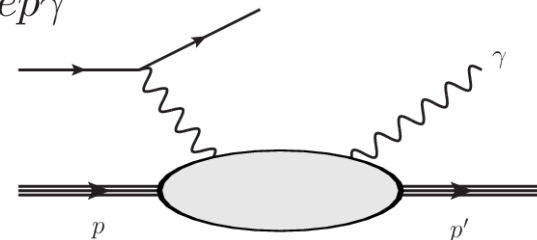
$$ep \rightarrow ehX$$



3D picture

Deeply Virtual Compton Scattering

$$ep \rightarrow ep\gamma$$



3D picture

Factorization



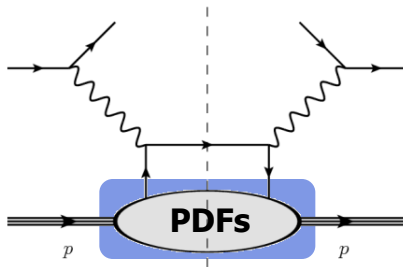
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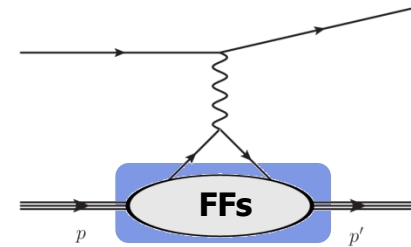
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1D picture

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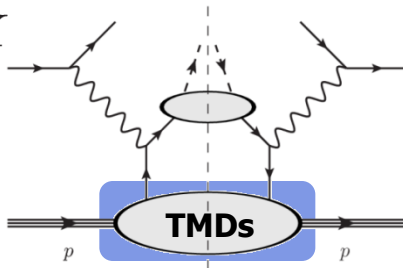
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2D picture

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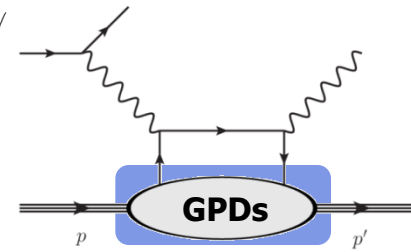
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3D picture

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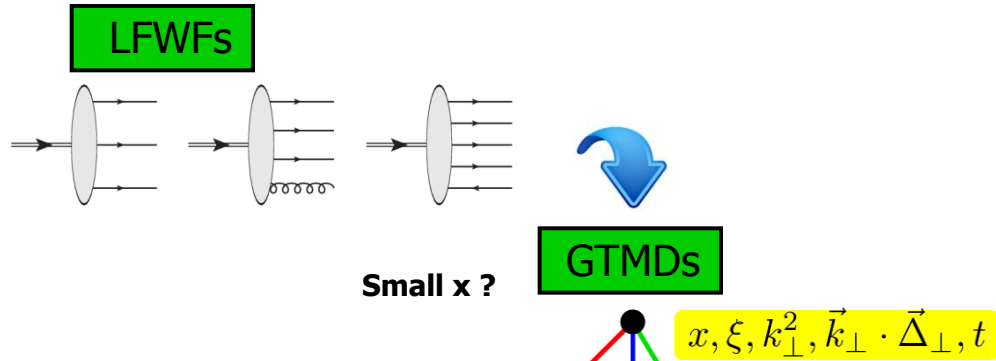
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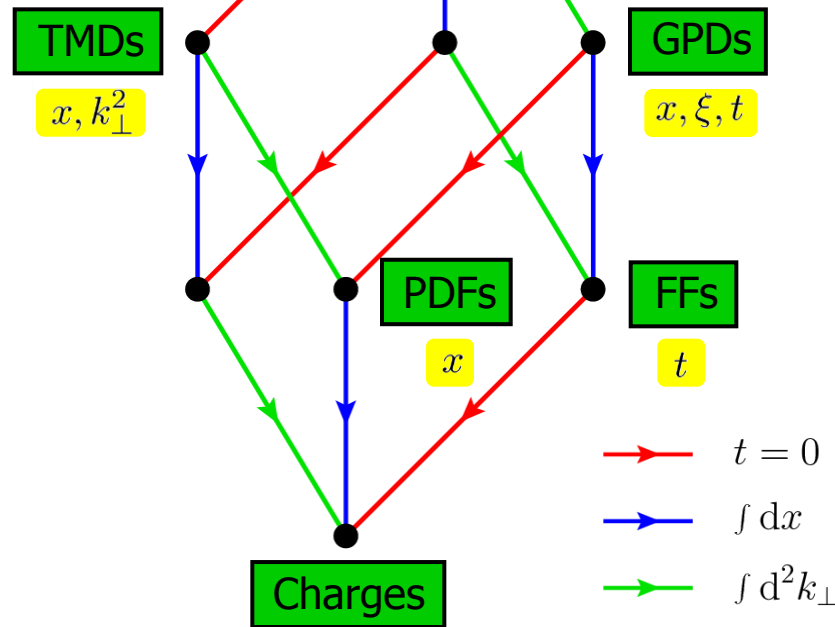
3D picture

Parton distributions (naive)

Theoretical tools

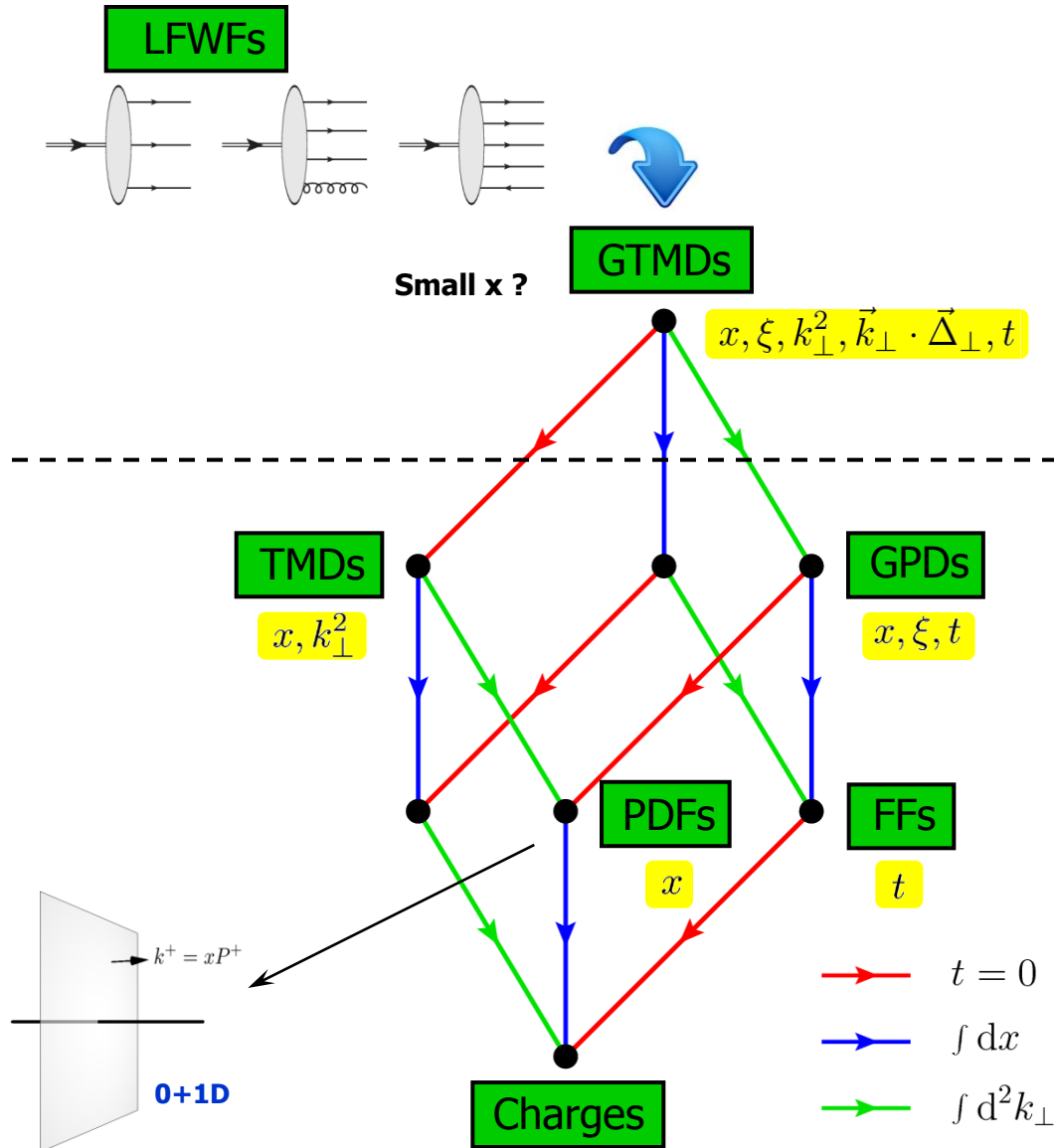


« Physical » objects



Parton distributions (naive)

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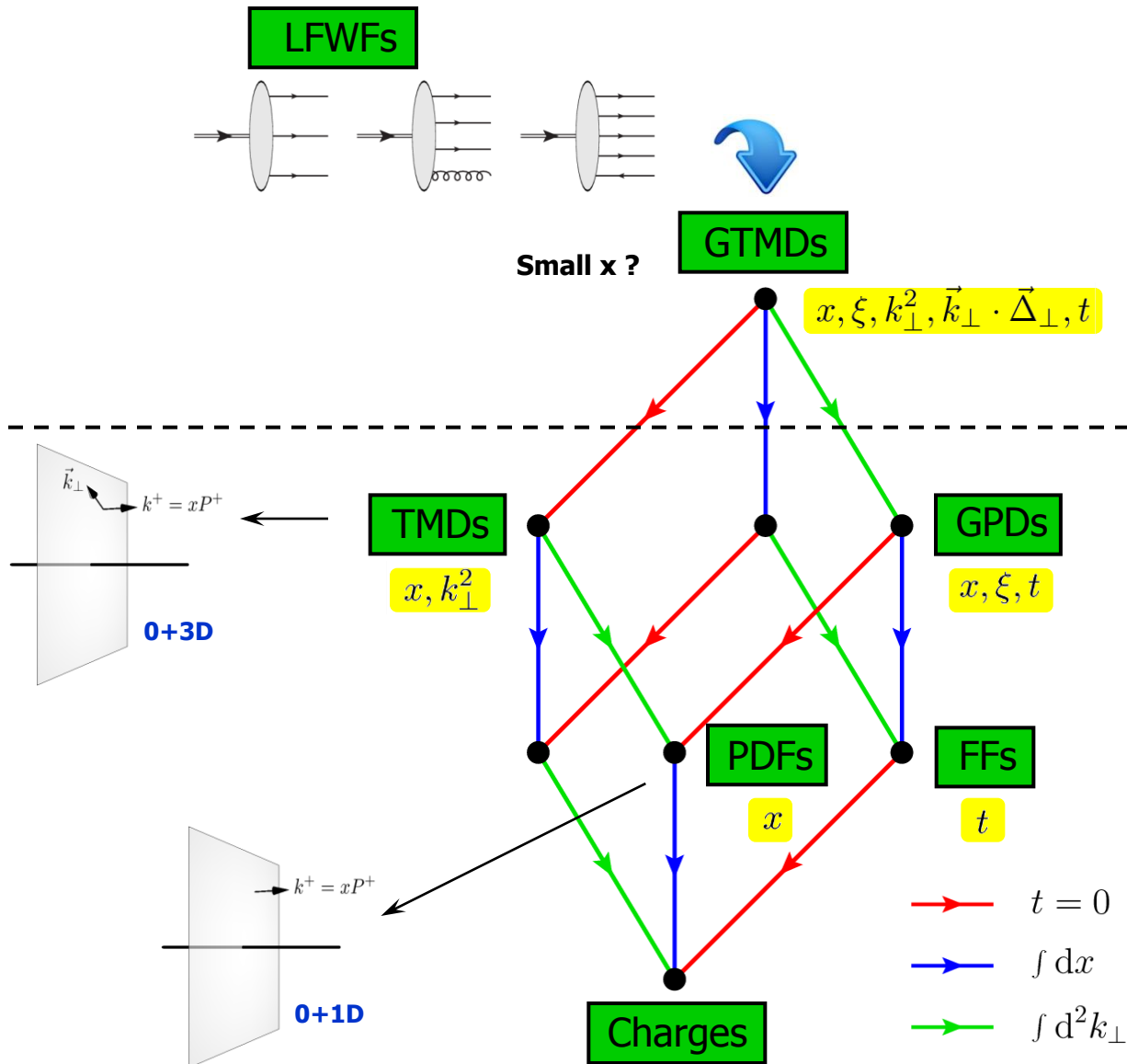


« Physical » objects

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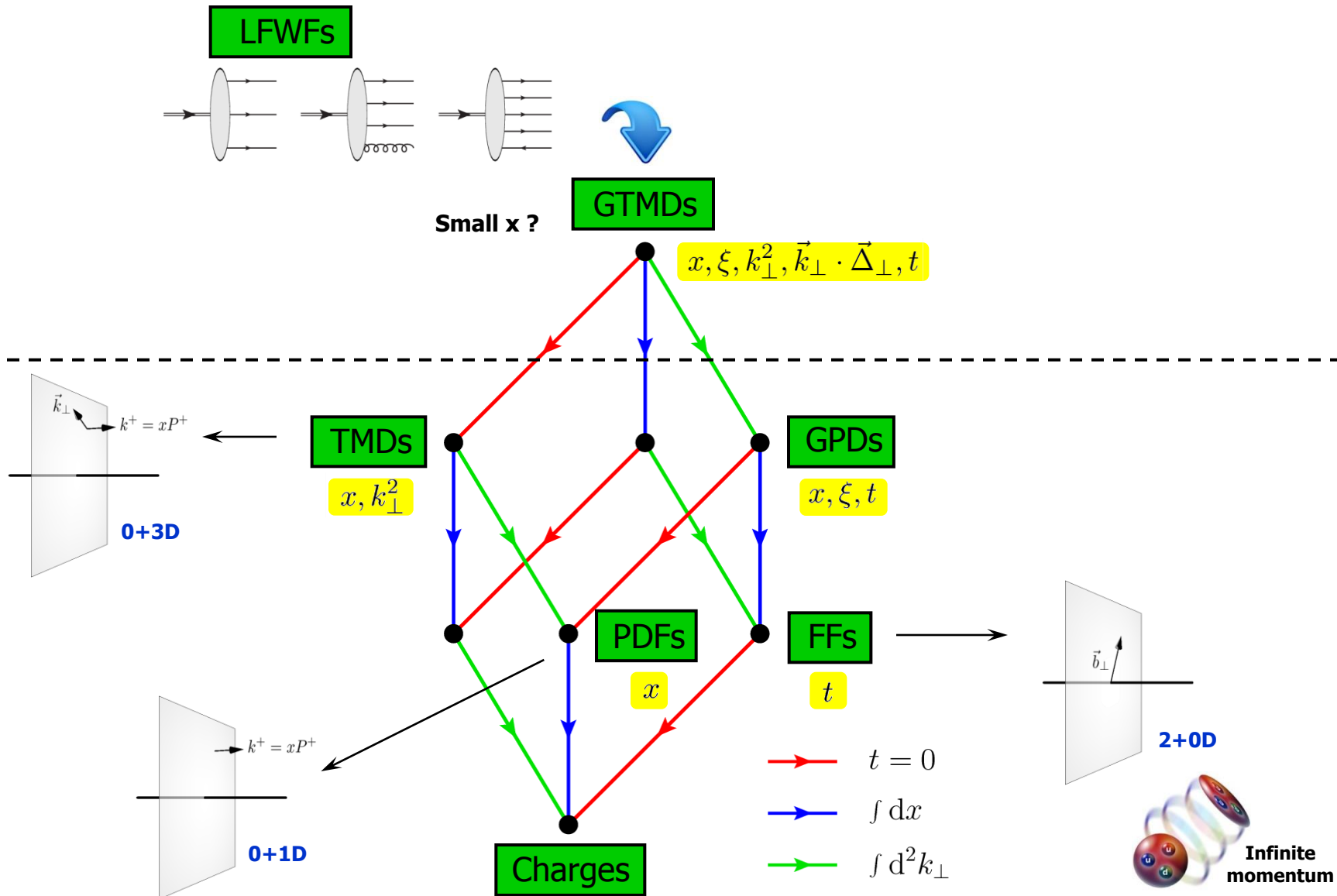
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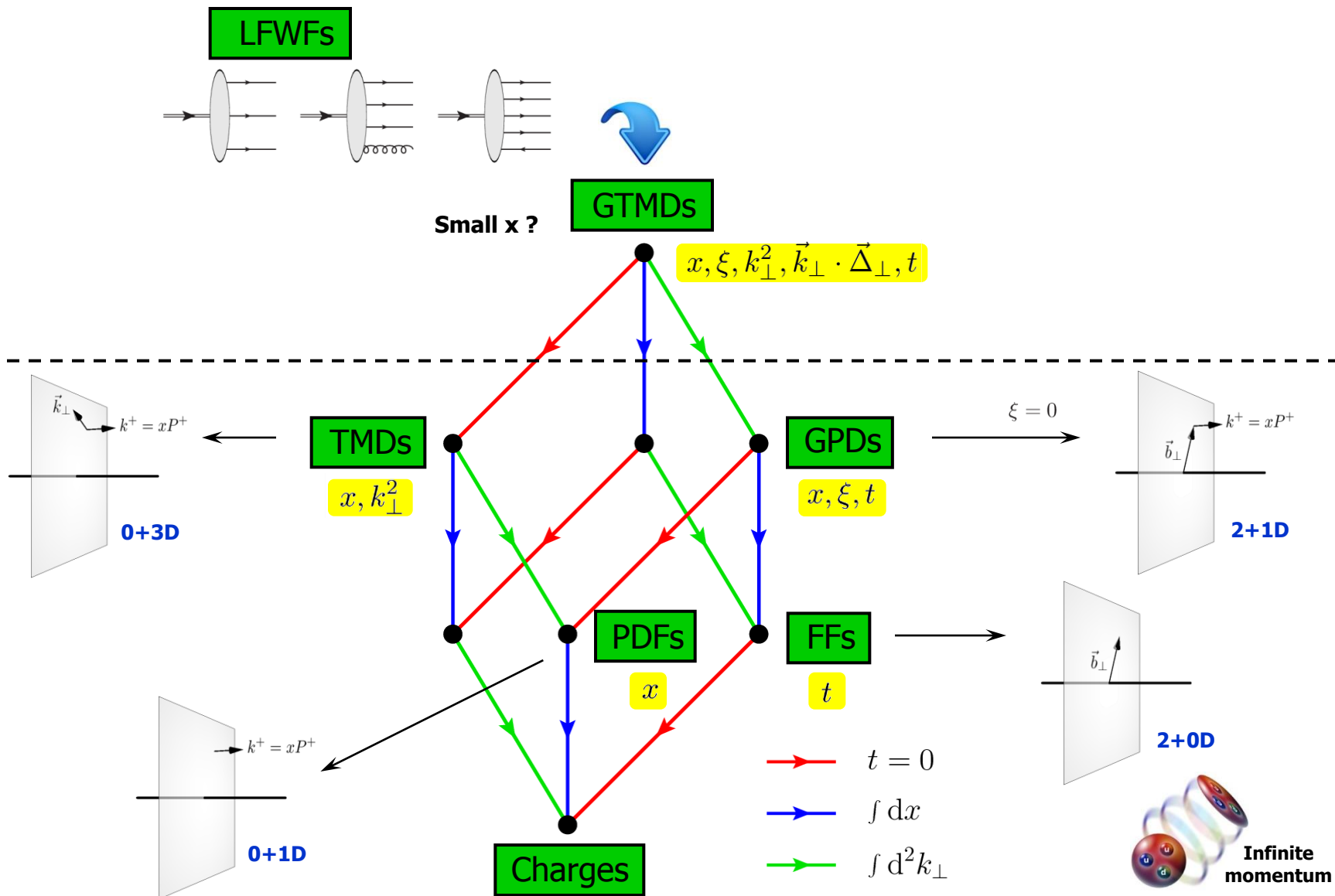
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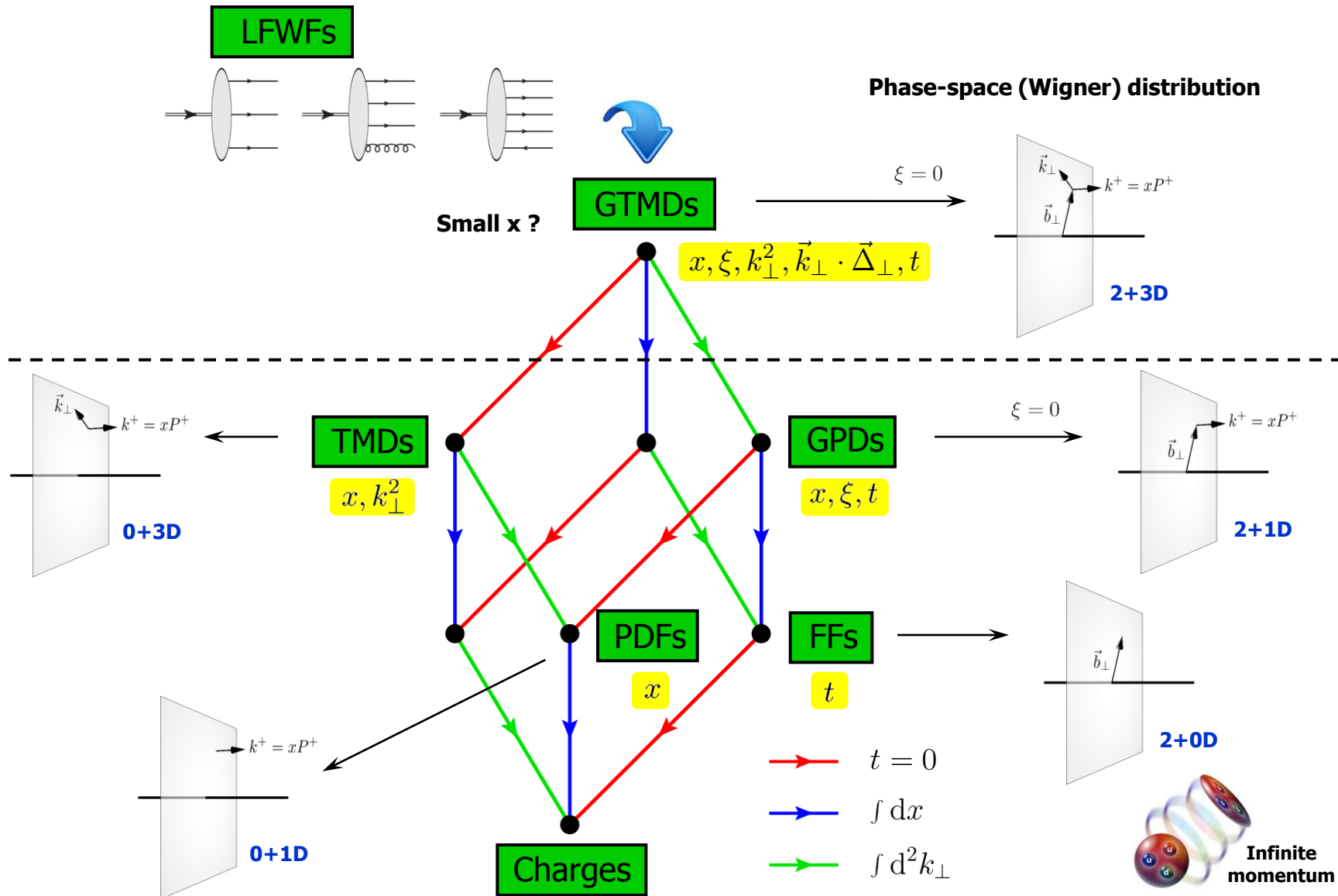
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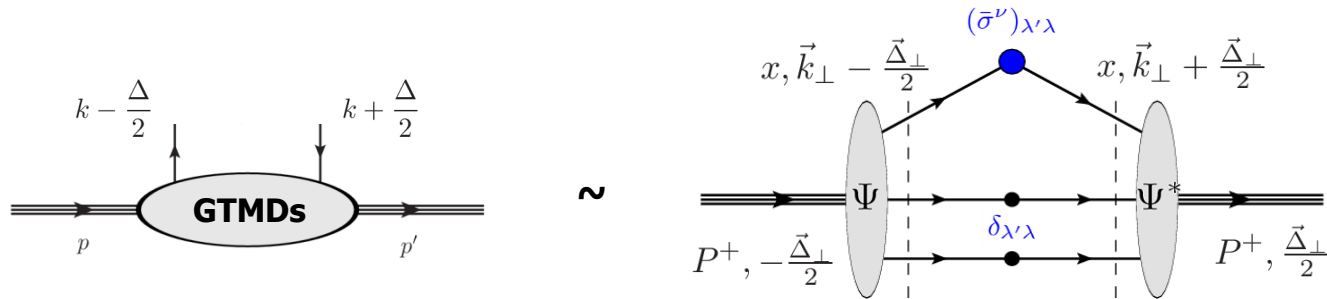
Parton distributions (naive)

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« Physical » objects



Light-front overlap representation



$$W_{\Lambda'\Lambda}^{[\Gamma]}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp) = \frac{1}{\sqrt{1-\xi^2}} \sum_{\beta', \beta} \int [dx]_3 [d^2 k_\perp]_3 \delta(\tilde{k}) \psi_{\Lambda'\beta'}^*(r') \psi_{\Lambda\beta}(r) M^{[\Gamma]\beta'\beta}$$

↑ **Momentum**
↑ **Polarization**

Light-front quark models

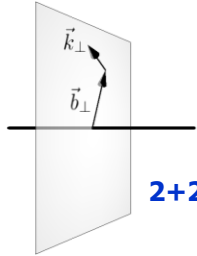
Wigner rotation

$$\psi_{\Lambda\beta}(r) = \mathcal{N} \Psi(r) \sum_{\sigma_i} \Phi_{\Lambda}^{\sigma_1 \sigma_2 \sigma_3} \prod_{i=1}^3 D_{\lambda_i \sigma_i}(\tilde{k}_i)$$

$$D(\tilde{k}) = \frac{1}{|\vec{K}|} \begin{pmatrix} K_z & K_L \\ -K_R & K_z \end{pmatrix}, \quad K_{R,L} = K_x \pm iK_y$$

Model	$\Psi(r)$	K_z	$\vec{K}_\perp$	κ_z
LFCQM	$\tilde{\psi}(r)$	$m + y\mathcal{M}_0$	$\vec{\kappa}_\perp$	$y\mathcal{M}_0 - \omega$
LF χ QSM	$\prod_{i=1}^3 \vec{K}_i $	$f_{\parallel}(y, \kappa_\perp)$	$\vec{\kappa}_\perp f_\perp(y, \kappa_\perp)$	$y\mathcal{M}_N - E_{\text{lev}}$

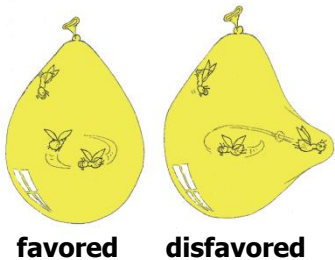
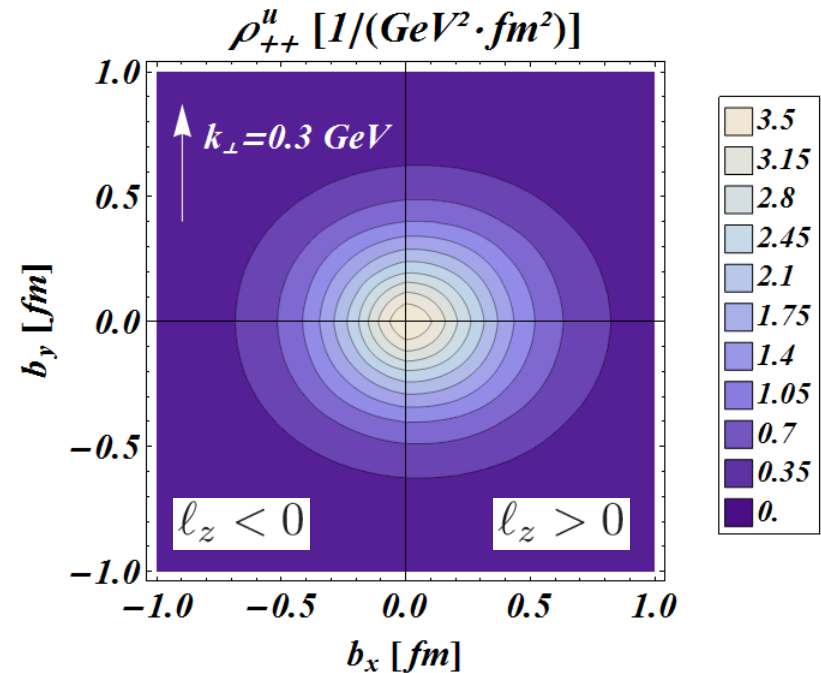
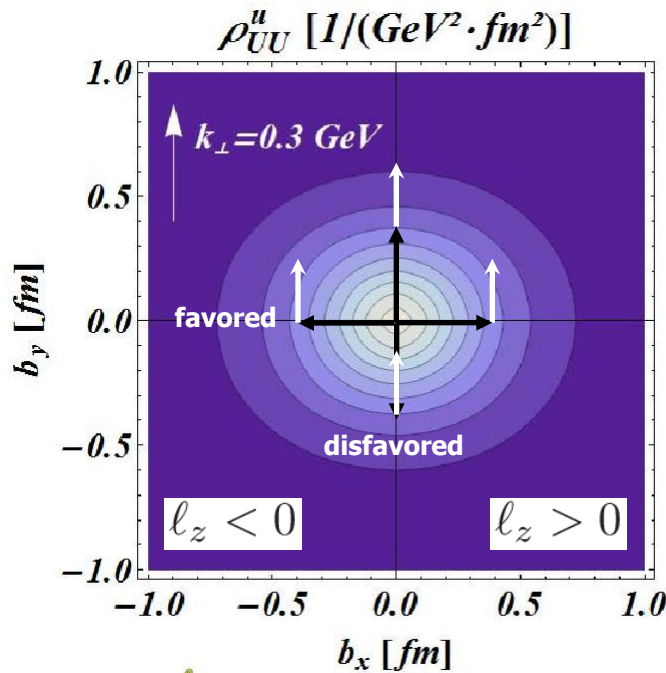
Model results



2+2D

$$\rho_{UU} = \frac{1}{2} \left(\rho_{++}^{[\gamma^+]} + \rho_{--}^{[\gamma^+]} \right) \propto F_{11}$$

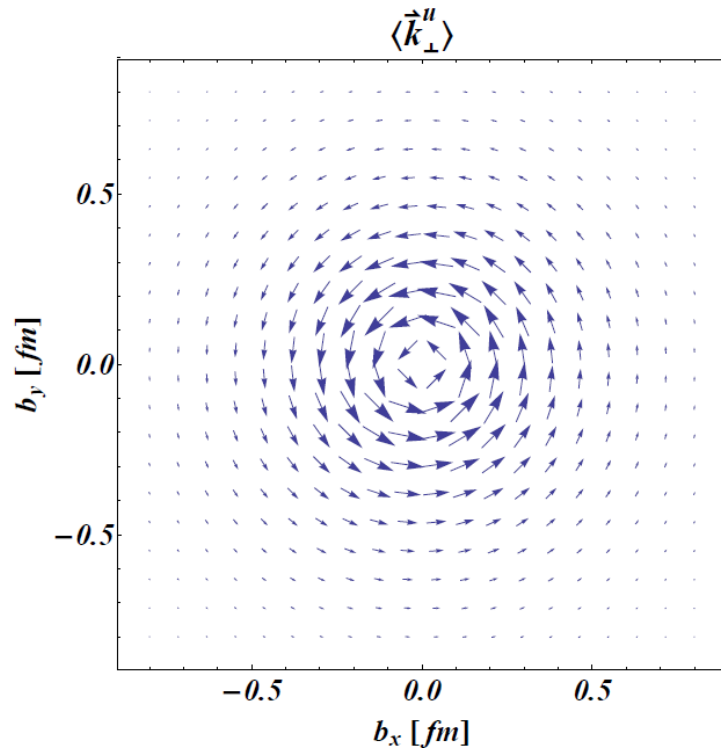
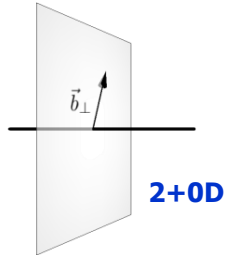
$$\rho_{++}^{[\gamma^+]} \propto F_{11}, F_{14}$$



$$\ell_z^{\text{tot}} = 0$$

$$\ell_z^{\text{tot}} > 0$$

Orbital angular momentum



Model result

Average quark momentum

$$\langle \vec{k}_\perp \rangle(\vec{b}_\perp) = \int dx d^2 k_\perp \vec{k}_\perp \rho_{++}^{[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

↑
Phase-space
density

Average quark OAM

$$\begin{aligned} \ell_z &= \int d^2 b_\perp \vec{b}_\perp \times \langle \vec{k}_\perp \rangle(\vec{b}_\perp) \\ &= - \int dx d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp) \end{aligned}$$

↑
« Vorticity »

[C.L., Pasquini (2011)]

[Hatta (2012)]

[C.L., Pasquini, Xiong, Yuan (2012)]

Parametrization @ twist-2 and $\xi=0$

Complete parametrizations : Quarks [Meissner, Metz, Schlegel (2009)]
 Quarks & gluons [C.L., Pasquini (2013)]

GTMDs

Quark polarization

	U	T_x	T_y	L
Target polarization	F_{11}	$\frac{i}{M} (k_y H_{11} + \Delta_y H_{12})$	$-\frac{i}{M} (k_x H_{11} + \Delta_x H_{12})$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
T_x	$\frac{i}{M} (k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11}))$	$\dots$	$\dots$	$\dots$
T_y	$-\frac{i}{M} (k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11}))$	$\dots$	$\dots$	$\dots$
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M} (k_x H_{17} + \Delta_x H_{18})$	$\frac{1}{M} (k_y H_{17} + \Delta_y H_{18})$	G_{14}

OAM

$$\vec{\Delta}_\perp = \vec{0}_\perp$$

$$\int d^2 k_\perp$$

TMDs

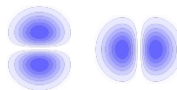
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDS

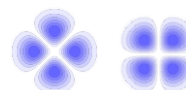
	U	T_x	T_y	L
U	H	$i \frac{\Delta_y}{2M} (2\tilde{H}_T + E_T)$	$-i \frac{\Delta_x}{2M} (2\tilde{H}_T + E_T)$	
T_x	$i \frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i \frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L				$\tilde{H}$



Monopole



Dipole



Quadrupole

Parametrization @ twist-2 and $\xi=0$

Complete parametrizations : Quarks [Meissner, Metz, Schlegel (2009)]
 Quarks & gluons [C.L., Pasquini (2013)]

GTMDs

Quark polarization

	U	T_x	T_y	L	
U	F_{11}	$\frac{i}{M} (k_y H_{11} + \Delta_y H_{12})$	$-\frac{i}{M} (k_x H_{11} + \Delta_x H_{12})$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$	???
T_x	$\frac{i}{M} (k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11}))$	$\dots$	$\dots$	$\dots$	
T_y	$-\frac{i}{M} (k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11}))$	$\dots$	$\dots$	$\dots$	
L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M} (k_x H_{17} + \Delta_x H_{18})$	$\frac{1}{M} (k_y H_{17} + \Delta_y H_{18})$	G_{14}	

OAM

$$\vec{\Delta}_\perp = \vec{0}_\perp$$

$$\int d^2 k_\perp$$

TMDs

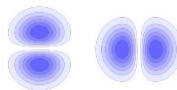
	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_1^\perp$	$-\frac{k_x}{M} h_1^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

GPDs

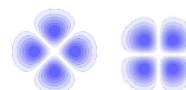
	U	T_x	T_y	L
U	H	$i \frac{\Delta_y}{2M} (2\tilde{H}_T + E_T)$	$-i \frac{\Delta_x}{2M} (2\tilde{H}_T + E_T)$	
T_x	$i \frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i \frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L				$\tilde{H}$



Monopole

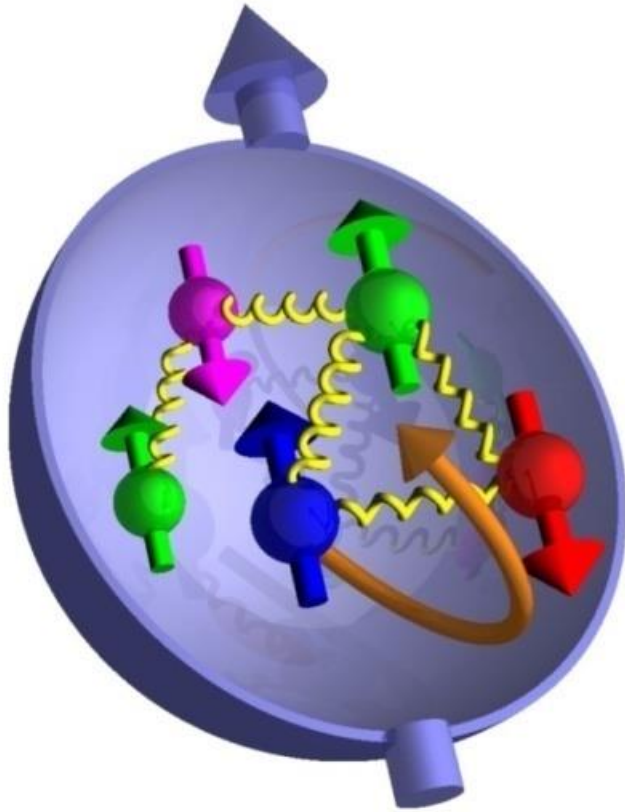


Dipole



Quadrupole

The outline

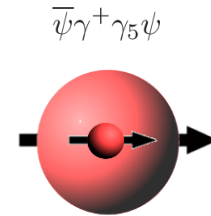


- Why spin and OAM ?
- Proton spin decompositions
- Phase-space distributions and OAM
- **Spin-orbit correlation**
- Conclusions

Proton spin structure

« Quark spin »

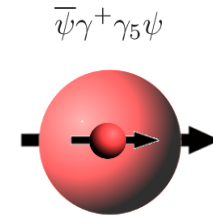
$$\langle\langle S_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda \lambda |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N S_z^q \rangle$$



Proton spin structure

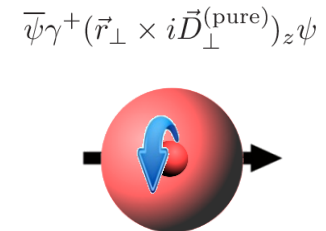
« Quark spin »

$$\langle\langle S_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda \lambda |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N S_z^q \rangle$$



« Quark OAM »

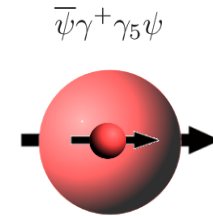
$$\langle\langle L_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N L_z^q \rangle$$



Proton spin structure

« Quark spin »

$$\langle\langle S_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda \lambda |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N S_z^q \rangle$$



« Quark OAM »

$$\langle\langle L_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N L_z^q \rangle$$

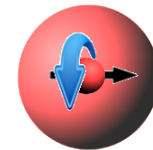
$$\bar{\psi} \gamma^+ (\vec{r}_\perp \times i \vec{D}_\perp^{(\text{pure})})_z \psi$$



Quark spin-orbit correlation

$$\langle\langle C_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^q L_z^q \rangle$$

$$\bar{\psi} \gamma^+ \gamma_5 (\vec{r}_\perp \times i \vec{D}_\perp^{(\text{pure})})_z \psi$$



OAM vs spin-orbit

OAM

$$L_z = \int d^3r \left[r^1 \langle \langle T^{+2} \rangle \rangle - r^2 \langle \langle T^{+1} \rangle \rangle \right]$$

Parametrization

$$\begin{aligned} T^{\mu\nu} &= \langle p', \Lambda' | \bar{\psi} \gamma^\mu \frac{i}{2} \overleftrightarrow{D}^\nu \psi | p, \Lambda \rangle \\ &= \bar{u}' \left[\frac{P^{\{\mu} \gamma^{\nu\}}}{2} A + \frac{P^{\{\mu} i \sigma^{\nu\} \Delta}}{4M} B \right. \\ &\quad \left. + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} C + M g^{\mu\nu} \bar{C} + \frac{P^{[\mu} \gamma^{\nu]}}{2} D \right] u \end{aligned}$$

Relations

$$L_z = \frac{1}{2} (A + B + D) \quad [\text{Shore, White (2000)}]$$

$$= \int dx \frac{1}{2} [x(H + E) - \tilde{H}] \quad [\text{Ji (1997)}]$$

$$= - \int dx \, x \, G_2 \quad [\text{Penttinen et al. (2000)}]$$

$$= - \int dx \, d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14} \quad [\text{C.L., Pasquini (2011)}]$$

$$[\text{Hatta (2012)}]$$

OAM vs spin-orbit

OAM

$$L_z = \int d^3r \left[r^1 \langle \langle T^{+2} \rangle \rangle - r^2 \langle \langle T^{+1} \rangle \rangle \right]$$

Parametrization

$$\begin{aligned} T^{\mu\nu} &= \langle p', \Lambda' | \bar{\psi} \gamma^\mu \frac{\overleftrightarrow{D}^\nu}{2} \psi | p, \Lambda \rangle \\ &= \bar{u}' \left[\frac{P^{\{\mu} \gamma^{\nu\}}}{2} A + \frac{P^{\{\mu} i \sigma^{\nu\} \Delta}}{4M} B \right. \\ &\quad \left. + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} C + M g^{\mu\nu} \bar{C} + \frac{P^{[\mu} \gamma^{\nu]}}{2} D \right] u \end{aligned}$$

Relations

$$\begin{aligned} L_z &= \frac{1}{2} (A + B + D) && \text{[Shore, White (2000)]} \\ &= \int dx \frac{1}{2} [x(H + E) - \tilde{H}] && \text{[Ji (1997)]} \\ &= - \int dx x G_2 && \text{[Penttinen *et al.* (2000)]} \\ &= - \int dx d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14} && \text{[C.L., Pasquini (2011)]} \\ &&& \text{[Hatta (2012)]} \end{aligned}$$

Spin-orbit

$$C_z = \int d^3r \left[r^1 \langle \langle T_5^{+2} \rangle \rangle - r^2 \langle \langle T_5^{+1} \rangle \rangle \right]$$

Parametrization

$$\begin{aligned} T_5^{\mu\nu} &= \langle p', \Lambda' | \bar{\psi} \gamma^\mu \gamma_5 \frac{\overleftrightarrow{D}^\nu}{2} \psi | p, \Lambda \rangle \\ &= \bar{u}' \left[\frac{P^{\{\mu} \gamma^{\nu\}} \gamma_5}{2} \tilde{A} + \frac{P^{\{\mu} \Delta^{\nu\}} \gamma_5}{4M} \tilde{B} \right. \\ &\quad \left. + \frac{P^{[\mu} \gamma^{\nu]} \gamma_5}{2} \tilde{C} + \frac{P^{[\mu} \Delta^{\nu]} \gamma_5}{4M} \tilde{D} + M i \sigma^{\mu\nu} \gamma_5 \tilde{F} \right] u \end{aligned}$$

Relations

$$\begin{aligned} C_z &= \frac{1}{2} (\tilde{A} + \tilde{C}) \\ &= \int dx \frac{1}{2} (x \tilde{H} - H) + \mathcal{O}(\frac{m_q}{M}) \\ &= - \int dx x (\tilde{G}_2 + 2\tilde{G}_4) \\ &= \int dx d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} G_{11} \end{aligned} \left. \vphantom{\begin{aligned} C_z &= \frac{1}{2} (\tilde{A} + \tilde{C}) \\ &= \int dx \frac{1}{2} (x \tilde{H} - H) + \mathcal{O}(\frac{m_q}{M}) \\ &= - \int dx x (\tilde{G}_2 + 2\tilde{G}_4) \end{aligned}} \right\} \begin{aligned} &\text{[C.L. (2014)]} \\ &\text{[C.L., Pasquini (2011)]} \\ &\text{[C.L. (2014)]} \end{aligned}$$

Some figures

Valence number

$$F_1^u(0) = 2$$

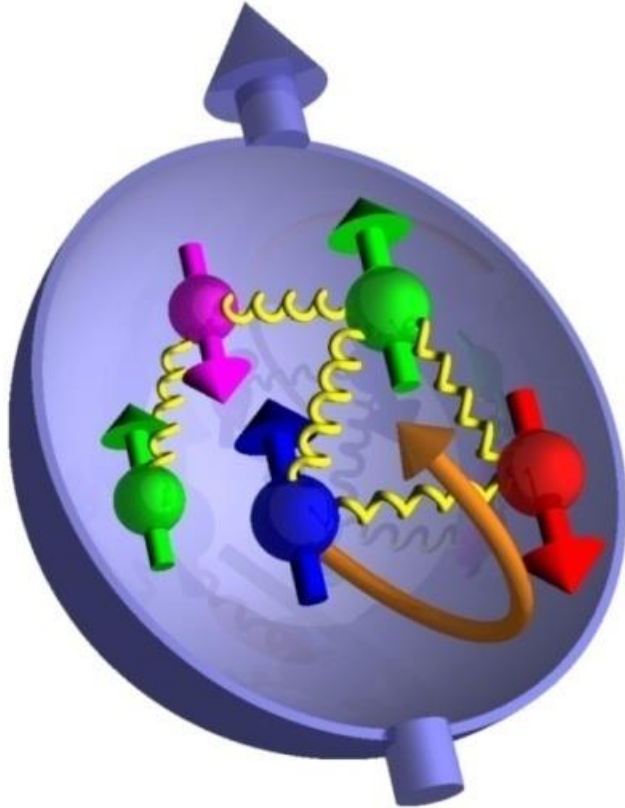
$$F_1^d(0) = 1$$

	$\int dx x \tilde{H}^u$	$\int dx x \tilde{H}^d$	C_z^u	C_z^d
NQM	4/9	-1/9	-7/9	-5/9
LFCQM	0.34	-0.09	-0.83	-0.54
LF χ QSM	0.39	-0.10	-0.80	-0.55
LSS2010	0.19	-0.06	-0.90	-0.53

Conclusion :

Spin and kinetic OAM of valence quarks are **anti-correlated !**

Conclusions



- Spin and OAM are keys to very interesting physics in a wide range of fields
- Many ambiguities coming from Lorentz and gauge symmetry but basically solved
- Ultimate goal is the quark/gluon phase-space distribution inside hadrons
- Also interesting for high-energy, low-x regime!
 - **Higgs spin and parity**
 - **CS vs CO mechanism for quarkonia**
 - **Momentum density vs energy flow**
 - **Quantum gravity**
 - ...