DIPOLAR DARK MATTER IN COSMOLOGY

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Based on

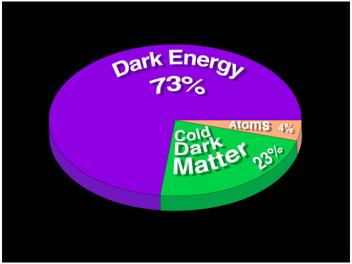
- L. Blanchet, Classical Quantum Gravity **24**, 3529 (2007)
- L. Blanchet & A. Le Tiec, Physical Review D **78**, 024031 (2008)
- L. Blanchet & A. Le Tiec, submitted (2009), arXiv:0901.3114

Outline of the talk

1. Phenomenology of dark matter
2. Modified Newtonian dynamics
3. Modified gravity theories
4. Modified matter approach
5. Dipolar dark matter and dark energy
6. Agreement with Λ-CDM and MOND

PHENOMENOLOGY OF DARK MATTER

Mass-energy content of the Universe



The total mass-energy of the Universe is made of

1. $\Omega_{\rm de} = 73\%$ of dark energy, maybe in the form of a cosmological constant Λ, as measured from the Hubble diagram of supernovas
2. $\Omega_{\rm dm} = 23\%$ of non-baryonic dark matter, a perfect fluid without pressure whose nature is unknown
3. $\Omega_{\rm b} = 4\%$ of baryonic matter, measured by the Big Bang nucleosynthesis and from CMB fluctuations

The concordance model Λ-CDM

This model brilliantly accounts for

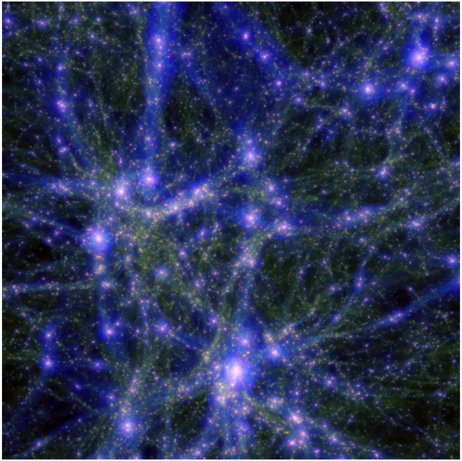
1. the mass discrepancy between the dynamical and luminous masses of clusters of galaxies
2. the precise measurements of the anisotropies of the cosmic microwave background (CMB)
3. the formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
4. the fainting of the light curves of distant supernovae

DM appears to be made by non-relativistic (cold) particles at large scales

Candidates include [Bertone, Hooper & Silk, 2004]

- the neutralino predicted by super-symmetric extensions of the standard model
- the axion introduced in an attempt to solve the problem of CP violation
- Kaluza-Klein states predicted by models with extra dimensions
- $\cdots$

Cosmic N-body simulations



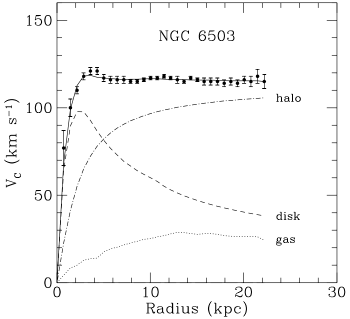
Thanks to high precision N-body simulations in cosmology the model Λ-CDM can be extrapolated and tested at the smaller scale of galaxies

Problems of CDM with galactic halos

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004; Famaey 2007]

1. Prediction of numerous but unseen satellites of large galaxies
2. Generic formation of cusps of DM in central regions of galaxies while the rotation curves seem to favor a constant density profile in the core
3. Evidence that tidal dwarf galaxies are dominated by DM contrary to CDM predictions [Bournaud *et al.* 2007; Gentile *et al.* 2007]
4. Failure to explain in a natural way Milgrom's law, that DM arises only in regions where gravity falls below some universal acceleration scale a_0
5. Difficulty at explaining in a natural way the flat rotation curves of galaxies and the Tully-Fisher relation

Rotation curves of galaxies are flat

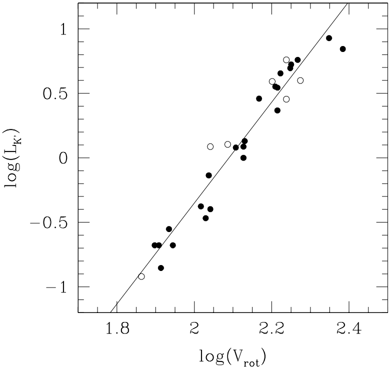


For a circular orbit $\quad v(r) = \sqrt{\frac{G\,M(r)}{r}}$

The fact that $v(r)$ is approximately constant implies that beyond the optical disk

$$M_{\text{halo}}(r) \approx r \qquad \rho_{\text{halo}}(r) \approx \frac{1}{r^2}$$

The Tully-Fisher empirical relation [Tully & Fisher 1977]



The relation between the asymptotic flat velocity and the luminosity of spirals is

$$v_{\text{flat}} \propto L^{1/4}$$

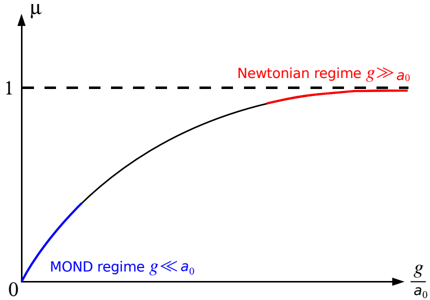
MODIFIED NEWTONIAN DYNAMICS

Modified Newtonian dynamics [Milgrom 1983]

MOND states that there is no dark matter and we witness a violation of the fundamental law of gravity in the regime of weak gravity

The Newtonian gravitational field is modified in an *ad hoc* way

$$\mu\left(\frac{g}{a_0}\right)\boldsymbol{g} = \boldsymbol{g}_{\text{Newton}}$$



In the MOND regime we have $\mu = g/a_0 + \mathcal{O}(g^2)$

Recovering flat rotation curves and the Tully-Fisher law

1. For a spherical mass $g_\mathrm{N} = \frac{G\,M}{r^2}$ hence $g \approx \frac{\sqrt{G\,M\,a_0}}{r}$
2. For circular motion $\frac{v^2}{r} = g$ thus v is constant and we get

$$\boxed{v_\mathrm{flat} \approx \left(G\,M\,a_0\right)^{1/4}}$$

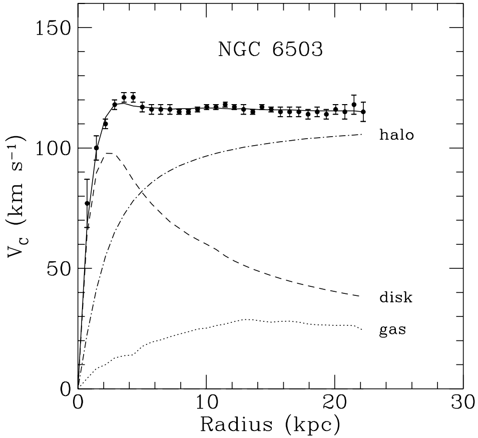
3. Assuming $L/M = \mathrm{const}$ one naturally explains the Tully-Fisher relation
4. The numerical value of the critical acceleration is measured as

$$\boxed{a_0 \approx 1.2\,10^{-10}\,\mathrm{m/s^2}}$$

This value of a_0 is very close to the acceleration scale associated with the cosmological constant Λ

$$a_0 \approx 1.3\,a_\Lambda \qquad \text{where} \qquad a_\Lambda \equiv \frac{1}{2\pi}\left(\frac{\Lambda}{3}\right)^{1/2}$$

The MOND fit of galactic rotation curves



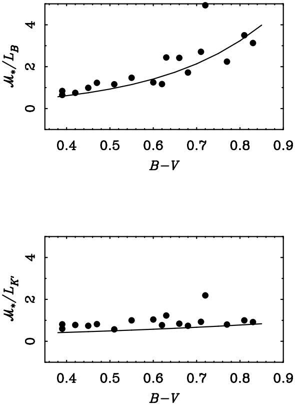
⟸ the solid line is the MOND fit

Many galaxies are accurately fitted with MOND [Sanders 1996, McGaugh & de Blok 1998]

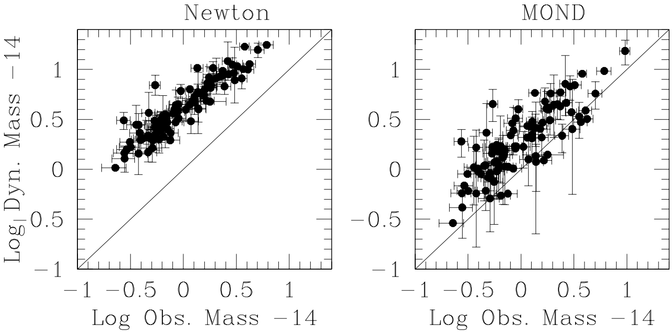
Fit of the mass-to-luminosity ratio

The fit of rotation curves is actually a one-parameter fit. The mass-to-luminosity ratio M/L of each galaxy is adjusted (and is therefore measured by MOND)

M/L shows the same trend with colour as is implied by models of population synthesis [Sanders & Verheijen 1998]

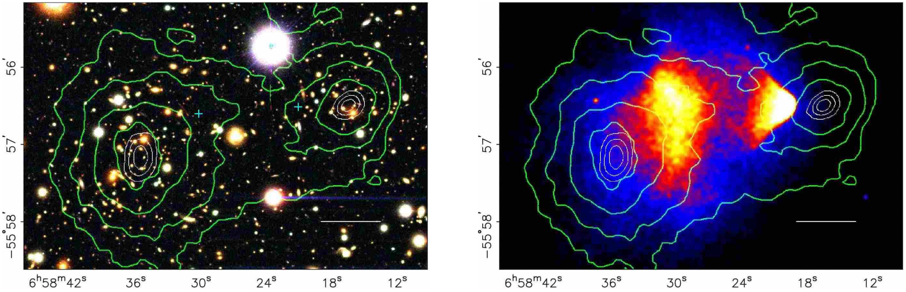


Problem with galaxy clusters [Gerbal, Durret *et al* 1992, Sanders 1999]



The mass discrepancy is $\approx 4 - 5$ with Newton and ≈ 2 with MOND

Collision of two clusters of galaxies [Clowe *et al* 2006]



The bullet cluster and more generally X-ray emitting galaxy clusters can be fitted with MOND and a component of baryonic dark matter and hot/warm neutrinos [Angus, Famaey & Buote 2008]

MODIFIED GRAVITY THEORIES

Different approaches to the DM problem

Faced with the "unreasonable effectiveness" of MOND, three solutions are possible

1. Standard: MOND could be explained within the CDM paradigm
2. Modified Gravity: There is a fundamental modification of the law of gravity in a regime of weak gravity (this is the traditional approach of MOND and its relativistic extensions like TeVeS)
3. Modified Matter: The law of gravity is not modified but DM is endowed with special properties which make it able to explain the phenomenology of MOND

Modified gravity or modified matter?

We consider that the Standard scenario (CDM) is excluded by observations

- To solve the problems of CDM in galactic halos one must invoke complicated astrophysical processes which have to be fine tuned for each galaxies
- No convincing mechanism has been found to incorporate in a natural way the acceleration scale a_0 in the simulated CDM halos

Only the Modified Gravity and Modified Matter approaches remain

- In these two solutions we shall have to explain why DM seems to be made of particles at cosmological scales

Modified gravity theory

- The MOND equation can be rewritten as the modified Poisson equation

$$\nabla \cdot \Big[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \Big] = -4\pi\, G\, \rho_{\text{b}}$$

where $\boldsymbol{g} = \nabla U$ is the gravitational field and ρ_{b} the density of ordinary matter

- This equation is derivable from the Lagrangian [Bekenstein & Milgrom 1984]

$$L = \frac{a_0^2}{8\pi}\, f\left(\frac{\nabla U^2}{a_0^2}\right) + \rho\, U$$

where f is related to the MOND function by $f'(x) = \mu(\sqrt{x})$.

Scalar-tensor theory [Bekenstein & Sanders 1994]

1. The tensor part is the usual Einstein-Hilbert action

$$L_g = \frac{1}{16\pi} R[g, \partial g, \partial^2 g]$$

2. The scalar field is given by an aquadratic kinetic term

$$L_\phi = \frac{a_0^2}{8\pi} F \left(\frac{g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}{a_0^2} \right)$$

3. The matter fields are universally coupled to gravity

$$L_\mathrm{m} = L_\mathrm{m}\Big[\Psi, \underbrace{\tilde{g}_{\mu\nu} \equiv e^{2\phi} g_{\mu\nu}}_{\text{physical metric}}\Big]$$

- Light signals do not feel the presence of the scalar field ϕ because the physical metric $\tilde{g}_{\mu\nu}$ is conformally related to the Einstein frame metric $g_{\mu\nu}$
- Since we observe huge amounts of dark matter by gravitational lensing (weak and strong) this theory is not viable

Tensor-vector-scalar theory [Bekenstein 2004, Sanders 2005]

1. The tensor part is the Einstein-Hilbert action L_g
2. The vector field part for $W_{\mu\rho} = \partial_\mu V_\nu - \partial_\nu V_\mu$ is

$$L_V = -\frac{1}{32\pi}\Big[K \underbrace{g^{\mu\nu}g^{\rho\sigma}W_{\mu\rho}W_{\nu\sigma}}_{\text{standard spin-1 action}} - 2\lambda \underbrace{\left(g^{\mu\nu}V_\mu V_\nu + 1\right)}_{\text{tells that } V^\mu \text{ is time-like and unitary}}\Big]$$

3. The scalar action reads

$$L_\phi = -\overbrace{\frac{1}{2}\left(g^{\mu\nu} - V^\mu V^\nu\right)\partial_\mu\phi\partial_\nu\phi}^{\text{dynamical scalar field } \phi} - \overbrace{\frac{\sigma^4}{4\ell^2}F\left(k\,\sigma^2\right)}^{\text{non-dynamical scalar field } \sigma}$$

4. Matter fields are coupled to the non-conformally related physical metric

$$\tilde{g}_{\mu\nu} = e^{-2\phi}\left(g_{\mu\nu} + V_\mu V_\nu\right) - e^{2\phi}V_\mu V_\nu$$

This theory has evolved recently toward Einstein-æther like theories [Jacobson & Mattingly 2001; Zlosnik, Ferreira & Starkman 2007; Halle, Zhao & Li 2008]

MODIFIED MATTER APPROACH

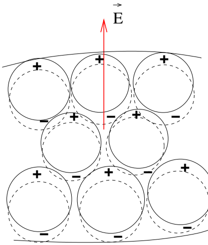
Why modified matter?

1. Keep the standard law of gravity namely general relativity
2. Use the phenomenology of MOND to guess what could be the nature of DM

This approach is based on a striking analogy between

- MOND which takes the form of a modified Poisson equation
- The electrostatics of dielectric media described by a modified Gauss equation

The electric field in a dielectric



- The atoms in a dielectric are modelled by electric dipole moments

$$\boldsymbol{p}_e = q\,\boldsymbol{\xi}$$

- The polarization vector is

$$\boldsymbol{P}_e = n\,\boldsymbol{p}_e$$

Density of polarization charges: $\rho_e^{\text{polar}} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}_e$

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = \frac{\rho_e + \rho_e^{\text{polar}}}{\varepsilon_0} \quad\Longleftrightarrow\quad \boldsymbol{\nabla}\cdot\Big(\overbrace{\varepsilon_0\,\boldsymbol{E} + \boldsymbol{P}_e}^{\boldsymbol{D}\text{-field}}\Big) = \rho_e$$

The polarization vector is aligned with the electric field

$$\boldsymbol{P}_e = \varepsilon_0\,\chi_e(E)\,\boldsymbol{E}$$

where $\chi_e(E)$ denotes the coefficient of electric susceptibility

Interpretation of MOND [Blanchet 2006]

The MOND equation in the form of a modified Poisson equation

$$\nabla \cdot \left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}} \boldsymbol{g} \right] = -4\pi\, G\, \rho_{\mathsf{b}}$$

is formally analogous to the equation of electrostatics inside a dielectric. We pose

$$\mu = 1 + \underbrace{\chi(g)}_{\substack{\text{gravitational}\\ \text{susceptibility}}} \qquad \text{and} \qquad \underbrace{\boldsymbol{\Pi}}_{\substack{\text{gravitational}\\ \text{polarization}}} = -\frac{\chi}{4\pi\, G}\, \boldsymbol{g}$$

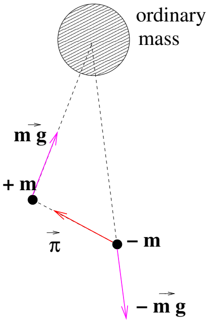
The MOND equation is equivalent to

$$\Delta U = -4\pi\, G\, \left(\rho_{\mathsf{b}} + \rho_{\text{polar}}\right)$$

In this interpretation the Newtonian law of gravity is not violated but we are postulating a new form of DM consisting of "polarization masses" with density

$$\rho_{\text{polar}} = -\nabla \cdot \boldsymbol{\Pi}$$

Microscopic description of the dipolar medium



The digravitational medium is modelled by individual dipole moments say

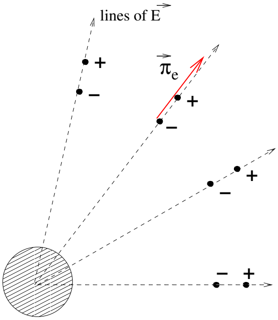
$$\boldsymbol{\pi} = m\,\boldsymbol{\xi}$$
$$\boldsymbol{\Pi} = n\,\boldsymbol{\pi}$$

Suppose that the dipole consists of a particle doublet with

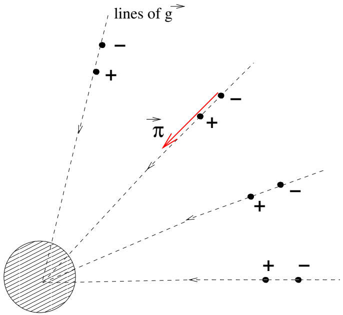
- opposite gravitational masses $m_\mathrm{g} = \pm m$
- positive inertial masses $m_\mathrm{i} = m$

- The dipoles tend to align in the same direction as the gravitational field thus $\chi < 0$ which is exactly what MOND predicts
- Since the constituents of the dipole will repel each other we need to invoke a non-gravitational force (i.e. a fifth force) to stabilize the dipolar medium

Electric screening versus gravitational anti-screening



electric charge
$Q > 0$



ordinary mass
$M > 0$

Screening by polarization charges

$$\chi_e > 0$$

Anti-screening by polarization masses

$$\chi < 0$$

Non viability of this model

The quasi-Newtonian model

- Suggests that the gravitational analogue of the electric polarization is possible
- Yields a simple and natural explanation of the MOND equation
- Requires the existence of a new non-gravitational force

BUT THIS MODEL IS NOT VIABLE

- Is not relativistic
- Involves negative gravitational masses so violates the equivalence principle

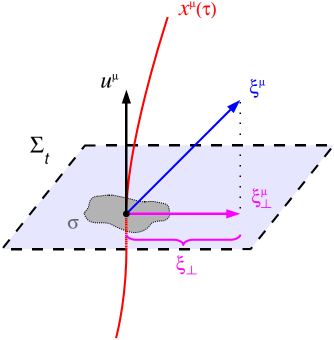
DIPOLAR DARK MATTER AND DARK ENERGY

Dipolar fluid in general relativity

The matter action in standard GR is of the type

$$S = \int d^4x \sqrt{-g}\, L\left[J^\mu, \xi^\mu, \dot{\xi}^\mu, g_{\mu\nu}\right]$$

where the current density J^μ and the dipole moment ξ^μ are two independent dynamical variables



- The current density $J^\mu = \sigma u^\mu$ is conserved

$$\nabla_\mu J^\mu = 0$$

- The covariant time derivative is denoted

$$\dot{\xi}^\mu \equiv \frac{\mathrm{D}\xi^\mu}{\mathrm{d}\tau} = u^\nu \nabla_\nu \xi^\mu$$

Lagrangian for the dipolar fluid [Blanchet & Le Tiec 2008; 2009]

We propose three terms

$$L = -\sigma + J^\mu \dot{\xi}_\mu - \mathcal{W}(\Pi_\perp)$$

1. A mass term σ in an ordinary sense (like for ordinary CDM)
2. An interaction term between the fluid's mass current $J^\mu = \sigma u^\mu$ and the dipole moment
3. A potential term $\mathcal{W}$ describing an internal force and depending on the norm of the polarization $\Pi_\perp = \sigma \, \xi_\perp$

One easily proves that the only dynamical degrees of freedom of the dipole moment are the space-like projection orthogonal to the velocity

$$\xi_\perp^\mu = \perp^\mu_\nu \xi^\nu \quad \text{where the projector is} \quad \perp^\mu_\nu = \delta^\mu_\nu + u^\mu u_\nu$$

Equations of motion and evolution

Variation with respect to ξ^μ

Equation of motion of the dipolar fluid

$$\underbrace{\dot{u}^\mu = -\mathcal{F}^\mu}_{\text{non-geodesic motion}} \qquad \text{where} \qquad \underbrace{\mathcal{F}^\mu \equiv \hat{\xi}_\perp^\mu \, \mathcal{W}'}_{\text{dipolar internal force}}$$

Variation with respect to J^μ

Evolution equation of the dipole moment

$$\dot{\Omega}^\mu = \frac{1}{\sigma}\nabla^\mu\left(\mathcal{W} - \Pi_\perp \mathcal{W}'\right) - \underbrace{\xi_\perp^\nu \, R^\mu{}_{\rho\nu\sigma} u^\rho u^\sigma}_{\text{coupling to Riemann curvature}}$$

$$\text{where} \quad \Omega^\mu \equiv \dot{\xi}_\perp^\mu + u^\mu\left(1 + 2\xi_\perp \mathcal{W}'\right)$$

Stress-energy tensor

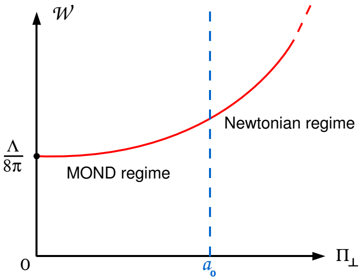
Variation with respect to $g_{\mu\nu}$

$$
\begin{aligned}
T^{\mu\nu} &= \Omega^{(\mu} J^{\nu)} && \Longleftarrow \text{monopolar DM} \\
&- \nabla_\rho \left(\left[\Pi_\perp^\rho u^{(\mu} - u^\rho \Pi_\perp^{(\mu} \right] u^{\nu)} \right) && \Longleftarrow \text{dipolar DM} \\
&- g^{\mu\nu} \left(\mathcal{W} - \Pi_\perp \mathcal{W}' \right) && \Longleftarrow \text{DE}
\end{aligned}
$$

The DM mass density is made of a monopolar term σ plus a dipolar term $-\nabla_\mu \Pi_\perp^\mu$ which appears as the relativistic analogue of the polarization mass density

$$
u_\mu u_\nu T^{\mu\nu} = \underbrace{\sigma - \nabla_\mu \Pi_\perp^\mu}_{\text{DM energy density}} + \underbrace{\mathcal{W} - \Pi_\perp \mathcal{W}'}_{\text{DE}}
$$

The internal potential



The potential $\mathcal{W}$ is phenomenologically determined through third order

$$\mathcal{W} = \frac{\Lambda}{8\pi} + 2\pi\,\Pi_\perp^2 + \frac{16\pi^2}{3a_0}\Pi_\perp^3 + \mathcal{O}\left(\Pi_\perp^4\right)$$

- The minimum of that potential is the cosmological constant Λ and the third-order deviation from the minimum contains the MOND scale a_0
- In this unification scheme the natural order of magnitude of the cosmological constant should be comparable with a_0

Order of magnitude of the cosmological constant

1. Introduce a purely numerical coefficient α such that

$$a_0 = \frac{1}{2\pi\,\alpha}\left(\frac{\Lambda}{3}\right)^{1/2} \qquad \left(\text{i.e.} \quad a_0 = \frac{a_\Lambda}{\alpha}\right)$$

2. Write the potential function as $\mathcal{W} = \frac{3\pi\,a_0^2}{2}\,f\left(\frac{\Pi_\perp}{a_0}\right)$ with

$$f(x) = \underbrace{\alpha^2 + \frac{4}{3}x^2 + \frac{32\pi}{9}x^3 + \mathcal{O}\left(x^4\right)}_{\text{some ``universal'' function of } x \,\equiv\, \Pi_\perp/a_0}$$

3. The numerical coefficients in $f(x)$ are expected to be of the order of one, hence the cosmological constant should be of the order of

$$\Lambda \sim a_0^2$$

in good agreement with observations (which give $\alpha \approx 0.8$)

AGREEMENT WITH Λ-CDM AND MOND

Cosmological perturbation at large scales

Consider a linear perturbation of the FLRW background. Since the dipole moment is space-like, it will break the spatial isotropy of the background, and must belong to the first-order perturbation

$$\xi_\perp^\mu = \mathcal{O}(1)$$

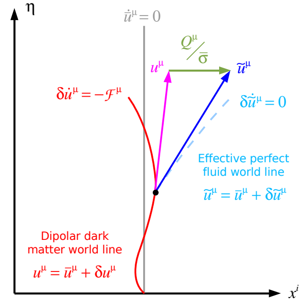
The stress-energy tensor reads $T^{\mu\nu} = T^{\mu\nu}_{\text{de}} + T^{\mu\nu}_{\text{dm}}$ where

1. the DE is given by the cosmological constant Λ
2. the DM takes the form of a perfect fluid with zero pressure

$$T^{\mu\nu}_{\text{dm}} = \rho\, \tilde{u}^\mu \tilde{u}^\nu + \mathcal{O}(2)$$

Here $\tilde{u}^\mu$ denotes an effective four-velocity field and $\rho \equiv \sigma - \nabla_\mu \Pi_\perp^\mu$ is the energy density of the DM fluid

Agreement with the Λ-CDM scenario



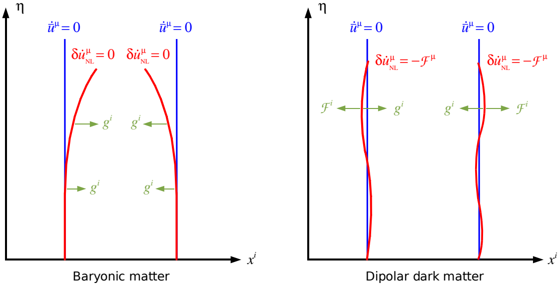
The dipolar fluid is undistinguishable from

- standard DE (a cosmological constant)
- standard CDM (a pressureless perfect fluid)

at the level of first-order cosmological perturbations

Adjusting Λ so that $\Omega_{\mathrm{de}} \simeq 0.73$ and $\bar{\sigma}$ so that $\Omega_{\mathrm{dm}} \simeq 0.23$ the model is consistent with CMB fluctuations

Weak clustering of dipolar DM



- Baryonic matter follows the geodesic equation $\dot{u}^\mu = 0$, therefore collapses in regions of overdensity
- Dipolar dark matter obeys $\dot{u}^\mu = -\mathcal{F}^\mu$, with the internal force $\boldsymbol{\mathcal{F}}$ balancing the gravitational field $\boldsymbol{g}$ created by an overdensity

The mass density of dipolar dark matter in a galaxy at low redshift should be smaller than the baryonic density and maybe close to its mean cosmological value

$$\sigma \approx \overline{\sigma} \ll \rho_\mathrm{b} \quad \text{and} \quad \boldsymbol{v} \approx \mathbf{0}$$

Non-relativistic limit of the model

The Lagrangian becomes in the limit $c \longrightarrow +\infty$

$$\mathcal{L}_{\mathsf{NR}} = \sigma\left(\frac{\boldsymbol{v}^2}{2} + U + \boldsymbol{g}\cdot\boldsymbol{\xi}_\perp + \boldsymbol{v}\cdot\frac{\mathrm{d}\boldsymbol{\xi}_\perp}{\mathrm{d}t}\right) - \mathcal{W}(\Pi_\perp)$$

where we recognize the gravitational analogue $\boldsymbol{g}\cdot\boldsymbol{\Pi}_\perp$ of the coupling of the polarization field to an exterior field

1. The equation of motion reads

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \boldsymbol{g} - \boldsymbol{\mathcal{F}}$$

2. The gravitational equation is

$$\boldsymbol{\nabla}\cdot\left(\boldsymbol{g} - 4\pi\,\boldsymbol{\Pi}_\perp\right) = -4\pi\left(\rho_{\mathsf{b}} + \sigma\right)$$

Recovering MOND in a galaxy at low red-shift

Crucial use is made of the weak clustering of dipolar DM

1. Using $\boldsymbol{v} \approx \mathbf{0}$ in the equation of motion

$$\boldsymbol{g} = \boldsymbol{\mathcal{F}} = \hat{\mathbf{\Pi}}_\perp \, \mathcal{W}' \implies \text{the dipolar medium is polarized}$$

2. Using $\sigma \ll \rho_\mathrm{b}$ in the field equation

$$\boldsymbol{\nabla} \cdot \Big[\boldsymbol{g} - 4\pi \, \mathbf{\Pi}_\perp\Big] = -4\pi \, \rho_\mathrm{b} \implies \text{the galaxy appears essentially baryonic}$$

Hence the MOND equation is recovered with MOND function $\mu = 1 + \chi$ such that

$$\boldsymbol{g} = \hat{\mathbf{\Pi}}_\perp \, \mathcal{W}' \quad \Longleftrightarrow \quad \mathbf{\Pi}_\perp = -\frac{\chi(g)}{4\pi} \, \boldsymbol{g}$$

Conclusions

This model

1. explains the phenomenology of MOND by the physical process of gravitational polarization
2. recovers the successful standard cosmological model Λ-CDM at linear perturbation order
3. makes a unification between dark energy in the form of Λ and dark matter *à la* MOND (with the interesting outcome that $\Lambda \sim a_0^2$)
4. but describes the dipolar medium in an effective way and is not related to microscopic fundamental physics

The model should be further tested in cosmology by

1. investigating second-order cosmological perturbations
2. computing the non-linear growth of perturbations
3. testing the intermediate scale of galaxy clusters