Plasma Instabilities and Isotropization in Heavy-Ion Collisions

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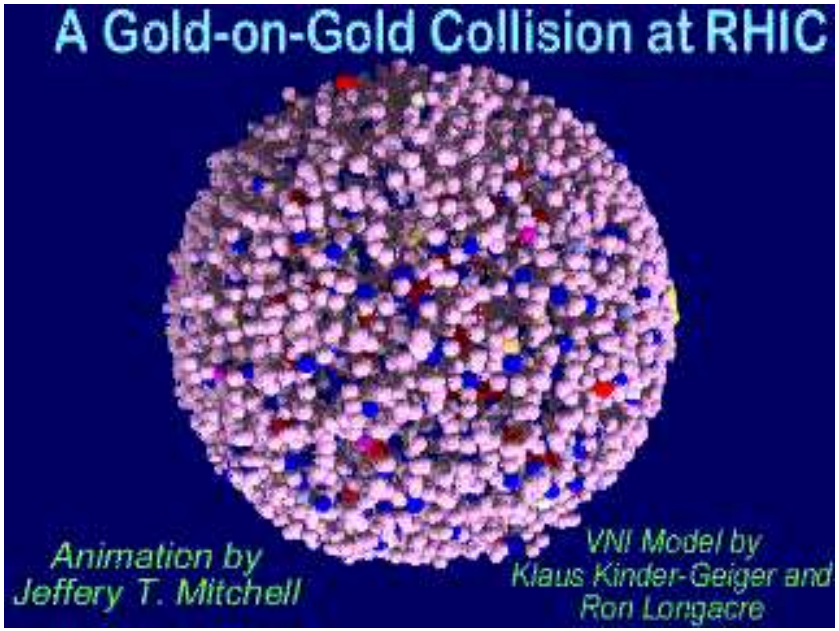
2nd June 2006

Outline

1. Introduction
   - Experimental Data
   - Hydrodynamics
2. Viscous Hydrodynamics
   - Set-up
   - First Results
3. Isotropization in Heavy-Ion Collisions
   - Non-Abelian Plasma Instabilities
   - Initial Conditions for HIC – The Glasma
   - The Unstable Glasma

Motivation: Au+Au Collisions at RHIC



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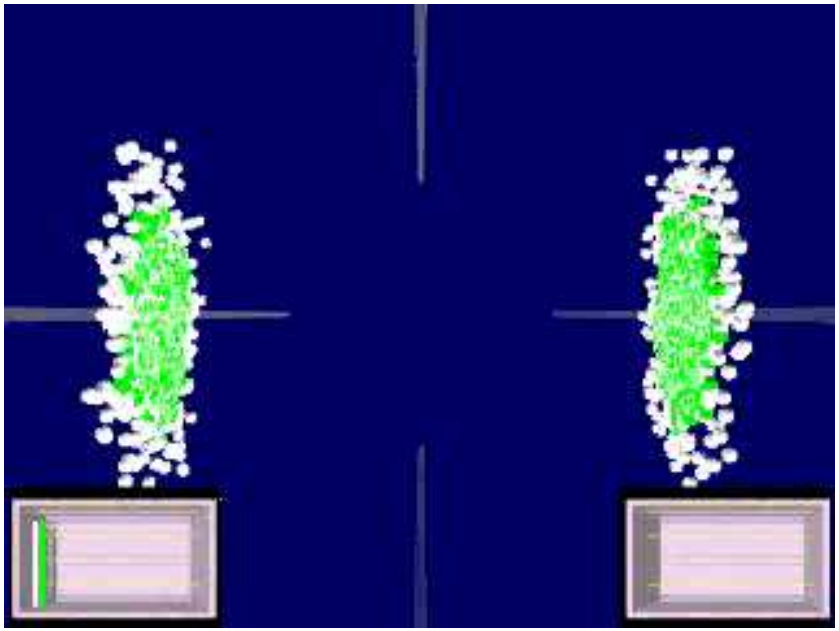

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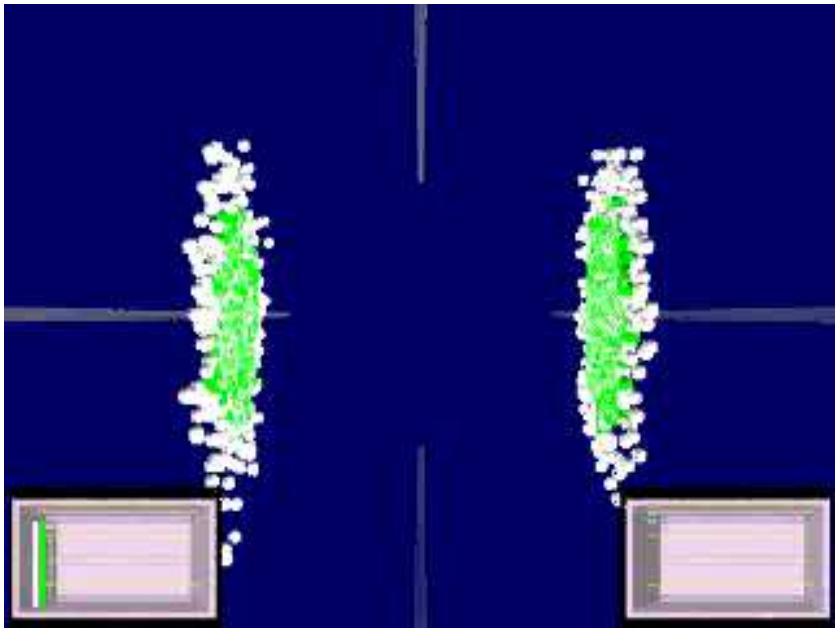

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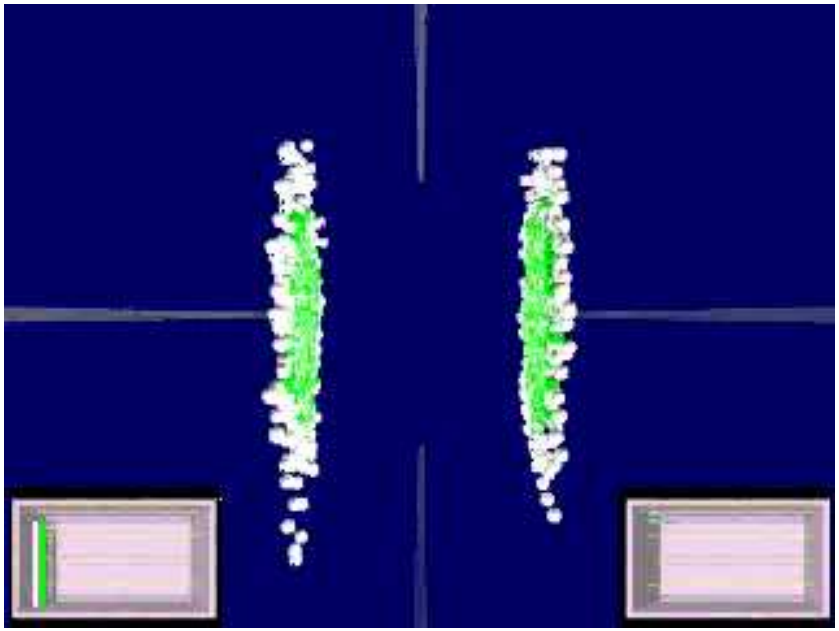

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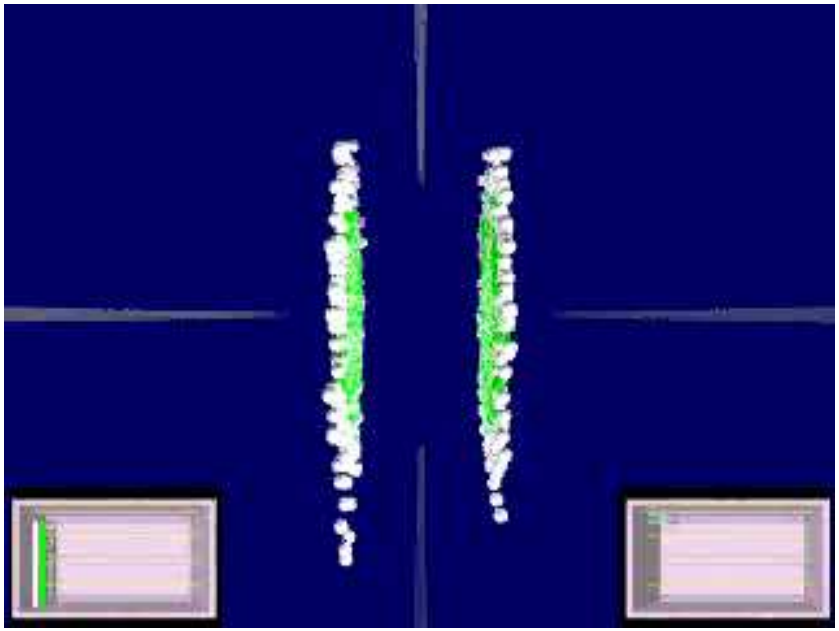

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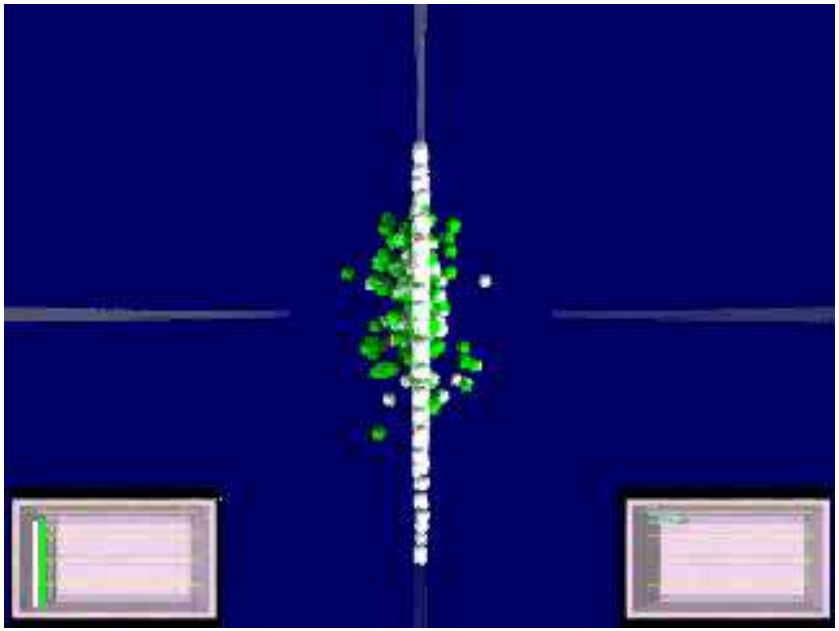

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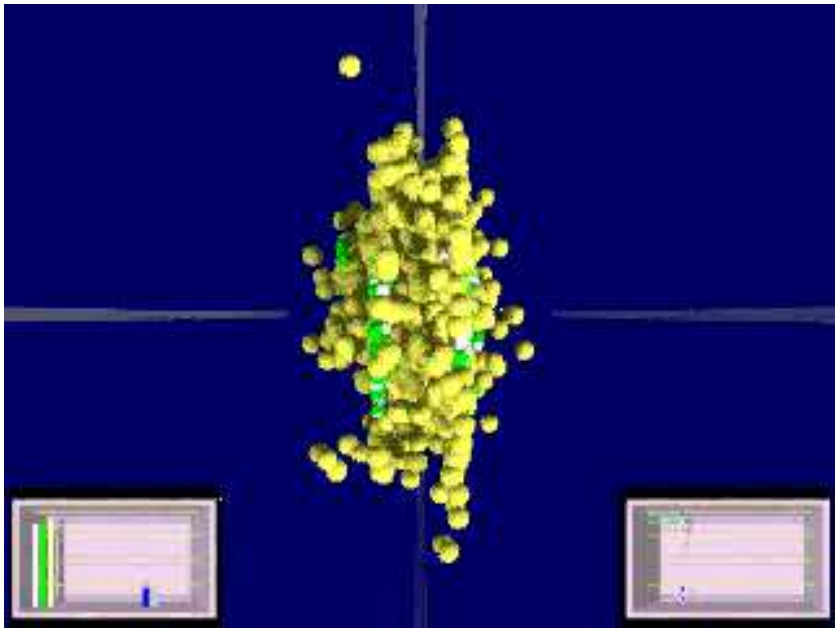

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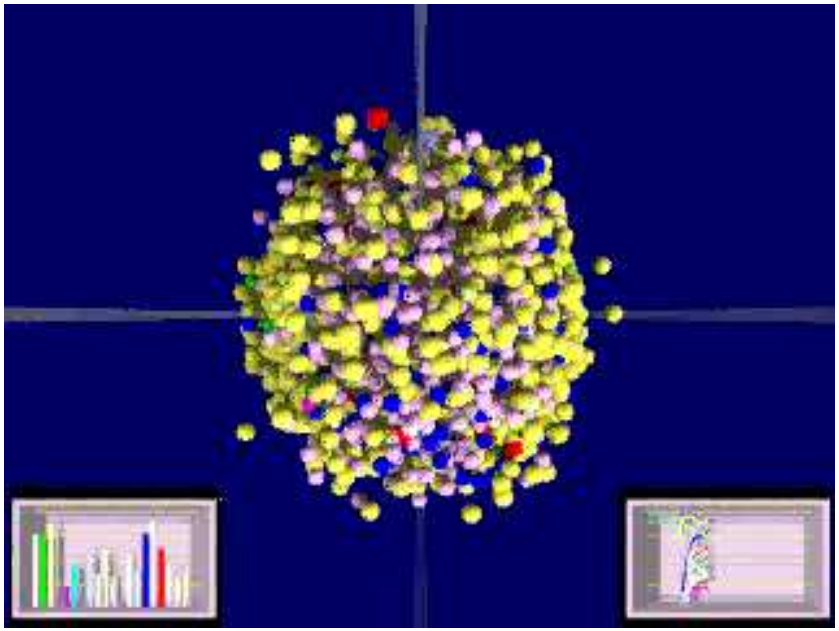
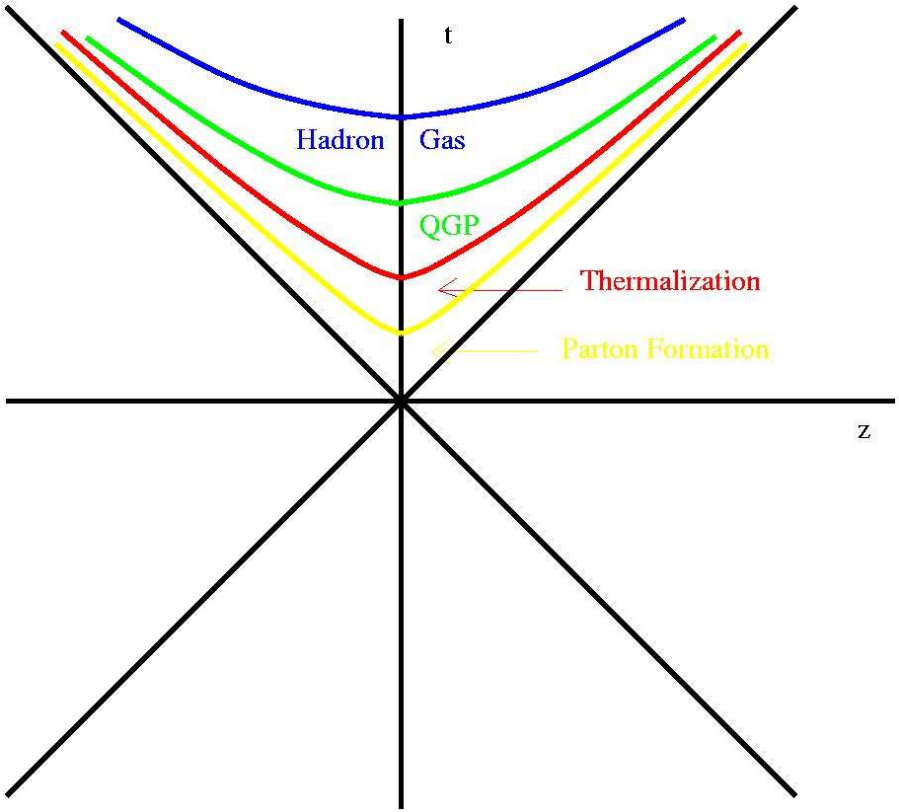


Space-time sketch of a HIC



E.Iancu & R.Venugopalan, hep-ph/0303204

HIC in Stages

- Stage I: 0 fm/c to $\sim$ 0.5 fm/c: Parton formation
- Stage II: 0.5 fm/c to $\sim$ 2 fm/c (?): Thermalization
- Stage III: 1 fm/c (?) to 10 fm/c: Ideal Hydro

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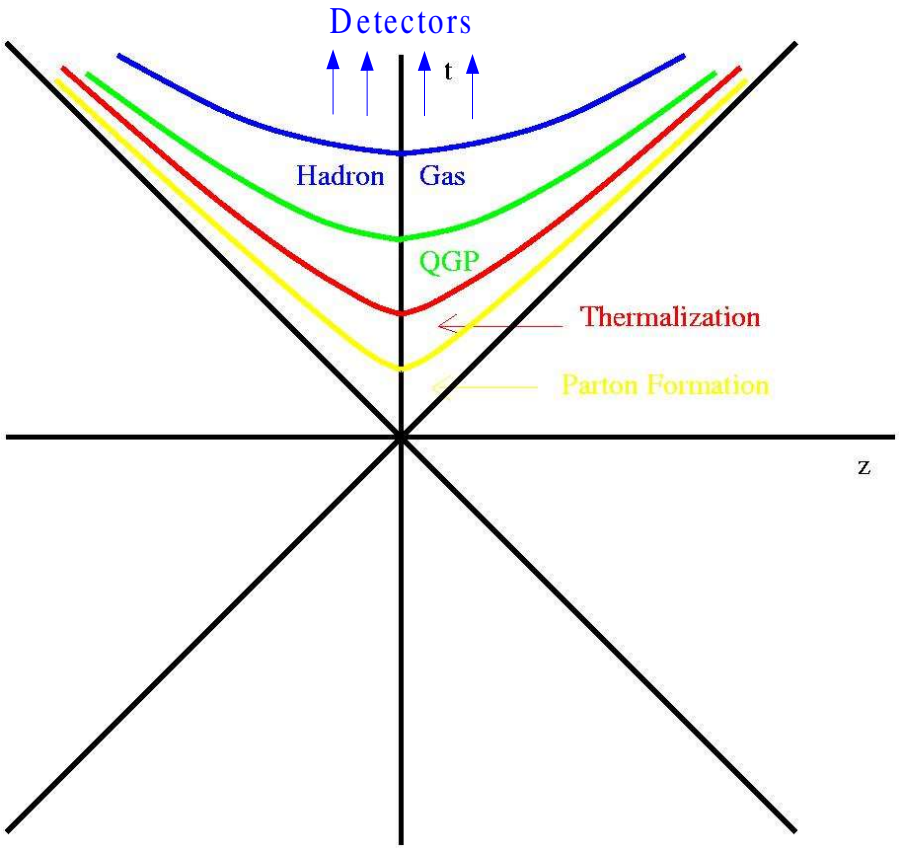
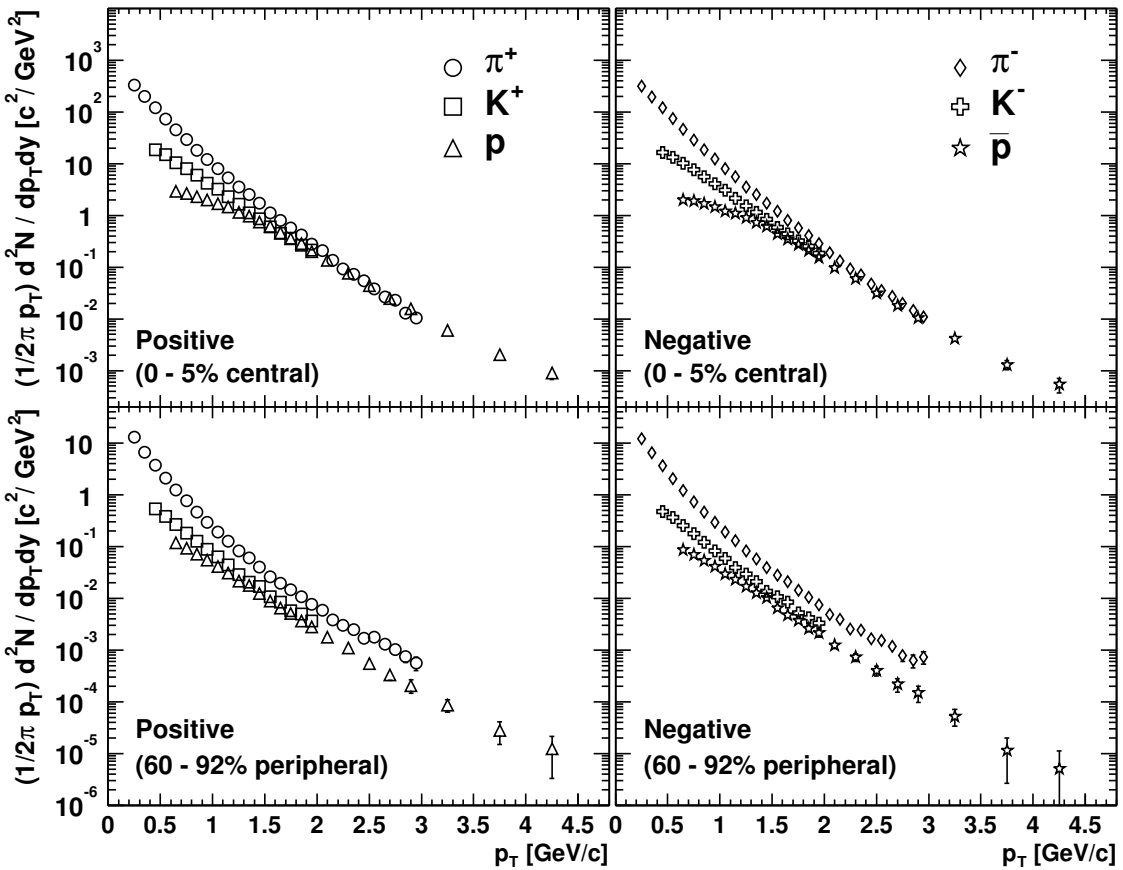


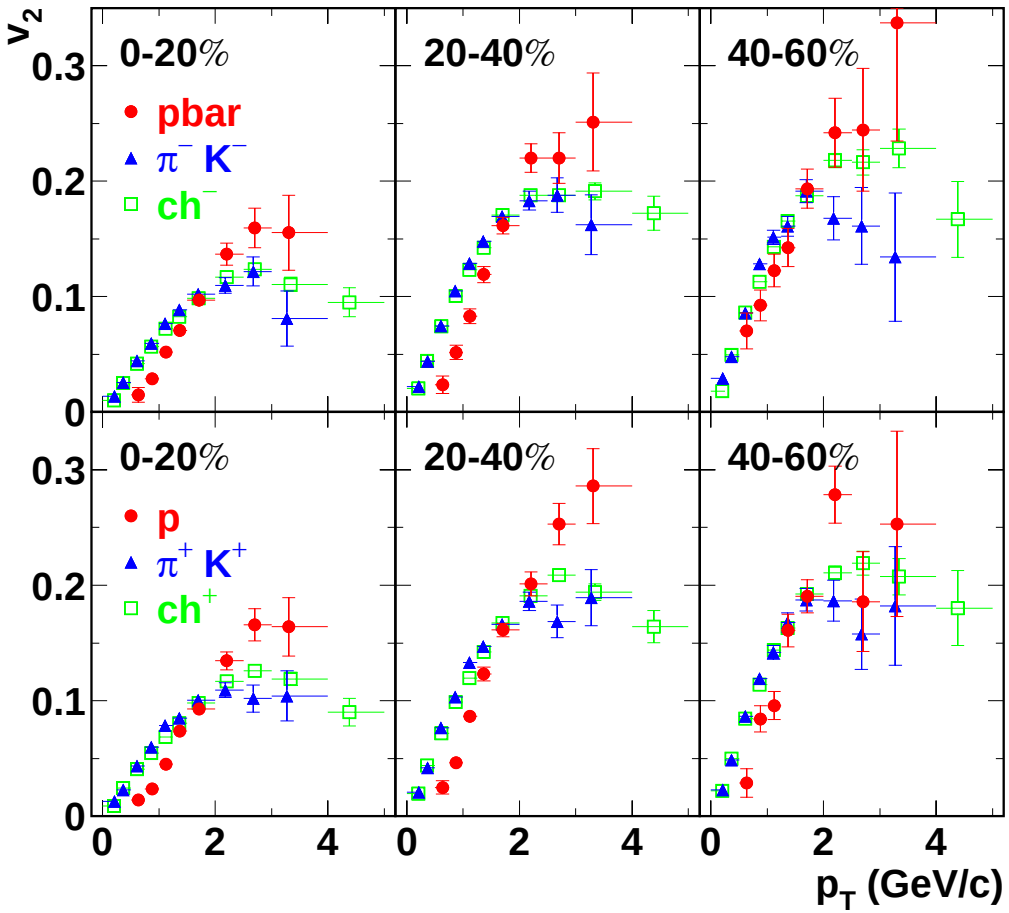
E.Iancu & R.Venugopalan, hep-ph/0303204

Hadron Spectra at RHIC $\sqrt{s} = 200$ GeV



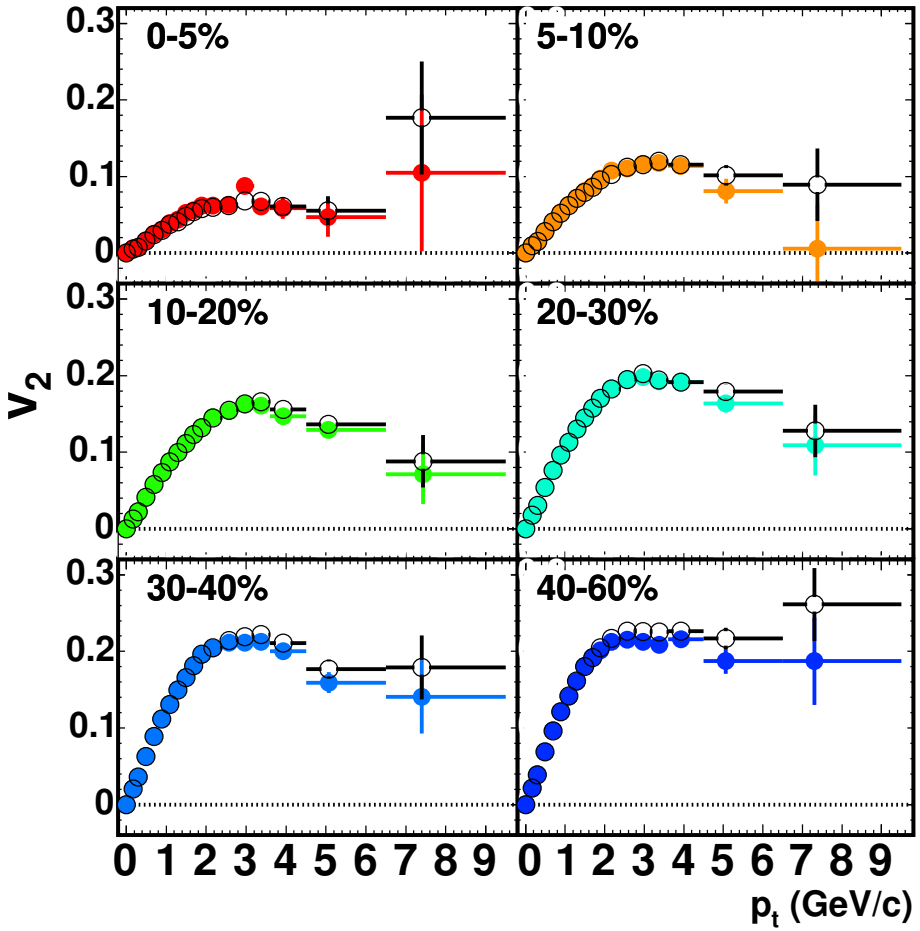
PHENIX, Phys.Rev.C69:034909,2004

Elliptic Flow at RHIC $\sqrt{s} = 200$ GeV



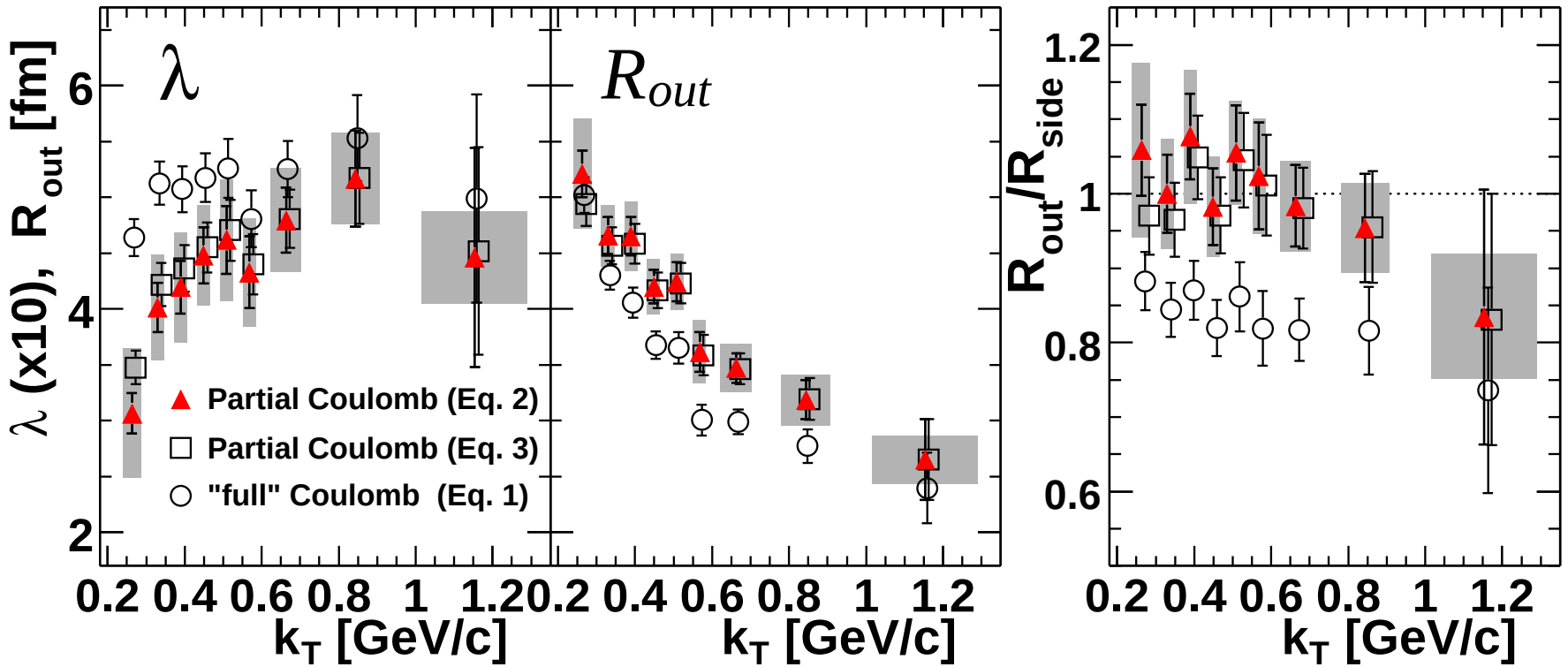
PHENIX, Phys.Rev.Lett. 91 182301 (2003)

Elliptic flow at RHIC $\sqrt{s} = 200$ GeV



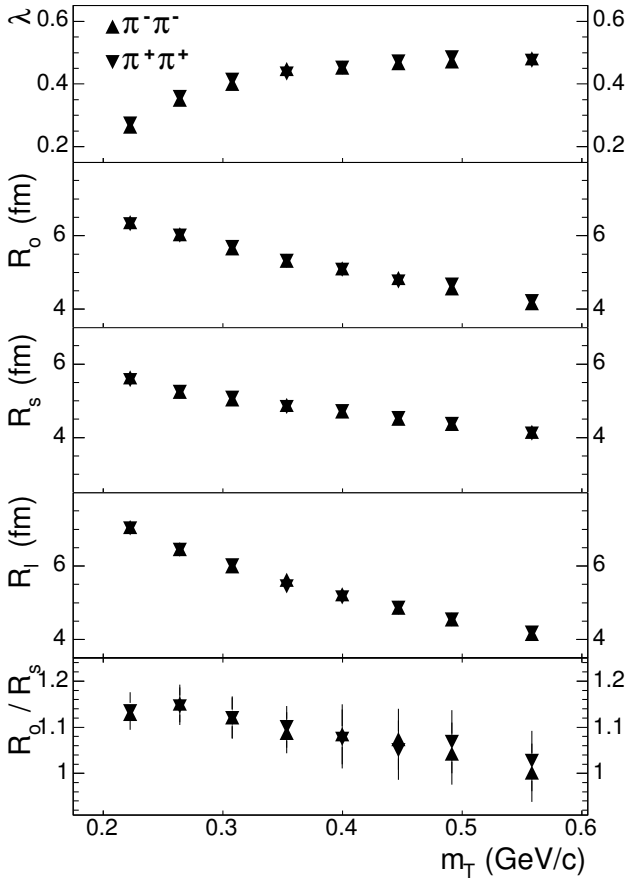
STAR, Phys. Rev. C 72 (2005) 014904

HBT Radii at RHIC $\sqrt{s} = 200$ GeV



PHENIX, Phys. Rev. Lett. 93, 152302 (2004)

HBT Radii at RHIC $\sqrt{s} = 200$ GeV



STAR, Phys. Rev. C 71 (2005) 044906

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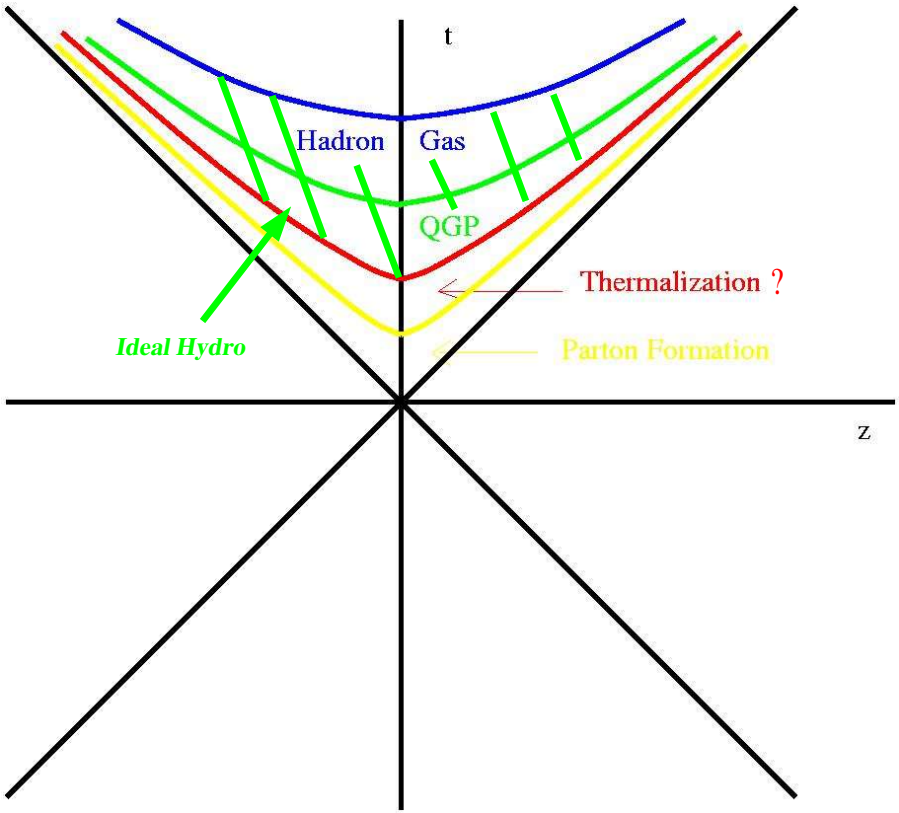
Space-time sketch of a HIC: Stage III



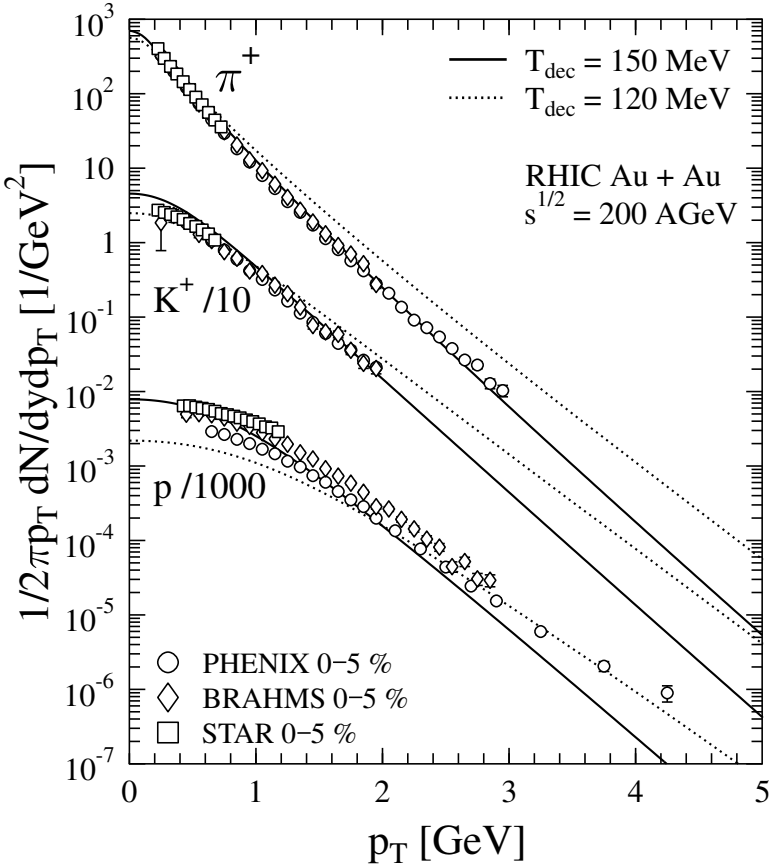
Assumptions for Ideal Hydro

- *Assume* isotropic system at $\tau < 1$ fm/c
- *Assume* the presence of an equation of state $\epsilon(p)$
- *Assume* interactions are strong enough to keep the system in equilibrium thereafter

In addition,

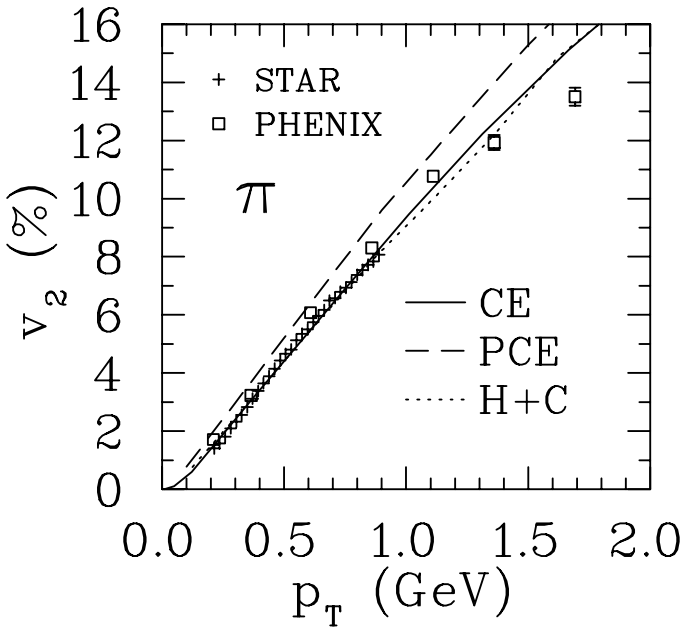
- Initial conditions for flow u^μ, temperature profile $T(\mathbf{x})$ as well as EOS are currently *chosen* to fit the data
- Freeze-out has to be modelled somewhat ad hoc

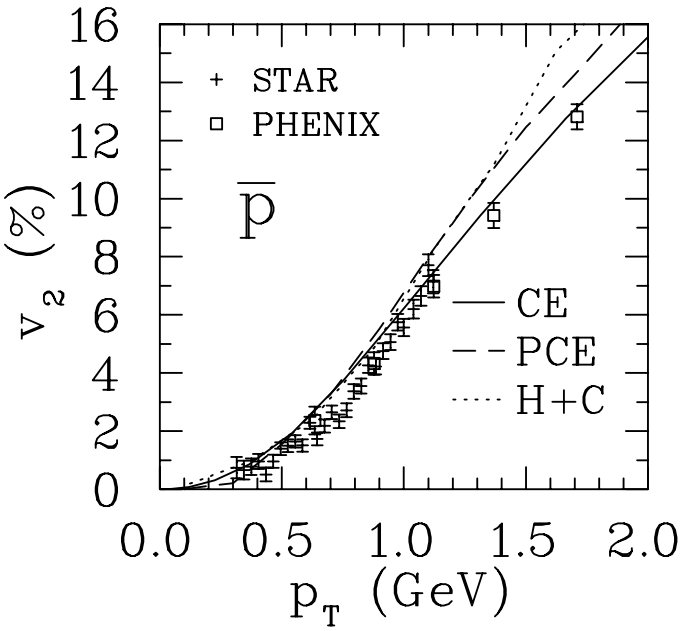
Hadron Spectra at RHIC $\sqrt{s} = 200$ GeV



P. Huovinen and P.V. Ruuskanen, nucl-th/0605008

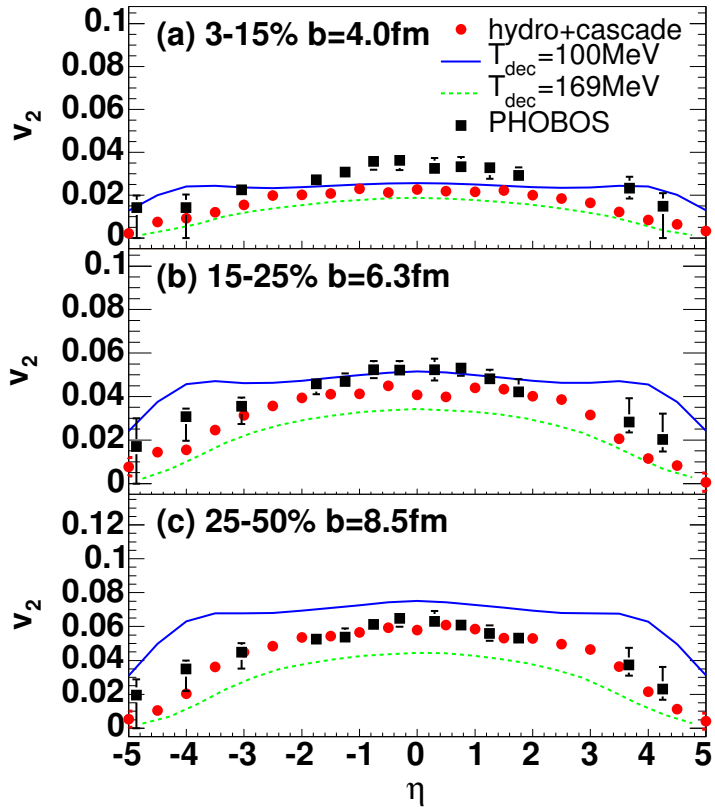
Elliptic Flow at RHIC $\sqrt{s} = 200$ GeV





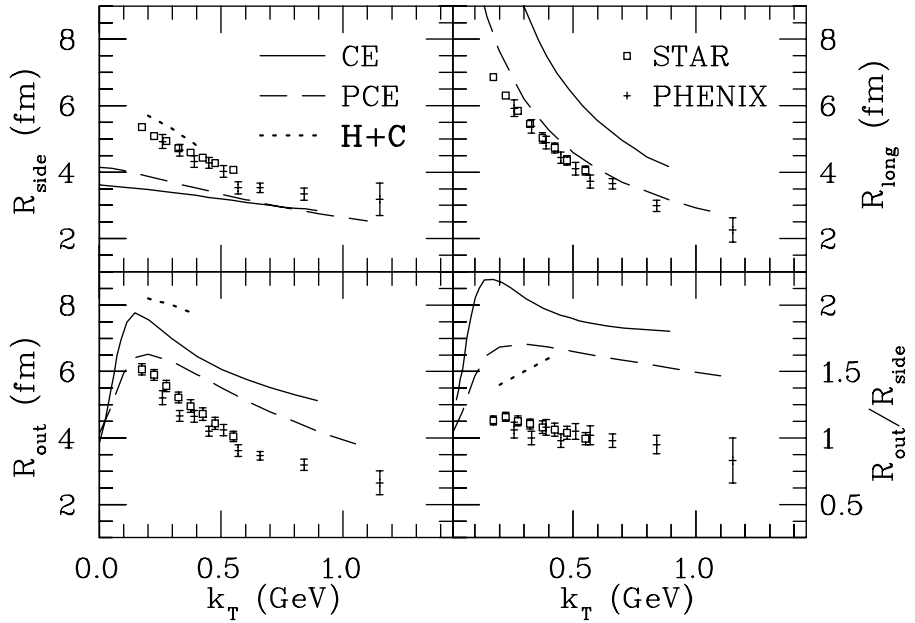
P. Huovinen and P.V. Ruuskanen, nucl-th/0605008

Elliptic flow(2) at RHIC $\sqrt{s} = 200$ GeV



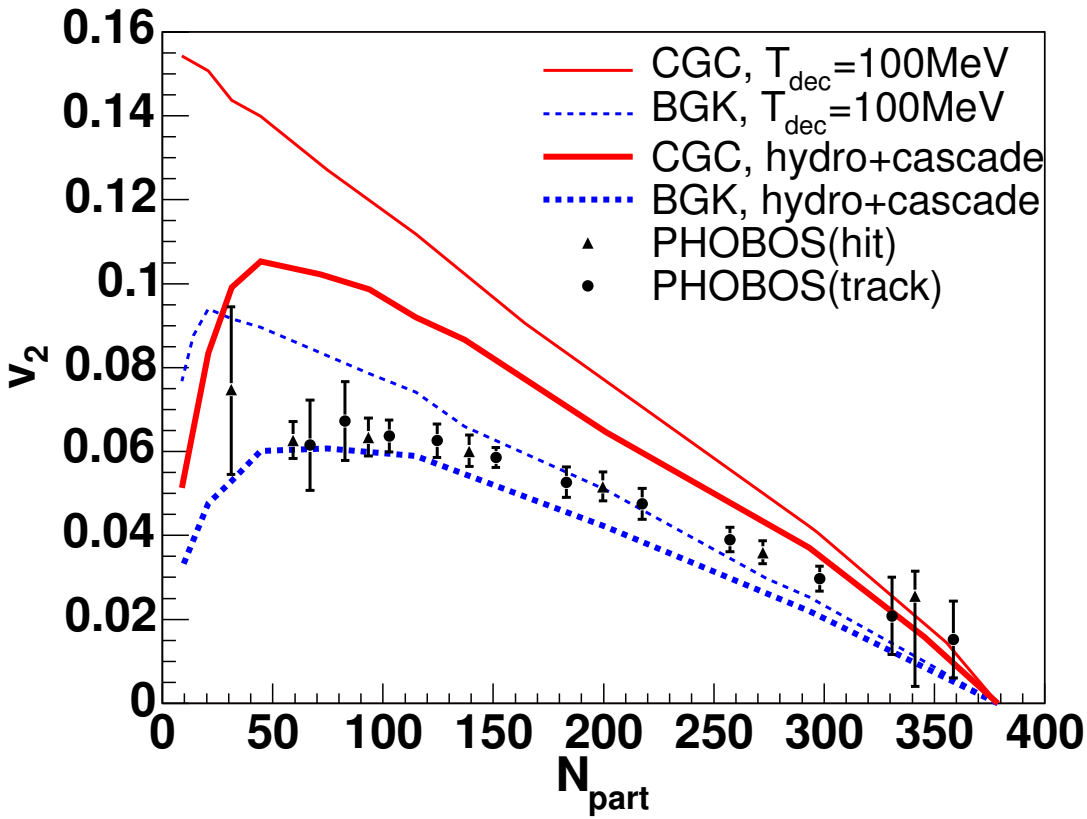
T. Hirano et al., Phys.Lett. B636 (2006) 299-304

HBT Radii at RHIC $\sqrt{s} = 200$ GeV



P. Huovinen and P.V. Ruuskanen, nucl-th/0605008

Elliptic flow(3) at RHIC $\sqrt{s} = 200$ GeV



T. Hirano et al., Phys.Lett. B636 (2006) 299-304

Ideal Hydro – Success or Failure?

Ideal Hydro...

- is able to describe part of experimental data
- describes data for very specific assumptions about initial conditions (and fails for others!)
- is *not* able to constrain value of shear viscosity
- depends strongly on isotropic system at $\tau < 1$ fm/c

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What is Viscous Hydro

- Ideal hydro is characterized by energy density $\epsilon(t, \mathbf{x})$ and flow components $u^i(t, \mathbf{x})$
- Energy-momentum tensor in ideal hydro:

$$T_0^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

- Viscous hydro allows departures from equilibrium
- Energy-momentum tensor in viscous hydro

$$T^{\mu\nu} = T_0^{\mu\nu} + \Pi^{\mu\nu}$$

What is that $\Pi^{\mu\nu}$?

Energy-momentum tensor for relativistic Navier-Stokes:

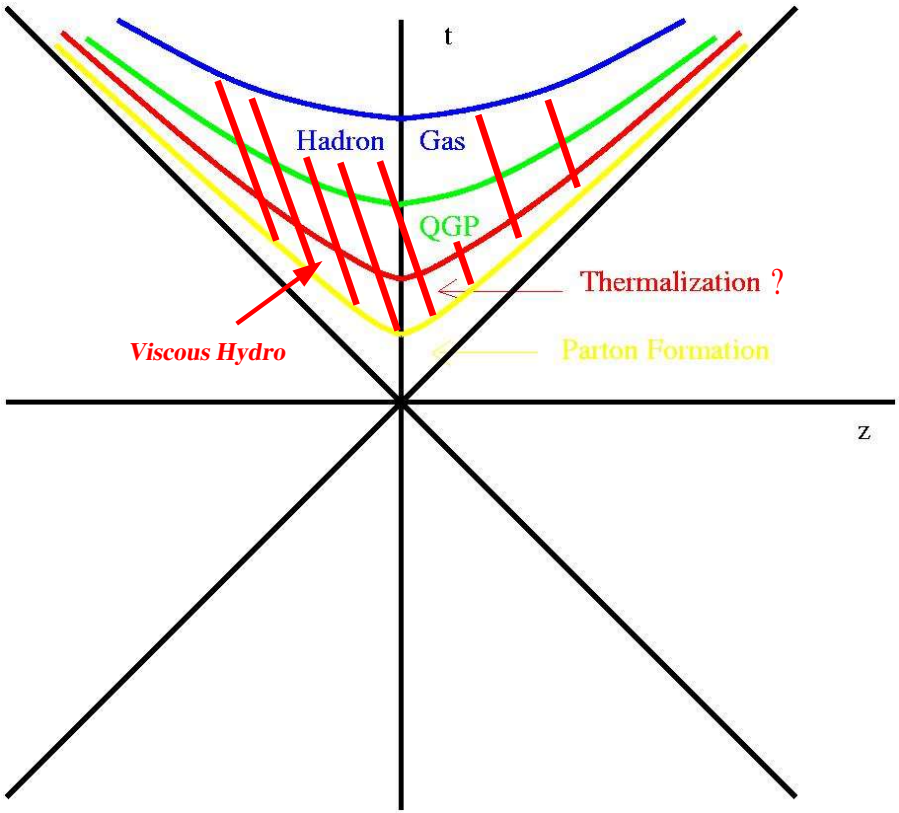
$$T^{\mu\nu} = \begin{array}{c} \\ t \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{cccc} t & x & y & z \end{array} \\ \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p + \frac{2}{3}\frac{\eta}{\tau} & 0 & 0 \\ 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & p - \frac{4}{3}\frac{\eta}{\tau} \end{pmatrix} \end{array},$$

Comparing $T^{xx} = p + \Pi^{xx}$ yields $\Pi^{xx} = \frac{2}{3}\frac{\eta}{\tau}$

Why Bother using Viscous Hydro

- Can constrain value of shear viscosity η
- Might be *better* in fitting experimental data than ideal hydro
- Does *not* require complete isotropy ($p_\perp \neq p_z$)

Space-time sketch of a HIC: Stage III(2)



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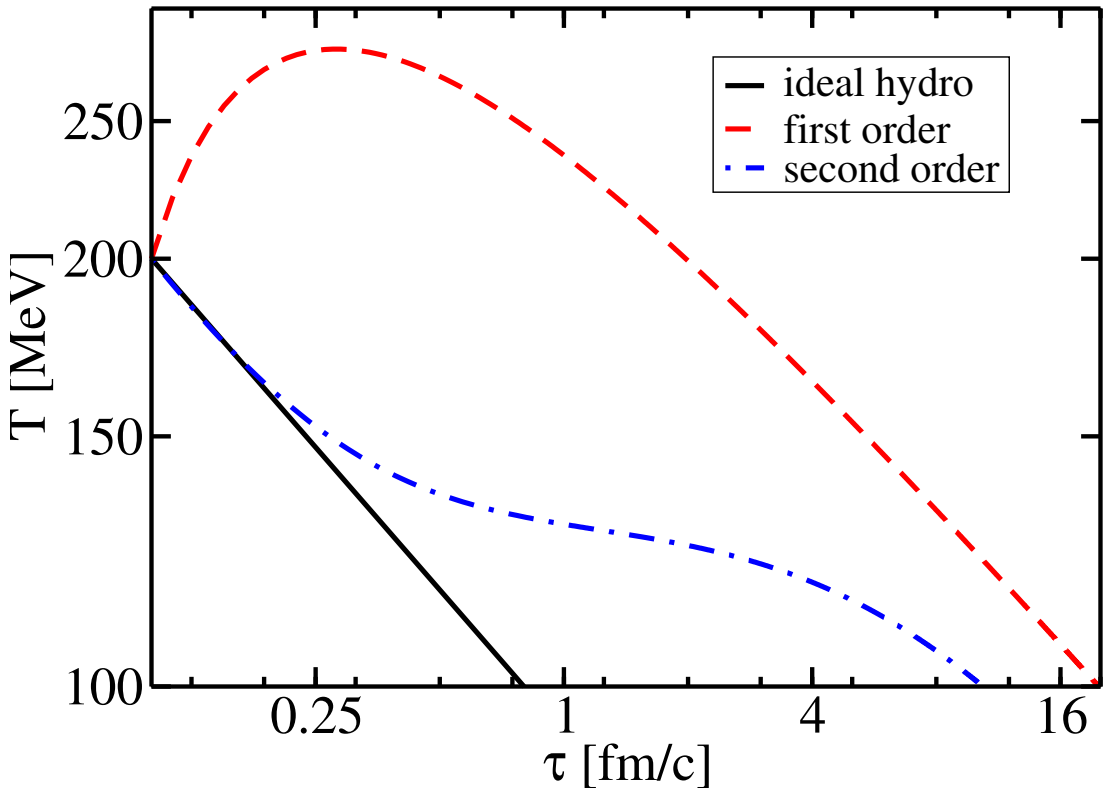
Why Viscous Hydro is Not Straightforward

- Easiest way to incorporate viscous effects is using Navier-Stokes equations
- Unfortunately, *relativistic* Navier-Stokes is *acausal*
- It can be shown that this is due to the *instantaneous* backreaction of viscosity on bulk variables (temperature,...) “first-oder theory”
- To cure this unphysical behavior, one has to include relaxation time (“second-order theory”, “Israel-Stewart”)

System Temperature Evolution

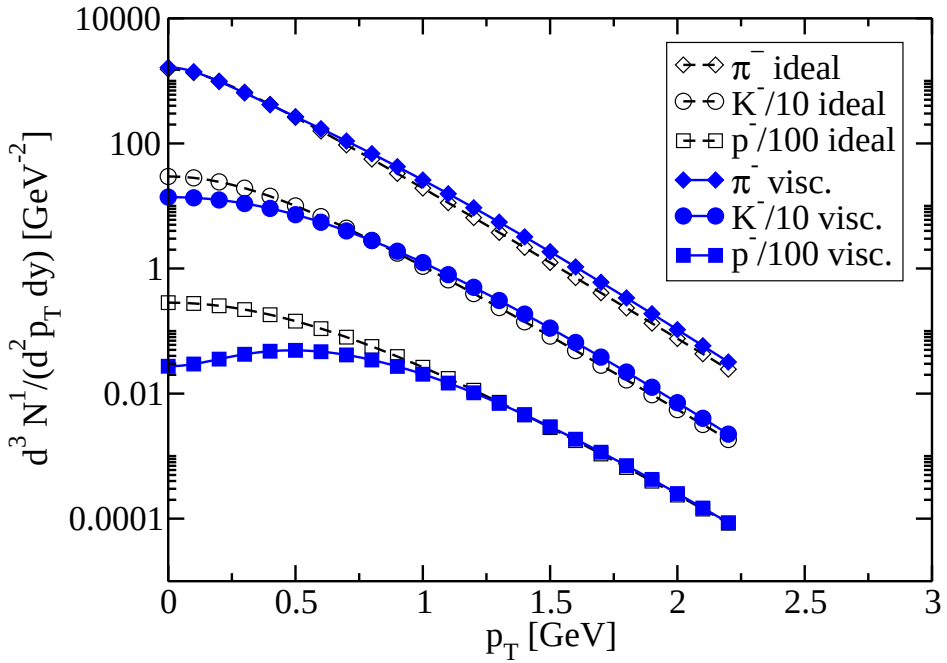
$\eta/s \sim 0.3$, Bjorken flow only



R. Baier, PR and U.A.Wiedemann, hep-ph/0602249

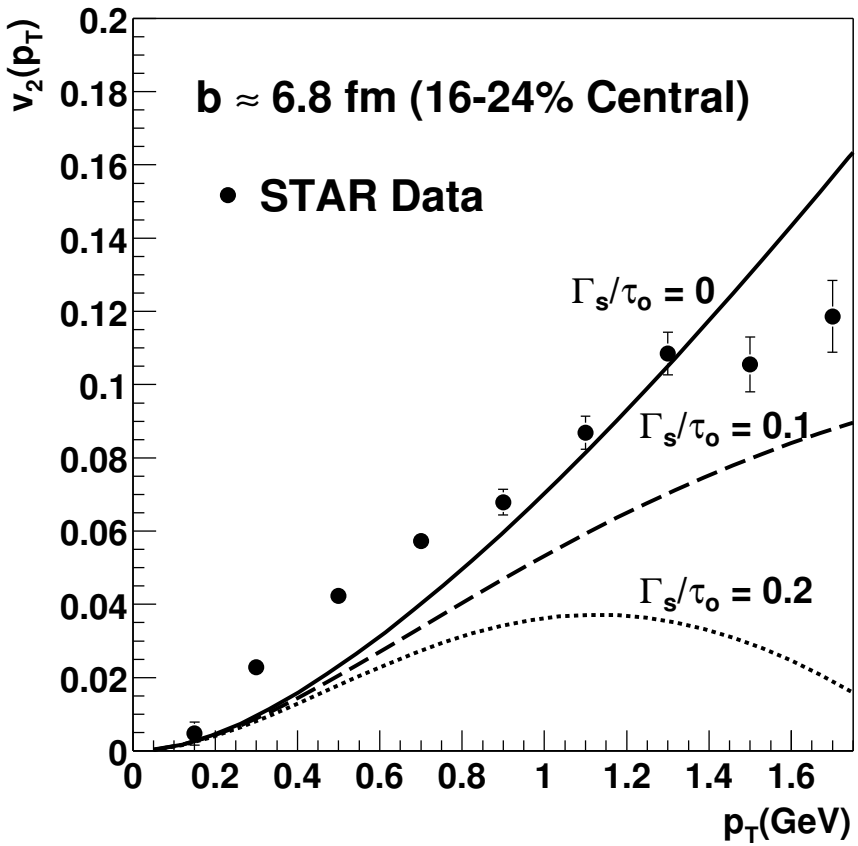
Causal Relativistic Viscous Hydro Particle Spectra

$\eta/s \sim 0.3$, Bjorken flow only



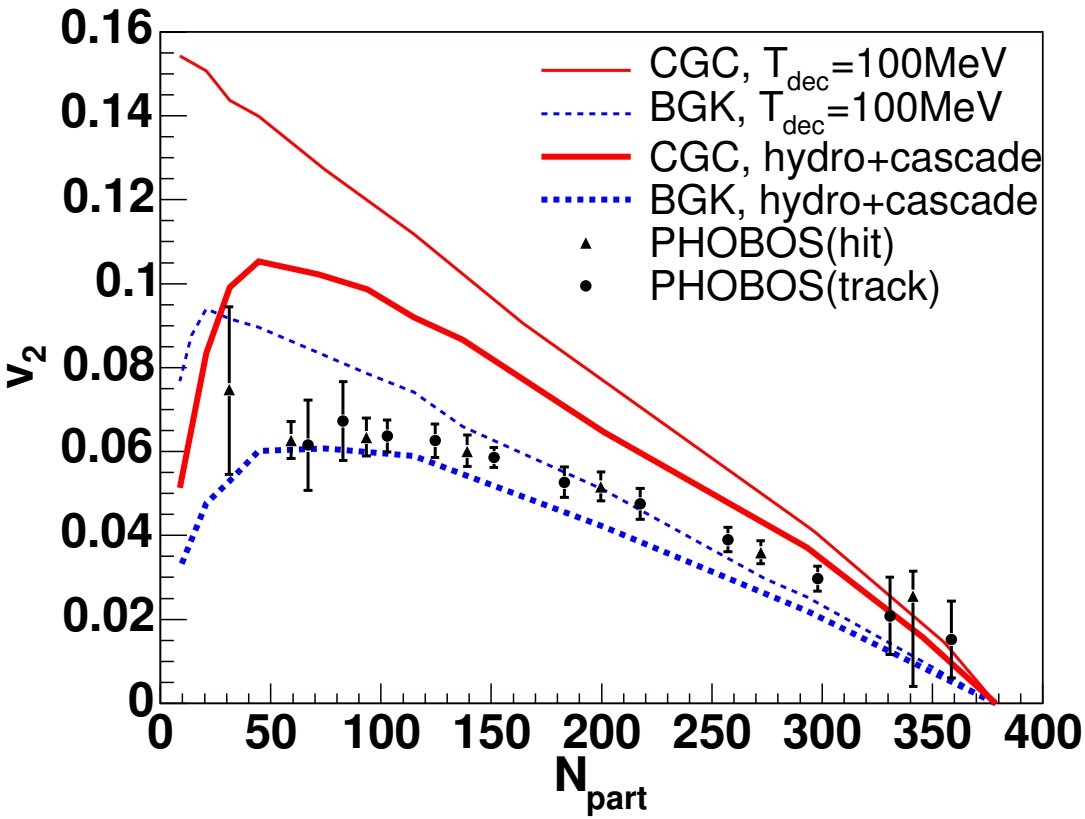
R. Baier, PR and U.A.Wiedemann, hep-ph/0602249

Isn't Viscosity ruled out from data on v_2?



D. Teaney, Phys.Rev.C68:034913,2003

No!



T. Hirano et al., Phys.Lett. B636 (2006) 299-304

Outlook on Viscous Hydro

Work in Progress on

- causal viscous hydro in 1+1 dimensions (+comparison to data on hadron spectra & HBT radii)
- causal viscous hydro in 2+1 dimensions (+comparison to data on v_2)
- (eventually) causal viscous hydro in 3+1 dimensions

Why does it take so long? Because it's hard!

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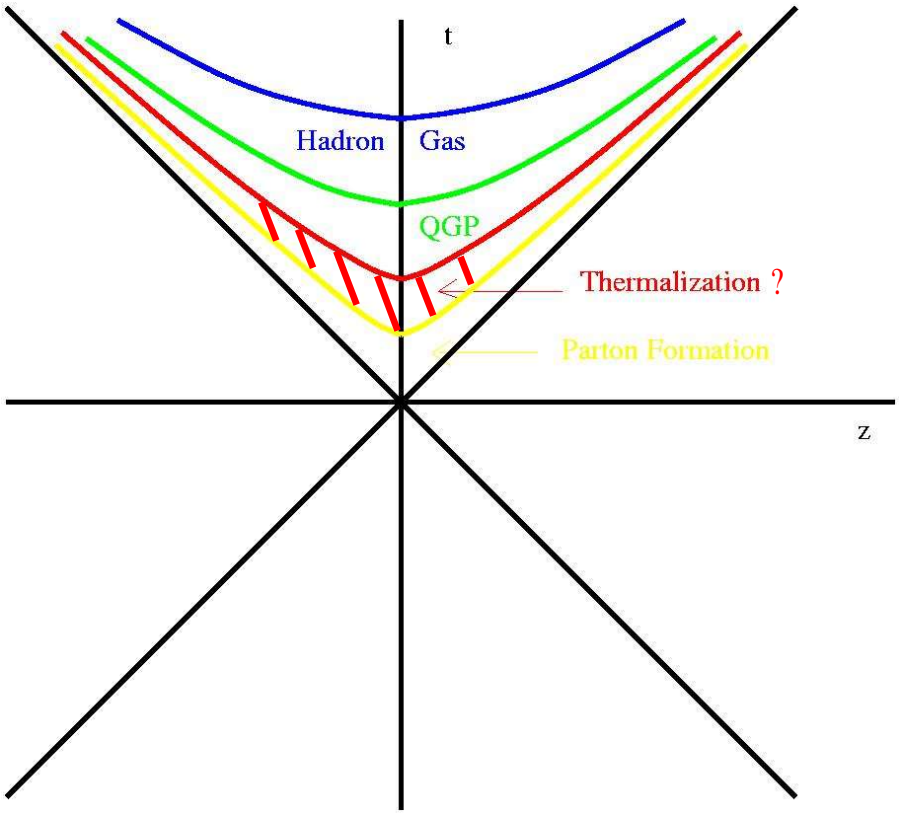
Space-time sketch of a HIC: Stage II



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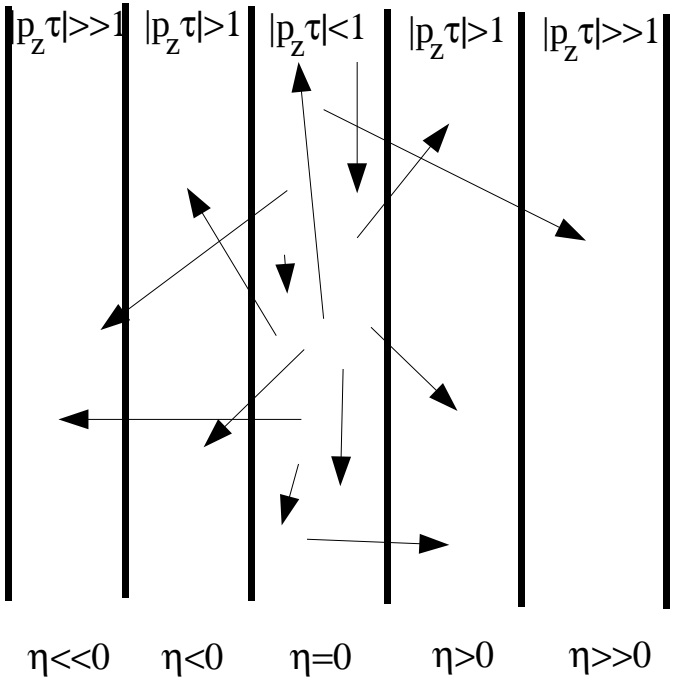
Momentum Space Anisotropy



Because of expansion, $p_z < \tau^{-1}$ at $\eta = 0$

Physics of Anisotropic Plasmas

- Momentum space anisotropy creates plasma instabilities
- These manifest themselves through exponentially growing magnetic fields $B(\mathbf{k}) \sim B_0 \exp(\Gamma(\mathbf{k})t)$ for soft momenta $\mathbf{k}$
- Since $|p_z| \ll |p_\perp|$ expect predominantly growth in $B_\perp$
- This may be important for isotropization because Lorentz force can “bend” particles out of transverse plane

Weibel-Instability – A Physical Picture

Current Filamentation

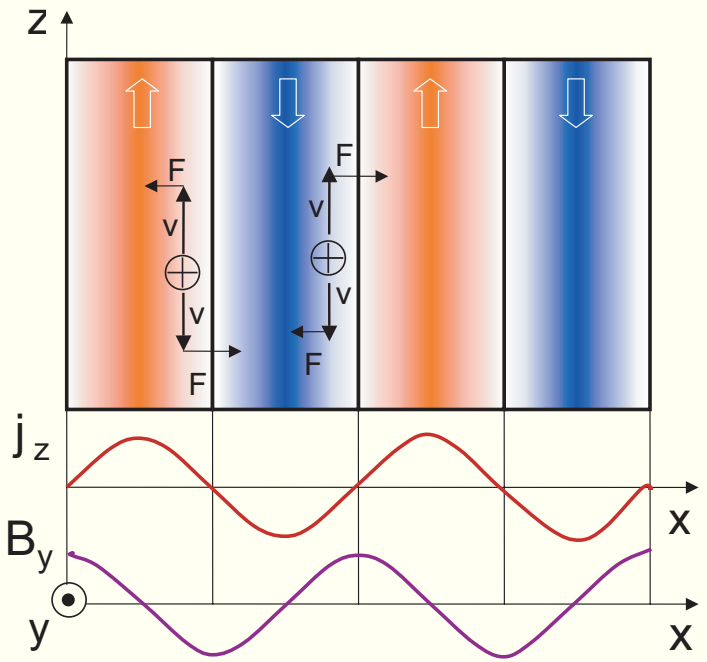


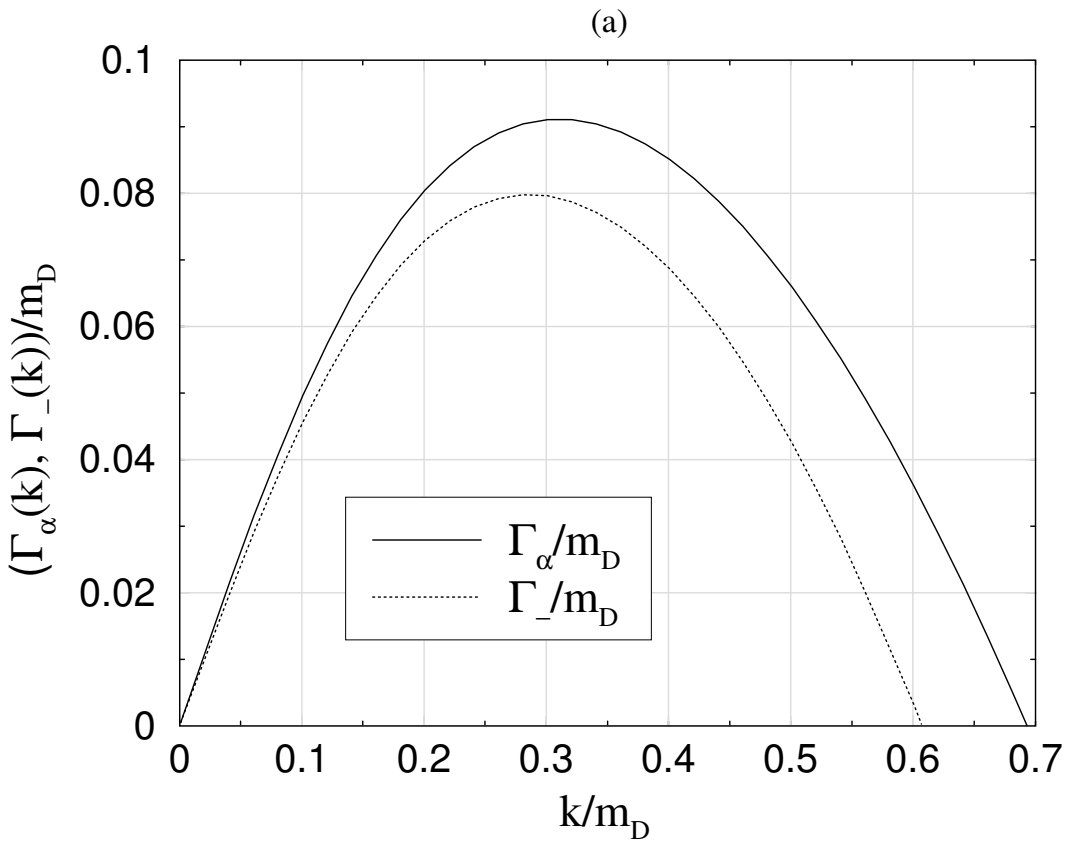
Figure courtesy S. Mrówczyński

Strong and Electroweak Matter 2006 – 12 May 2006 – p. 4/31

M.Strickland, SEWM06

Non-Abelian Plasma Instabilities

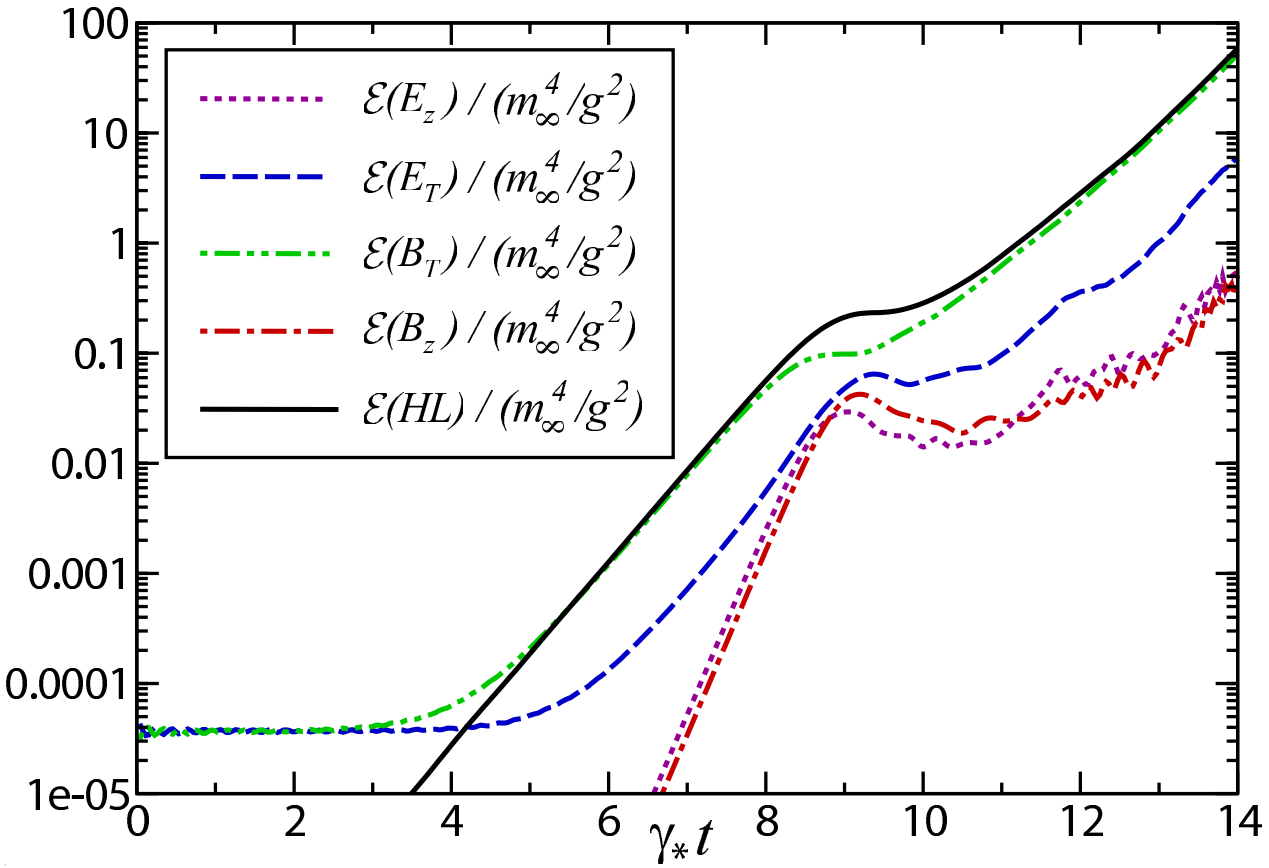
Theoretical Results – Growth Rate in Static System



PR+M.Strickland, PRD 68 (2003)

Non-Abelian Plasma Instabilities

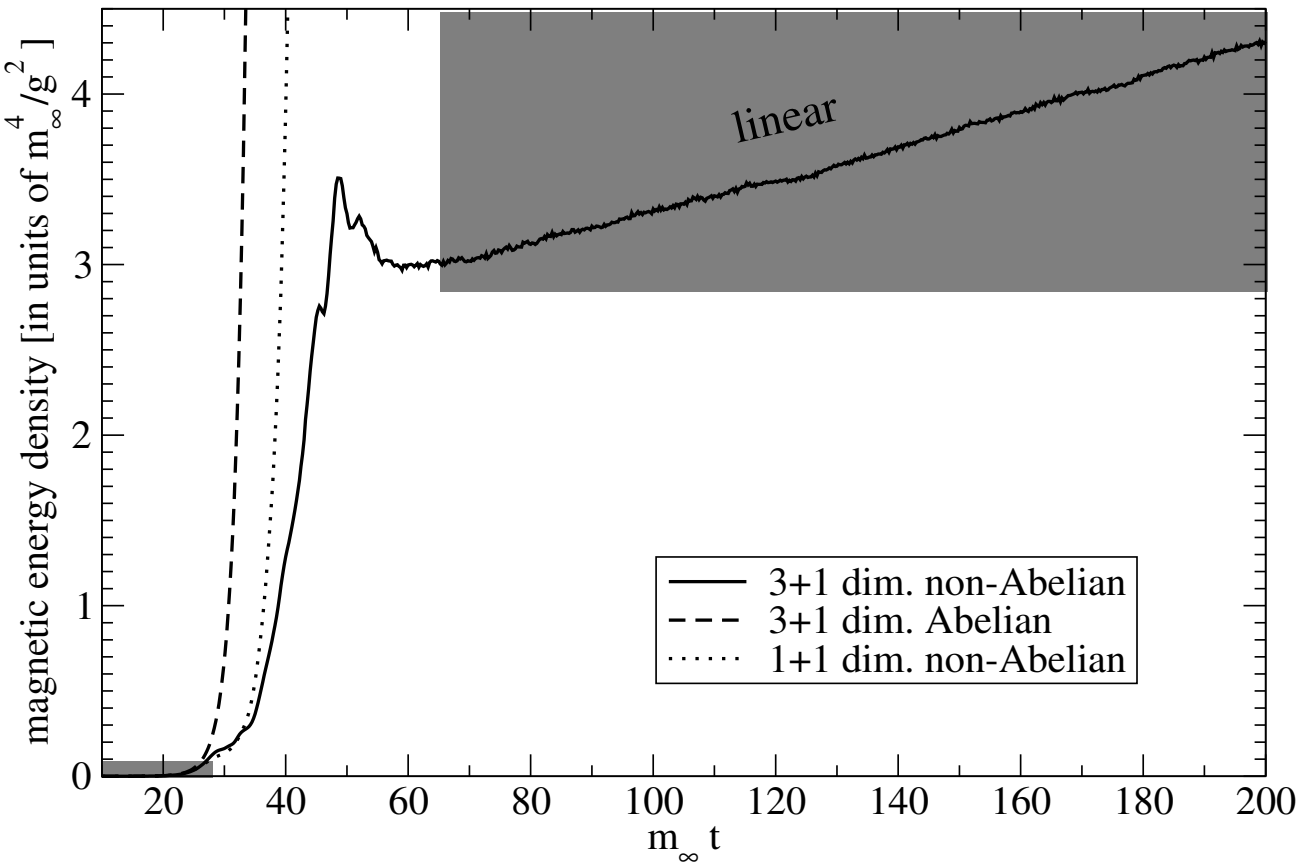
Theoretical Results – Numerical Simulations



A.Rebhan, PR, M.Strickland, Phys.Rev.Lett.94:102303,2005

Non-Abelian Plasma Instabilities

Theoretical Results – Nonabelian Saturation

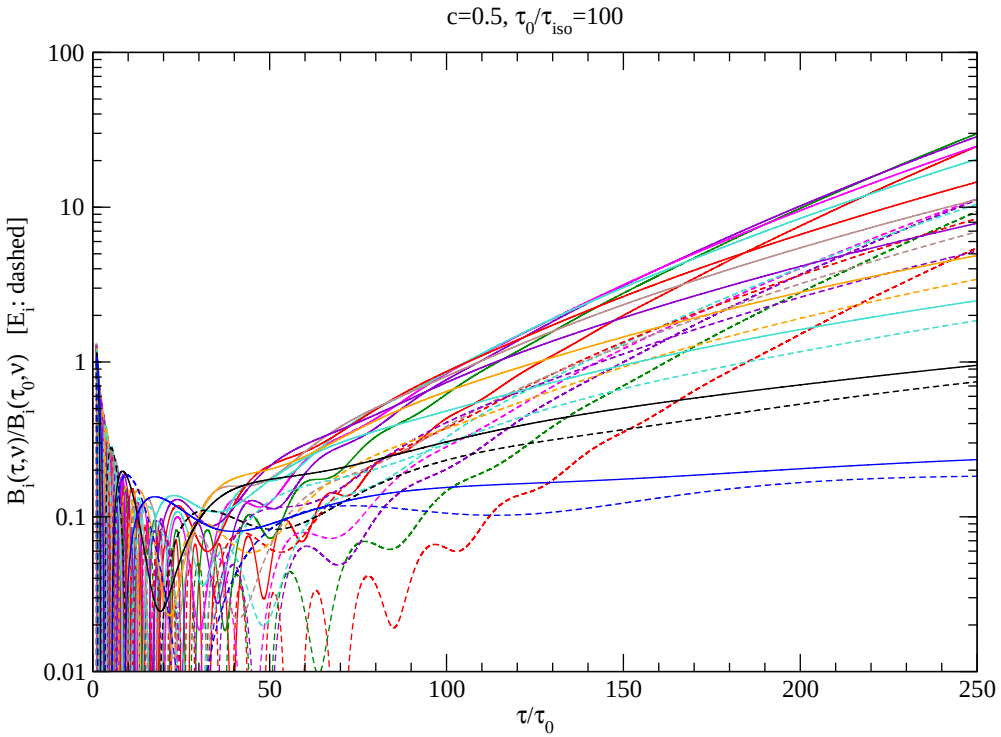


P. Arnold, G.D. Moore, L. G. Yaffe, Phys.Rev.D72:054003,2005

Non-Abelian Plasma Instabilities

Theoretical Results – Effect of Expansion

Magnetic and electric field strength for each Fourier mode in rapidity



PR and A. Rebhan, adapted from hep-ph/0605064

Non-Abelian Plasma Instabilities

Summary

- PI's are generically present if a plasma is formed after a HIC
- The basic features of PI's have been identified qualitatively
- Quantitative results depend strongly on initial conditions (the output from Stage I); e.g. $\frac{d^3N}{d^3k}$ from Stage I decides whether PI's can achieve an isotropic system at $\tau < 1$fm/c or not

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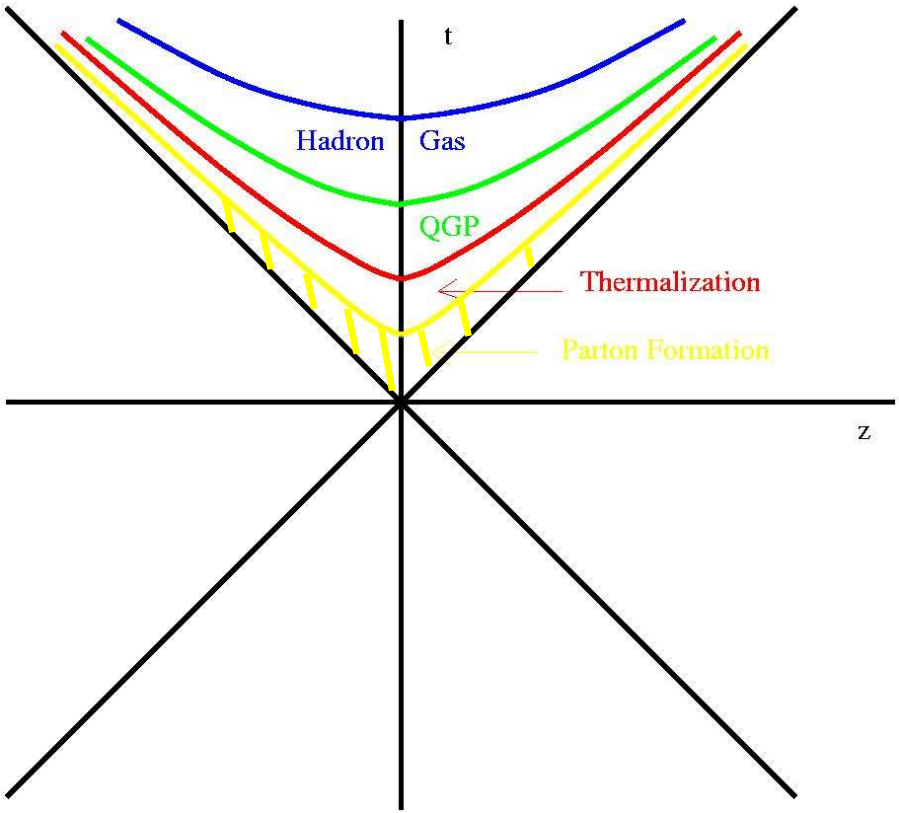
Space-time sketch of a HIC: Stage I



What the CGC provides
McLerran-Venugopalan-Model

- Color source for a large nucleus moving (nearly) with $v \sim c$

$$J_a^\mu = \delta_+^\mu \rho_a(x_\perp)\delta(x^-)$$

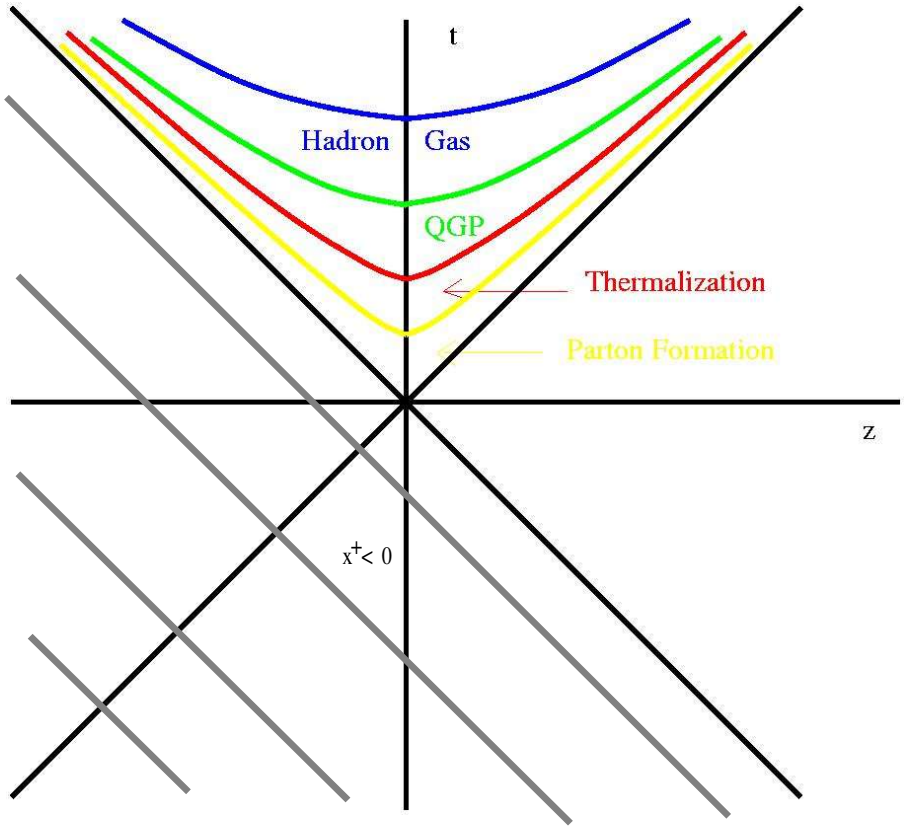
where $x^\pm = (t \pm z)/\sqrt{2}$.

- Color charges ρ_a in the McLerran-Venugopalan-Model:

$$< \rho_a(x_\perp)\rho_b(y_\perp) >= g^2\mu^2\delta_{ab}\delta^2(x_\perp - y_\perp)$$

where $g^2\mu \sim Q_s \sim 1$ GeV at RHIC

Space-time sketch of a HIC: “Stage 0”



Model of a Heavy-Ion Collision

- Consider two infinitely large nuclei

$$J_a^\mu = \delta_+^\mu \rho_a^{(1)}(x_\perp)\delta(x^-) + \delta_-^\mu \rho_a^{(2)}(x_\perp)\delta(x^+)$$

- Nuclei interact only at $\tau = \sqrt{2x^+x^-} = 0$
- Property of $A^\mu(\tau = 0)$: independent of rapidity $\eta = \operatorname{arctanh}\frac{z}{t}$
- "Boost-Invariance of the fields:" Can solve YM-equations on 2+1 D lattice in forward lightcone (Krasnitz, Venugopalan, Nara, Lappi)

The Glasma

- This earliest stage following a HIC has been termed "Glasma" = part Color Glass Condensate and Quark Gluon Plasma
- It can be studied by numerically solving classical Yang-Mills equations in the forward lightcone
- Using the Color Glass Condensate model as initial condition *before the collision* this provides $\frac{d^3N}{d^3k}$ for Stage II
- Results assuming *exact* boost-invariance of the fields exist, but. . .

Initial Conditions for a Heavy-Ion Collision

More Realistic Model

- Exact boost-invariance not achievable because of
- geometric effects (cannot accelerate Au to $v = c$)
- quantum effects (BFKL predicts sizeable rapidity fluctuations)
- => Sources will not be confined to the light-cone (no exact δ-functions)
- There will be initial ($\tau \ll 1$fm/c) fluctuations in rapidity (though size& spectrum currently unknown)

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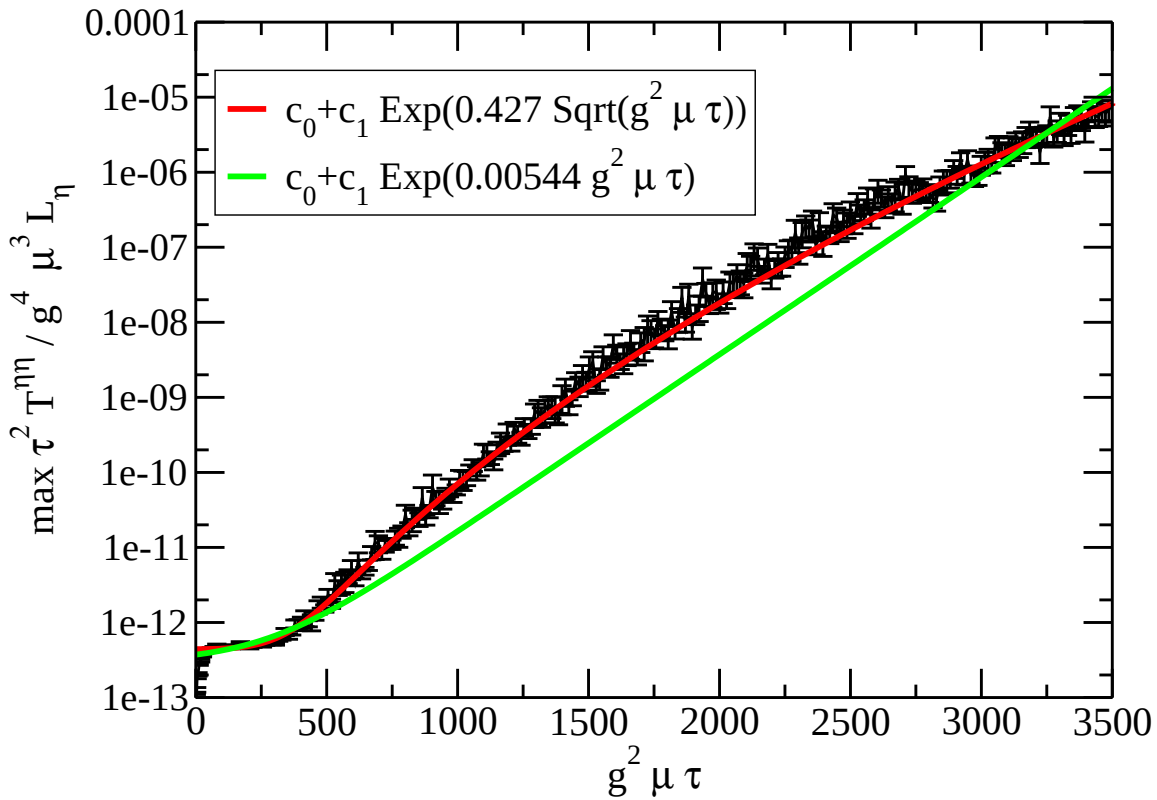
Outline

1. Introduction
   - Experimental Data
   - Hydrodynamics
2. Viscous Hydrodynamics
   - Set-up
   - First Results
3. Isotropization in Heavy-Ion Collisions
   - Non-Abelian Plasma Instabilities
   - Initial Conditions for HIC – The Glasma
   - The Unstable Glasma

Plasma Instabilities in the Glasma

- Expansion is incorporated in numerical simulations of classical YM
- Expansion leads to anisotropic momentum space distributions
- Anisotropic momentum space distributions lead to plasma instabilities
- Can instabilities occur in purely classical (non-abelian) gluon dynamics?

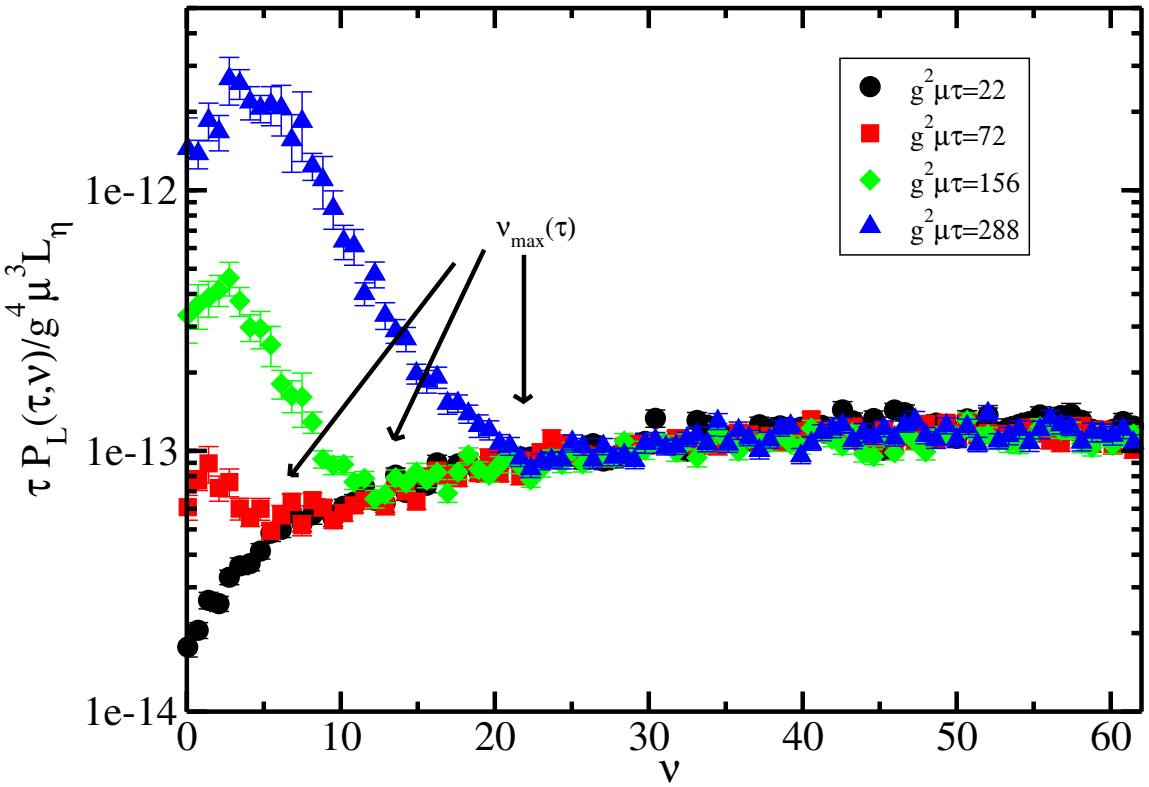
Yes!



PR+R.Venugopalan, PRL96 (2006)

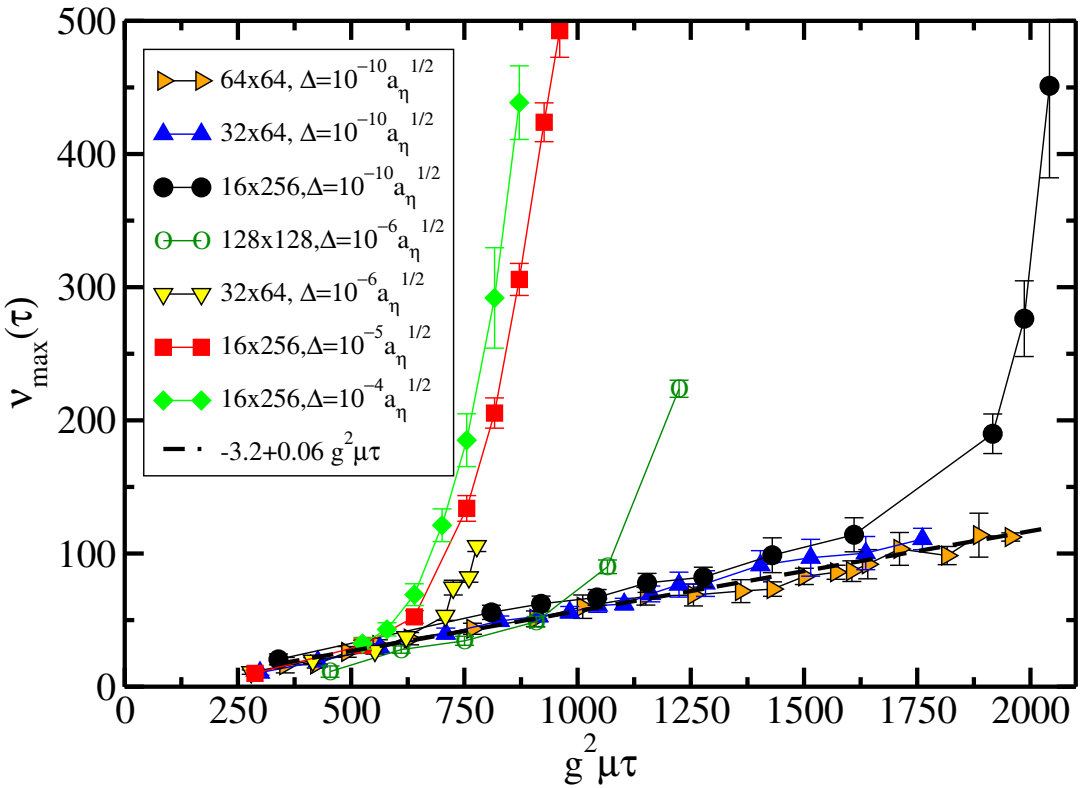
Simulating the Glasma in 3+1D

Longitudinal Mode Spectrum



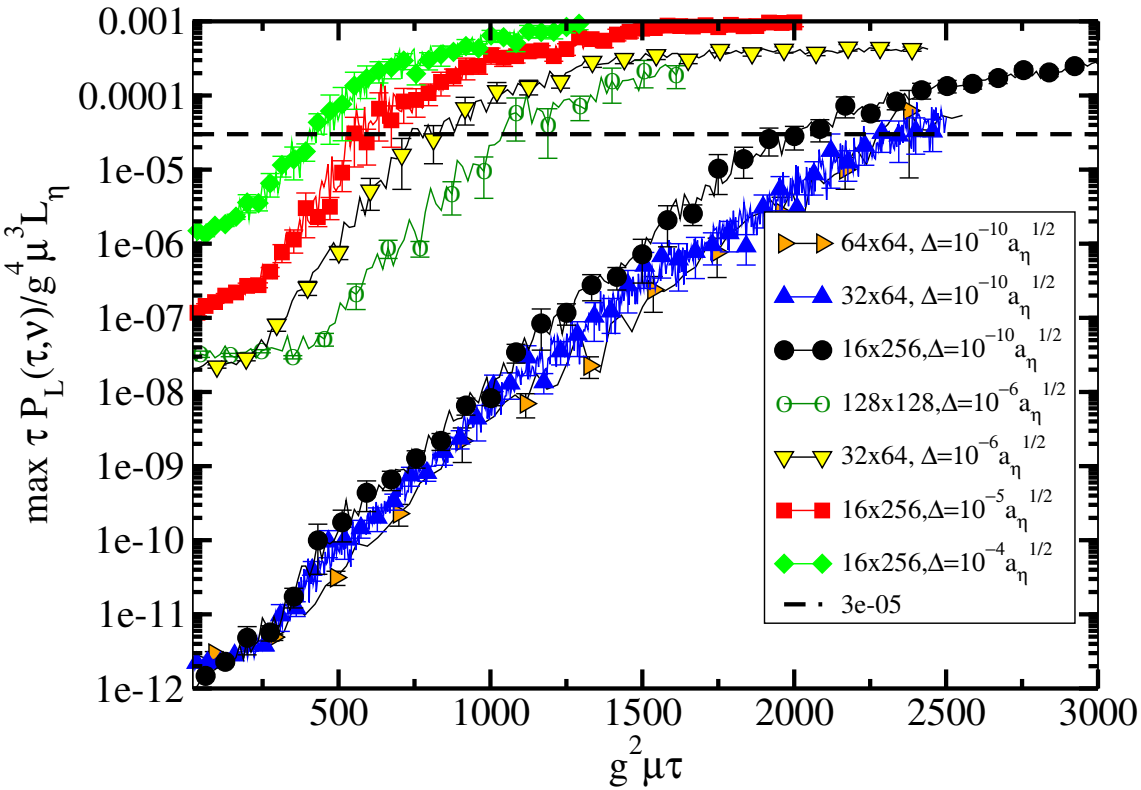
PR+R.Venugopalan, hep-ph/0605045

Time Evolution of $\nu_{\max}$



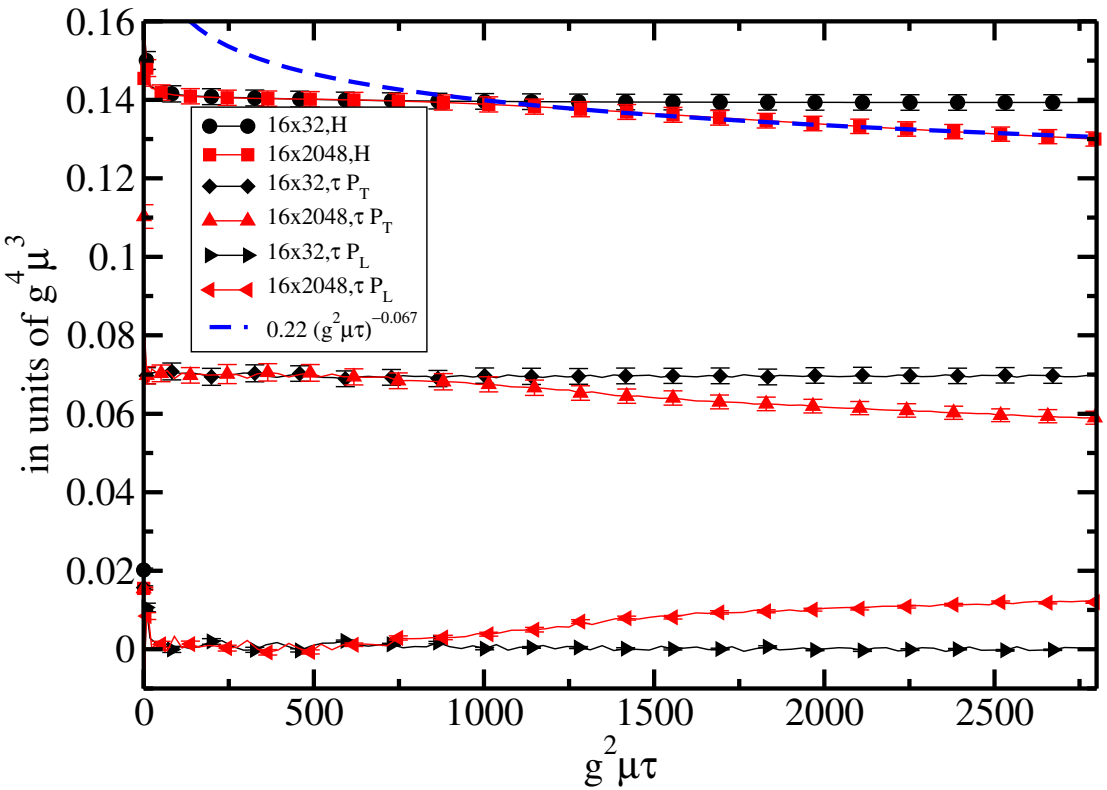
PR+R.Venugopalan, hep-ph/0605045

Relation to Fluctuation Amplitudes



PR+R.Venugopalan, hep-ph/0605045

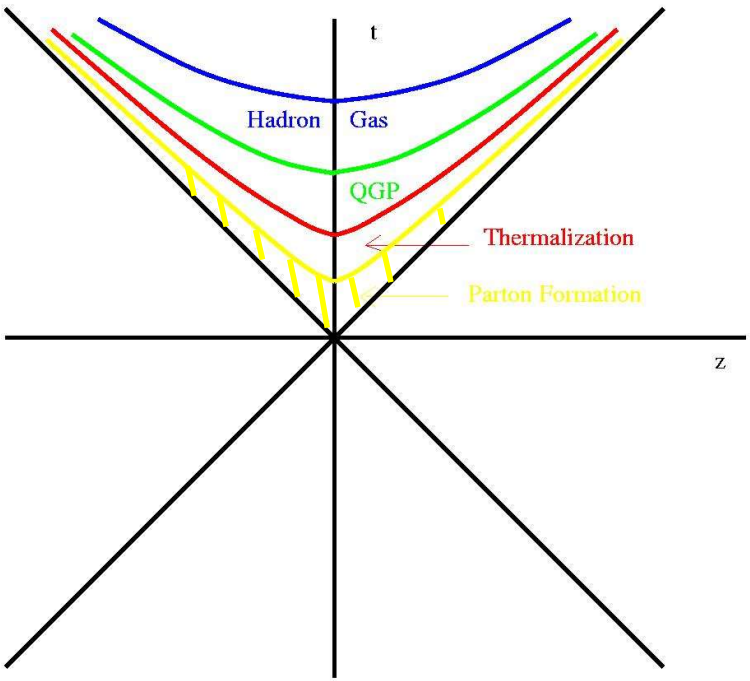
Towards Isotropy



PR+R.Venugopalan, hep-ph/0605045

Summary (1/5)

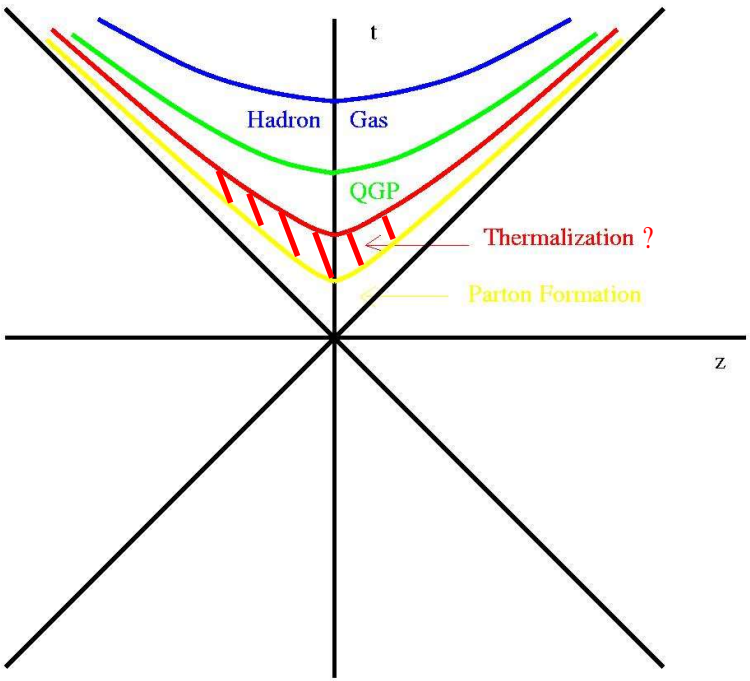
Stage I: Parton Formation – From CGC to the Glasma



- YM on the Lattice: consistent framework, shows PI's + isotropization, provides $\frac{d^3N}{d^3k}$ for stage II
- Open problems: $\frac{dN}{dy}$ at $\tau \ll 1$ fm/c currently unknown

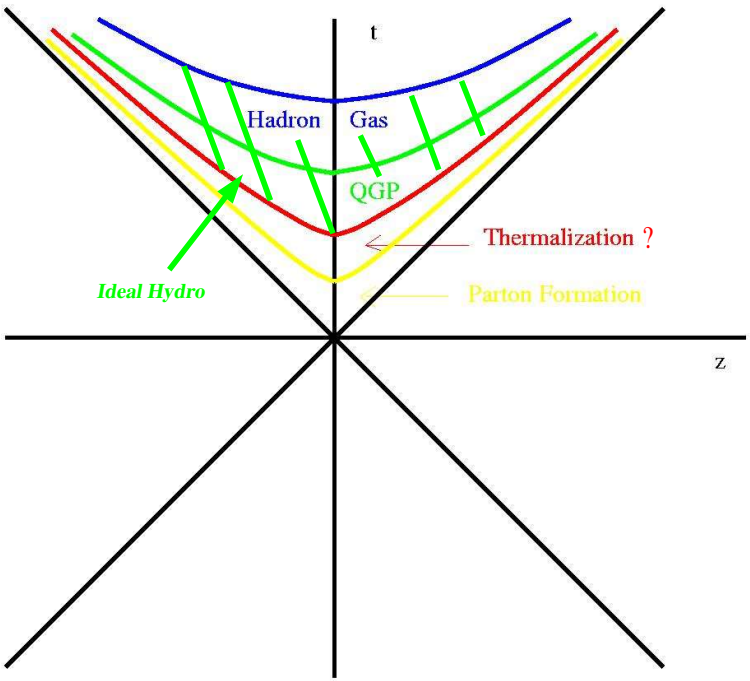
Summary (2/5)

Stage II: Thermalization – From Glasma to the QGP



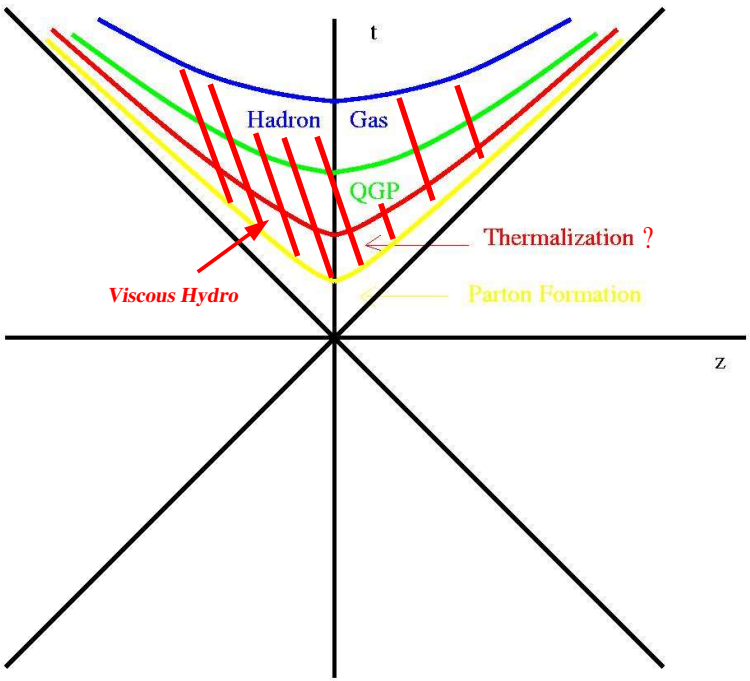
- Plasma Instabilities: may provide fast isotropization + other interesting physics
- Open problems: $\frac{d^3N}{d^3k}$ input from Stage I unknown, sets time-scale

Summary (3/5)
Stage III (1): QGP to Freeze-out



- Ideal Hydro: partly fits experimental data
- Open problems: Stage II does not seem to lead to fast isotropy as required by hydro; CGC seems to destroy agreement with data

Summary (4/5)
Stage III (2): Glasma to Freeze-out



- Viscous Hydro: works also for non-equilibrium systems
- Open problems: still debates about formulation, no comparison to data yet

Summary (5/5)

- Equilibration in HIC is still not fully understood (but we're working on it!)
- Main current difficulty: $\frac{dN}{dy}$ at $\tau \ll 1$ fm/c undetermined
- My personal bet: no early $\tau < 1$ fm/c equilibration (not even with PI's!)
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- Bad news for experimentalists: no prediction from Stage I/II that can be compared to data directly (but maybe soon?)
- Good news for theorists: there is still work to do!

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