Thermalization of (mini-) jets in a quark-gluon plasma

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Work in progress with A. H. Mueller, E. Iancu · · ·

Outline

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- **Thermalization of (mini-jet) in a QGP**
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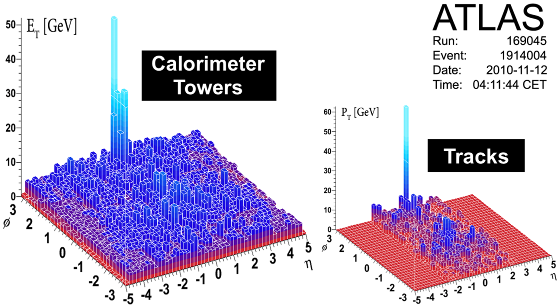
  3. Analytical solution in the simplified case
  4. Numerical studies
- **Summary and perspective**

1.1 Motivations

- **Experimental observation: Dijet asymmetry**

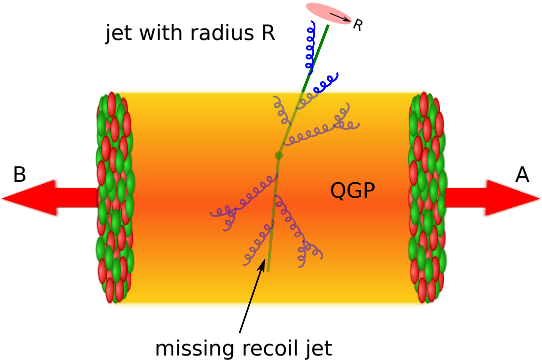


One jet with $E_T > 100$ GeV and no evident recoiling jet!

G. Aad et al. [ATLAS Collaboration], Phys. Rev. Lett. **105***, 252303 (2010) [arXiv:1011.6182 [hep-ex]].*

1.1 Motivations

- **Theoretical explanation: Parton energy loss**



Solid green lines represent either a high-energy quark or gluon.

1.2 Radiative parton energy loss

- **Medium-induced gluon spectrum at LO**

- **Parametric result** ($\omega \ll E$)

$$\omega \frac{dI}{d\omega dt} \sim \alpha_s N_c \frac{1}{t_{form}(\omega)} \sim \frac{1}{t_{br}(\omega)}$$

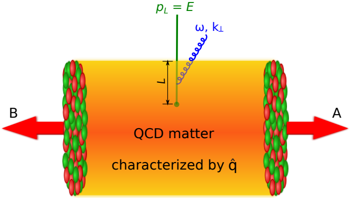
$$\sim \alpha_s N_c \sqrt{\frac{\hat{q}}{\omega}},$$

where the formation time is

$$t_{form}(\omega) \sim \frac{\omega}{\langle k_\perp^2 \rangle} = \frac{\omega}{\hat{q}\, t_{form}(\omega)}$$

$$\Rightarrow t_{form}(\omega) \sim \sqrt{\frac{\omega}{\hat{q}}}$$

with $\hat{q}$ the transport coefficient.

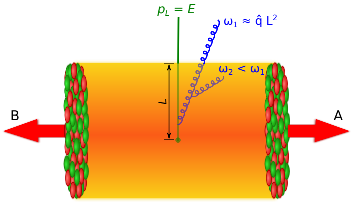


$$\Delta E_1 \sim \alpha_s N_c \omega_c = \alpha_s N_c \hat{q} L^2$$

R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B **483**, *291 (1997). B. G. Zakharov, JETP Lett.* **63** *(1996) 952; JETP Lett.* **65** *(1997) 615.· · ·*

Higher order calculation doable in the following TWO limits!

- **Double logarithmic correction to ΔE at NLO**



Radiative $p_\perp$-broadening: *Bin Wu, JHEP* **1110***, 029 (2011); T. Liou, A. H. Mueller and B. Wu, Nucl. Phys. A* **916***, 102 (2013).* **Renormalization of $\hat{q}$:** *J. P. Blaizot and Y. Mehtar-Tani, Nucl. Phys. A* **929***, 202 (2014); E. Iancu, JHEP* **1410***, 95 (2014).* **Average energy loss:** *Bin Wu, JHEP* **1412***, 081 (2014) [arXiv:1408.5459].*

- **Radiative $\langle p_\perp^2 \rangle$ and energy loss**
  - Parametric result

$$\Delta E \sim \alpha_s N_c \frac{\omega_c L}{t_{form}(\omega_c)} = \alpha_s N_c L \langle p_\perp^2 \rangle$$
$$= \alpha_s N_c L \left(\hat{q} L + \langle p_\perp^2 \rangle_{rad} \right).$$

where $\omega_c = \hat{q} L^2$ and

$$\langle p_\perp^2 \rangle_{rad} = \frac{\alpha_s N_c}{8\pi} \hat{q} L \ln^2 \left(\frac{L}{l_0} \right)^2$$

with l_0 the size of constituents of the matter.

  - Exact result

$$\Delta E_2 = \frac{\alpha_s N_c}{12} L \langle p_\perp^2 \rangle_{rad}.$$

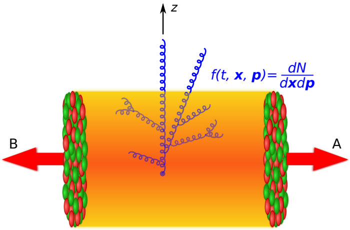
Bin Wu, JHEP **1412***, 081 (2014) [arXiv:1408.5459].*

- **Uncorrelated gluon branching**

  R. Baier, Y. L. Dokshitzer, A. H. Mueller and D. Schiff, JHEP **0109***, 033 (2001) [hep-ph/0106347].*
  R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B **502***, 51 (2001)*

- **The (phase-space) distribution function**

$$f(t, \mathbf{x}, \mathbf{p}) \equiv \frac{dN}{d\mathbf{x}d\mathbf{p}}.$$



- **The branching rate**

$$\frac{dP}{dpdt} \equiv \frac{dI}{dpdt} \sim \frac{1}{p^{\frac{3}{2}}} \text{ for soft gluons.}$$

In the following, a simplified quantities shall be used

$$f(t, z, p_z) \equiv \frac{dN}{dzdp_z} \equiv \int d^2x_\perp d^2p_\perp f(t, \mathbf{x}, \mathbf{p}).$$

1.2 Radiative parton energy loss

- **Democratic branching**

  J. P. Blaizot, E. Iancu and Y. Mehtar-Tani, Phys. Rev. Lett. **111**, *052001 (2013).*

  - **A scaling solution at small p_z resulted from the LPM spectrum**

$$f(t, z, p_z) \sim \frac{1}{p_z^{\frac{3}{2}}}.$$

See also *R. Baier, A. H. Mueller, D. Schiff and D. T. Son, Phys. Lett. B* **502**, *51 (2001)*

  - **Thermalization is missing**

    Unphysical at $p_z = 0$,

$$p_z f(t, z, p_z) = \epsilon_0(t)\delta(p_z),$$

since the LPM spectrum is valid only for $p \gtrsim T$.

Another motivation to study thermalization of jets.

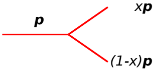
2.1 Combining thermalization in jet evolution

- **The rate for a gluon to split into two**

$$\frac{dI(p)}{dxdt} \simeq \underbrace{\frac{\alpha_s N_c}{\pi}\left(\frac{\hat{q}}{p}\right)^{\frac{1}{2}}}_{\frac{1}{t_{br}(p)}} \frac{(1-x+x^2)^{\frac{5}{2}}}{[x(1-x)]^{\frac{3}{2}}} \equiv \frac{1}{t_{br}(p)} K(x)$$

- **The loss term from branching**

$$C_{loss} \quad = \quad$$

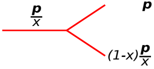


$$= \frac{1}{2t_{br}(p)} \int dx K(x) f(t, \vec{x}, \vec{p})$$

- **The gain term from branching**

$$C_{gain} \quad = \quad \int \frac{d^3p'}{(2\pi)^3} \int dx \frac{dI(p')}{dxdt} (2\pi)^3 \delta(x\vec{p}' - \vec{p}) f(t, \vec{x}, \vec{p}')$$

$$= \quad \frac{1}{t_{br}(p)} \int \frac{dx}{x^{\frac{5}{2}}} K(x) f(t, \vec{x}, \vec{p}) =$$



2.1 Combining thermalization in jet evolution

- **By adding diffusion and drag terms, we have**

$$\left(\frac{\partial}{\partial_t} + \vec{v}\cdot\nabla_x\right) f_{\vec{p}} = \underbrace{\frac{1}{4}\hat{q}\nabla_p\cdot\left[\left(\nabla_p + \frac{\vec{v}}{T}\right) f_{\vec{p}}\right]}_{C_{el}[f]} + \underbrace{\frac{1}{t_{br}(p)}\int dx K(x)\left[\frac{1}{x^{\frac{5}{2}}} f_{\frac{\vec{p}}{x}} - \frac{1}{2} f_{\vec{p}}\right]}_{C_{inel}[f]}$$

with

$$\begin{aligned} \hat{q} &= 8\pi\alpha_s^2 N_c \log\left(\frac{\langle k_{max}^2\rangle}{m_D^2}\right)\int \frac{d^3\vec{p}}{(2\pi)^3}\left[N_c f_{eq}(1+f_{eq}) + N_f F_{eq}(1-F_{eq})\right] \\ &= \frac{2\pi\alpha_s^2 N_c}{3} T^3 \log\left(\frac{\langle k_{max}^2\rangle}{m_D^2}\right)(2N_c+N_f) \sim \alpha_s^2 T^3. \end{aligned}$$

J. P. Blaizot, B. Wu and L. Yan, Nucl.Phys. A930 (2014) 139-162 arXiv:1402.5049 [hep-ph].

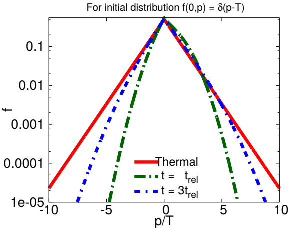
- **By integrating out $p_\perp$ and $x_\perp$, for $|p_z| \gtrsim p_\perp$**

$$\left(\frac{\partial}{\partial_t} + v_z\frac{\partial}{\partial p_z}\right) f(t,z,p_z) = \underbrace{\frac{1}{4}\hat{q}\frac{\partial}{\partial_{p_z}}\left[\left(\frac{\partial}{\partial_{p_z}} + \frac{v_z}{T}\right) f(t,z,p_z)\right]}_{\text{ultra-relativisitc Fokker-Planck (FP) equation: thermalization}}$$

$$\underbrace{+\frac{1}{t_{br}(p_z)}\int dx K(x)\left[\frac{1}{x^{\frac{1}{2}}} f(t,z,\frac{p_z}{x}) - \frac{1}{2} f(t,z,p_z)\right]}_{\text{Jet evolution}}.$$

2.2 Parametric estimate

- **Elastic scattering**
  - **Relaxation time**



$$t_{rel} \equiv \frac{4T^2}{\hat{q}} \sim \frac{1}{\alpha_s^2 T}$$

- **Collisional energy loss (drag)**
  - Collisional energy loss per unit length:

$$-\frac{dE_{col}}{dt} = \frac{\hat{q}}{4T} \sim \frac{T}{t_{rel}} \sim \alpha_s T^2.$$

  - Thermalization time by elastic scattering:

$$t_{el} \sim \frac{E}{\frac{dE_{col}}{dt}} = \frac{4ET}{\hat{q}} \sim \frac{E}{T} t_{rel}.$$

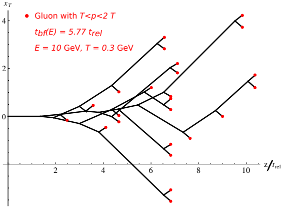
- **Inelastic scattering (parton splitting)**
  - **Quenching time:**

$$t_{quench} \sim t_{br}(E) = \sqrt{\frac{E}{T}} t_{br}(T) \sim \sqrt{\frac{E}{T}} t_{rel}.$$

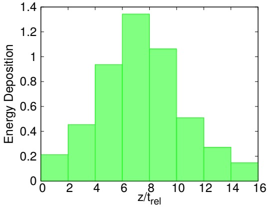
- **As a result:**
  1. **Elastic scattering (diffusion + drag): thermalization**
  2. **Branching: high-energy jet to gluons with $p \sim T$**

2.3 Analytical solutions in the simplified case

- **Motivation**
  - **A typical event from Monte Carlo generator**



  - **Energy deposition averaged over 10 events**



- **A simplified model**

$$\left(\frac{\partial}{\partial_t} + v\frac{\partial}{\partial_p}\right) f(t,z,p) = \frac{1}{4}\hat{q}\frac{\partial}{\partial_p}\left[\left(\frac{\partial}{\partial_p} + \frac{v}{T}\right) f(t,z,p)\right] + \delta(t-z)\delta(p-p_0).$$

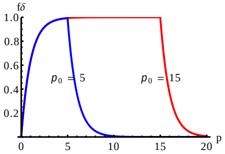
Here and in the following the subscript z is omitted.

- **The stationary solution**

$$f = \begin{cases} f_s(p,p_0)\delta(t-z) + \left[\frac{1}{4}\mathrm{erf}\left(\frac{\sqrt{t-z}}{2\sqrt{2}}\right) + \frac{1}{4} + \frac{e^{\frac{z-t}{8}}}{\sqrt{2\pi}\sqrt{t-z}}\right] e^{-p} & \text{for } p \geq 0, \\ \frac{1}{4}e^{p}\left[\mathrm{erf}\left(\frac{2p+t-z}{2\sqrt{2}\sqrt{t-z}}\right)+1\right] + \frac{e^{-\frac{(-2p+t-z)^2}{8(t-z)}}}{\sqrt{2\pi}\sqrt{t-z}} & \text{for } p \leq 0, \end{cases}$$

which satisfies

$$\begin{cases} 0 = (f' + f)' + \delta(x^-)\delta(p-p_0) & \text{for } p > 0, \\ 2\frac{\partial}{\partial x^-} f = (f' - f)' & \text{for } p < 0, \end{cases}$$

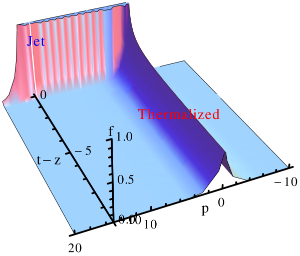


Here,

$$f_s(p,p_0) \equiv \left[e^{-p}\left(e^{p_0}-1\right)\theta(p-p_0) + \left(1-e^{-p}\right)\theta(p_0-p)\right],$$

and the natural unit $T = 1$ and $t_{rel} = 1$ have been taken here.

- **Jet and its thermalized tail**



Here, Dirac δ function is regulated by $\delta_\epsilon(t-z) = \frac{1}{\sqrt{\epsilon\pi}} e^{\frac{(t-z)2}{\epsilon}}$.

- **Jet shape is broadened**

  Described by f_s.

- **Additional energy loss deposited in the QGP**

$$f = f_{eq}(p) \equiv \frac{1}{2} e^{-|p|} \text{ at } z \ll t.$$

This also explains why there exist such a stationary solution.

- **General solution**

$$f(t,z,p) = \int dp_0 dz_0 f_G(t, z-z_0, p, p_0) f_0(z_0, p_0) + \int dp_0 dz_0 \int_0^t dt' f_G(t-t', z-z_0, p, p_0)\mathcal{F}(t', z_0, p_0)$$

satisfies

$$\left(\frac{\partial}{\partial_t} + v\frac{\partial}{\partial_p}\right) f(t,z,p) = \frac{1}{4}\hat{q}\frac{\partial}{\partial_p}\left[\left(\frac{\partial}{\partial_p} + \frac{v}{T}\right) f(t,z,p)\right] + \mathcal{F}(t,z,p).$$

with $f(0,z,p) = f_0(z_0, p_0)$.

- **The Green function:**

Solution to ultra-relativistic FP equation with $f_G(0,z,p) = \delta(z)\delta(p-p_0)$.

- **The Green function:**

$$f_G(t,z,p) = \frac{e^{-\frac{p_0-p}{2}-\frac{t}{4}}}{2\sqrt{\pi}\sqrt{t}}\left[e^{-\frac{(p-p_0)^2}{4t}} - e^{-\frac{(p+p_0)^2}{4t}}\right]\delta(t-z)$$

$$+\frac{e^{-\frac{(p+p_0-z)^2}{4t}-p}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right]\operatorname{erfc}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p+p_0}{t+z}-1\right)\right)$$

$$+\frac{(t+z)e^{-\frac{(p+p_0)^2}{2(t+z)}+\frac{p_0-p}{2}-\frac{t}{4}}(p+p_0+t-z)}{4\pi t^2\sqrt{(t-z)(t+z)}} \qquad \text{for } p\geq 0,$$

$$=\frac{e^{p-\frac{(p+p_0-z)^2}{4t}}\left[t(t+2)-(p+p_0-z)^2\right]\operatorname{erf}\left(\frac{1}{2}\sqrt{t-\frac{z^2}{t}}\left(\frac{p}{t-z}-\frac{p_0}{t+z}+1\right)\right)}{8\sqrt{\pi}t^{5/2}}$$

$$+\frac{e^{p-\frac{(p+p_0-z)^2}{4t}}}{8\sqrt{\pi}t^{5/2}}\left[t(t+2)-(p+p_0-z)^2\right]$$

$$+\frac{p(z-t)+(t+z)(p_0+t-z)}{4\pi t^2\sqrt{t^2-z^2}}e^{-\frac{p^2}{2(t-z)}+\frac{p+p_0}{2}-\frac{p_0^2}{2(t+z)}-\frac{t}{4}} \qquad \text{for } p\leq 0.$$

2.3 Analytical solutions in the simplified case

- **At large t**

$$f_G \to \frac{e^{-|p|-\frac{z^2}{4t}}}{4\sqrt{\pi t}}.$$

The longitudinal width of the gluon distribution $\propto \sqrt{2t}$

- **The equivalent Langevin equation**

$$\frac{d}{dt}p = -v + \xi(t), \qquad \frac{d}{dt}x = v$$

with a Gaussian white noise

$$\langle \xi(t)\xi(t') \rangle = 2\delta(t - t').$$

This gives us

$$\frac{d}{dt}\langle (x+p)^2 \rangle = 2,$$

that is,

$$\langle (x(t) + p(t))^2 \rangle = 2t + \langle (x(0) + p(0))^2 \rangle.$$

2.4 Numerical studies

- **Spatially homogeneous case**
  - **Scaling solution from branching:**
    By taking $f = \frac{1}{p^{\beta}}$ in C_{inel}

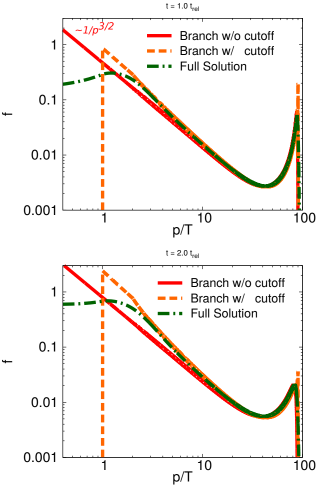
$$0 = \frac{1}{x^{\frac{1}{2}}} f(p/x) - x f(p)$$
$$= (x^{\beta - \frac{1}{2}} - x) \frac{1}{p^{\beta}} \Rightarrow \beta = \frac{3}{2}.$$

  - **The initial condition**

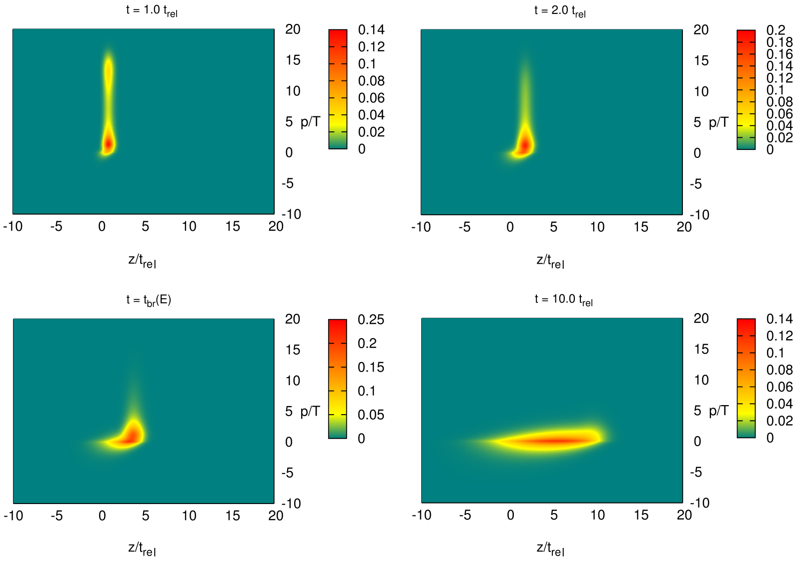
$$f(0,p) = e^{-10(p-90.0)^2},$$

    That is, $E \simeq 90T$.
  - **The lower cutoff scale: T**
    Validity of the LPM spectrum only for $p \gtrsim T$.

A small deviation of the scaling solution!

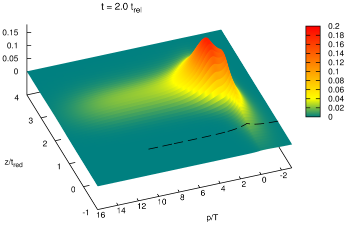


- **Preliminary result** $\left(f(0,z,p) = e^{-10(p-15)^2/T^2 - 10z^2/t_{rel}^2}\right)$

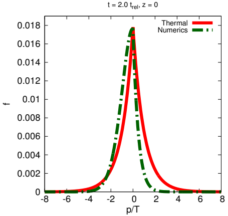


2.4 Numerical studies

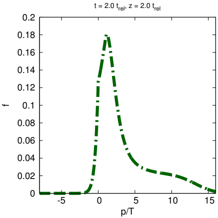
- **General features at $t = 2.0t_{rel}$**



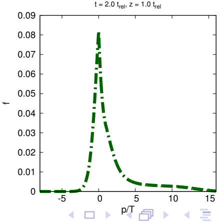
- **The tail: nearly thermal**



- **The front: Pile-up**



- **At $z = t_{rel}$**



Conclusions and Perspectives

- **Detailed evolution of jet in a QGP**
  1. **Characterized by three time scales**

$$\text{Relaxation time: } t_{rel} \equiv \frac{4T^2}{\hat{q}} \sim \frac{1}{\alpha_s^2 T},$$

$$\text{Collisional energy loss: } t_{el} \equiv \frac{4ET}{\hat{q}} \sim \frac{E}{T}\frac{1}{\alpha_s^2 T},$$

$$\text{Branching: } t_{br}(E) = \sqrt{\frac{E}{T}}\, t_{br}(T) \sim \sqrt{\frac{E}{T}}\frac{1}{\alpha_s^2 T}.$$

  2. **The front: Pile-up**
  3. **The diffusion tail: energy deposited in the QGP**
- **Many things to do**
  1. **Better understand the pile-up and other quantitative feathers**
  2. **Confrontation with experimental data**
  3. **Combination with thermalization of bulk matter** $\cdots$

     For example, replace the static QGP by time-dependent bulk matter in

T. Epelbaum and F. Gelis, Phys. Rev. Lett. **111***, 232301 (2013) [arXiv:1307.2214 [hep-ph]].*

Thank you and happy sheep year!