

# Large anisotropies in the Little Bang

Heavy ion seminar  
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*Li Yan, JYO, arXiv:1312.6555, PRL 112 (2014) 082301*

*Li Yan, JYO, Art Poskanzer, in preparation*

# Anisotropic flow

- Particles are emitted with a *probability distribution* that is not isotropic in azimuthal angle

$$P(\phi) = 1 + 2 \sum_{n>0} v_n \cos(n(\phi - \psi_n))$$

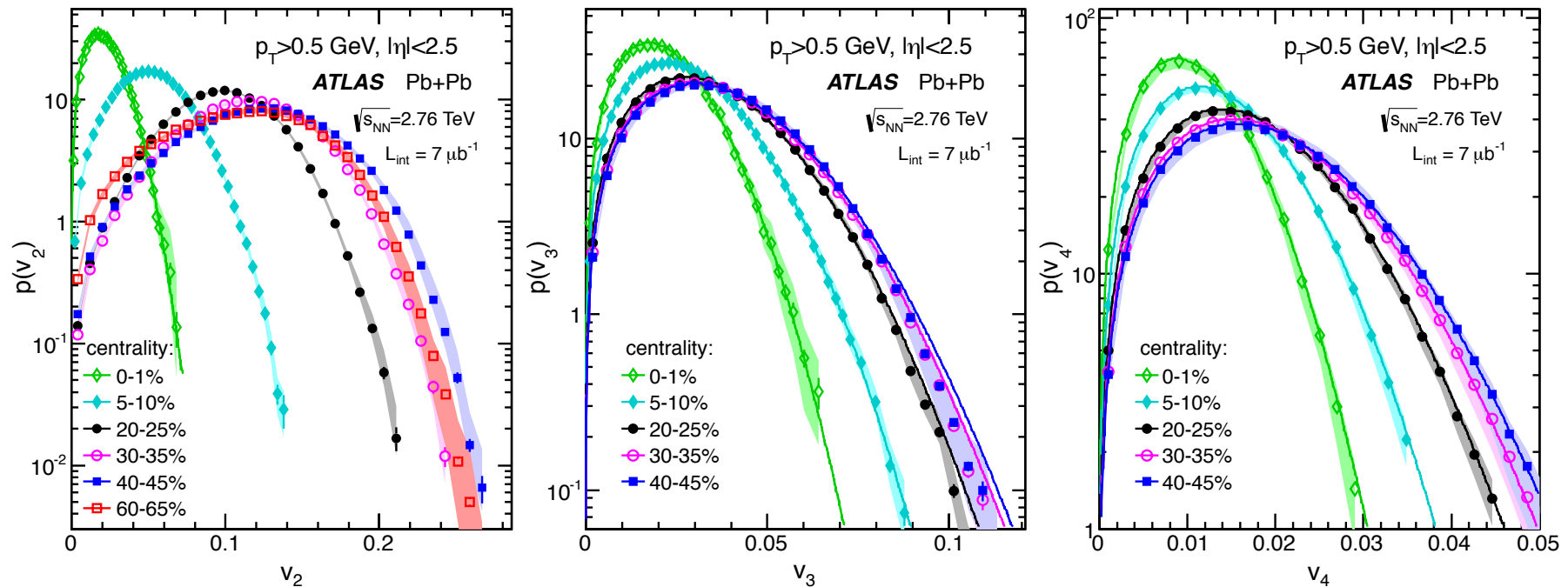
- $v_n \equiv$  anisotropic flow  
 $v_2 \equiv$  elliptic flow  
 $v_3 \equiv$  triangular flow...
- Finite number of particles  $\rightarrow$  trivial anisotropies from statistical fluctuations.
- $v_n$  can be measured only after statistical fluctuations are subtracted (“unfolded”)

# Flow fluctuations

- $v_n$  fluctuates event to event (PHOBOS, 2005)
- $v_n$  itself has a *probability distribution* for a given system and centrality.

# New data in Pb-Pb

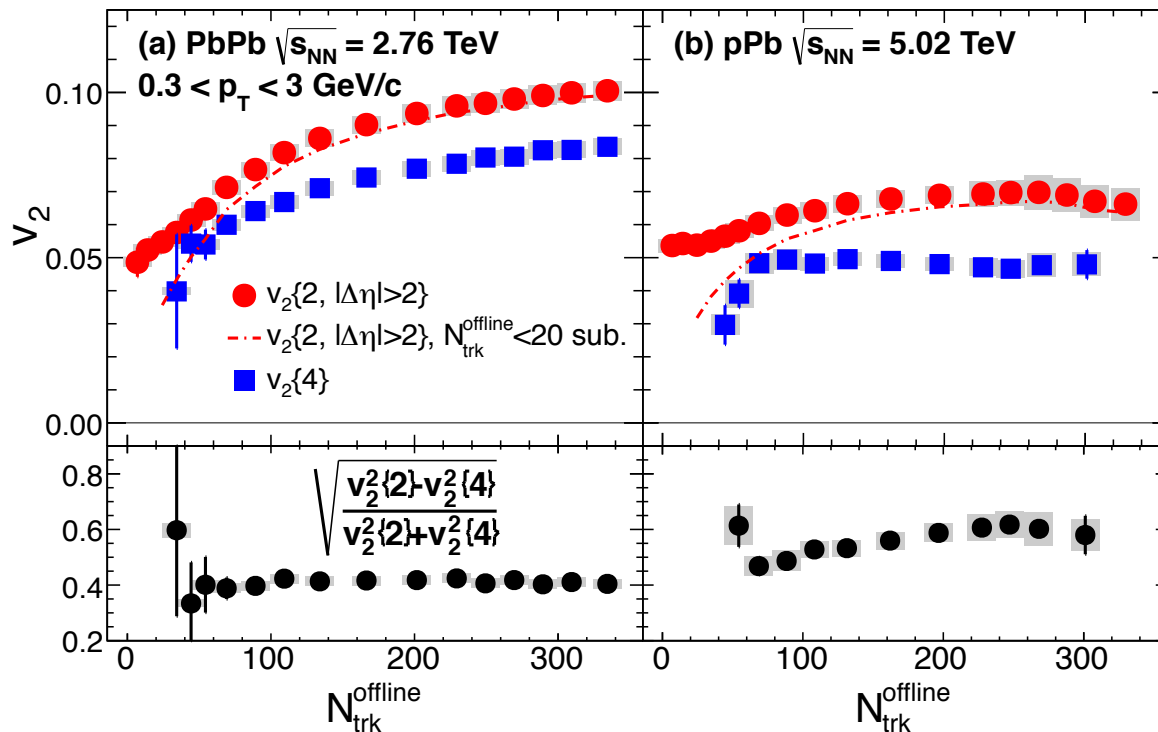
The probability distribution of  $v_2$ ,  $v_3$ ,  $v_4$  for various centralities



ATLAS 1305.2942

# New data in p-Pb

First 2 cumulants of the distribution of  $v_2$   
(less detailed than the full distribution)



$$v_2\{2\} \equiv (\langle v_2^2 \rangle)^{1/2}$$

$$v_2\{4\} \equiv (2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle)^{1/4}$$

If  $v_2$  doesn't fluctuate,  
 $v_2\{2\} = v_2\{4\} = v_2$

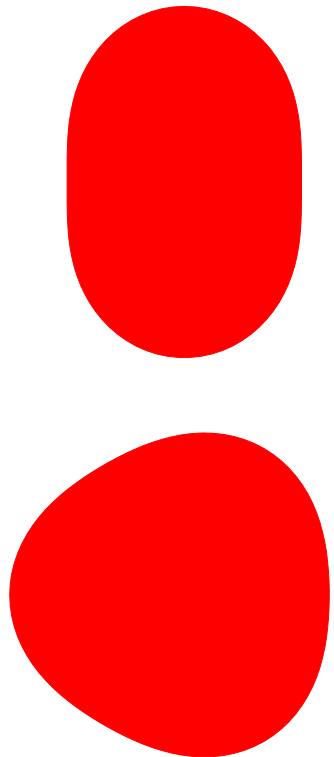
In general  $v_2\{4\} < v_2\{2\}$

*CMS 1305.0609*

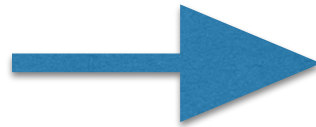
- Do we understand these new data?
- What can we learn from them?

# The origin of anisotropic flow

*Initial* transverse  
density profile



Expansion



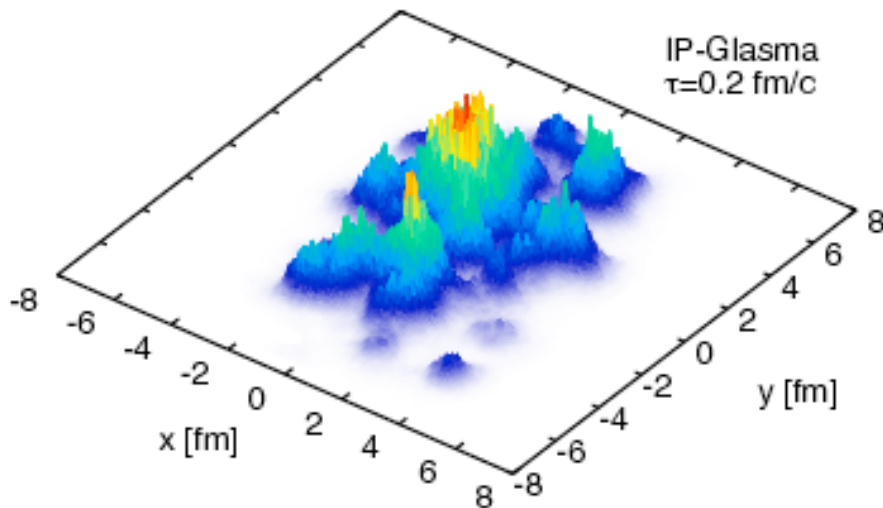
*Final* distribution

Elliptic flow  $v_2$

Triangular flow  $v_3$

# Initial anisotropies

= Fourier decomposition of the initial density profile  $\rho(x,y)$



*Gale Jeon Schenke 1301.5893*

$$\epsilon_n \equiv \frac{\left| \int r^n e^{in\phi} \rho(r,\phi) r dr d\phi \right|}{\int r^n \rho(r,\phi) r dr d\phi}$$

$\epsilon_2 \equiv$  initial eccentricity

$\epsilon_3 \equiv$  initial triangularity

$|\epsilon_n| < 1$  by definition



# Anisotropic flow $\approx$ initial anisotropy

$$v_2 \approx K_2 \epsilon_2$$

$$v_3 \approx K_3 \epsilon_3$$

response coefficients

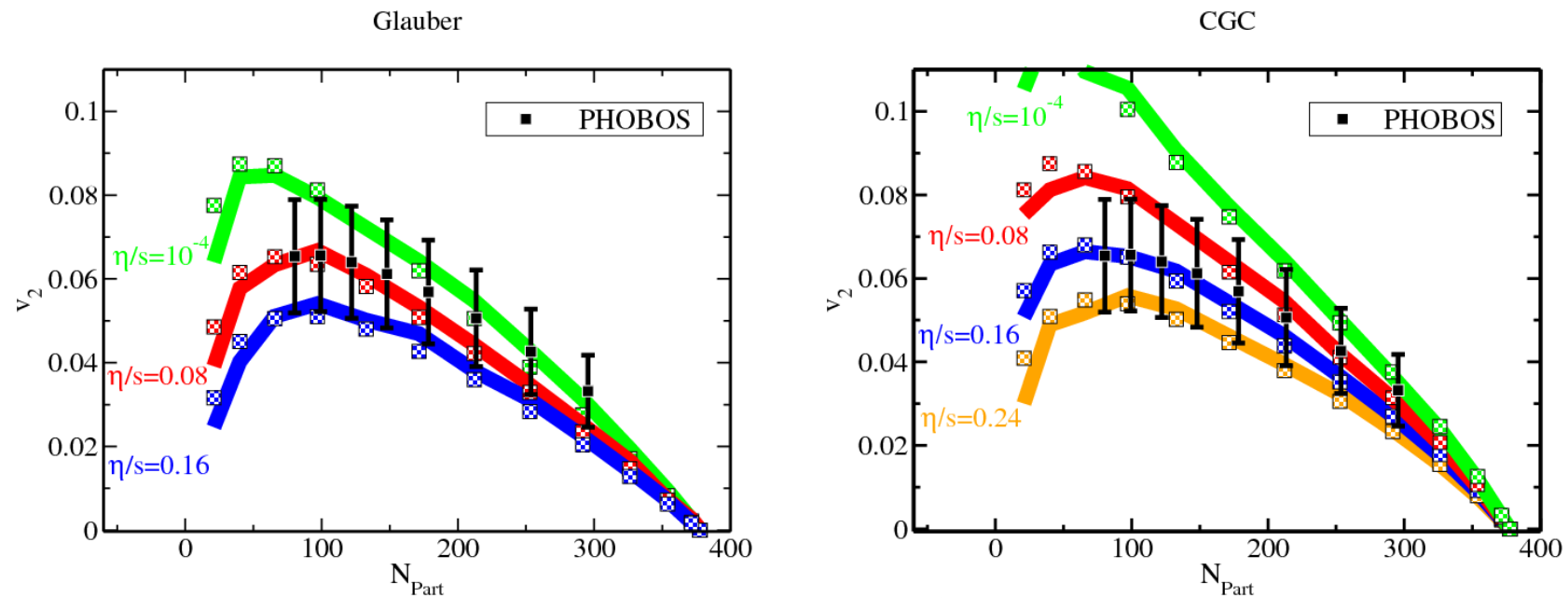
fluctuate event to event.

*depend on system and centrality*

*in hydro, depend on viscosity*

$v_n$  fluctuations are due to  $\epsilon_n$  fluctuations

# Problem: can we disentangle the initial anisotropy from the response?



*Luzum Romatschke 0804.4015*

A long-standing problem in heavy-ion physics:  
for any model of initial conditions (Glauber and CGC), i.e.,  
for any  $\epsilon_n$ , one can tune the viscosity — the response  $K_n$  —  
to match the observed  $v_n$

# Is there a general law that describes anisotropy fluctuations?

- If we know the statistics of the initial  $\epsilon_n$ , then the distribution of observed  $v_n$  is the distribution of  $\epsilon_n$ , rescaled by the response  $K_n$
- State of the art (as of 2013): Gaussian fluctuations  
 $P(\epsilon_n) \propto \epsilon_n \exp(-\epsilon_n^2/\sigma^2)$  *Voloshin et al 0708.0800*
- Then the distribution of  $v_n$  is also a Gaussian, of width  $K_n \times \sigma$ : we are still unable to disentangle the initial state from the response.

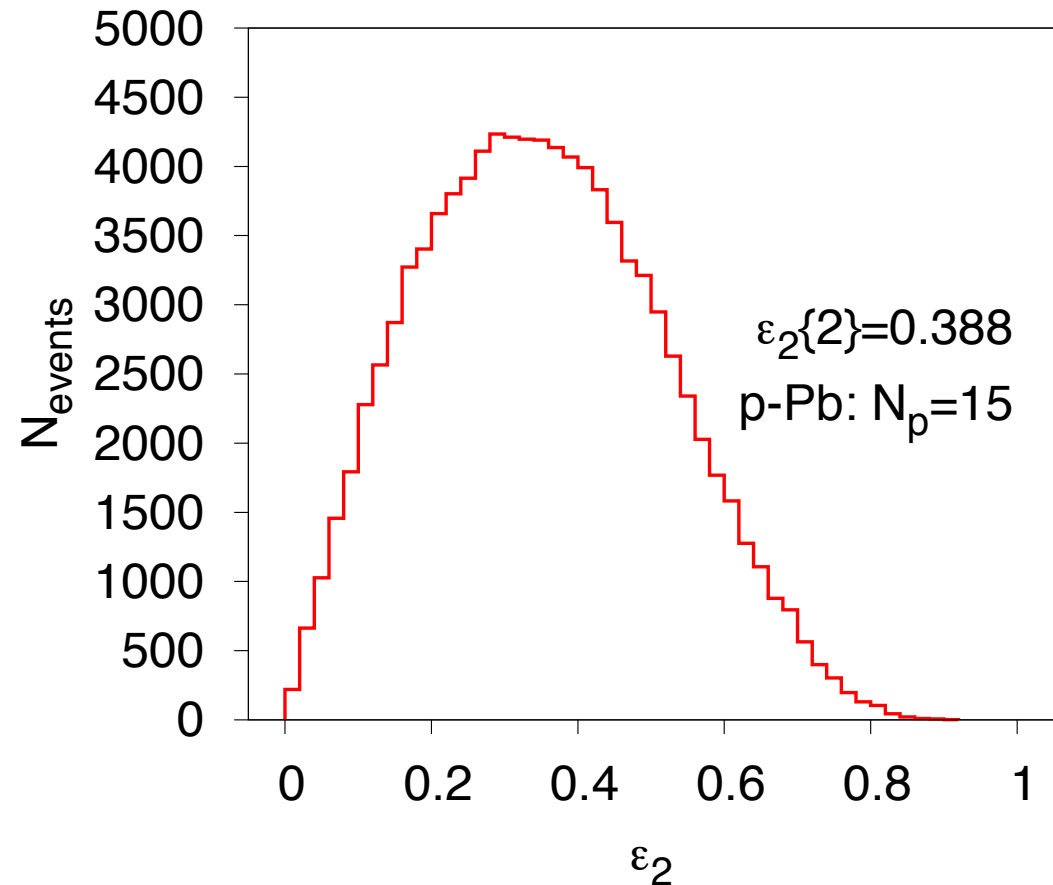
# The statistics of initial fluctuations

$$\epsilon_2 \equiv \frac{|\int r^2 e^{2i\phi} \rho(r, \phi) r dr d\phi|}{\int r^2 \rho(r, \phi) r dr d\phi}$$

central p+Pb collision:  
initial density  $\rho(r, \phi) =$   
independent of  $\phi$  up to  
fluctuations

small system: *large*  
fluctuations & *anisotropies*

Monte-Carlo Glauber simulation



*Is there a simple law that describes this distribution?*

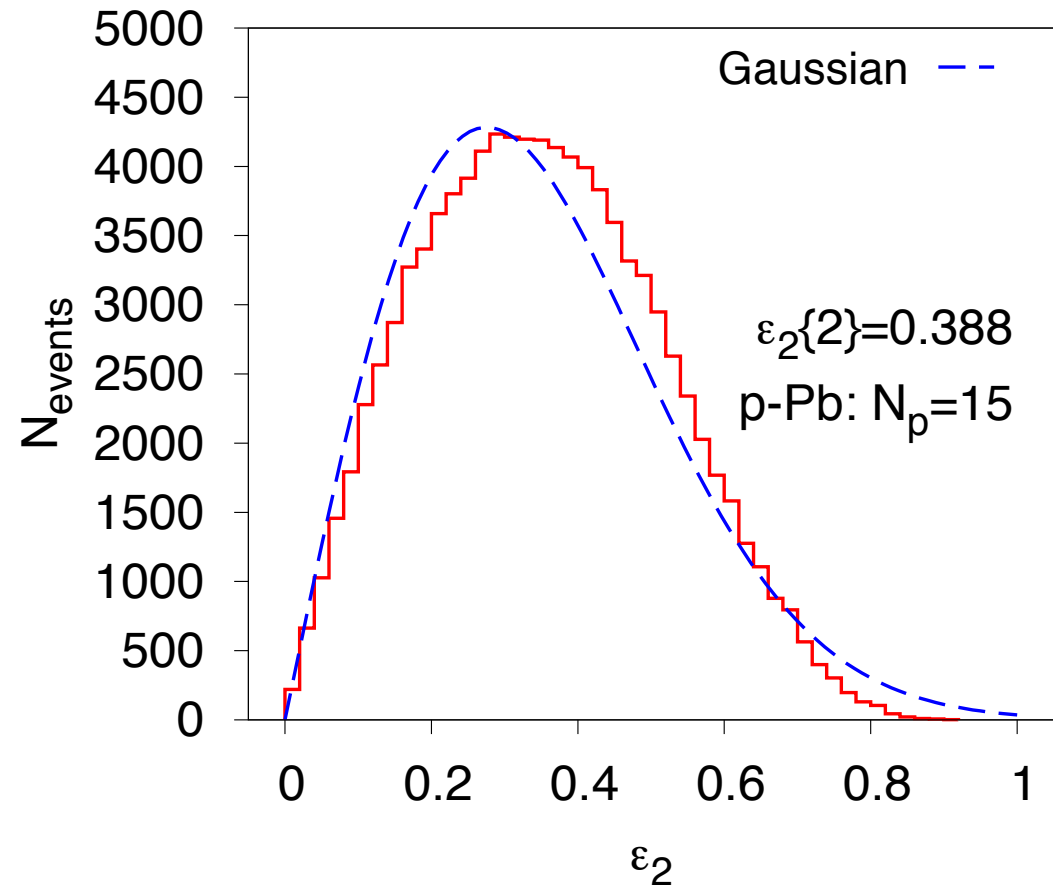
# Gaussian?

Central limit theorem

$$P(\epsilon_2) = 2(\epsilon_2/\sigma^2) \exp(-\epsilon_2^2/\sigma^2)$$

Not a good fit.

Does not implement  
the condition  $\epsilon_2 < 1$



# New “Power” distribution

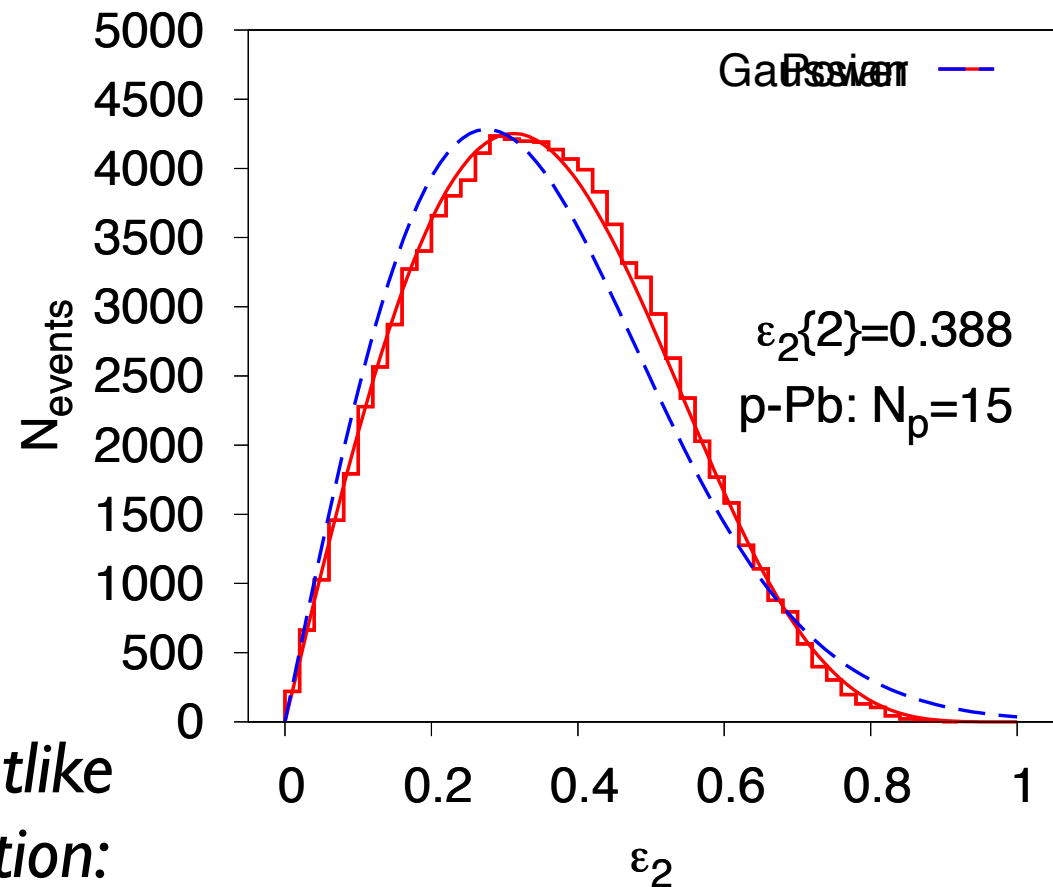
$$P(\epsilon_2) = 2\alpha\epsilon_2(1-\epsilon_2^2)^{\alpha-1}$$

Equivalent to Gaussian for  
 $\alpha \gg 1$

Naturally implements the  
condition  $\epsilon_2 < 1$ .

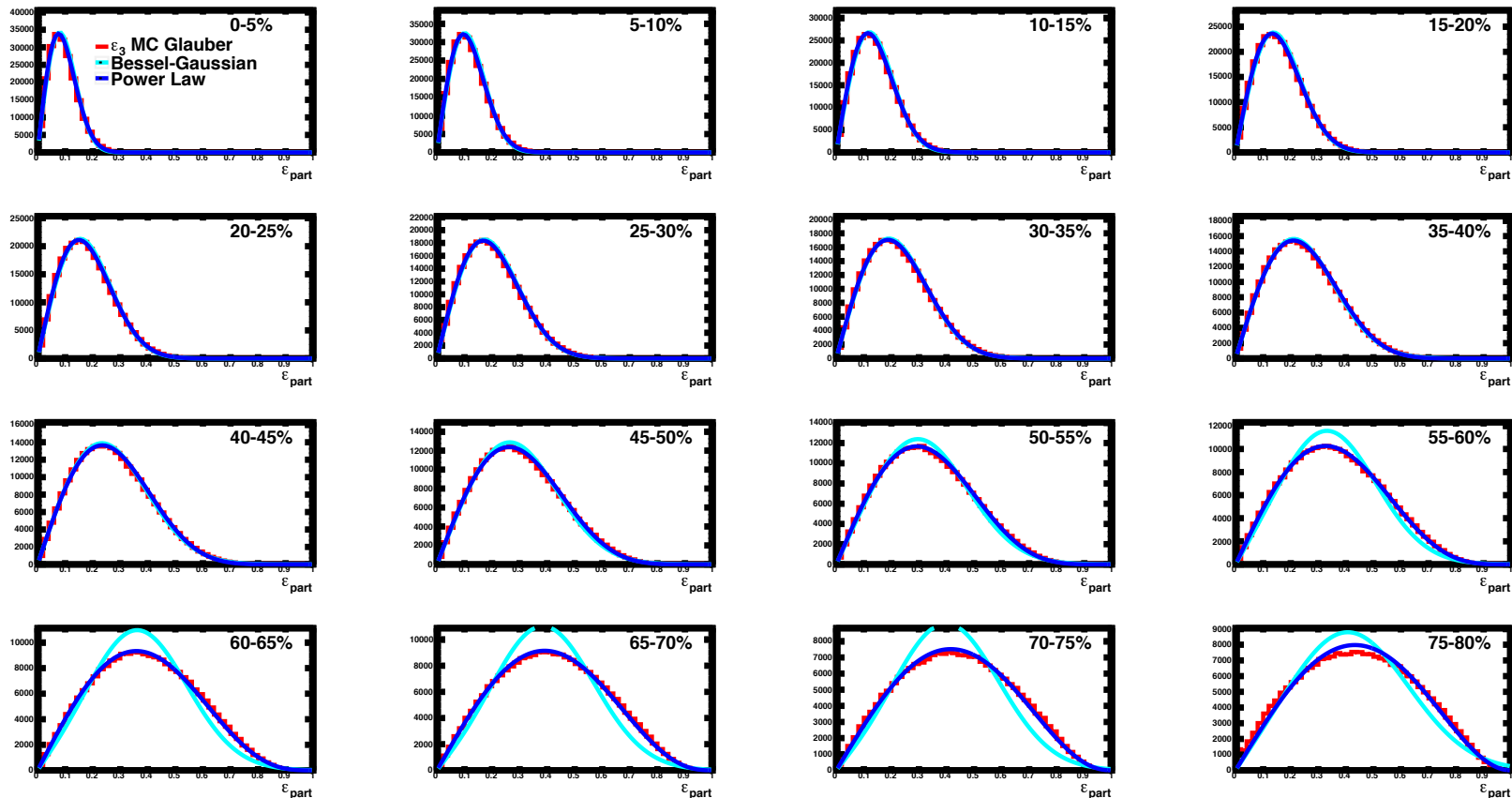
Exact result for  $N=2\alpha+1$  pointlike  
sources with Gaussian distribution:

*JYO, PRD46(1992)229*



**Much better fit to Monte-Carlo results!**

# Testing the *Power* distribution for $\epsilon_3$ in Au-Au collisions



*fits to Monte-Carlo Glauber by Art Poskanzer*

# Universality of initial anisotropy fluctuations

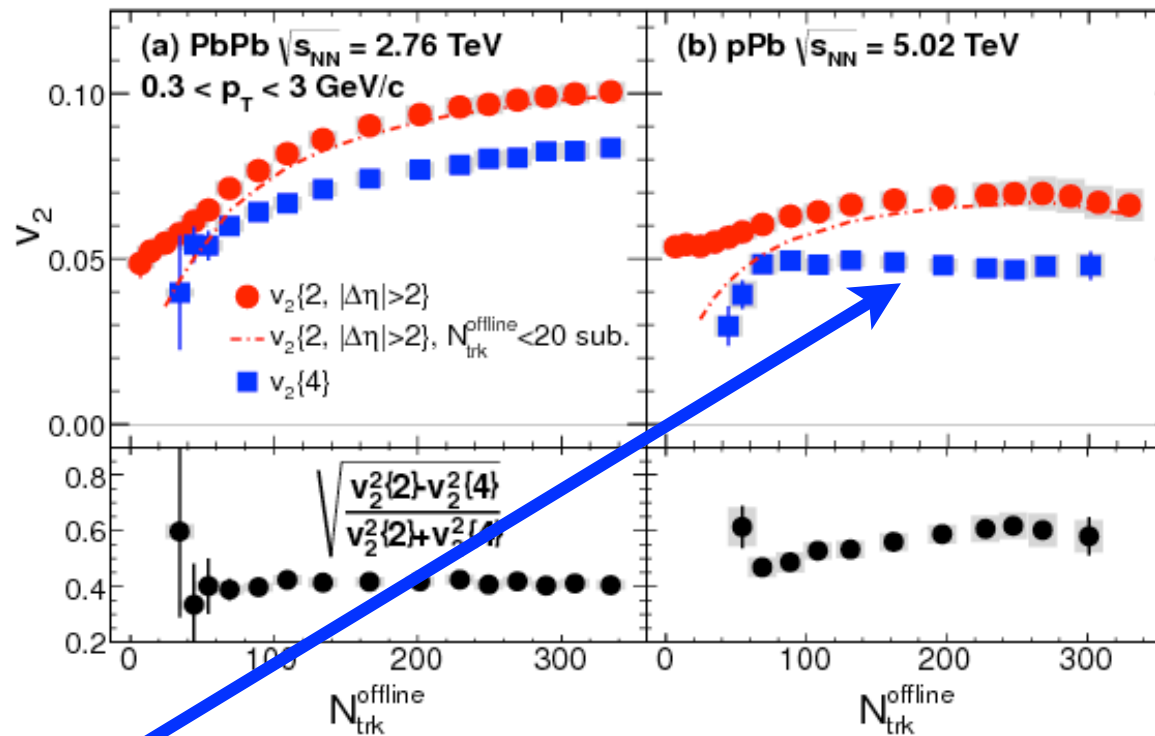
- The *Power* distribution fits several models of initial conditions (MC Glauber, MC KLN, IP-Glasma, DIPSY) when the anisotropy is solely created by fluctuations:  $\epsilon_2$  in p-p collisions,  $\epsilon_2$  and  $\epsilon_3$  in p-Pb collisions,  $\epsilon_3$  in Pb-Pb or Au-Au collisions.

*Li Yan, JYO, PRL 112 (2014) 082301*

- We postulate that it is universal, to a good approximation.



# Natural explanation for $v_2\{4\}$ in pPb

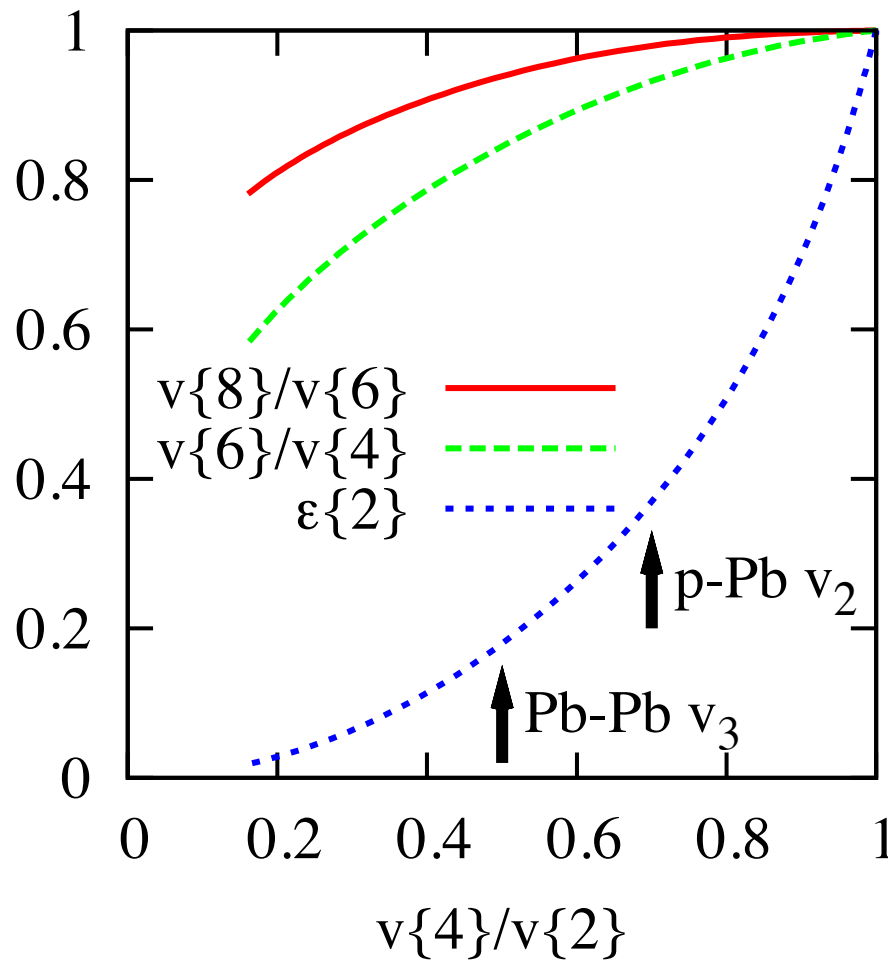


*CMS 1305.0609*

Gaussian fluctuations give  $v_2\{4\} = 0$ .

Our new *Power* distribution naturally predicts a large  $v_2\{4\}$  in p-Pb.

# Consequences & predictions



- Using as input the experimentally measured ratio  $v_n\{4\}/v_n\{2\}$
- Quantitative prediction for higher-order cumulants  $v_n\{6\}$  and  $v_n\{8\}$
- We can read off the rms anisotropy  $\varepsilon_n\{2\}$ , a property of the initial state, directly from experimental data

# Generalization to $\varepsilon_2$ in Pb-Pb

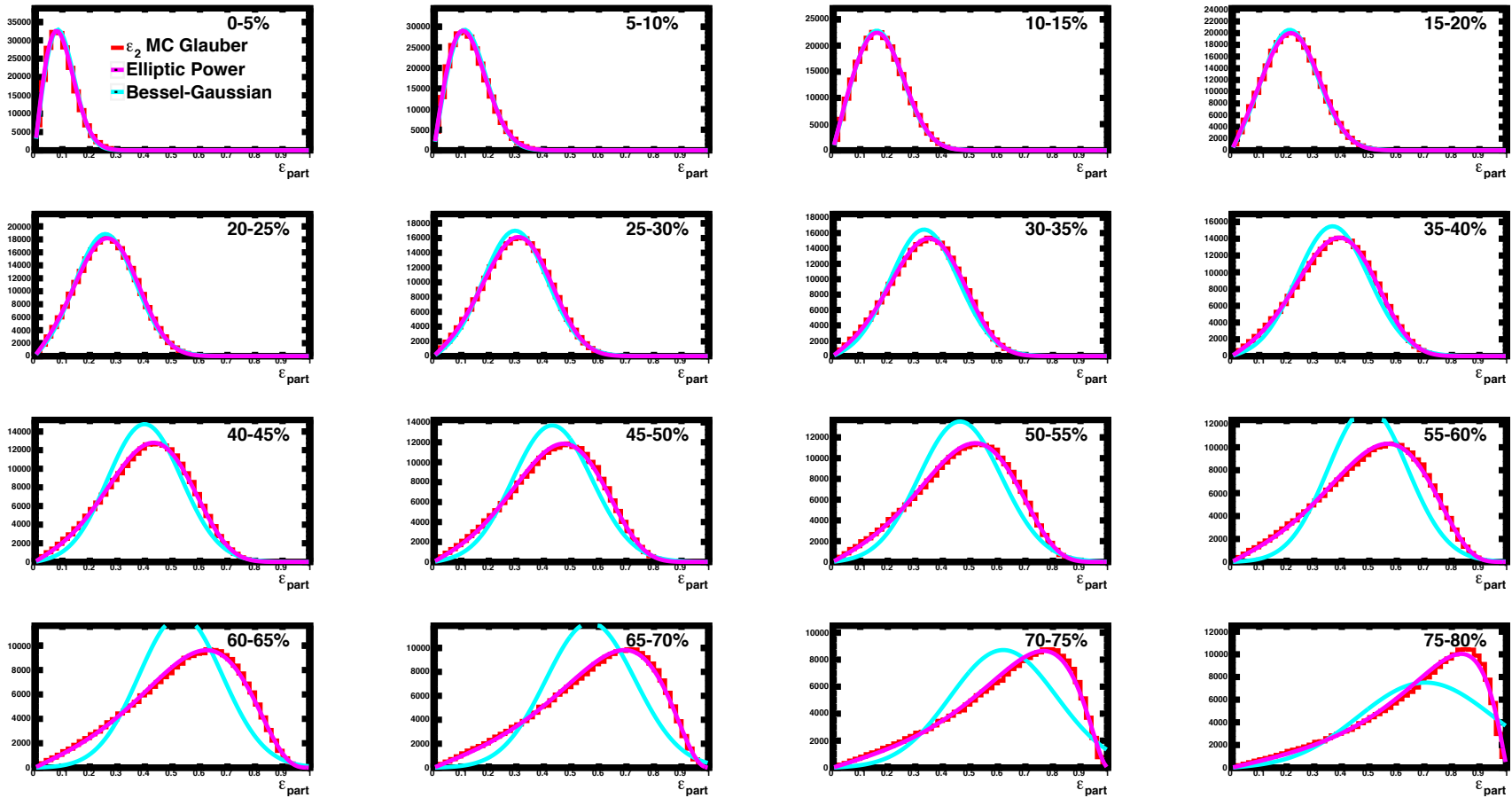
- For  $\varepsilon_2$  in non-central Pb-Pb or Au-Au collisions, there is a **mean anisotropy in the reaction plane** in addition to **fluctuations**: requires a generalized distribution with 1 extra parameter: the *Elliptic Power* distribution

$$\frac{dn}{d\varepsilon} = \frac{2}{\pi} \varepsilon \alpha (1 - \varepsilon^2)^{(\alpha-1)} (1 - \varepsilon_0^2)^{(\alpha+1/2)} \int_0^\pi (1 - \varepsilon_0 \varepsilon \cos \phi)^{-(1+2\alpha)} d\phi$$

Reduces to the Power distribution for  $\varepsilon_0 = 0$

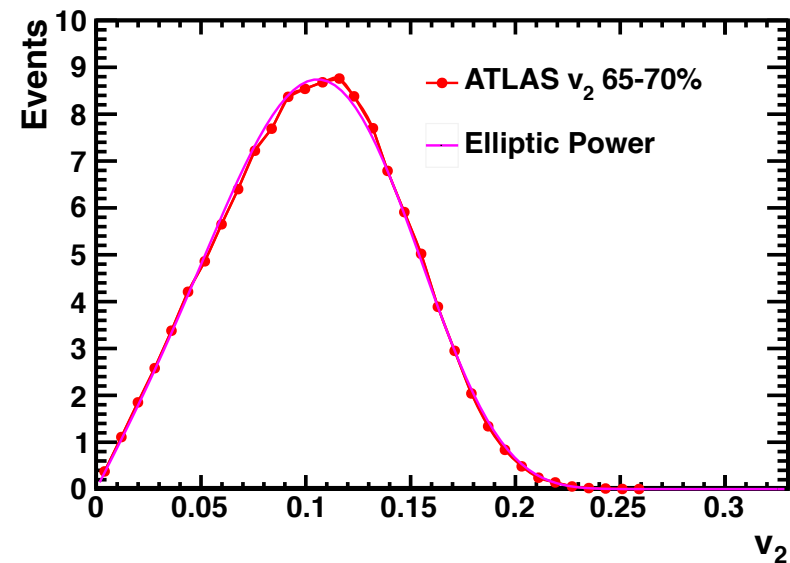
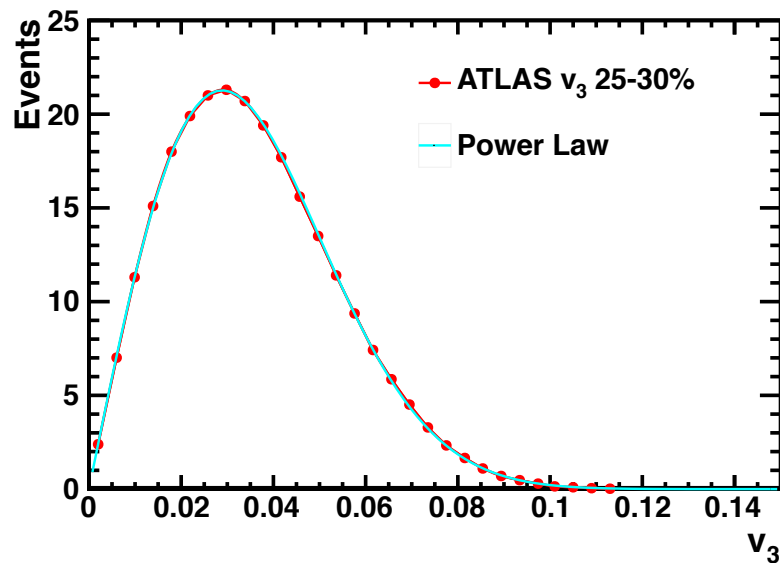
*Li Yan, JYO, Art Poskanzer, in preparation*

# Testing the *Elliptic Power* distribution for $\epsilon_2$



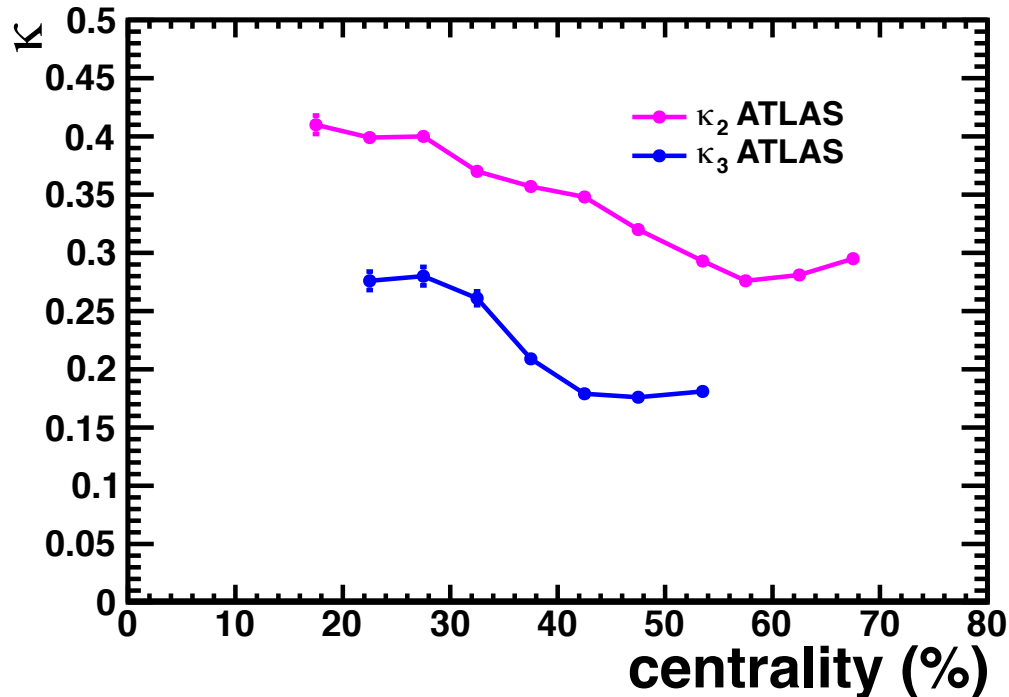
*fits to Monte-Carlo Glauber by Art Poskanzer*

# Fitting ATLAS $v_3$ and $v_2$ distributions with rescaled *Power* and *Elliptic-Power*



We obtain good fits to ATLAS data for  $v_2$  and  $v_3$  for all centralities

# Extracting the hydro response from ATLAS data

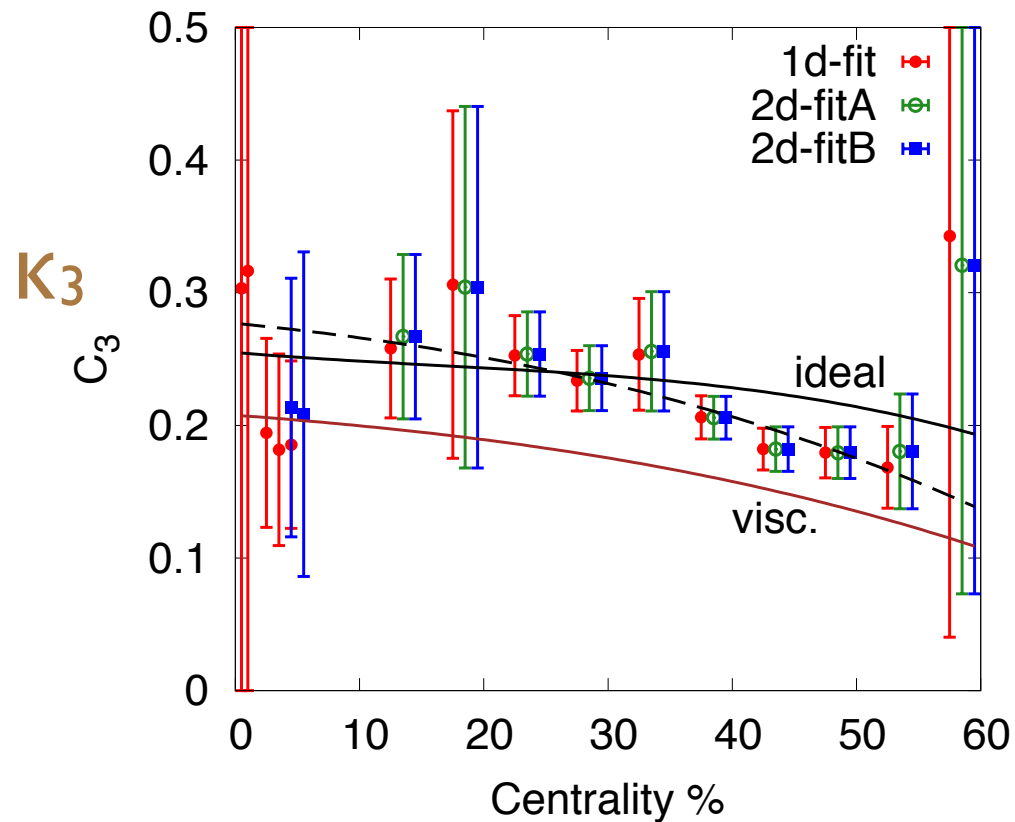


*Art Poskanzer,  
preliminary  
(stat errors only)*

As expected from viscous effects, the response decreases for more peripheral collisions.

As expected, viscous decrease is faster for  $v_3$  than for  $v_2$

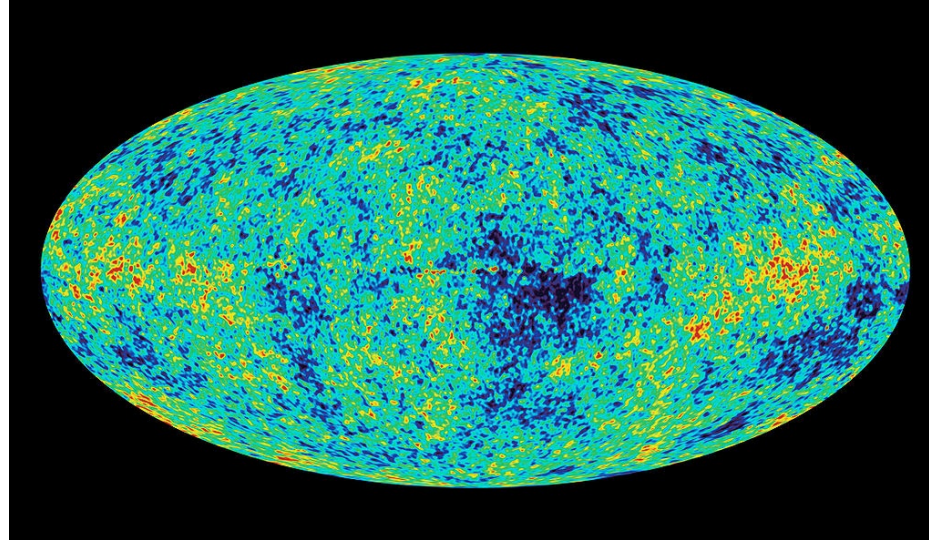
# Extracting the QGP viscosity from ATLAS data



*Li Yan (preliminary)  
with syst errors*

Viscous hydro fits return a value of  $\eta/s$  close to 0.1.

# Big Bang versus Little Bang



WMAP

**Small** anisotropies observed in the cosmic microwave background are thought to originate from quantum fluctuations in the early Universe.

Anisotropic flow at RHIC and LHC is a similar phenomenon, occurring within a **tiny** system with **large** fluctuations.

The **non-Gaussianity** of these fluctuations, and the fact that they are **universal**, allows us to **disentangle initial fluctuations from the response**.

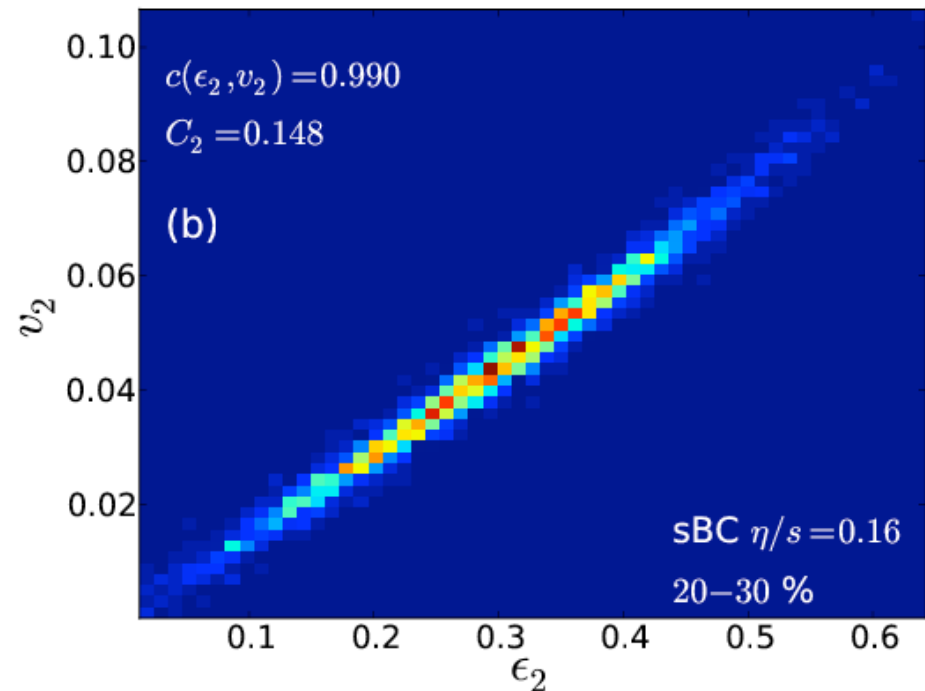
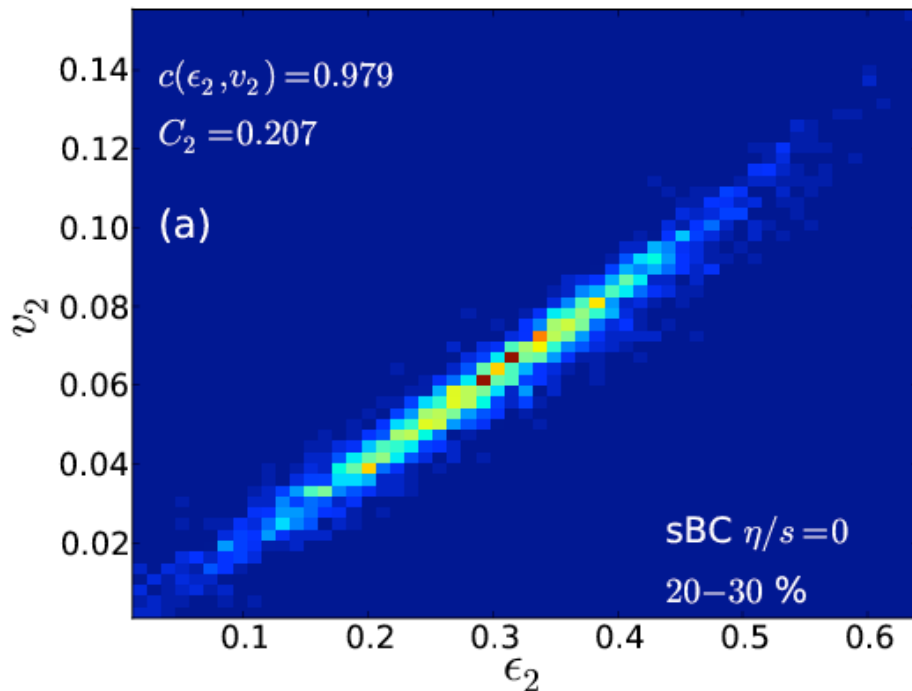


# Conclusions, perspectives

- Direct evidence from experimental data that anisotropic flow in p-Pb and Pb-Pb collisions is driven by **large anisotropies** in the initial state: the statistics of  $\varepsilon_n$  *hits the boundary*  $\varepsilon_n < 1$
- The statistics of large fluctuations is not described by the central limit theorem but nevertheless **universal** to a good approximation
- We can extract both the initial anisotropy and the “hydrodynamic” response  $K_n$  from experimental data without any prior assumption about the initial state. Toward the first robust measurement of the viscosity of the QGP (work in progress).

# Backup

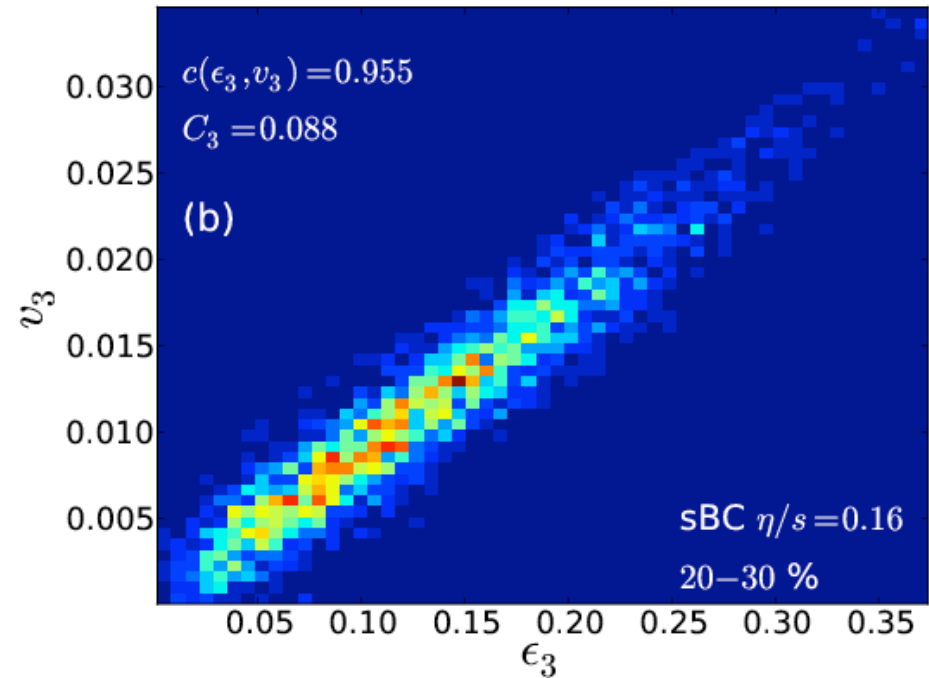
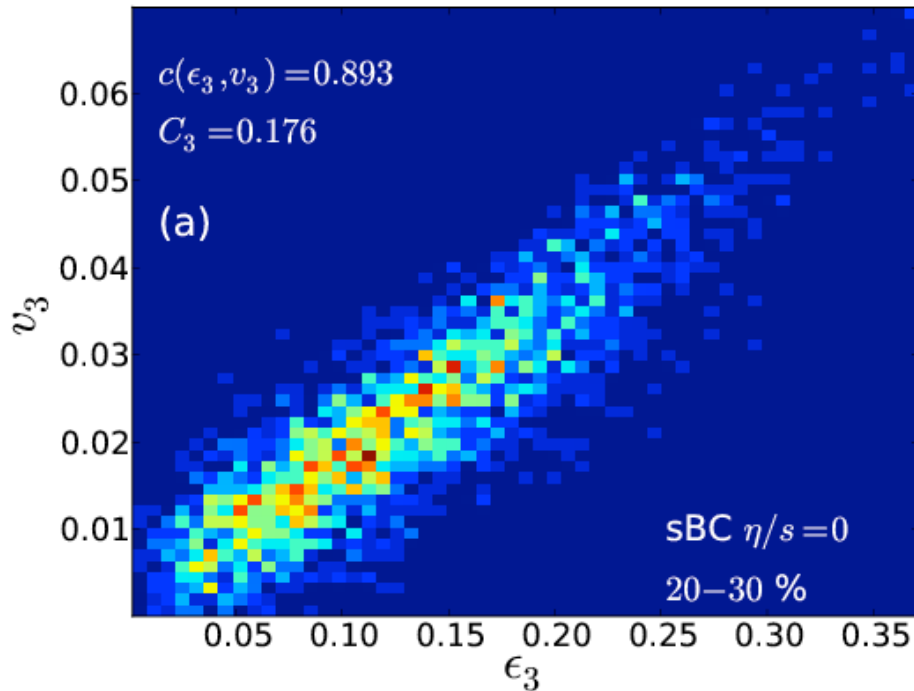
# Elliptic flow $v_2$ versus initial eccentricity $\epsilon_2$



*Niemi Denicol Holopainen Huovinen 1212.1008*

Each point=different initial density profile.  
 $v_2$  is almost perfectly linear in  $\epsilon_2$

# Triangular flow $v_3$ versus initial triangularity $\epsilon_3$



*Niemi Denicol Holopainen Huovinen 1212.1008*

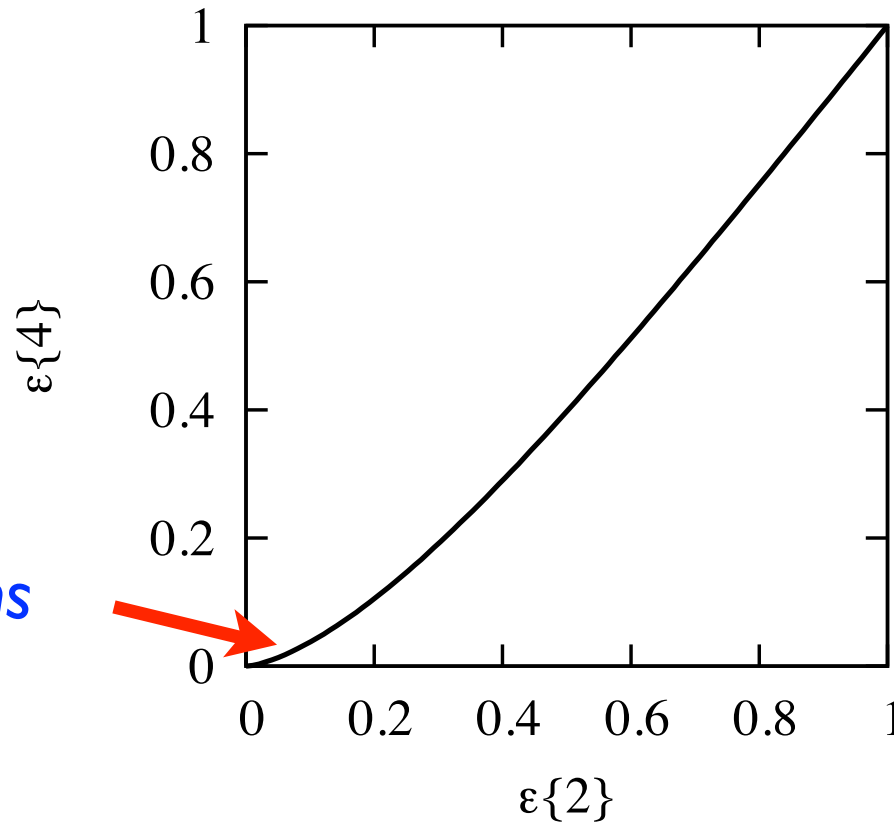
$v_3$  is also strongly correlated with  $\epsilon_3$

# Cumulants

- 2-dimensional Gaussian: Wick's theorem  
 $\langle \varepsilon^4 \rangle = 2 \langle \varepsilon^2 \rangle^2$  where  $\langle \dots \rangle \equiv$  average over events
- Define  $\varepsilon\{2\} \equiv \langle \varepsilon^2 \rangle^{1/2}$  (rms anisotropy)  
$$\varepsilon\{4\} \equiv (2 \langle \varepsilon^2 \rangle^2 - \langle \varepsilon^4 \rangle)^{1/4}$$
- $\varepsilon\{4\} = 0$  for Gaussian.
- The *power* distribution predicts a universal, relation between  $\varepsilon\{4\}$  and  $\varepsilon\{2\}$

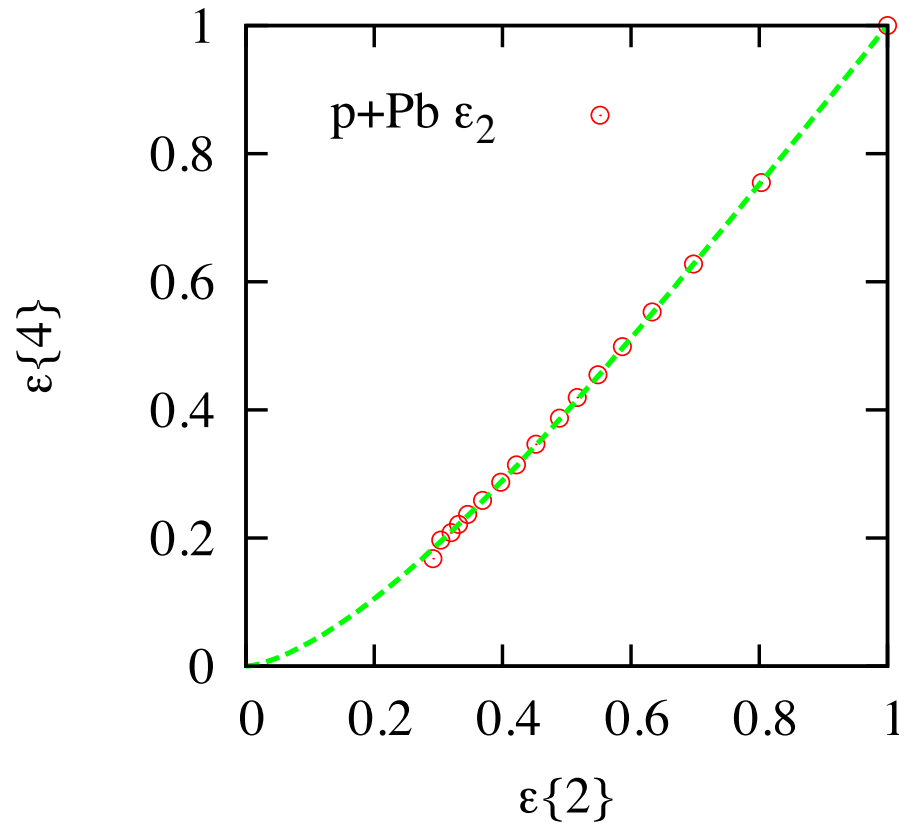
# Testing universality with cumulants

Central limit:  
large system,  
small fluctuations  
 $\varepsilon\{2\} \ll 1$  and  
 $\varepsilon\{4\} \ll \varepsilon\{2\}$



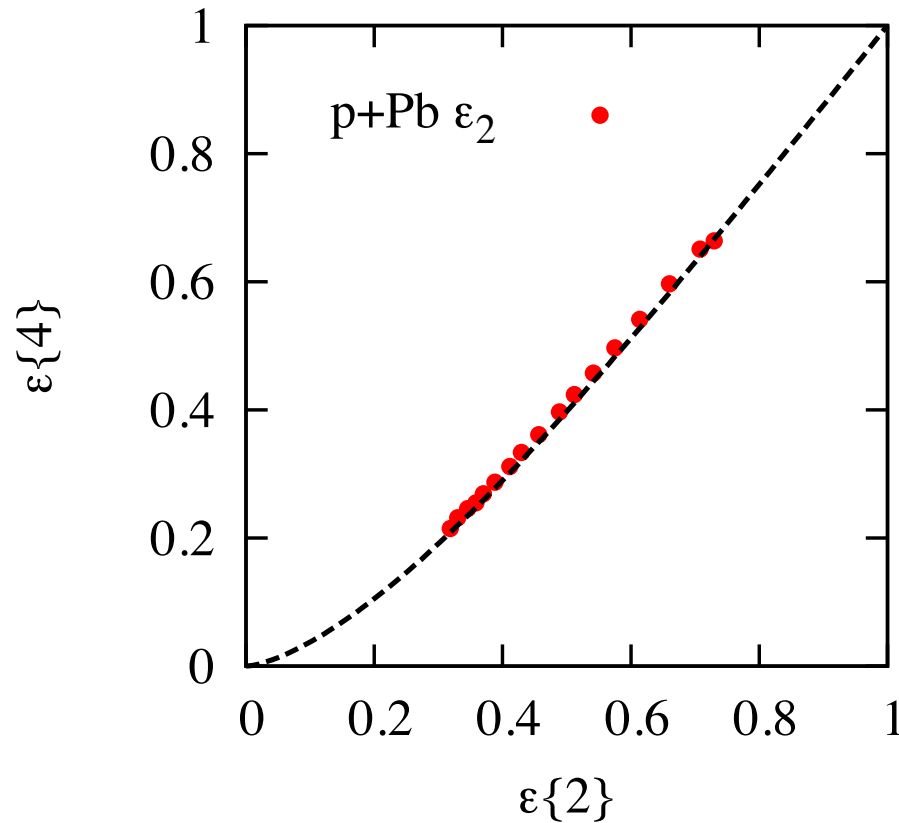
Prediction of the *power* distribution

# Testing universality with cumulants



Pointlike sources with Gaussian distribution:  
power distribution=exact=test of Monte-Carlo

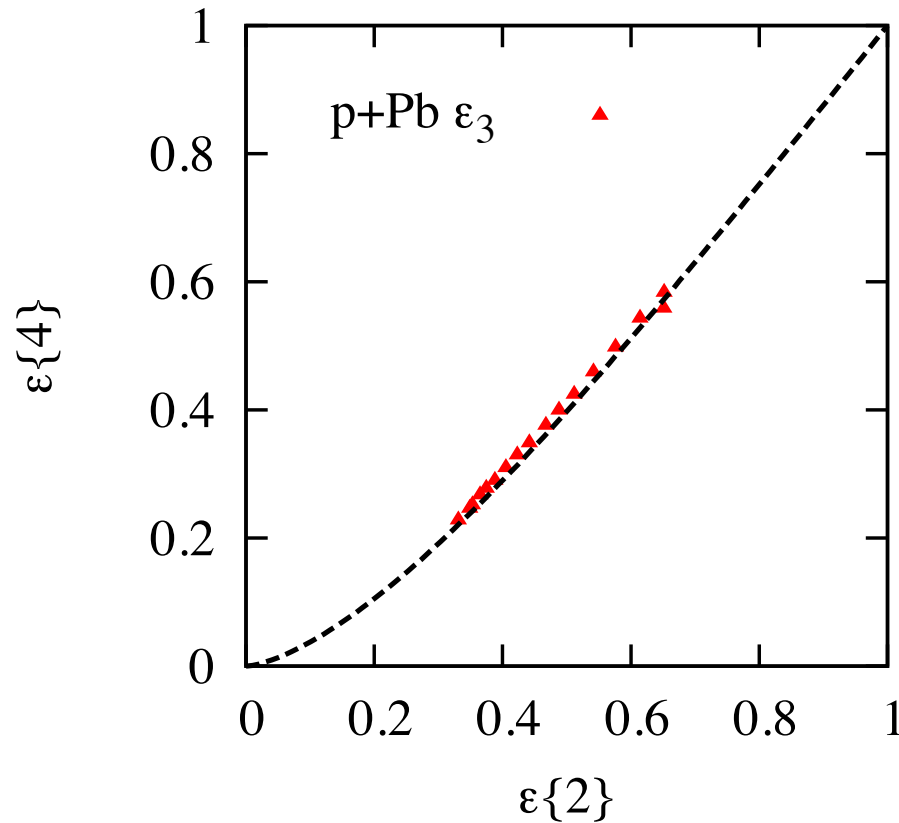
# Testing universality with cumulants



Each point: different number of hit nucleons in target

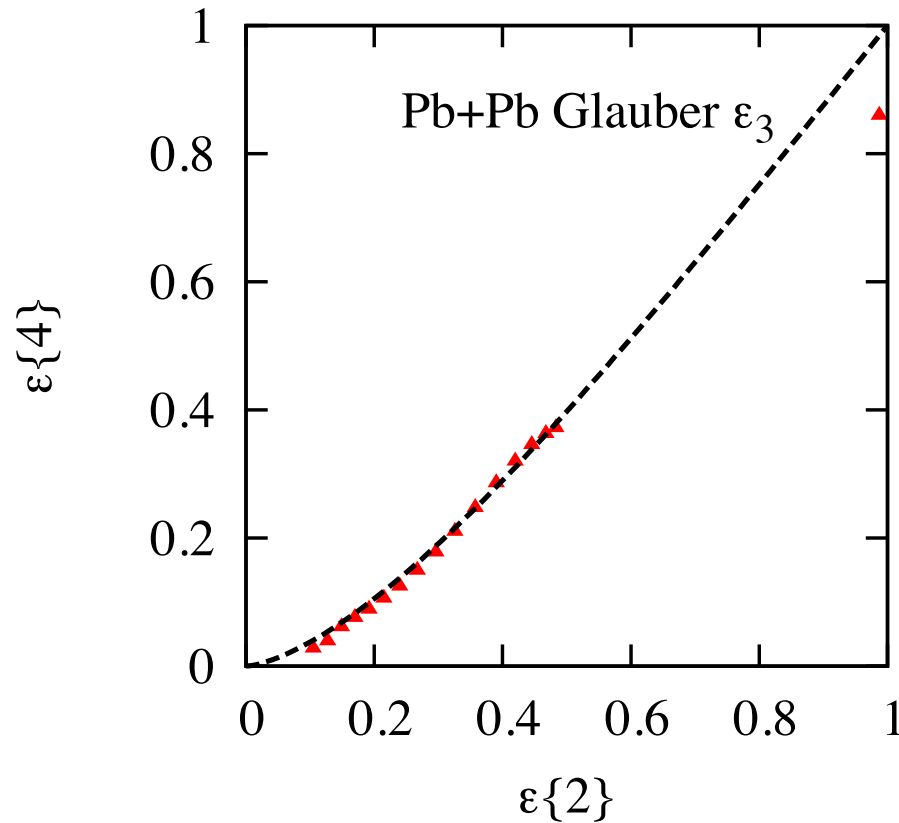


# Testing universality with cumulants



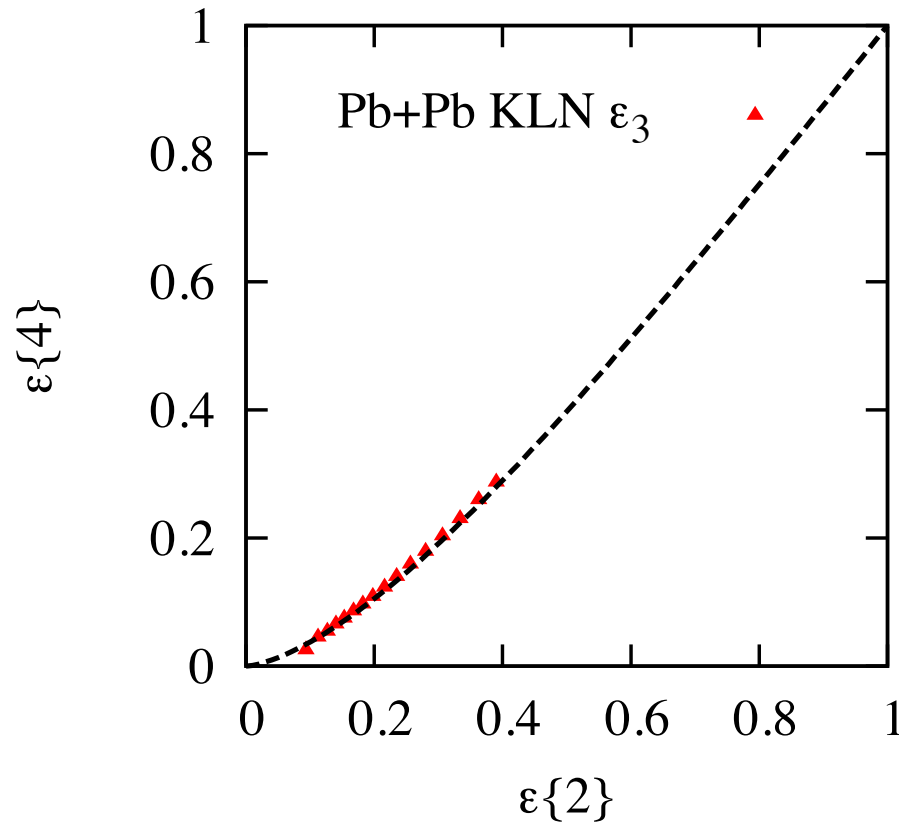
Each point: different number of hit nucleons in target

# Testing universality with cumulants



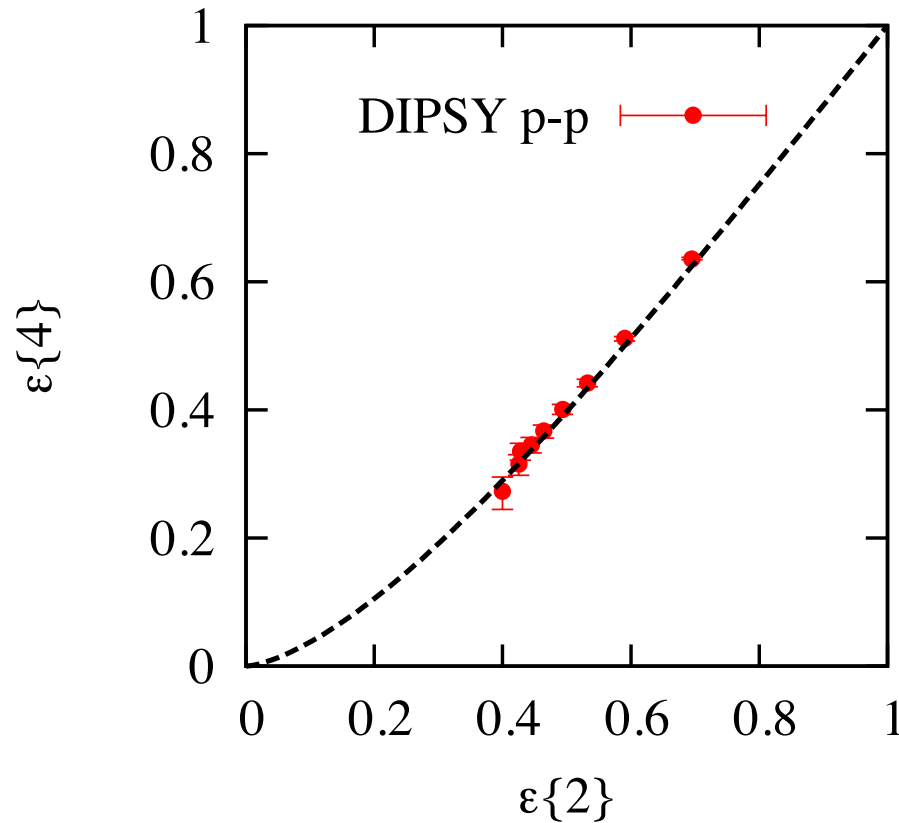
Each point: different centrality  
Pb-Pb: Larger system: smaller anisotropies

# Testing universality with cumulants



Each point: different centrality  
Pb-Pb: Larger system: smaller anisotropies

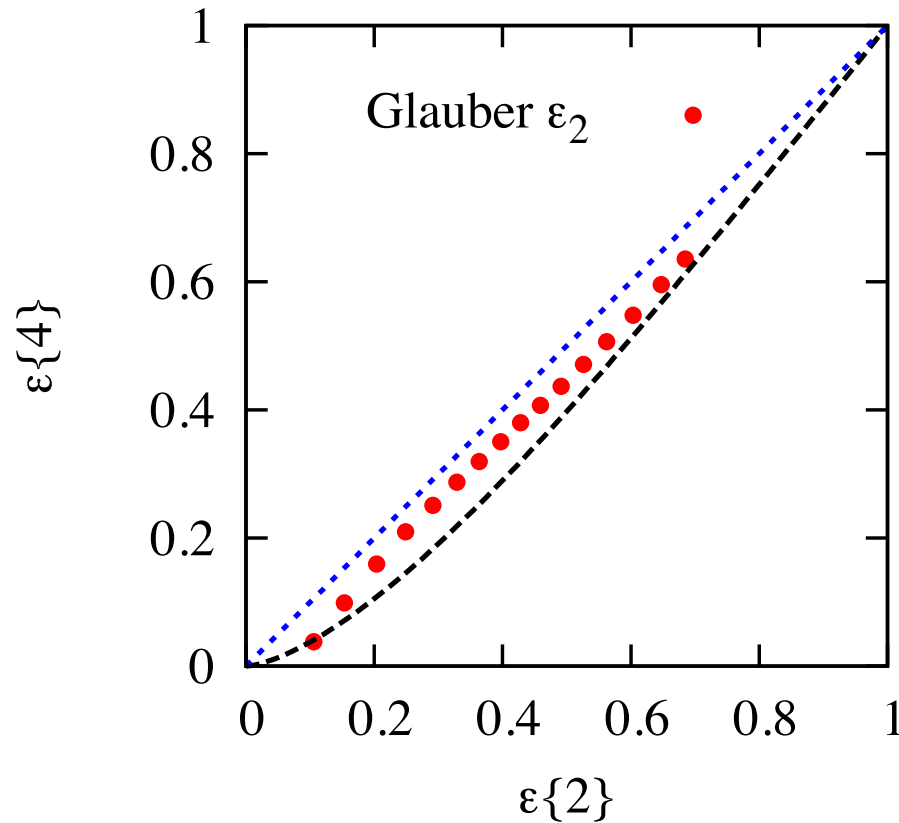
# Testing universality with cumulants



*data from Avsar Flensburg Hatta JYO Ueda 1009.5643*

Each point: different parton multiplicity

# Elliptic anisotropy in Pb-Pb



Driven by almond shape of overlap area, not fluctuations:  
Deviates from the power distribution

# Applying the power distribution to experimental data

If  $v_n = K_n \epsilon_n$ , with constant  $K_n$

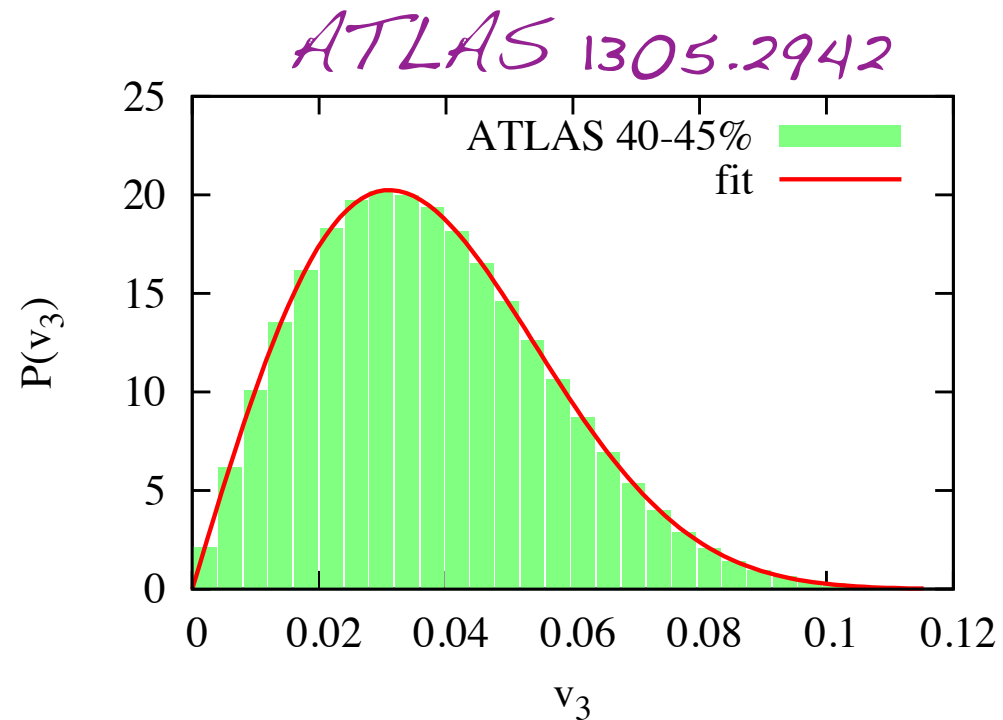
then  $v_n\{4\}/v_n\{2\} = \epsilon_n\{4\}/\epsilon_n\{2\}$

we can read off the parameter  $\alpha$  from the experimentally-measured ratio  $v_n\{4\}/v_n\{2\}$

# Fitting the distribution of $v_n$

The ATLAS distribution has published the distribution of  $v_n$  with  $n=2,3,4$  in Pb-Pb collisions. We can fit these data assuming  $v_3 \approx K_3 \epsilon_3$  and a power distribution for  $\epsilon_3$

The fit returns  
 $K_3 = 0.18 \pm 0.02$ ,  
in agreement with  
viscous hydrodynamics



*Yan, Poskanzer, JY0, in preparation*

# Simple predictions from eccentricity scaling

- Experimentally, one can measure moments (or cumulants) of the distribution of  $v_n$ .

- Eccentricity scaling implies that, e.g.

$$\langle v_n^4 \rangle / \langle v_n^2 \rangle^2 = \langle \epsilon_n^4 \rangle / \langle \epsilon_n^2 \rangle^2$$

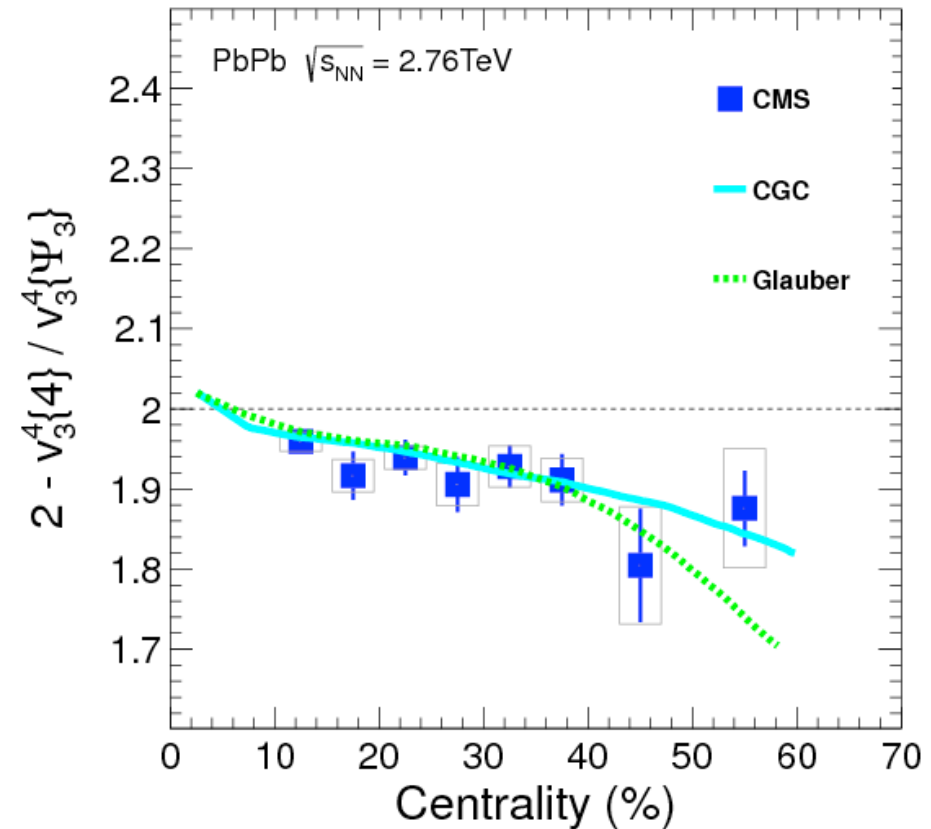
- Thus one can check if a particular model of the initial state is compatible with data.



# Eccentricity scaling versus data

data:  $\langle v_3^4 \rangle / \langle v_3^2 \rangle^2$

models:  $\langle \epsilon_3^4 \rangle / \langle \epsilon_3^2 \rangle^2$



*CMS 1310.8651*

# Higher-order cumulants

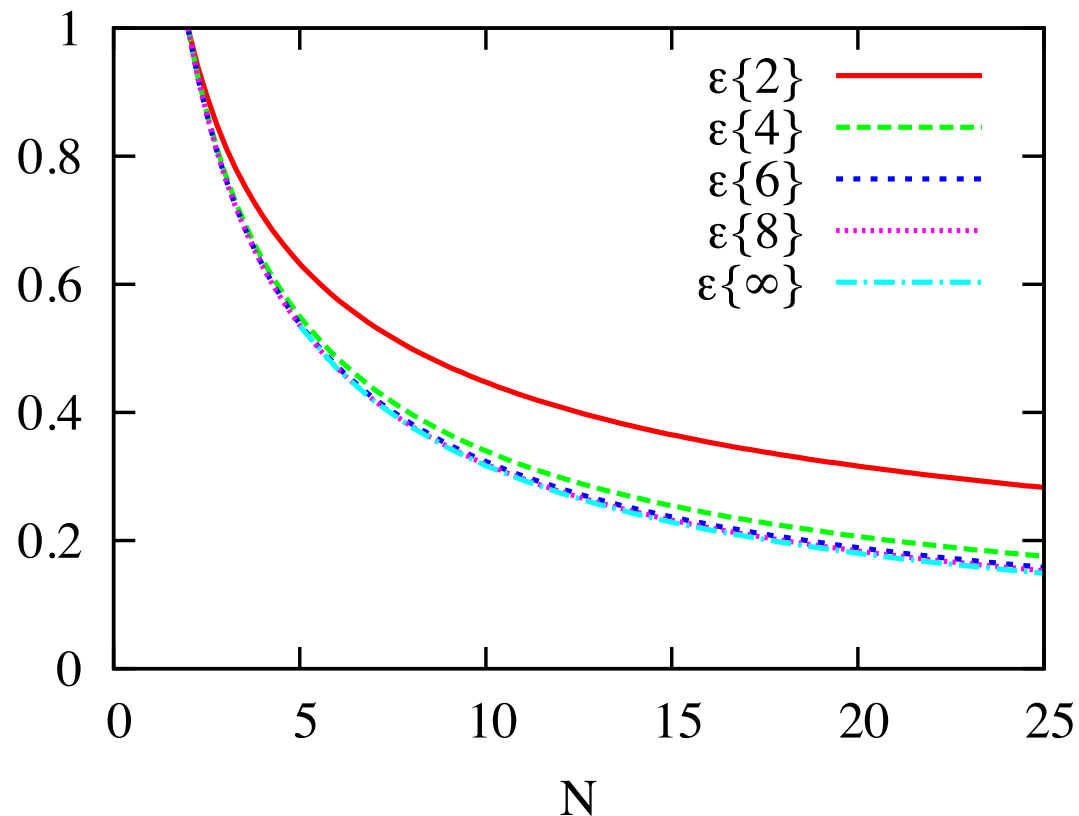
Expand the generating function

$$G(k) = \ln \langle \exp(i k \cdot \varepsilon) \rangle$$

where  $k$  and  $\varepsilon$  are 2-d vectors in the transverse plane, to order  $k^{2n}$ .

Asymptotic behavior for large  $n$  = singularity of  $G(k)$  = zero of the Fourier transform of the distribution of  $\varepsilon$ .

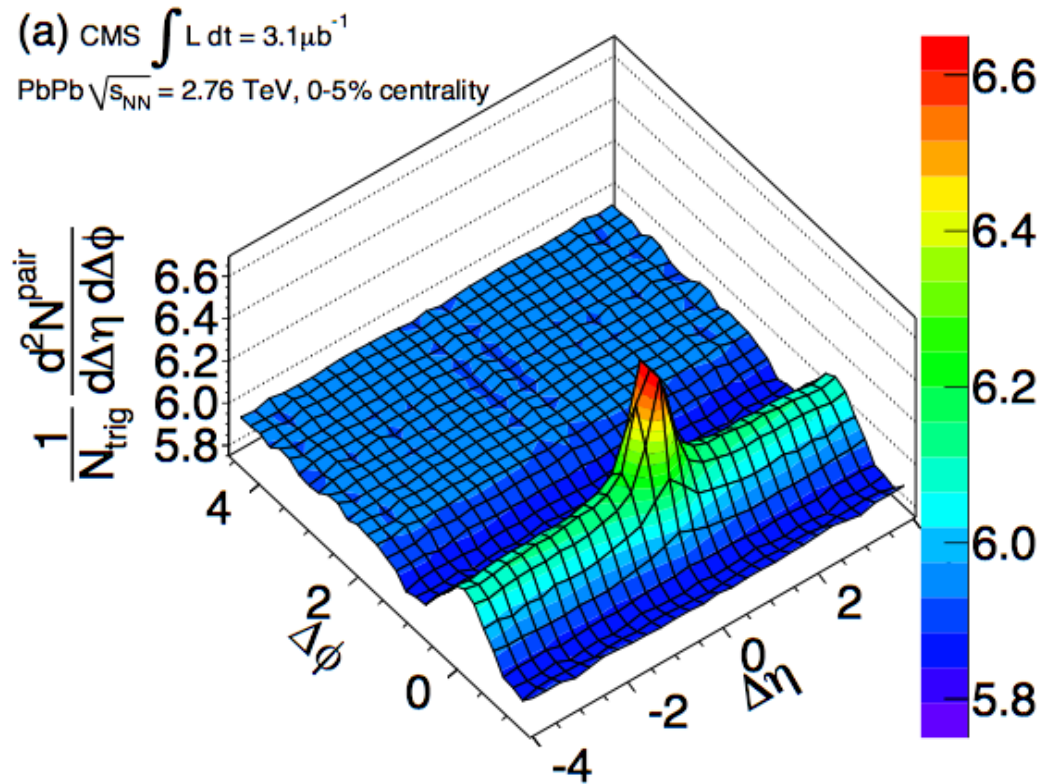
# Higher-order cumulants (predicted by the power distribution)



$\varepsilon\{n\}$  quickly converges as order  $n$  increases

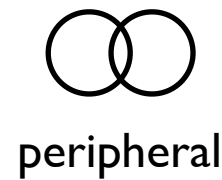
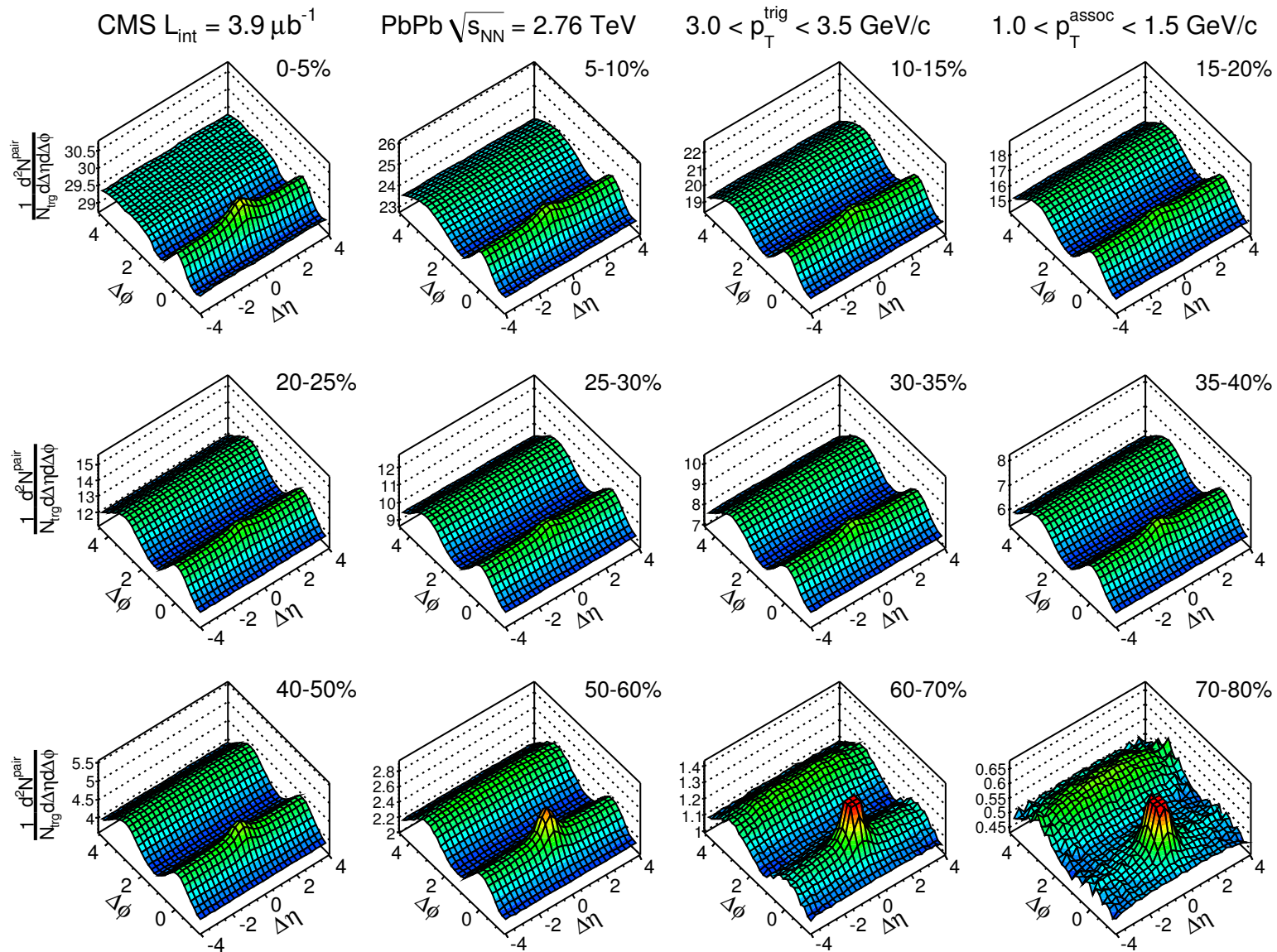
# Anisotropic flow

⇒ correlations at large  $\Delta\eta$



*CMS arXiv:1105.2438*

# Anisotropic flow



CMS 1201.3158