Experimental Aspects of Deep-Inelastic Scattering
-
Kinematics, Techniques and Detectors

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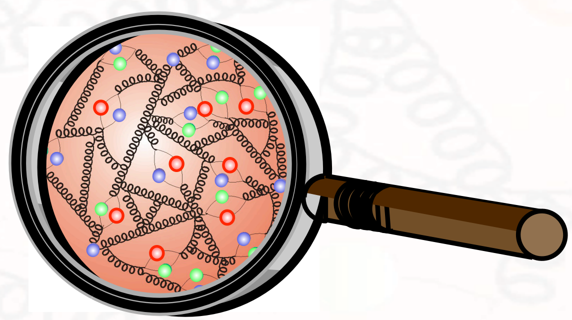
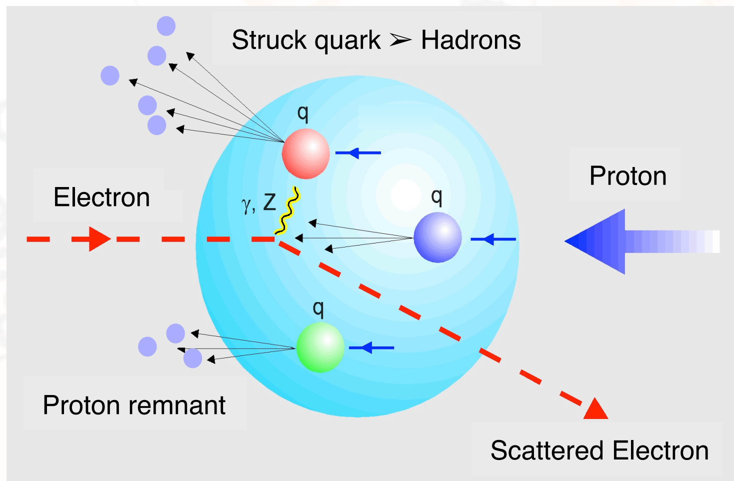


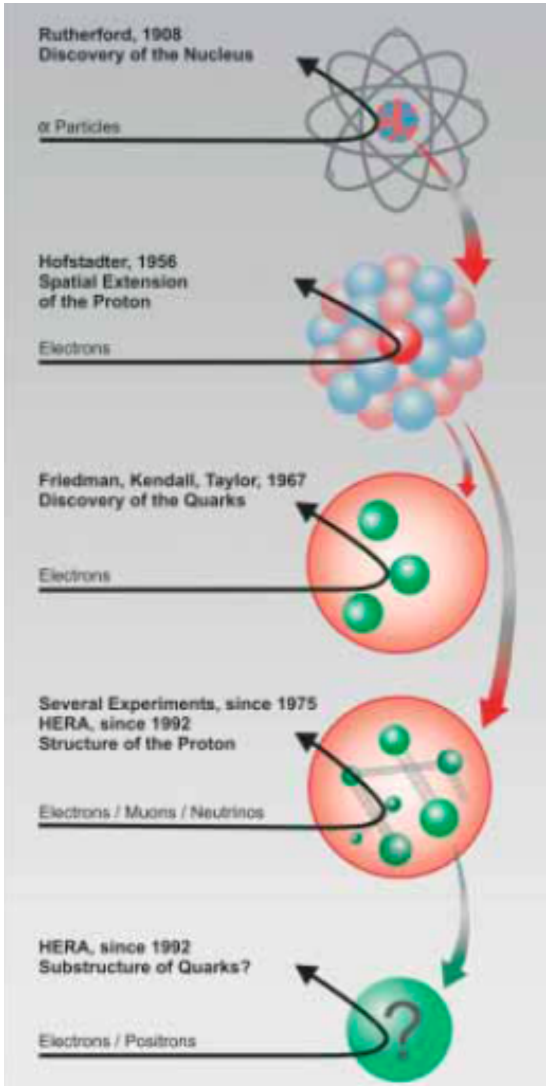
Outline

- DIS Kinematics
- DIS Structure Function Measurements
- DIS process description
  - Dirac Cross-Section
  - Mott Cross-Section
  - Rutherford Cross-Section
  - DIS Cross-Section
- DIS Collider Detectors
- DIS Introduction
- Summary and Outlook

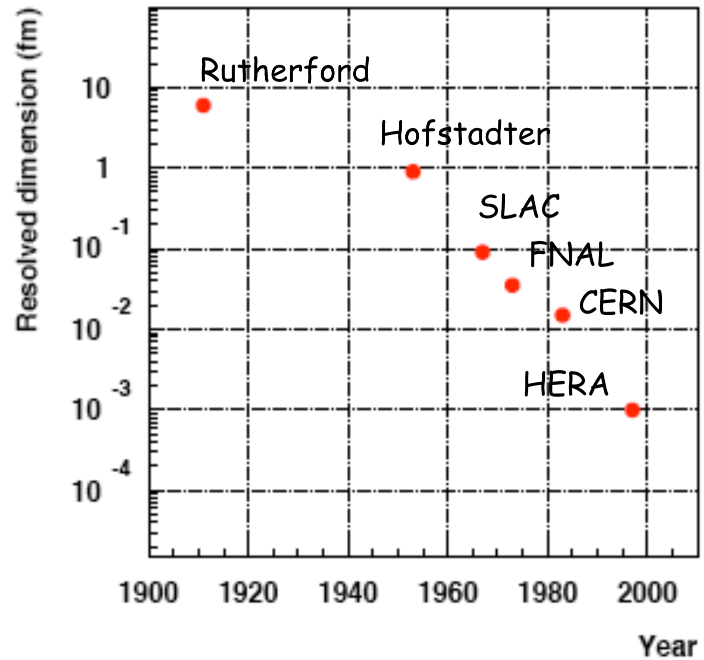


DIS Introduction

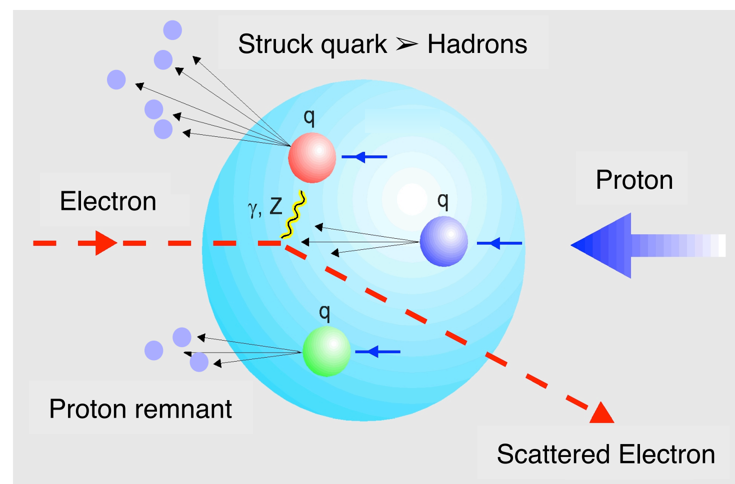
General considerations on scattering experiments



Probing smaller distances requires larger momentum transfer q (small wavelength)

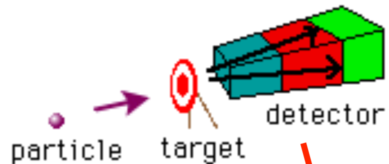


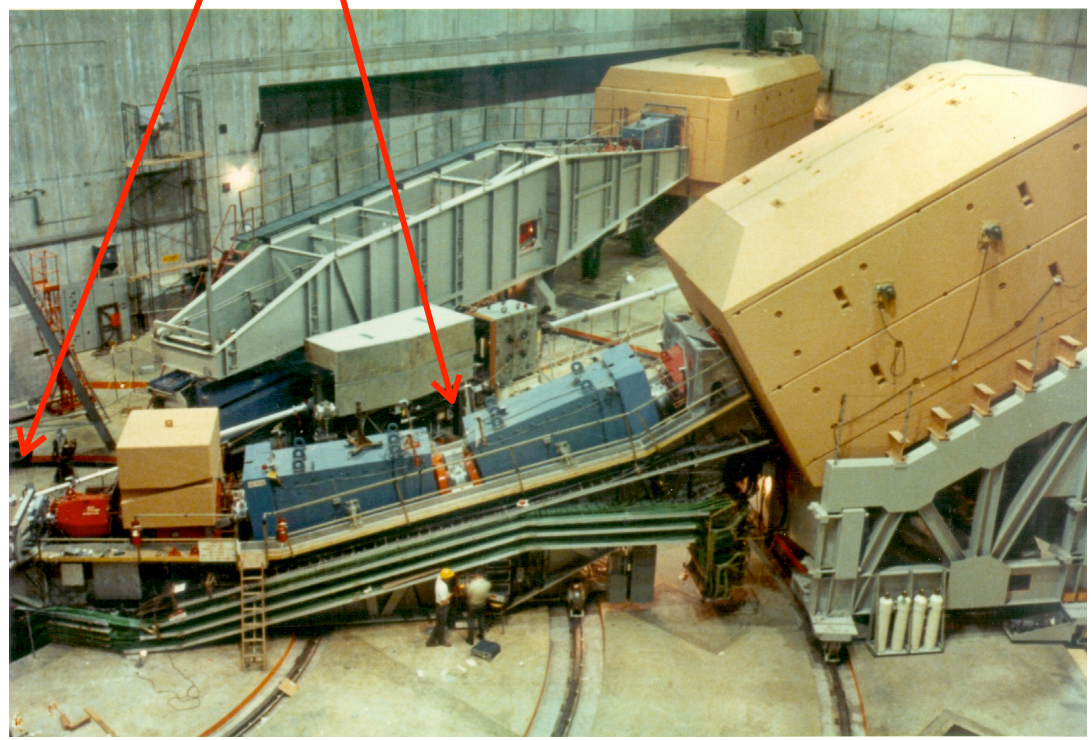
- Measurement of the final-state (e.g. scattered electron): $\Rightarrow$ Structure of target!
- Scatter point-like probe onto object (target)



DIS Introduction

- Fixed-target experiment: Endstation A at Stanford Linear Accelerator Center (SLAC)



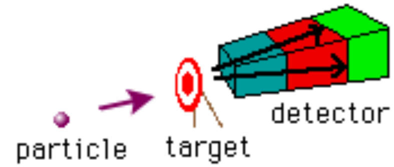


DIS Introduction

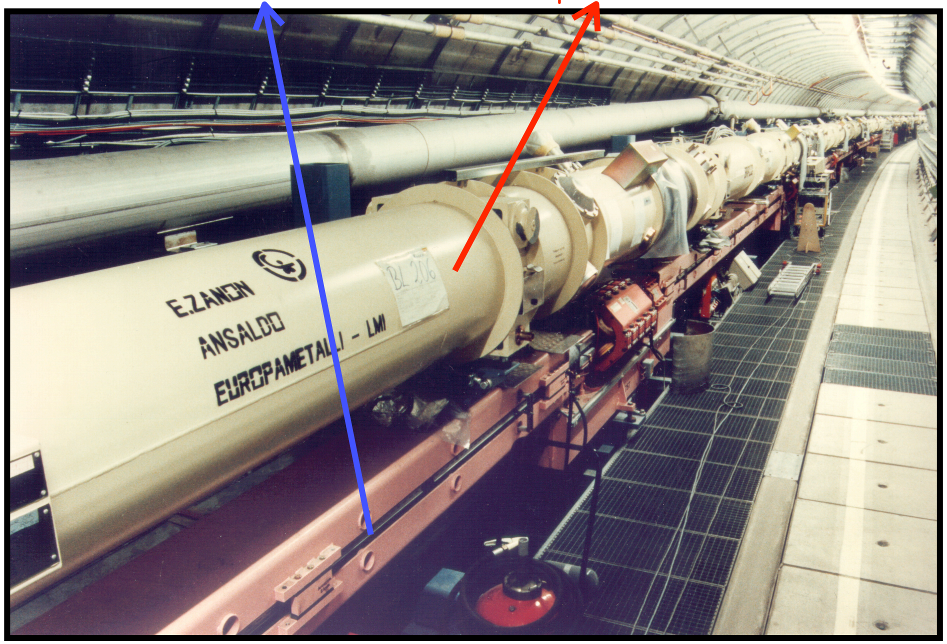
- Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)

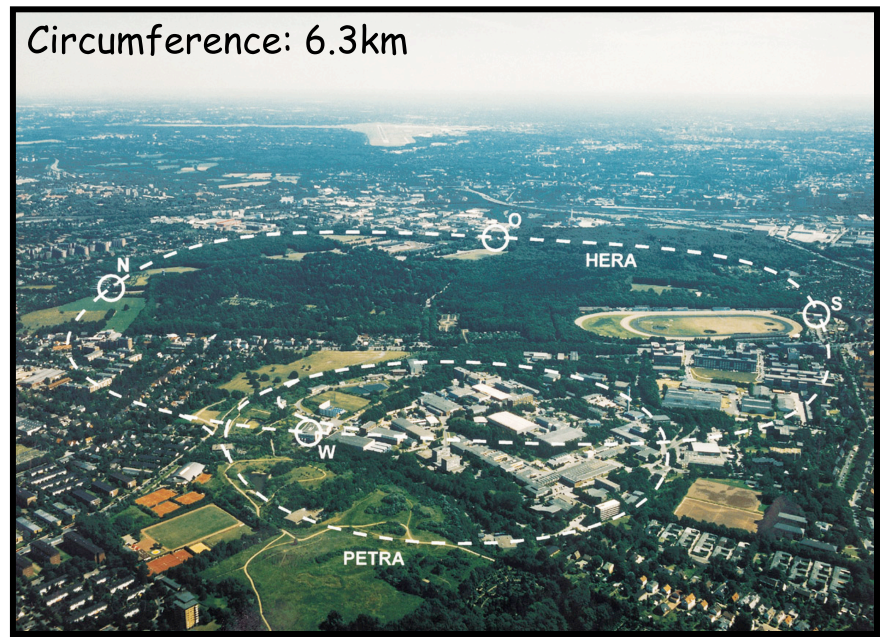
Equivalent to fixed target of

E_e = 50600 GeV:



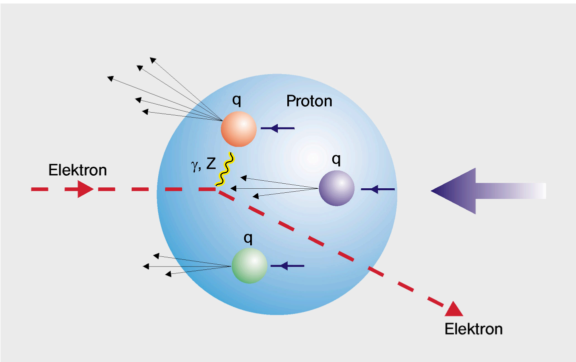
E_e = 27.5 GeV E_p = 920 GeV





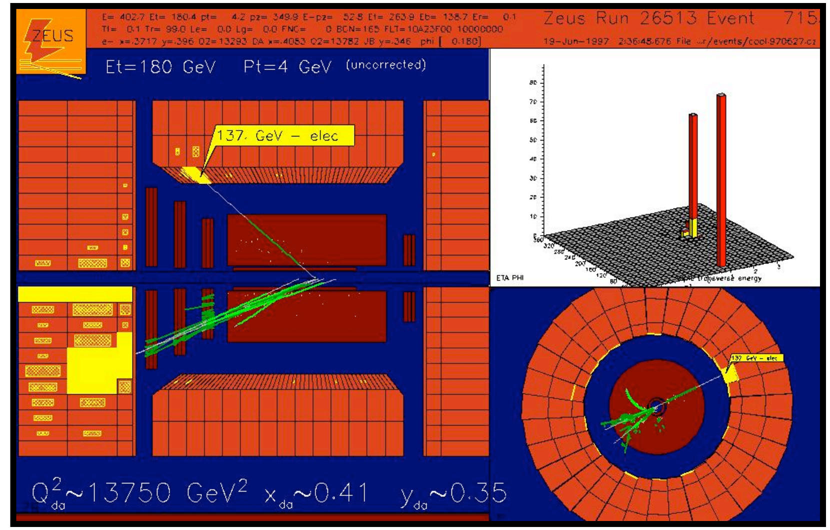
DIS Introduction

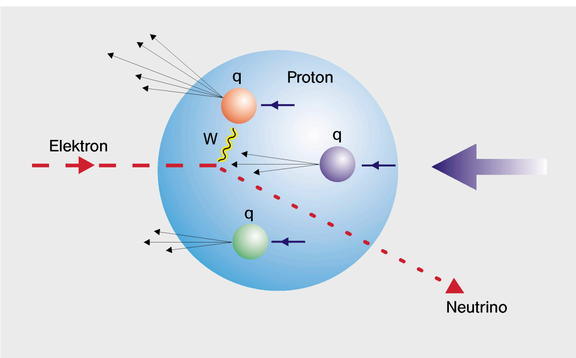
- DIS major event classes



Neutral-current event

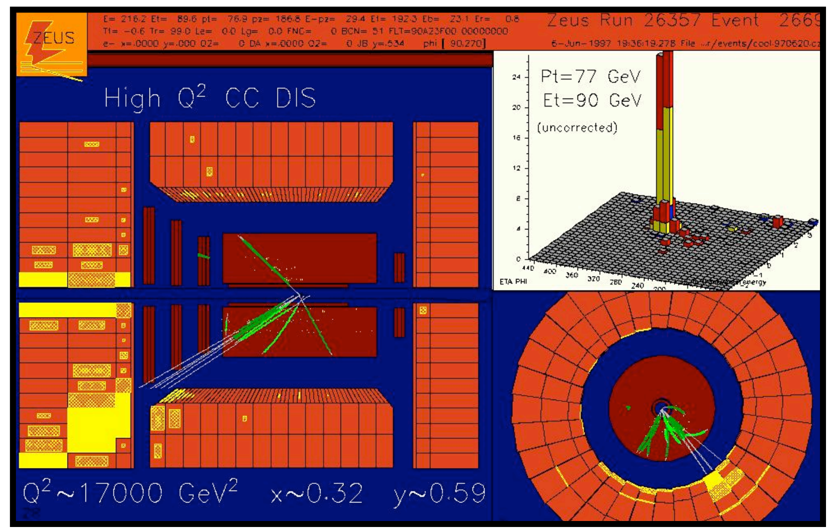
$e\,p \rightarrow e\,X$





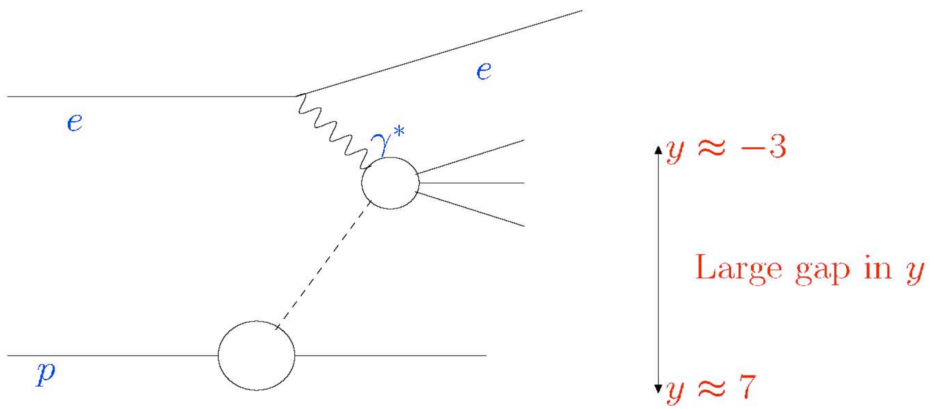
Charged-current event

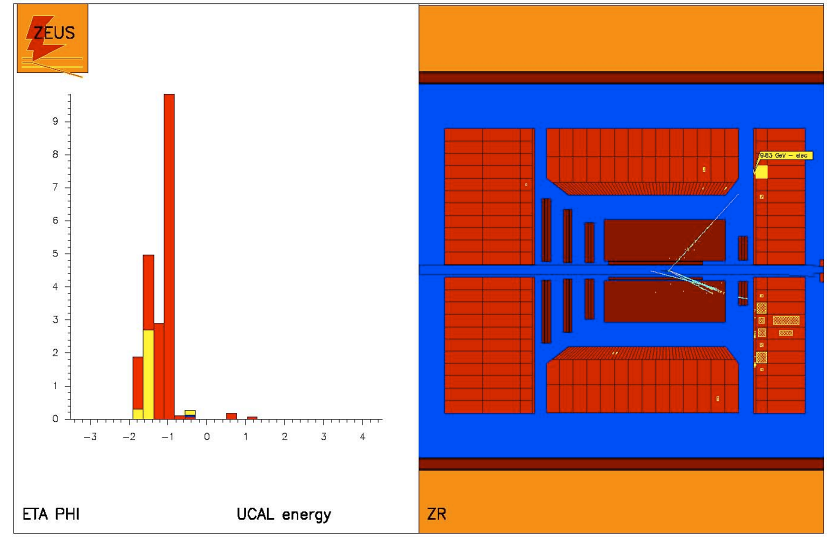
$e\,p \rightarrow \nu\,X$



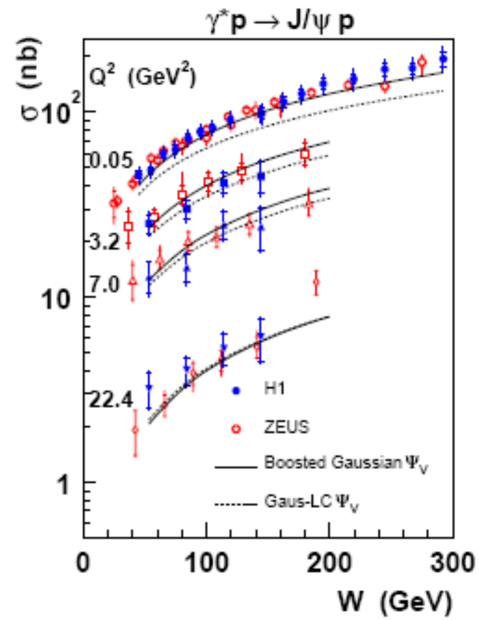
DIS Introduction

- Diffractive events





- Ratio of diffractive to total cross-section (200<W<245GeV): 15% at Q^2=4GeV^2
- Dipole models: Successful description of inclusive and various diffractive measurements (e.g. Ratio of diffractive to inclusive cross-section, Diffractive Vector-Meson production)



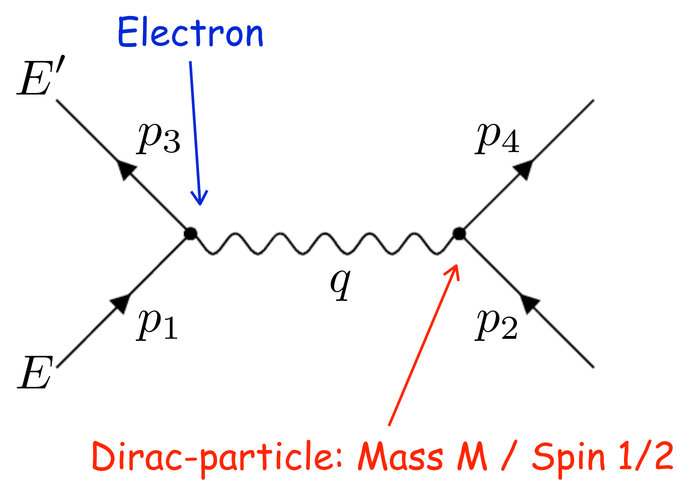
$$\propto \alpha_s^2 \left[g(x,Q^2)\right]^2$$

DIS Process Description

- Dirac Cross-Section (Electron-Mass M (Spin 1/2) Particle scattering)
  - Recoil: E' is kinematically determined by E and θ

$$E' = \frac{E}{1 + (2E/Mc^2)\sin^2(\theta/2)}$$

$$M \to \infty \quad E' = E$$



  - Scattering process: 2 $\to$ 2

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{\hbar E'}{8\pi M c E}\right)^2 \langle|\mathcal{M}|^2\rangle$$

  - Dirac Cross-Section:

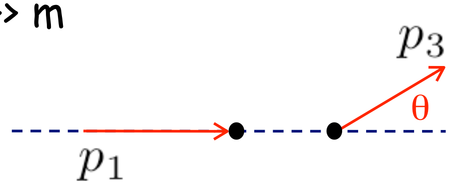
$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2\hbar^2c^2}{4E^2\sin^4(\theta/2)}\left(\frac{E'}{E}\right)\left\{\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2c^2}\sin^2\left(\frac{\theta}{2}\right)\right\}$$

Impact of electron-spin

Impact of target-spin (mass M)

DIS Process Description

- Mott Cross-Section (Electron-Heavy Mass M (Spin 1/2) Particle scattering)
  - Mott condition: Electron scatters off a much heavier spin 1/2 particle of mass M >> m



$$|\vec{p}_1| = |\vec{p}_3| = |\vec{p}| \qquad \vec{p}_1 \cdot \vec{p}_3 = \vec{p}^2 \cos\theta$$

$$p_1 = \left(\frac{E}{c}, \vec{p}_1\right) \quad p_2 = \left(Mc, \vec{0}\right) \longrightarrow p_3 = \left(\frac{E}{c}, \vec{p}_3\right) \quad p_4 = \left(Mc, \vec{0}\right)$$

  - Calculate four-vector combinations:

$$Q^2 = -q^2 = -(p_1 - p_3)^2 = 4\vec{p}^2 \sin^2\frac{\theta}{2}$$

$$(p_1 \cdot p_3) = m^2c^2 + 2\vec{p}^2 \sin^2\frac{\theta}{2}$$

$$(p_1 \cdot p_2)(p_3 \cdot p_4) = (p_1 \cdot p_4)(p_2 \cdot p_3) = (ME)^2$$

$$(p_2 \cdot p_4) = (Mc)^2$$

$$\langle|\mathcal{M}|^2\rangle = \frac{8g_e^4}{(p_1 - p_3)^4} \times [(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_3)(Mc)^2 - (p_2 \cdot p_4)(mc)^2 + 2(mMc^2)^2]$$

DIS Process Description

Summary

- Re-write Dirac cross-section using the Mott cross-section

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2 \hbar^2 c^2}{4E^2 \sin^4(\theta/2)} \left(\frac{E'}{E}\right) \left\{ \cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2c^2} \sin^2\left(\frac{\theta}{2}\right) \right\}$$

with: $q^2 = -4(E/c)(E'/c)\sin^2(\theta/2)$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(\frac{E'}{E}\right) \left\{ 1 - \frac{q^2}{2M^2c^2} \tan^2\left(\frac{\theta}{2}\right) \right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cos^2(\theta/2) \left(\frac{E'}{E}\right) \left\{ 1 - \frac{q^2}{2M^2c^2} \tan^2\left(\frac{\theta}{2}\right) \right\}$$

Note:

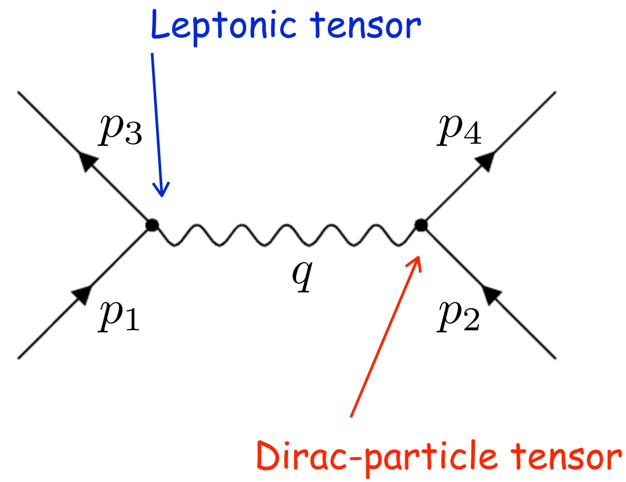
- This is the result we would obtain if the proton would be a Dirac particle: $M=m_p$
- Modifications to Rutherford cross-section: Relativistic effects (SPIN effects: Probe and Target (Negligible for very heavy target)!

DIS Process Description

- Elastic ep scattering (1)
  - Recall:

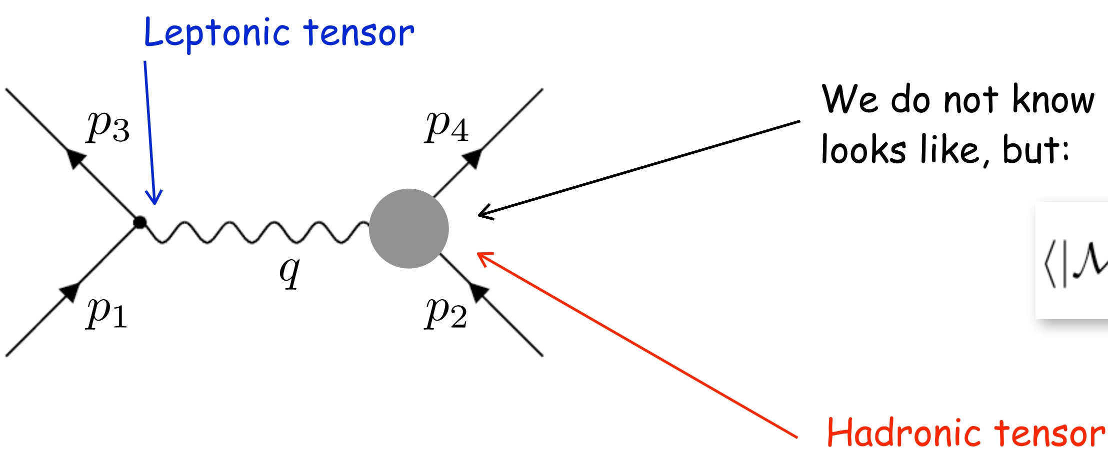
$$\langle|\mathcal{M}|^2\rangle = \frac{(4\pi\alpha)^2}{q^4} L^{\mu\nu}_{\text{electron}} L_{\mu\nu,\text{Dirac particle M}}$$

Feynman graph:



Note: The proton cannot be just a Dirac particle (g=2)!

Therefore: ep scattering



We do not know exactly how this tensor looks like, but:

$$\langle|\mathcal{M}|^2\rangle = \frac{(4\pi\alpha)^2}{q^4} L^{\mu\nu}_{\text{electron}} K_{\mu\nu,\text{proton}}$$

DIS Process Description

Elastic ep scattering (2)

- High-energy limit:

$$\langle |\mathcal{M}|^2 \rangle = \left(\frac{(4\pi\alpha c)^2}{4EE' \sin^4(\theta/2)} \right) \left\{ 2K_1 \sin^2\left(\frac{\theta}{2}\right) + K_2 \cos^2\left(\frac{\theta}{2}\right) \right\}$$

With:

$$E' = \frac{E}{1 + (2E/Mc^2)\sin^2(\theta/2)}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{\hbar E'}{8\pi M c E}\right)^2 \langle |\mathcal{M}|^2 \rangle$$

Dirac particle result:

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2\hbar^2c^2}{4E^2\sin^4(\theta/2)} \left(\frac{E'}{E}\right) \left\{ \cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2c^2} \sin^2\left(\frac{\theta}{2}\right) \right\}$$

For: $K_2 = (2Mc)^2$ $K_1 = -q^2$

We find:

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{\alpha\hbar}{4ME\sin^2(\theta/2)}\right)^2 \frac{E'}{E} \left(K_2 \cos^2\left(\frac{\theta}{2}\right) + 2K_1 \sin^2\left(\frac{\theta}{2}\right) \right)$$

Rosenbluth formula

Form factors!

DIS Process Description

- Elastic ep scattering (3)

$$\frac{K_2}{4M^2} = \frac{G_E^2 + \tau G_M^2}{1+\tau} \qquad \frac{2K_1}{4M^2} = 2\tau G_M^2 \qquad \tau = -q^2/4M^2$$

  - Summary Rosenbluth formula

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \cos^2\left(\frac{\theta}{2}\right) \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right)$$

Rutherford cross section

Recoil term!

Mott cross section

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right)$$

Note: For heavy target: Contribution from G^2_M term negligible:

$$\tau = -q^2/4M^2 \to 0$$

For $G^2_M = 1$ and $G^2_E = 1$ this reduces to the well-known Dirac particle case::

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \frac{E'}{E} \cos^2\left(\frac{\theta}{2}\right) \left(1 + 2\tau \tan^2\left(\frac{\theta}{2}\right)\right)$$

DIS Process Description

Elastic ep scattering (4)

- Scattering of electron (Spin 1/2) on point-charge charge (Spin 0): Mott cross-section

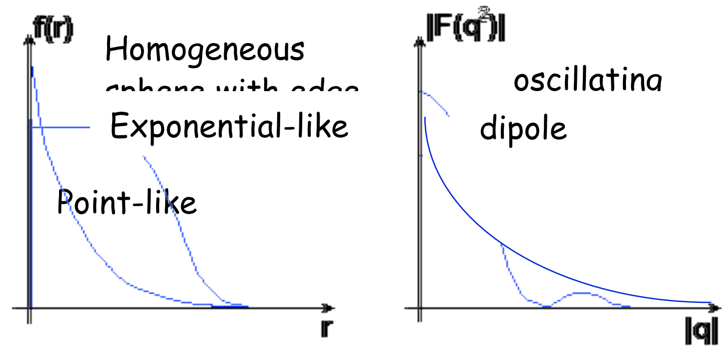
$$\left(\frac{d\sigma}{d\Omega}\right)^*_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \cos^2(\theta/2)$$

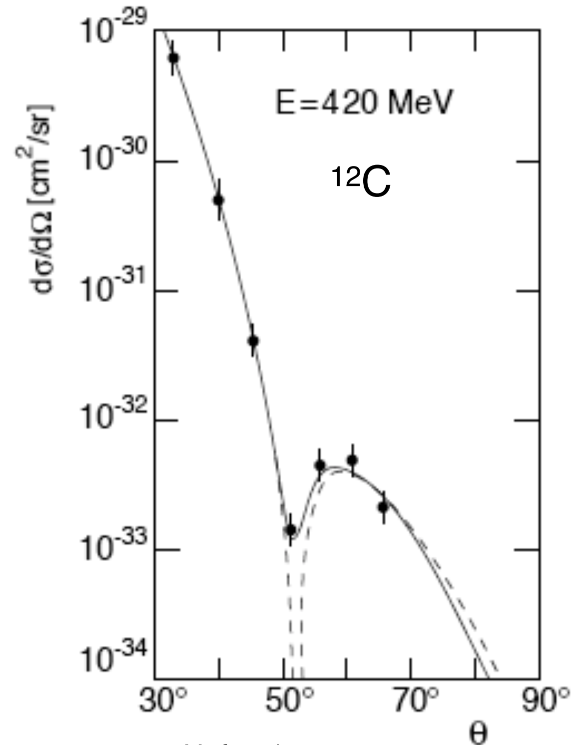
- Take into account finite charge distribution: Form factor

$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)^*_{Mott} \cdot \left|F(q^2)\right|^2$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)^*_{Mott} \frac{E'}{E}$$

} Ignore recoil!





Hofstadter, 1953

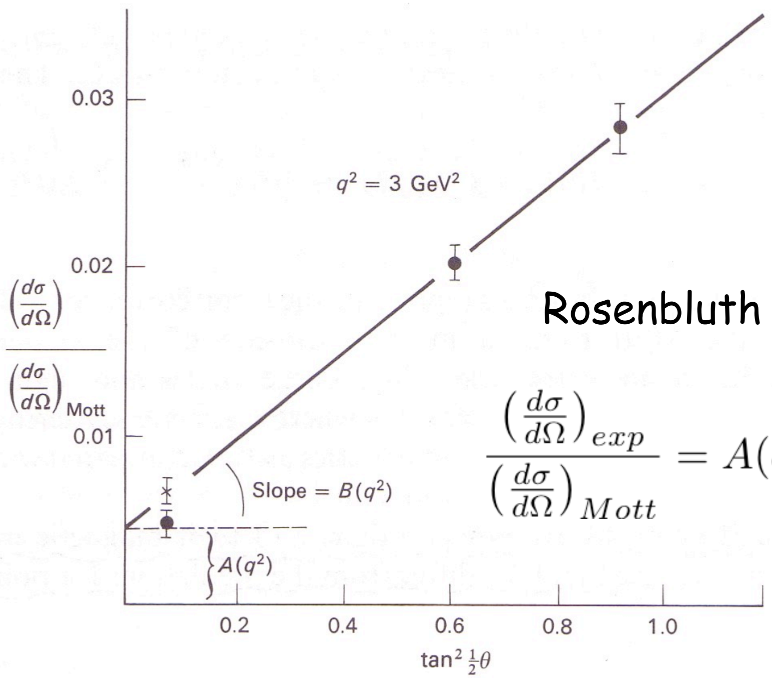
DIS Process Description

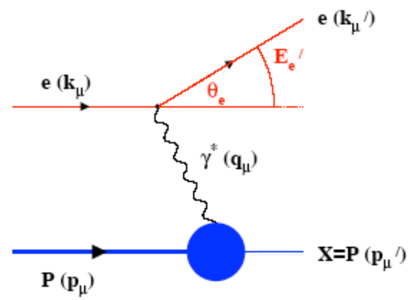
Elastic ep scattering (5)

Scattering of electron (Spin 1/2) on proton (Spin 1/2)

$$\left(\frac{d\sigma}{d\Omega}\right)_{point} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \{1 + 2\tau \tan^2(\theta/2)\}$$

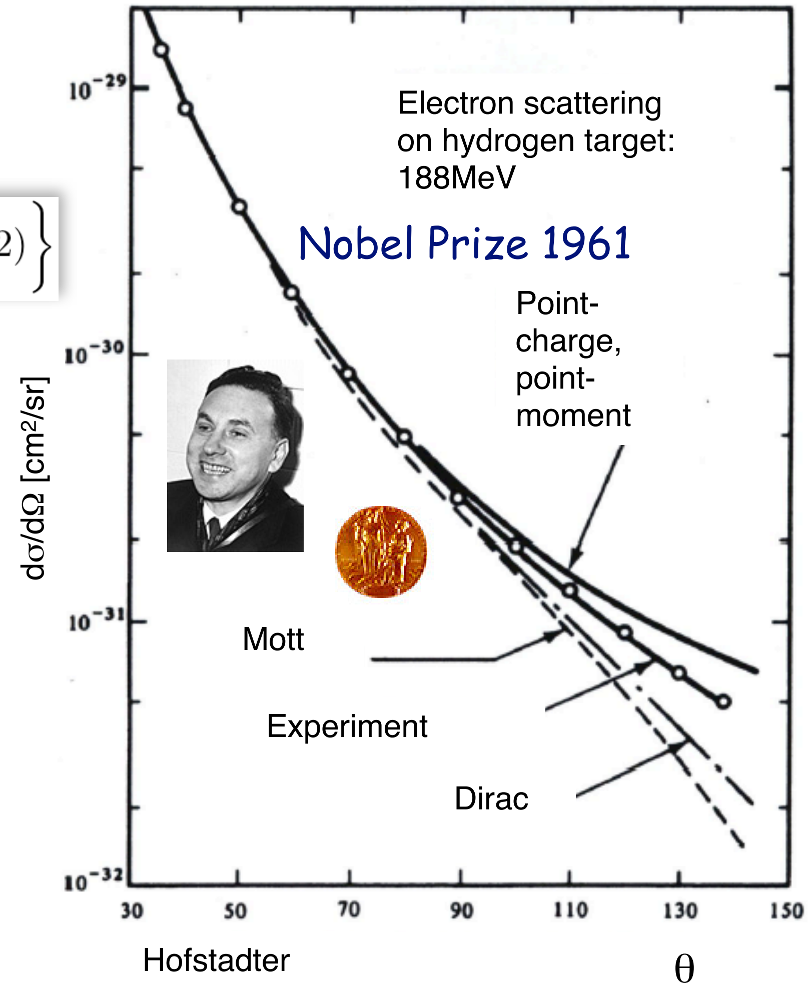
$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left\{\frac{G_E^2(q^2) + \tau G_M^2(q^2)}{1+\tau} + 2\tau G_M^2(q^2)\tan^2(\theta/2)\right\}$$





Rosenbluth separation method:

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)_{exp}}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = A(q^2) + B(q^2)\tan^2(\theta/2)$$



DIS Process Description

- Inelastic ep scattering (1)
  - Simplify this: Massless electron (m=0) of energy E strikes a stationary proton of mass M:

$$\sqrt{(p_1 \cdot p_2) - (m_1 m_2 c^2)^2} = ME$$

$$|\vec{p}_3| = E'/c$$

$$d^3\vec{p}_3 = |\vec{p}_3|^2 d|\vec{p}_3| d\Omega$$

$$E' = E_3$$

$$\left(\frac{d\sigma}{dE' d\Omega}\right) = \left(\frac{\alpha\hbar}{cq^2}\right)^2 \left(\frac{E'}{E}\right) L^{\mu\nu} W_{\mu\nu}$$

Leptonic tensor ($L^{\mu\nu}$) — Hadronic tensor ($W_{\mu\nu}$)

  - Note:
    - E' is no longer kinematically determined by E and θ
    - The total hadronic momentum can vary and is no longer constrained, i.e.:

$$p_{tot}^2 \neq M^2 c^2$$

DIS Process Description

Inelastic ep-scattering (2)

- Goal: Measure differential cross section in a particular energy range dE'
- Leptonic tensor: $L^{\mu\nu}_{\text{electron}} = 2\{p_1^\mu p_3^\nu + p_1^\nu p_3^\mu + g^{\mu\nu}[(mc)^2 - (p_1 \cdot p_3)]\}$
- Ansatz for hadronic tensor: $W_{\mu\nu,\,\text{proton}} = -W_1 g_{\mu\nu} + \frac{W_2}{(Mc)^2} p_\mu p_\nu + \frac{W_4}{(Mc)^2} q_\mu q_\nu + \frac{W_5}{(Mc)^2}(p_\mu q_\nu + p_\nu q_\mu)$
- With $q_\mu W^{\mu\nu} = 0$
- We get: $W_4 = \frac{(Mc)^2}{q^2} W_1 + \left(\frac{p \cdot q}{q^2}\right)^2 W_2 \qquad W_5 = -\frac{q \cdot p}{q^2} W_2$
- Therefore: $W_{\mu\nu,\,\text{proton}} = W_1 \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{W_2}{(Mc)^2}\left[p_\mu + \left(\frac{q \cdot p}{q^2}\right) q_\mu\right]\left[p_\nu + \left(\frac{q \cdot p}{q^2}\right) q_\nu\right]$
- Contraction with leptonic tensor yields: $\left(\frac{d^2\sigma}{dE' d\Omega}\right) = \frac{(\alpha\hbar)^2}{4E^2 \sin^4(\theta/2)}\left(W_2 \cos^2\left(\frac{\theta}{2}\right) + 2W_1 \sin^2\left(\frac{\theta}{2}\right)\right)$
- Note:
  - W_1 and W_2 are functions of q^2 and $q \cdot p$
  - G_E and G_M are functions of q^2 only!

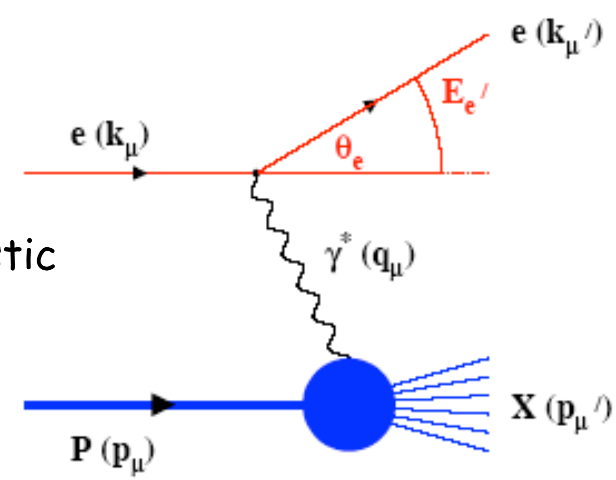
$Q^2 = -q^2$ } Negative four-momentum transfer squared!

$x \equiv \frac{Q^2}{2q \cdot p}$ } Bjorken scaling variable

DIS Process Description

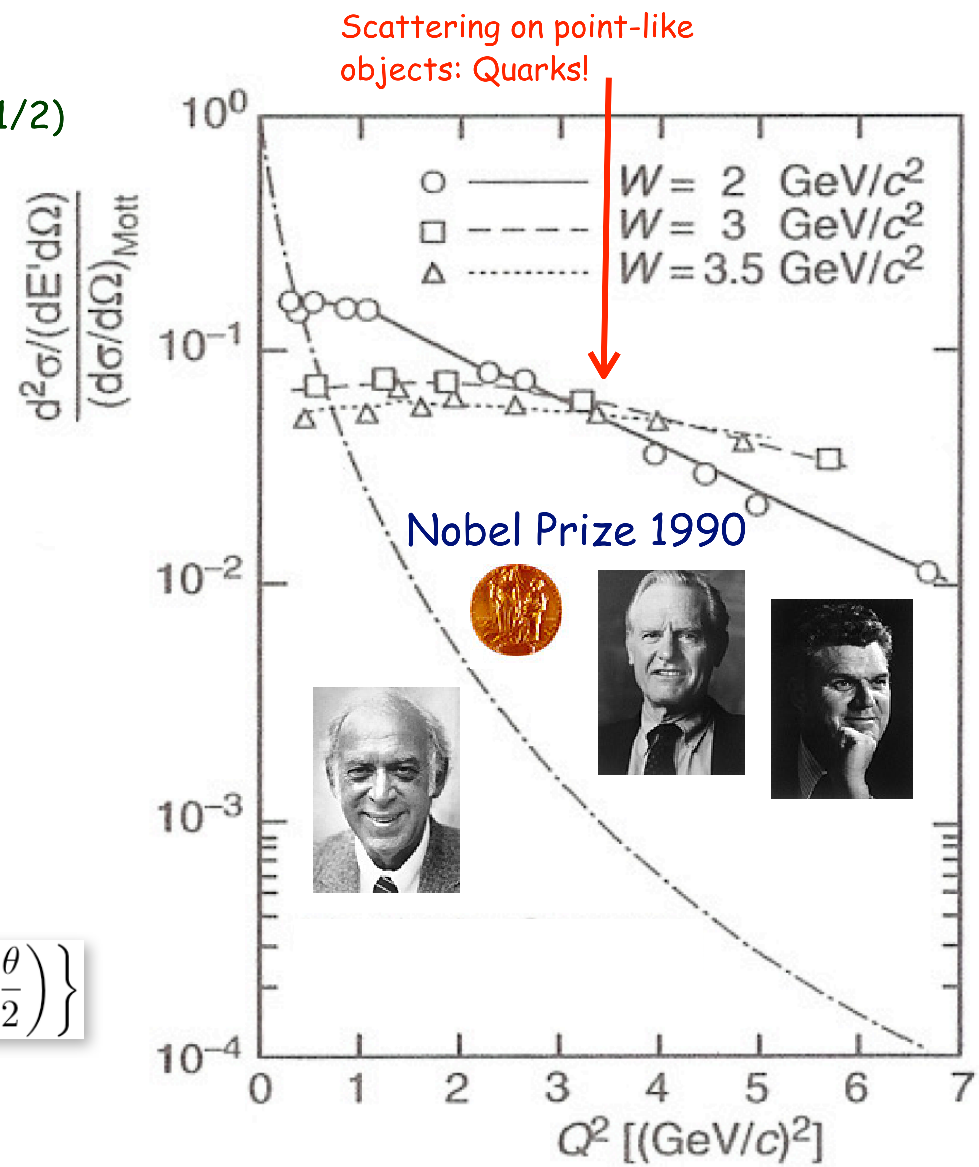
- Inelastic ep scattering (3)
  - Scattering of electron (Spin 1/2) on proton (Spin 1/2)

Here: Deep-inelastic scattering (DIS)



$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E'}{E}\left\{1 + 2\tau \tan^2\left(\frac{\theta}{2}\right)\right\}$$

$$\left(\frac{d^2\sigma}{dE'd\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left\{W_2(Q^2,x) + 2W_1(Q^2,x)\tan^2\left(\frac{\theta}{2}\right)\right\}$$



Friedman, Kendall and Taylor00

DIS Process Description

- Summary

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E'}{E}\left\{1 + 2\tau\tan^2\left(\frac{\theta}{2}\right)\right\} \qquad \tau = -q^2/4M^2$$

Dirac cross-section

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E'}{E}\left\{\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2\tan^2\left(\frac{\theta}{2}\right)\right\}$$

Elastic ep cross-section

$$\tau = -q^2/4M^2$$

$$\left(\frac{d^2\sigma}{dE'd\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left\{W_2(Q^2,x) + 2W_1(Q^2,x)\tan^2\left(\frac{\theta}{2}\right)\right\}$$

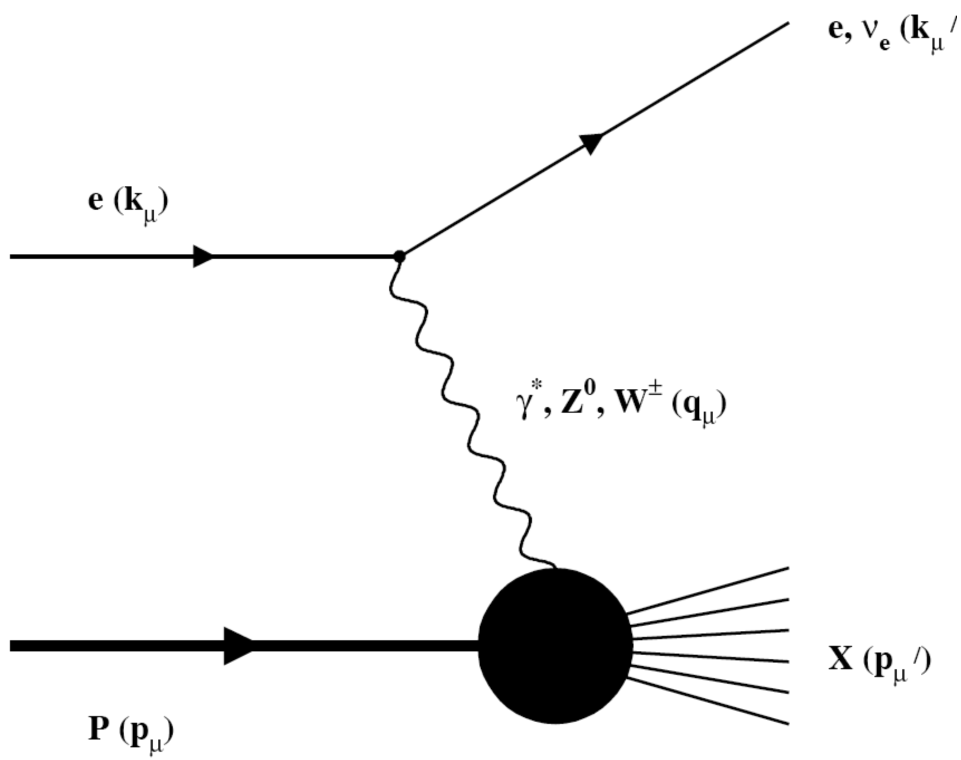
Inelastic ep cross-section

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \cos^2(\theta/2) \qquad \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{4\alpha^2 E'^2}{q^4}$$

DIS Kinematics

- General considerations

$$e(k) + P(p) \rightarrow l(k^{'}) + X(p^{'})$$



- Four vectors: $k, \;\; p, \;\; k', \;\; p'$
- Neutral current exchange (NC): γ^*, Z^0
- Charged current exchange (CC): $W^{\pm}$
- Measurement of structure functions:
  - NC: Scattered electron and/or hadronic final state
  - CC: Hadronic final state (neutrino escapes detection)

} Determine kinematics!

DIS Kinematics

- Kinematic variables (1)
  - Mandelstam variables:
    - s,t,u
    - Centre-of-mass energy: √s

$$s = (k+p)^2 \simeq 4E_e E_P$$

$$t = (p-p^{'})^2$$

$$u = (k^{'}-p)^2$$

$$Q^2 = -(k-k^{'})^2 = -(p-p^{'})^2 = -t = -q^2$$

  - Q^2
    - Negative square of the momentum transfer q
    - Determines wavelength of photon and therefore the size Δ which can be resolved
    - $Q^2_{max} = s$

$$x = \frac{Q^2}{2(p \cdot q)} \simeq -\frac{t}{u+s} \qquad 0 \leq x \leq 1$$

  - x
    - Bjorken scaling variable: Fraction of the proton momentum carried by the struck parton (Quark-Parton model)!
  - W^2
    - Invariant mass squared of the hadronic final state system
    - Small x refers to large W^2

'$\simeq$' ...refers to the case where masses are ignored!

$$W^2 = (p+q)^2 = (p^{'})^2 = m_p^2 + \frac{Q^2}{x}(1-x) \simeq s+t+u$$

DIS Kinematics

Kinematic variables (2)

- ν
  - Energy of the exchanged boson in the proton rest frame

$$\nu = \frac{p \cdot q}{m_p}$$

$$\nu = \frac{p \cdot q}{m_p} = \frac{m_p(E_e - E_e^{'})}{m_p} = (E_e - E_e^{'})$$

- y
  - Fraction of the incoming electron carried by the exchanged gauge boson also known as inelasticity in the proton rest

$$y = \frac{p \cdot q}{p \cdot k} \simeq \frac{u+s}{s} \qquad 0 \leq y \leq 1$$

- t
  - Momentum transfer at the hadronic vertex

$$t = (p - p^{'})^2$$

- Note the above variables are connected by:

$$Q^2 \simeq s \cdot x \cdot y$$

- For fixed x and y, ep collider allows to reach much large values in Q2
- For fixed y and Q2, ep collider allows to reach much smaller values in x

DIS Kinematics

- Collider kinematics (1)

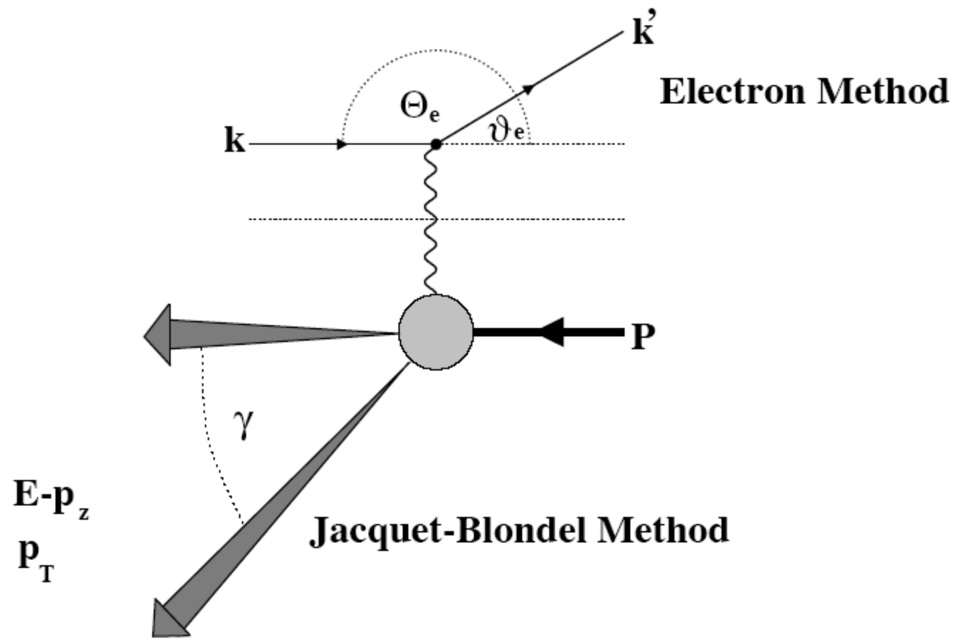
$$k = \begin{pmatrix} E_e \\ 0 \\ 0 \\ -E_e \end{pmatrix} p = \begin{pmatrix} E_P \\ 0 \\ 0 \\ E_P \end{pmatrix}$$

$$k' = \begin{pmatrix} E_e' \\ E_e' \sin\theta_e' \cos\phi_e' \\ E_e' \sin\theta_e' \sin\phi_e' \\ E_e' \cos\theta_e' \end{pmatrix} p' = \begin{pmatrix} \sum_h E_h \\ \sum_h p_{X,h} \\ \sum_h p_{Y,h} \\ \sum_h p_{Z,h} \end{pmatrix}$$

$$x_e = \frac{Q_e^2}{s y_e} = \frac{E_e' \cos^2 \frac{\theta_e'}{2}}{E_p(1 - \frac{E_e'}{E_e} \sin^2 \frac{\theta_e'}{2})}$$

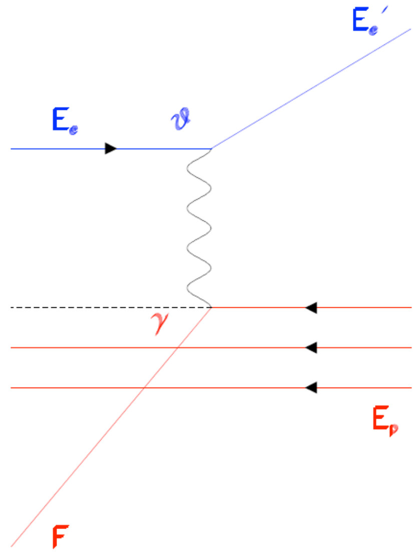
$$y_e = 1 - \frac{E_e'}{2E_e}(1 - \cos\theta_e') = 1 - \frac{E_e'}{E_e} \sin^2 \frac{\theta_e'}{2}$$

$$Q_e^2 = 2E_e E_e'(1 + \cos\theta_e') = 4E_e E_e' \cos^2 \frac{\theta_e'}{2} = \frac{p_{T,e}^2}{1 - y_e}$$



DIS Kinematics

Collider kinematics (2)



$$F = \frac{p_{T,h}^2 + (E - p_z)_h^2}{2\,(E-p_z)_h}$$

$$\cot\gamma = \frac{p_{T,h}^2 - (E - p_z)_h^2}{p_{T,h}^2 + (E - p_z)_h^2}$$

- Electron method: scattered electron

$$x_e = \frac{Q_e^2}{s y_e} = \frac{E_e' \cos^2\left(\frac{\theta_e'}{2}\right)}{E_p\left(1 - \frac{E_e'}{E_e}\sin^2\left(\frac{\theta_e'}{2}\right)\right)}$$

$$y_e = 1 - \frac{E_e'}{2E_e}\left(1 - \cos\theta_e'\right) = 1 - \frac{E_e'}{E_e}\sin^2\left(\frac{\theta_e'}{2}\right)$$

$$Q_e^2 = 2E_eE_e'\left(1+\cos\theta_e'\right) = 4E_eE_e'\cos^2\left(\frac{\theta_e'}{2}\right) = \frac{p_{T,e}^2}{1-y_e}$$

- Jacquet-Blondel method: hadronic final state

$$x_{JB} = \frac{Q_{JB}^2}{s y_{JB}}$$

$$y_{JB} = \frac{(E-p_z)_h}{2E_e}$$

$$Q_{JB}^2 = \frac{p_{T,h}^2}{1-y_{JB}}$$

$$p_{T,h}^2 = \left(\sum_h p_{x,h}\right)^2 + \left(\sum_h p_{y,h}\right)^2$$

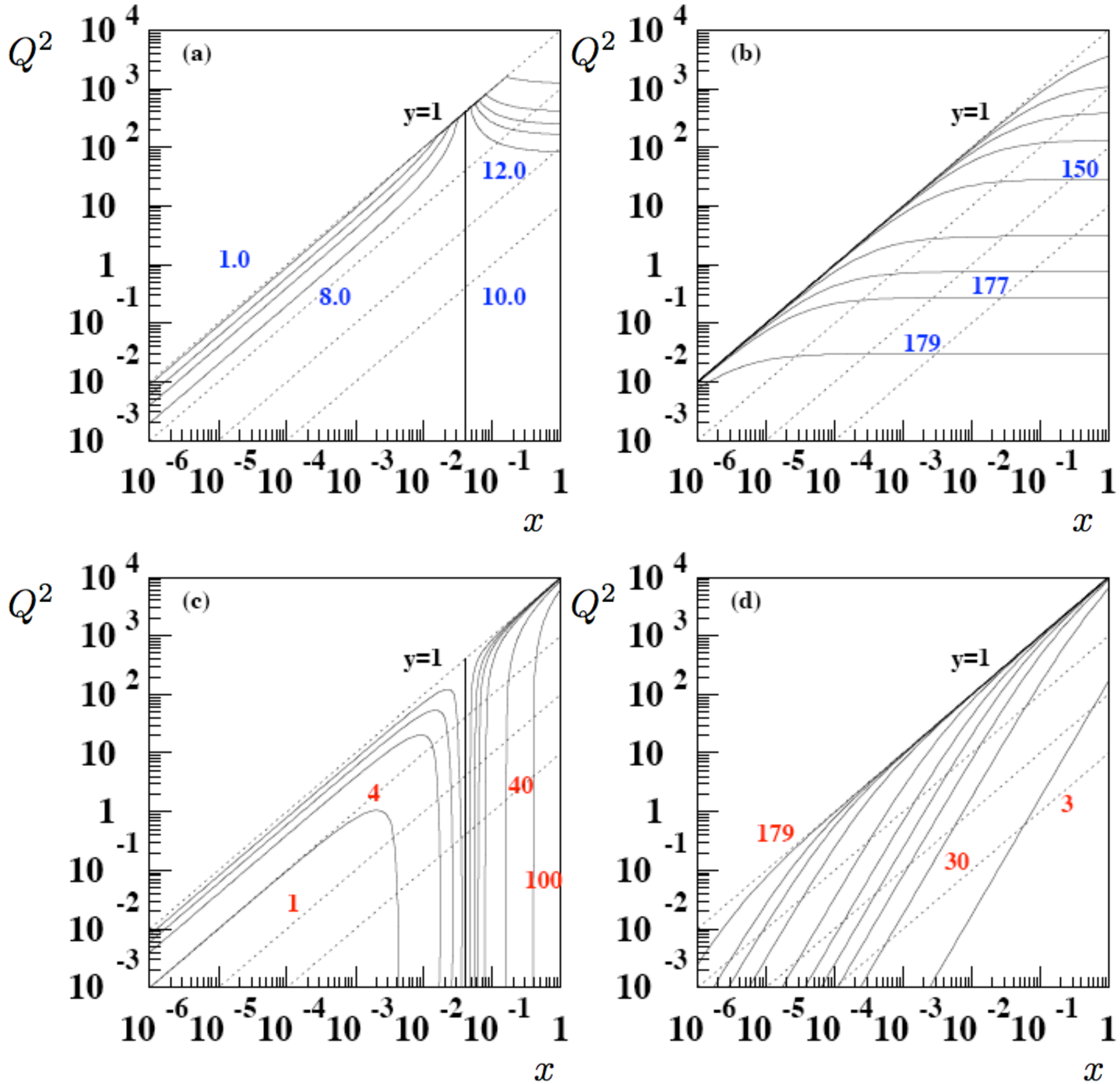
$$(E-p_z)_h = \sum_h (E_h - p_{z,h})$$

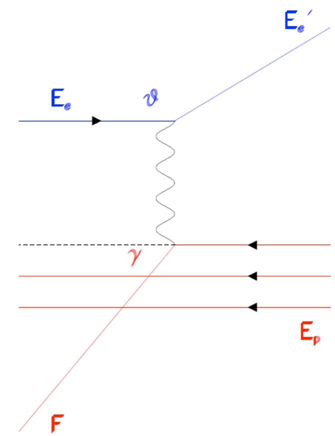
DIS Kinematics

- Collider kinematics (3)

Lines of constant electron energy (E'_e)

Lines of constant electron angle (ϑ'_e)





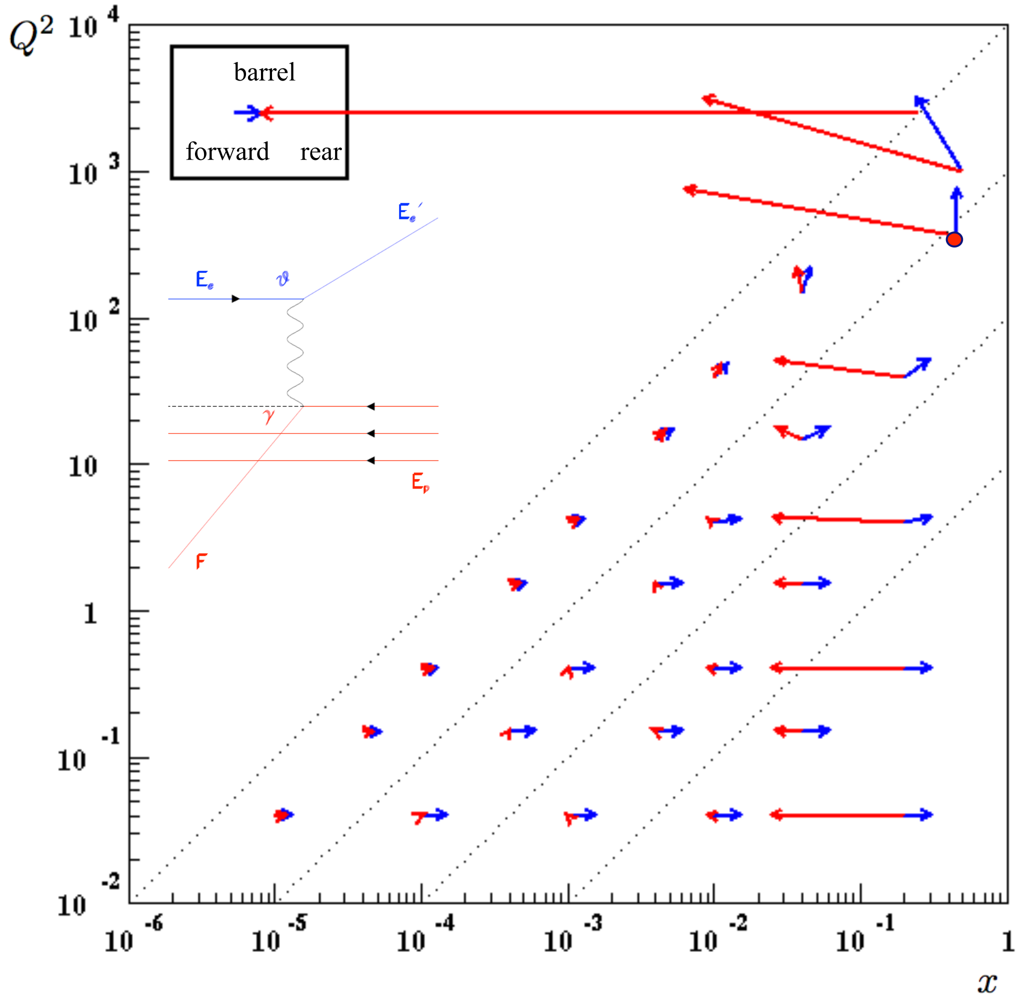
Lines of constant hadron energy (F)

Lines of constant hadron angle (γ)

DIS Kinematics

Collider kinematics (4)

- Low-x-low Q^2: Electron and current jet (low energy) predominantly in rear direction
- High-x-low Q^2: Electron in rear and current jet (High energy) in forward direction



- High-x-high Q^2: Electron predominantly in barrel/forward direction (High energy) and current jet in forward direction (High energy)

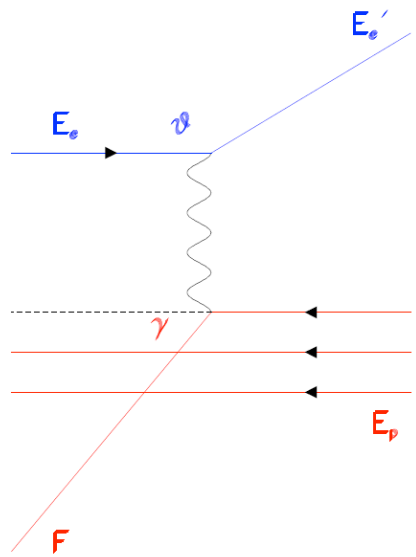
$Q^2 = 361 GeV^2 \quad x = 0.45$

$E_e' = 18 GeV \quad F = 104 GeV$

$\vartheta_e' = 90° \quad \vartheta_h = 10°$

DIS Kinematics

- Collider kinematics (5)



$$F = \frac{p_{T,h}^2 + (E-p_z)_h^2}{2\,(E-p_z)_h}$$

$$\cot\gamma = \frac{p_{T,h}^2 - (E-p_z)_h^2}{p_{T,h}^2 + (E-p_z)_h^2}$$

- Electron method: scattered electron

$$\left(\frac{\delta x_e}{x_e}\right) = \left(\frac{1}{y_e}\right)\frac{\delta E_e'}{E_e'} \otimes \left[\frac{x_e}{E_e/E_p} - 1\right]\tan\left(\frac{\theta_e'}{2}\right)\delta\theta_e'$$

$$\left(\frac{\delta y_e}{y_e}\right) = \left(1 - \frac{1}{y_e}\right)\frac{\delta E_e'}{E_e'} \otimes \left[\frac{1}{y_e} - 1\right]\cot\left(\frac{\theta_e'}{2}\right)\delta\theta_e'$$

$$\left(\frac{\delta Q_e^2}{Q_e^2}\right) = \frac{\delta E_e'}{E_e'} \otimes \tan\left(\frac{\theta_e'}{2}\right)\delta\theta_e'$$

- Jacquet-Blondel method: hadronic final state

$$\left(\frac{\delta x_{JB}}{x_{JB}}\right) = \left(\frac{1}{1-y_{JB}}\right)\frac{\delta F}{F} \otimes \left[2\cot\gamma + \left(\frac{2y_{JB}-1}{1-y_{JB}}\right)\cot\left(\frac{\gamma}{2}\right)\right]\delta\gamma$$

$$\left(\frac{\delta y_{JB}}{y_{JB}}\right) = \frac{\delta F}{F} \otimes \cot\left(\frac{\gamma}{2}\right)\delta\gamma$$

$$\left(\frac{\delta Q_{JB}^2}{Q_{JB}^2}\right) = \left(\frac{2-y_{JB}}{1-y_{JB}}\right)\frac{\delta F}{F} \otimes \left[2\cot\gamma + \left(\frac{y_{JB}}{1-y_{JB}}\right)\cot\left(\frac{\gamma}{2}\right)\right]\delta\gamma$$

DIS Structure Function Measurements

- Relativistic Invariant Cross-Section
  - In terms of laboratory variables:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \cos^2(\theta/2)$$

$$\left(\frac{d^2\sigma}{dE'd\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left\{W_2(Q^2,x) + 2W_1(Q^2,x)\tan^2\left(\frac{\theta}{2}\right)\right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{4\alpha^2 E'^2}{q^4}$$

  - Formulate this now in relativistic invariant quantities: $\theta'_e, \quad E'_e \rightarrow y_e, \quad Q^2_e$

  - Instead of W_1 and W_2, use: F_1 and F_2:

$$F_1 = m_p W_1 \qquad F_2 = \nu W_2$$

$$\left(\frac{d^2\sigma}{dydQ^2}\right) = \frac{2\pi\alpha^2 Y_+}{yQ^4}\left(F_2 - \frac{y^2}{Y_+}F_L\right)$$

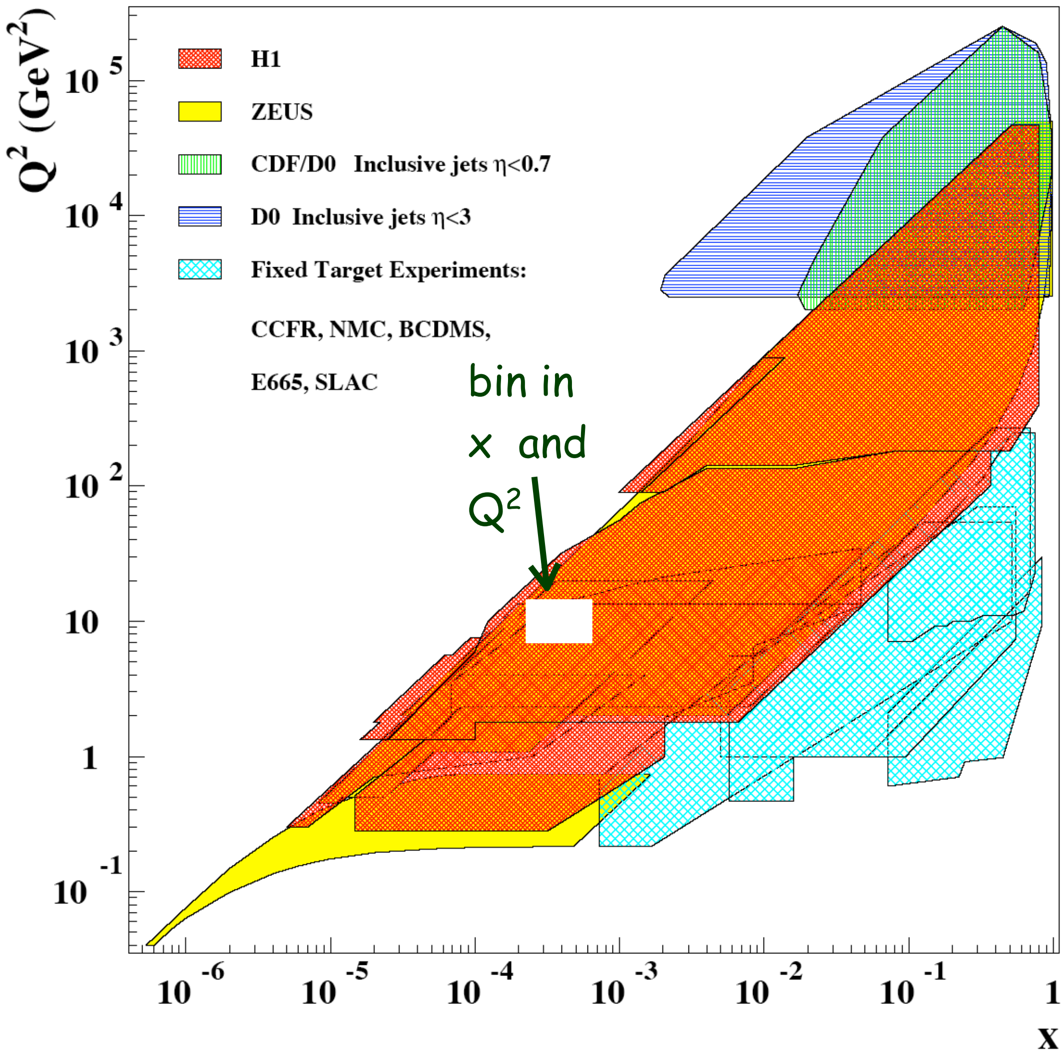
Longitudinal structure function: F_L

$$F_L = F_2 - 2xF_1$$

$$Y_+ = 1 + (1-y)^2$$

DIS Structure Function Measurements

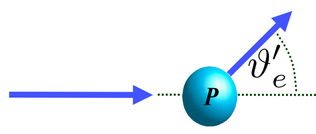
- Essential idea



1. Determination of kinematics (e.g. electron method):

$$Q^2 = 4E'_e E_e \sin^2\left(\frac{\vartheta'_e}{2}\right)$$

$$y = 1 - \frac{E'_e}{E_e}\cos^2\left(\frac{\vartheta'_e}{2}\right)$$



$$x = \frac{Q^2}{sy}$$

2. Determination of cross-section and extraction of F_2:

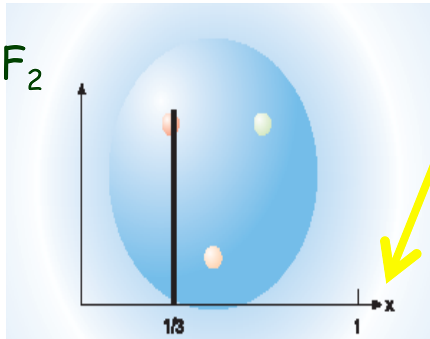
Number of selected events → N; Background → B; Luminosity → L; Efficiency → ϵ

$$\frac{d^2\sigma}{dxdQ^2} = \frac{(N-B)}{L\cdot\epsilon} \propto F_2$$

DIS Structure Function Measurements

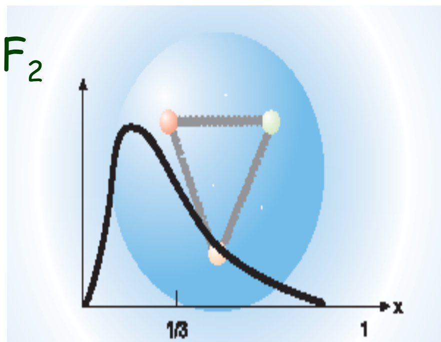
- Results

Three valence quarks



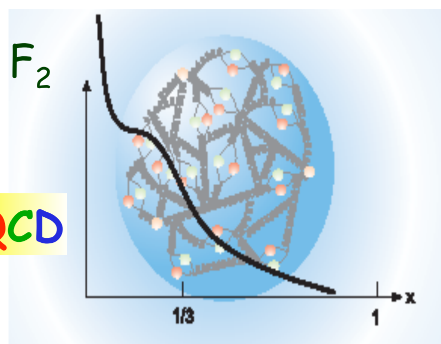
x: Momentum fraction of struck quark

Three bound valence quarks

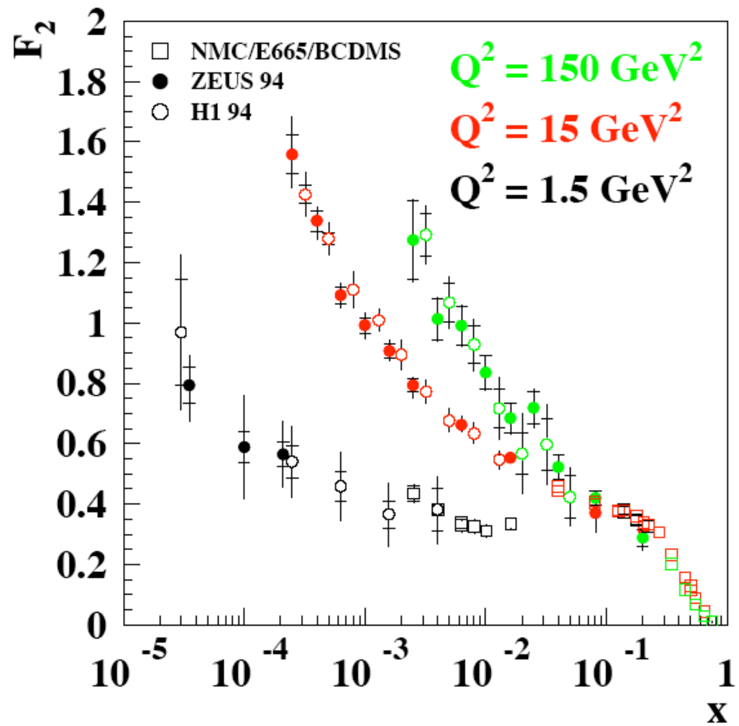


Three valence quarks and sea quarks + gluons

QCD



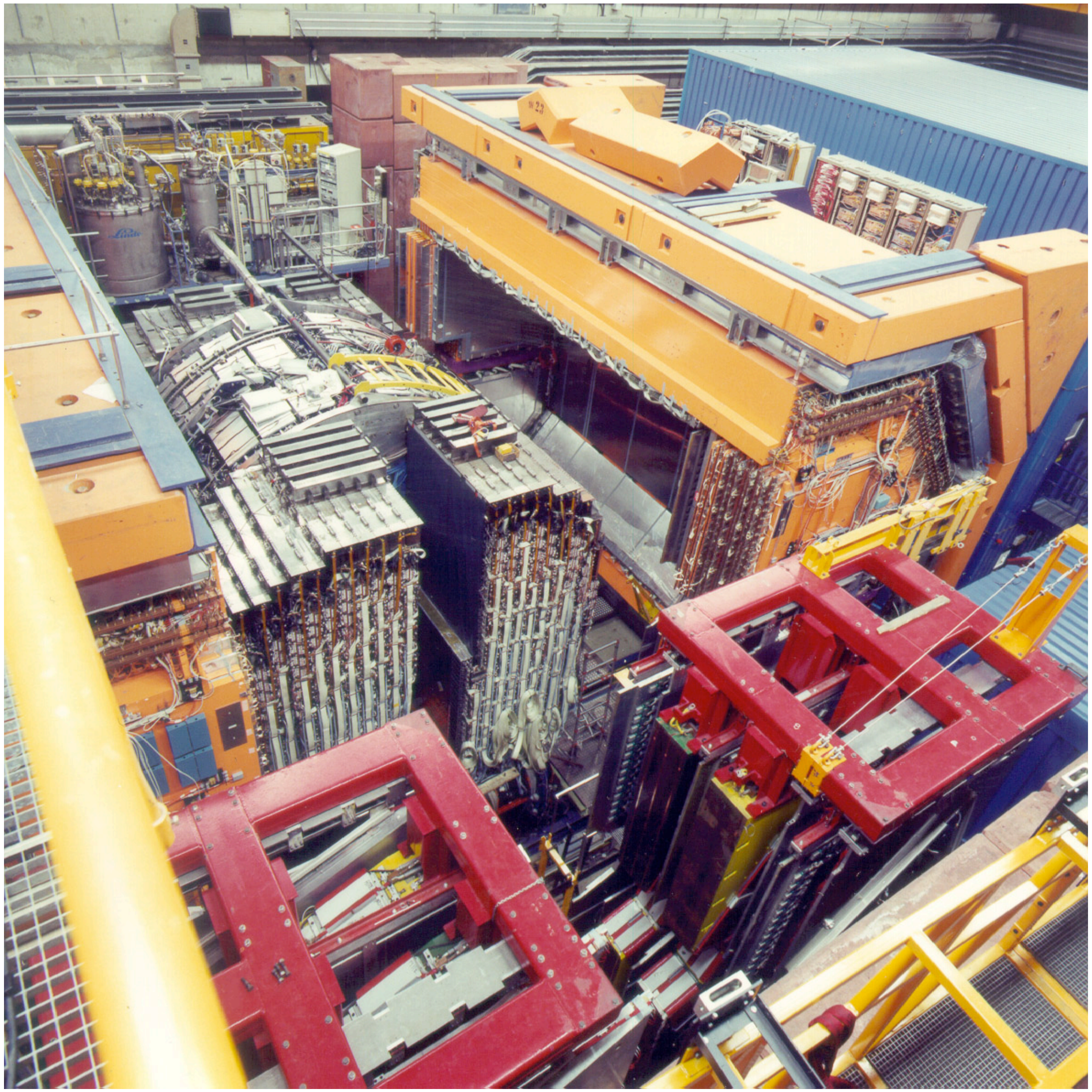
Valence quarks and QCD sea



Discovery of asymptotic freedom in the theory of strong interaction (Quantum Chromo Dynamics): Nobel prize in physics 2004

DIS Collider Detectors

- ep detector system: Here ZEUS Detector



- Central-Tracking detector:

$$\frac{\delta p_T}{p_T} = 0.0059\, p_T \otimes 0.0065$$

$\Rightarrow$ Inside superconducting solenoid of 1.43T

- Uranium calorimeter (barrel, rear and forward sections):
  - electromagnetic part:

$$\frac{\delta E}{E} = \frac{18\%}{\sqrt{E}}$$

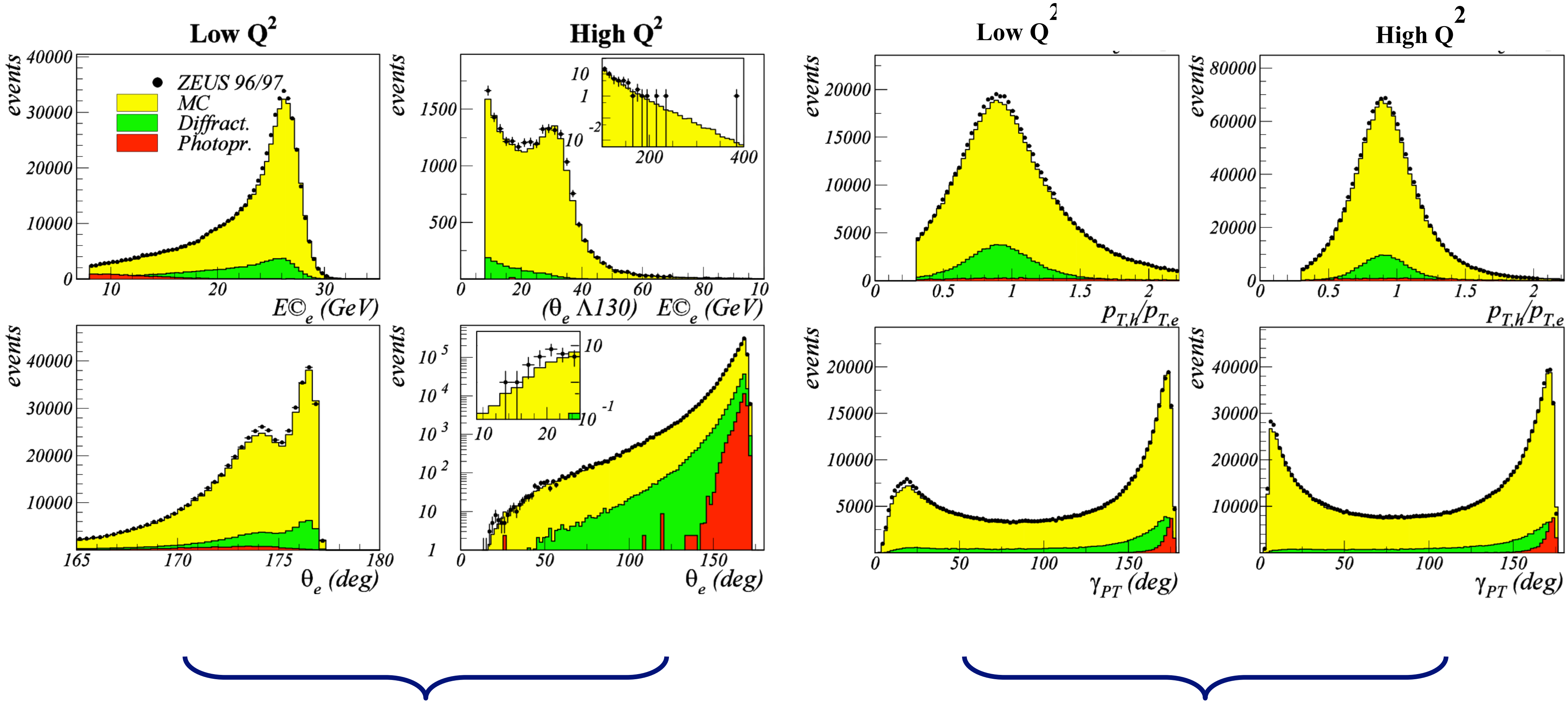
  - hadronic part:

$$\frac{\delta E}{E} = \frac{35\%}{\sqrt{E}}$$

- Muon detection system in barrel, rear and forward direction

DIS Collider Detectors

- ZEUS detector - Kinematic variable measurement



Electron variables

Hadronic final-state variables

Summary

- Connection of DIS cross-section and Dirac / Mott / Rutherford cross-sections

- Collider kinematics: Reconstruction of kinematics through electron or hadron method or combination of both

- Literature:

  - Review on ep physics: Bernd Surrow, Eur. Phys. J. direct C1:2, 1999.

  - Textbook on DIS: Robin Devenish and Amanda Cooper-Sarkar - Deep Inelastic Scattering

Backup - DIS Process Description

- Basic aspects of scattering theory (1)

  - Scattering process: $a_1(p_1) + ... + a_n(p_n) \rightarrow b_1(p'_1) + ... + b'_m(p'_m)$

  - Initial state: $\lim_{t\rightarrow -\infty} |t\rangle = |i\rangle = |a_1(p_1) + ... + a_n(p_n)\rangle$

  - Final state: $\lim_{t\rightarrow +\infty} |t\rangle = |f\rangle = |b_1(p'_1) + ... + b'_m(p'_m)\rangle$

  - Scattering amplitude: $S_{fi} = \langle b_1(p'_1) + ... + b'_m(p'_m)|S|a_1(p_1) + ... + a_n(p_n)\rangle$

$$\sum_f |\langle f|S|i\rangle|^2 = \sum_f \langle i|S^\dagger|f\rangle\langle f|S|i\rangle = \langle i|S^\dagger S|i\rangle = 1$$

Backup - DIS Process Description

- Basic aspects of scattering theory (2)

Amplitude M: Dynamics

$$1 + 2 \rightarrow 3 + 4 + ... + n$$

$$d\sigma = |\mathcal{M}|^2 \frac{\hbar S}{4\sqrt{(p_1 \cdot p_2)^2 - (m_1 m_2 c^2)^2)}} \left[\left(\frac{c d^3\vec{p}_3}{(2\pi)^3 2E_3} \right) \cdot \left(\frac{c d^3\vec{p}_4}{(2\pi)^3 2E_4} \right) \cdots \left(\frac{c d^3\vec{p}_n}{(2\pi)^3 2E_n} \right) \right]$$

$$\times (2\pi)^4 \cdot \delta^4(p_1 + p_2 - p_3 - p_4 - ... - p_n)$$

Phase space: Kinematics

Delta function enforces conservation of energy and momentum!

Note:

$p_i = (E_i/c, \vec{p}_i)$ Four-momentum of the i[th] particle

S Statistical factor: 1/j! for each group of j identical particles in the final state

Backup - DIS Process Description

- Basic aspects of scattering theory (3)

  - Amplitude: Electron-Mass M (Spin 1/2) Particle scattering: Dirac scattering

$$\langle|\mathcal{M}|^2\rangle = \frac{8g_e^4}{(p_1 - p_3)^4} \times$$

$$[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) - (p_1 \cdot p_3)(Mc)^2 - (p_2 \cdot p_4)(mc)^2 + 2(mMc^2)^2]$$

Momentum transfer: $q = (p_1 - p_3)$

  - Approximation:
    - Laboratory frame with the target particle of mass M at rest
    - Electron with energy E scatters at an angle emerging with energy E'
    - Assumption: E,E' >> mc^2 (m=0)

  - Spin averaged amplitude:

$$\langle|\mathcal{M}|^2\rangle = \frac{(4\pi)^2\alpha^2c^2(2Mc)^2}{4EE'\sin^4(\theta/2)}\left\{\cos^2\left(\frac{\theta}{2}\right) - \frac{q^2}{2M^2c^2}\sin^2\left(\frac{\theta}{2}\right)\right\}$$

Backup - DIS Process Description

- Mott Cross-Section (1)
  - Cross-section result:

$$\langle|\mathcal{M}|^2\rangle = \left(\frac{g_e^2 Mc}{\vec{p}^2 \sin^2(\theta/2)}\right)^2 \left((mc)^2 + \vec{p}^2 \cos^2\frac{\theta}{2}\right)$$

Impact of target spin for very heavy target drops out: Mott cross section: Scattering of spin 1/2 particle on heavy spin 0 heavy target

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi Mc}\right)^2 \langle|\mathcal{M}|^2\rangle$$

Spin-averaged matrix element squared and cross-section for: M >> m

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{2\vec{p}^2 \sin^2(\theta/2)}\right)^2 \left((mc)^2 + \vec{p}^2 \cos^2\frac{\theta}{2}\right)$$

Mott cross-section

Backup - DIS Process Description

Mott Cross-Section (2)

- Further simplification: m=0

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar}{2\vec{p}^2\sin^2(\theta/2)}\right)^2 \vec{p}^2\cos^2\frac{\theta}{2} \qquad E' = |\vec{p}|c \qquad Q^2 = -q^2 = -(p_1 - p_3)^2 = 4\vec{p}^2\sin^2\frac{\theta}{2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{4\alpha^2\hbar^2 E'^2}{q^4c^2}\cos^2\frac{\theta}{2}$$

- Multiply by q^2:

$$q^2\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = (\alpha\hbar)^2\frac{\cos^2(\theta/2)}{\sin^2(\theta/2)}$$

Scaling behavior!

Rutherford cross section!

Backup - DIS Process Description

- Rutherford scattering (1)

  - Non-relativistic limit: Incident electron is non-relativistic $\vec{p}^2 \ll (mc)^2$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\hbar^2 m^2 c^2}{4\vec{p}^4 \sin^4(\theta/2)} \left(1 + \frac{\vec{p}^2}{m^2c^2}\cos^2\frac{\theta}{2}\right)$$

Consequence of spin 1/2 nature of incoming probe particle!

$\cos^2\theta$ term drops out in non-relativistic limit:

This can also be written as: $p = mv$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{4m^2v^4\sin^4(\theta/2)}$$

Or:

$$E' \simeq mc^2$$

$$\frac{d\sigma}{d\Omega} = \frac{4\alpha^2\hbar^2 m^2 c^2}{q^4}$$

Rutherford cross section!

$$Q^2 = -q^2 = -(p_1 - p_3)^2 = 4\vec{p}^2\sin^2\frac{\theta}{2}$$

Backup - DIS Process Description

Rutherford scattering (2)

- In natural units: $\hbar = c = 1$

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{4\alpha^2 E'^2}{q^4}$$

- Note:
  - The Rutherford cross section is obtained from the Mott cross section assuming we are working in the non-relativistic limit:

    ⇒ Spin effects of probe and target particle are negligible!
  - The Mott cross section is obtained for the case of a target particle at rest (Heavy target): No recoil!

    ⇒ Impact of spin 1/2 of probe particle taken into account - Spin effects of target particle negligible: Result is identical to scattering of spin 1/2 on spin 0 target!

- Difference between Rutherford and Mott cross section: $\cos^2(\theta/2)$ factor

- Factor is a consequence of angular momentum conservation:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} \cdot \cos^2\frac{\theta}{2}$$

  - Helicity conservation for massless particles ($\beta \rightarrow 1$): Scattering by 180° requires spin flip (Impossible for spin 0 target)!