Introduction to QCD

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Outline

- Fundamentals of Quantum Chromodynamics (QCD)
- Why should we believe QCD?
- EIC as the next QCD machine

Quantum Chromodynamics (QCD)

= A quantum field theory of quarks and gluons =

- ☐ **Fields:** $\psi_i^f(x)$

Quark fields: spin-½ Dirac fermion (like electron)
Color triplet: $i = 1, 2, 3 = N_c$
Flavor: $f = u, d, s, c, b, t$

$A_{\mu,a}(x)$

Gluon fields: spin-1 vector field (like photon)
Color octet: $a = 1, 2, ..., 8 = N_c^2 - 1$

- ☐ **QCD Lagrangian density:**

$$\begin{aligned}\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[(i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij}\right] \psi_j^f \\ -\frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c}\right]^2 \\ + \text{gauge fixing} + \text{ghost terms}\end{aligned}$$

- ☐ **Color matrices:**

$$[\,t_a, t_b\,] = i\,C_{abc}\,t_c$$

Generators for the fundamental representation of SU3 color

Physical observables

- ☐ **Cross section:**

Scattering amplitude square – Probability – Positive definite

A function of in-state and out-state variables: momentum, spin, …

- ☐ **Spin-averaged cross section:**

$$\sigma = \frac{1}{2}\left[\sigma(\vec{s}) + \sigma(-\vec{s})\right]$$ – Positive definite

- ☐ **Asymmetries or difference of cross sections:**

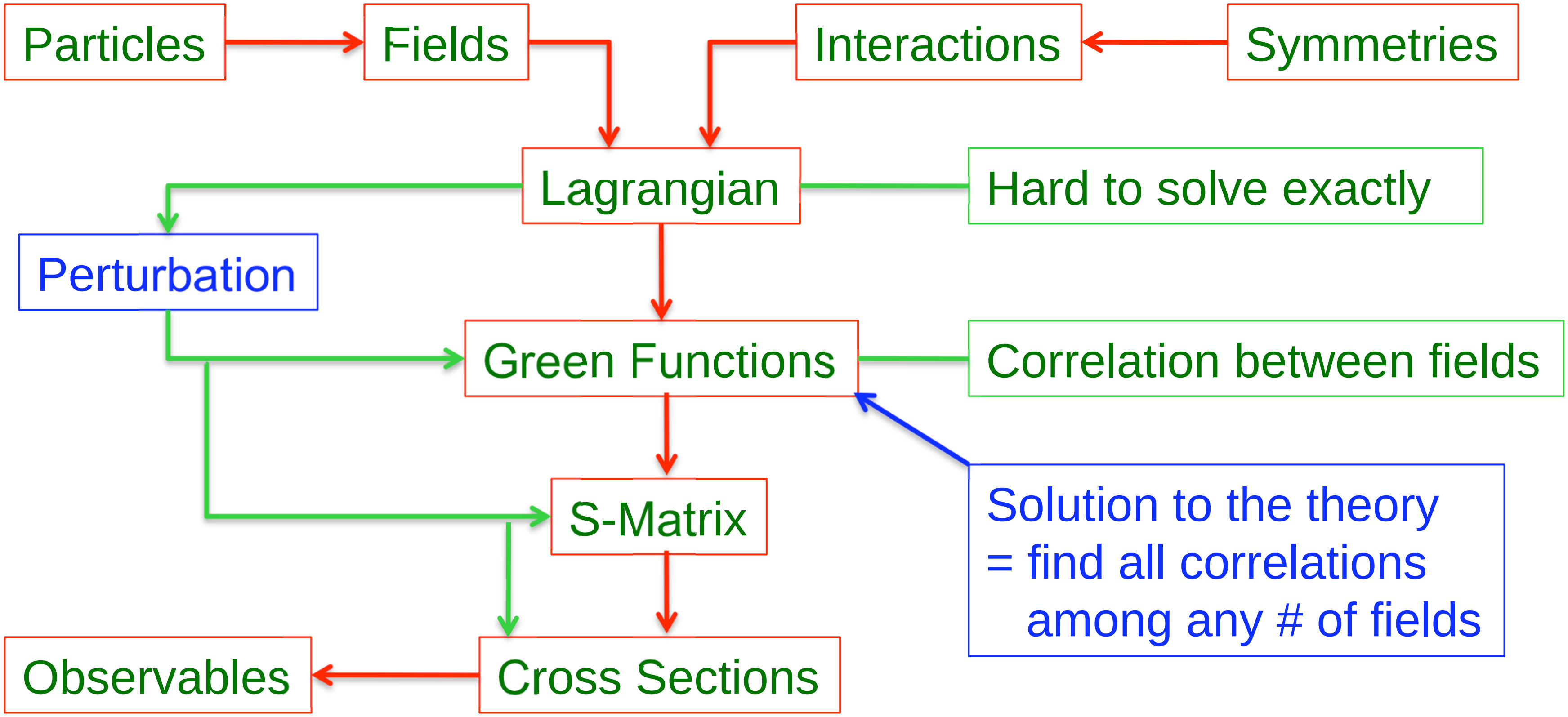
$$A(\vec{s}) = \frac{\Delta\sigma(\vec{s})}{\sigma} = \frac{\sigma(\vec{s}) - \sigma(-\vec{s})}{\sigma(\vec{s}) + \sigma(-\vec{s})} \qquad \Delta\sigma(\vec{s}) = \frac{1}{2}\left[\sigma(\vec{s}) - \sigma(-\vec{s})\right]$$

Not necessary positive!

Chance to see quantum interference directly

From Lagrangian to Cross Section

- ☐ **Theorists:** Lagrangian = "complete" theory
- ☐ **Experimentalists**: Cross Section ➡ Observables
- ☐ **A road map – from Lagrangian to Cross Section:**



Particles

Fields

Interactions

Symmetries

Lagrangian

Hard to solve exactly

Perturbation

Green Functions

Correlation between fields

S-Matrix

Solution to the theory
= find all correlations
among any # of fields

Observables

Cross Sections

The Question

- ☐ **We measure:**

  Cross sections of hadrons and leptons – Observables

- ☐ We believe:

  Hadrons are bound states of quarks and gluons

- ☐ **But,**

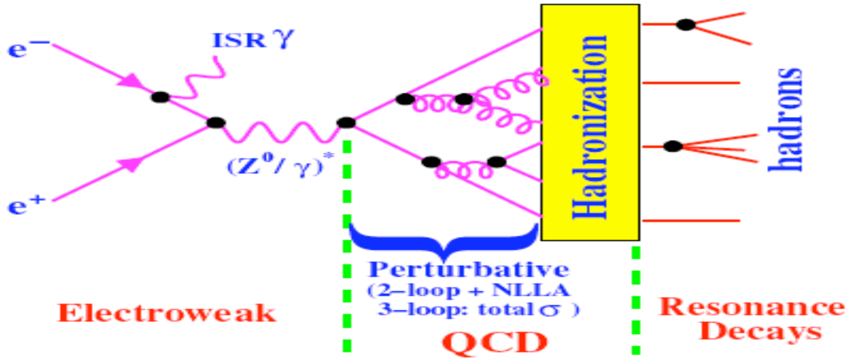
  We have not been able to solve QCD analytically
  to understand the confinement

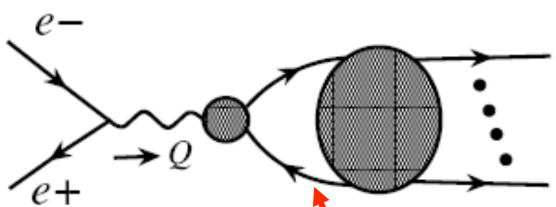
- ☐ **Question:**

  How to test QCD – a theory of quarks and gluons
  without seeing the quarks and gluons?

Observables not sensitive to hadronization

☐ e^+e^- ➔ hadron total cross section – no identified hadron!





Hadrons
“n”

Partons “m”

If there is no quantum interference between partons and hadrons,

$$\sigma^{\rm tot}_{e^+e^-\to\rm hadrons} \propto \sum_n \mathrm{P}_{e^+e^-\to n} = \sum_n \boxed{\sum_m \mathrm{P}_{e^+e^-\to m}\mathrm{P}_{m\to n}} = \sum_m \mathrm{P}_{e^+e^-\to m}\boxed{\sum_n \mathrm{P}_{m\to n}} = 1$$

$$\sigma^{\rm tot}_{e^+e^-\to\rm partons} \propto \sum_m \mathrm{P}_{e^+e^-\to m}$$

Unitarity

$$\Longrightarrow \boxed{\sigma^{\rm tot}_{e^+e^-\to\rm hadrons} = \sigma^{\rm tot}_{e^+e^-\to\rm partons}}$$

Finite in perturbation theory – KLN theorem

“Local” – of order of 1/Q

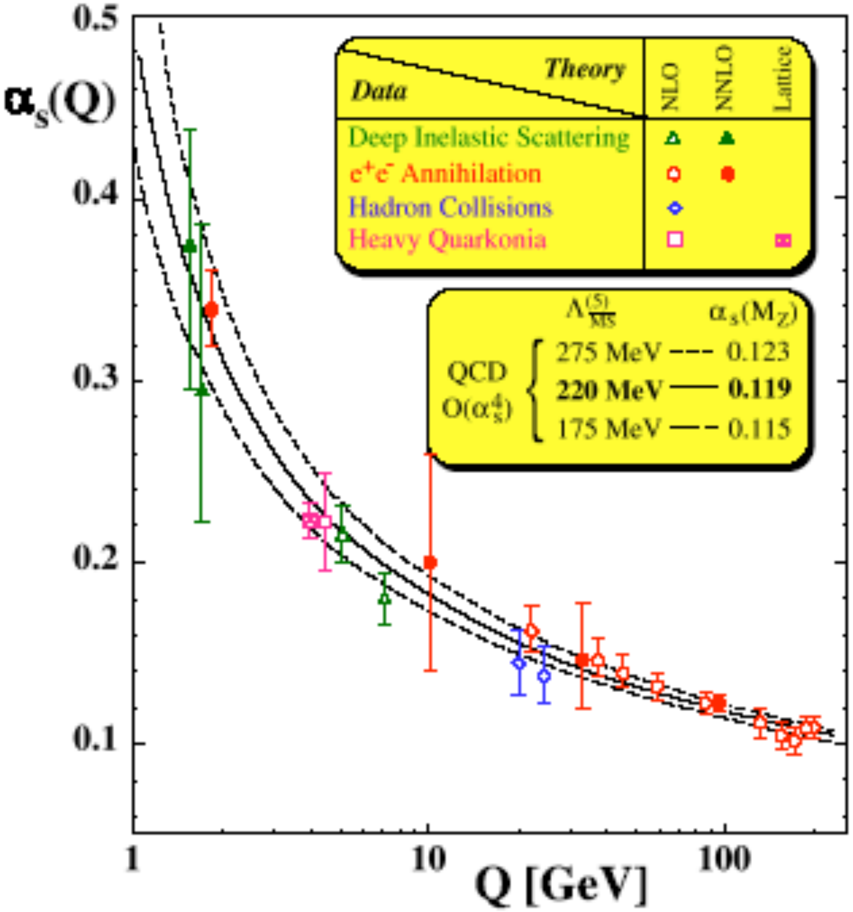
☐ **Inclusive** $R = \dfrac{\sigma_{e^+e^-\to\rm Hadrons}}{\sigma_{e^+e^-\to\mu^+\mu^-}}$

Test of QCD dynamics, color = 3, heavy quark mass threshold, …

QCD Asymptotic Freedom

- **QCD is a renormalizable theory**
- **Running coupling:** $\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda_{\rm QCD}^2}\right)}$

μ_2 and μ_1 not independent



Asymptotic Freedom ⇔ antiscreening

QCD: $\frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$

Compare

QED: $\frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

2004 Nobel Prize in Physics

Effective Quark Mass

- ☐ **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\exp\left[-\int_{\mu_1}^{\mu_2}\frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))]\right]$$

Quark mass depend on the renormalization scale!

- ☐ **QCD running quark mass:**

$$m(\mu_2) \Rightarrow 0 \qquad \text{as } \mu_2 \to \infty \qquad \text{since} \quad \gamma_m(g(\lambda)) > 0$$

- ☐ **Choice of renormalization scale:**

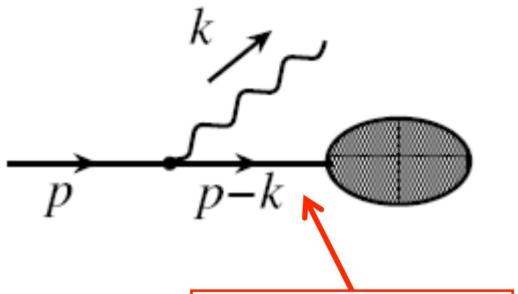
$\mu \sim Q$ for small logarithms in the perturbative coefficients

- ☐ **Light quark mass:** $m_f(\mu) \ll \Lambda_{\rm QCD} \quad \text{for } f = u, d, \text{ even } s$

QCD perturbation theory (Q>>$\Lambda_{\rm QCD}$) is effectively a massless theory

Infrared and Collinear Divergence

❑ Consider a general diagram:



Singularity

$p^2 = 0, \quad k^2 = 0$ for a massless theory

✧ $k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$

$\Longrightarrow$ Infrared (IR) divergence

✧ $k^\mu \;||\; p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \;\; 0 < \lambda < 1$

$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2\, p^2 = 0$

$\Longrightarrow$ Collinear (CO) divergence

IR and CO divergences are generic problems of a massless perturbation theory

Infrared (IR) Safety

- ☐ **Infrared safety:**

$$\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right) \Rightarrow \hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right) + \mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = $\kappa > 0$

Asymptotic freedom is useful

only for

quantities that are infrared safe

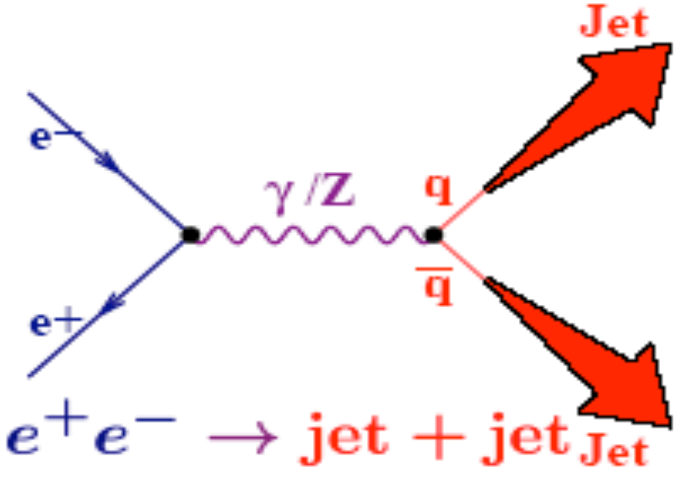
$$\sigma^{\rm Total}_{e^+e^-\to{\rm Partons}}(s=Q^2) = \sigma_0\left[1+\frac{\alpha_s(Q^2)}{\pi}+...\right]$$

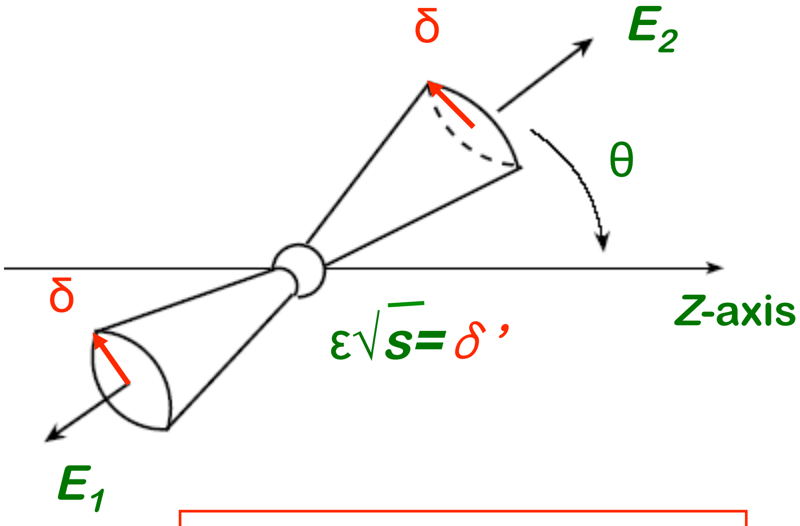
is IR safe

- ☐ **Go beyond the inclusive total cross section?**

Jets in e^+e^- - Trace of Partons

- **Jets – Inclusive x-section with a limited phase-space**
- **Q: will IR cancellation be completed?**
  - ✧ Leading partons are moving away from each other
  - ✧ Soft gluon interactions should not change the direction of an energetic parton → a “jet” – “trace” of a parton
- **Many Jet algorithms**



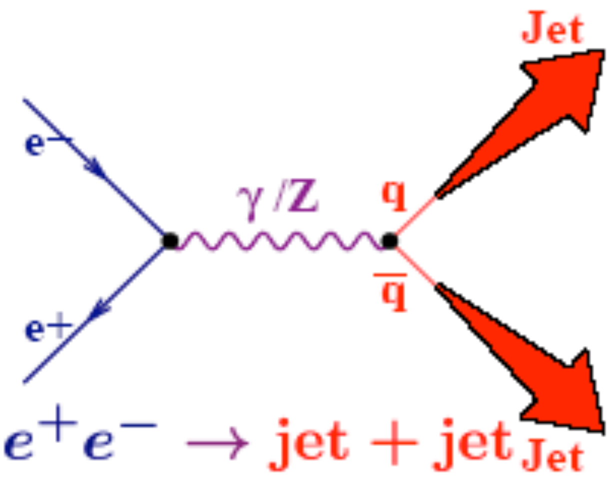


Sterman-Weinberg Jet

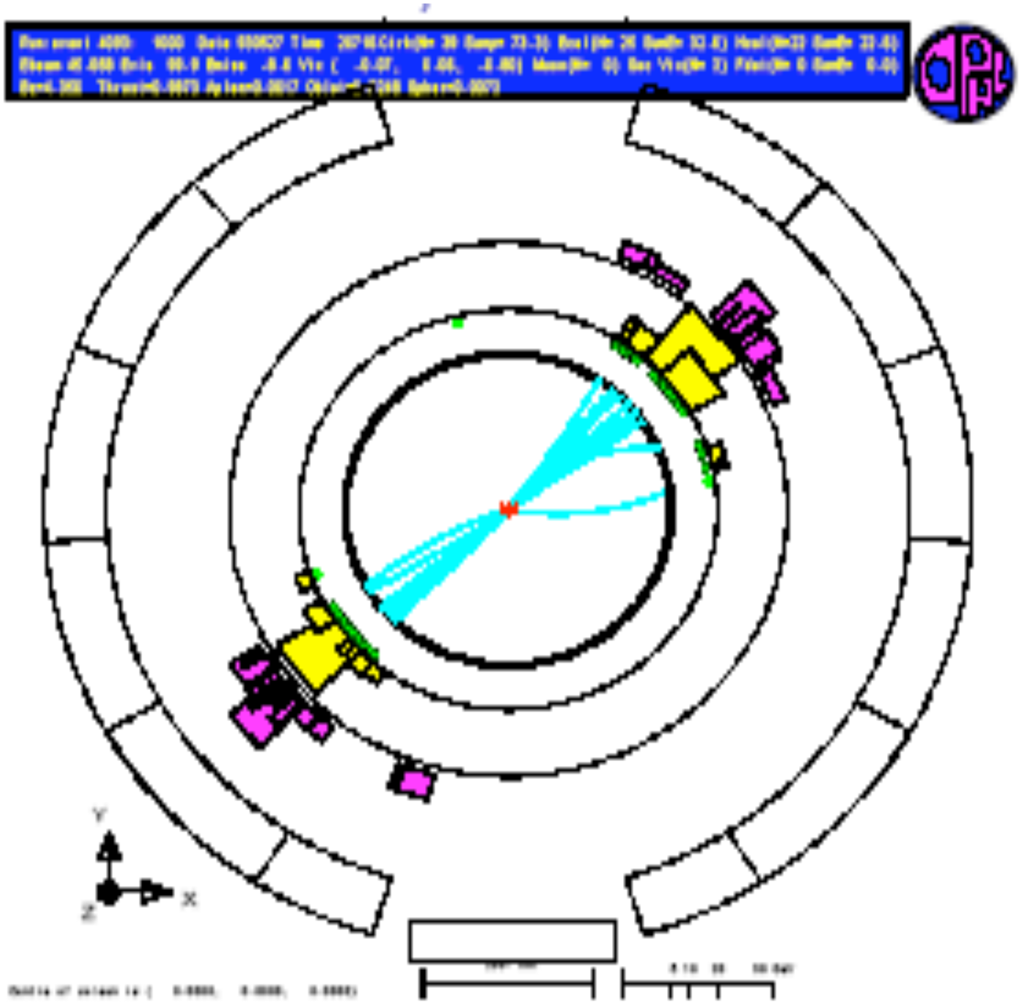
A Clean Two-jet Event

Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):

LEP ($\sqrt{s} = 90 - 205$ GeV)



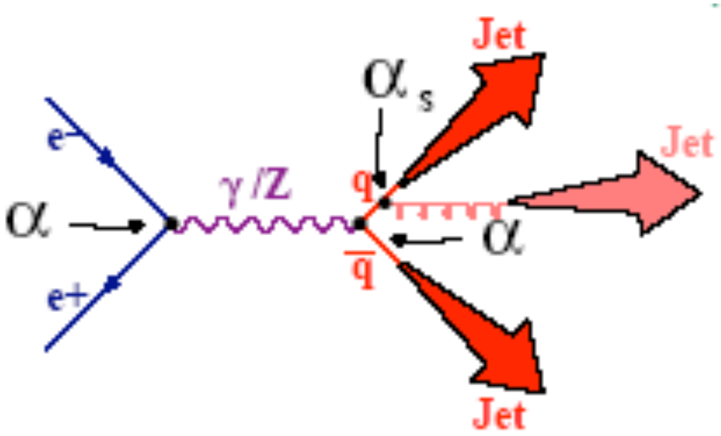
$e^+e^- \rightarrow \text{jet} + \text{jet}$



A clean trace of two partons – a pair of quark and antiquark

Discovery of a Gluon Jet

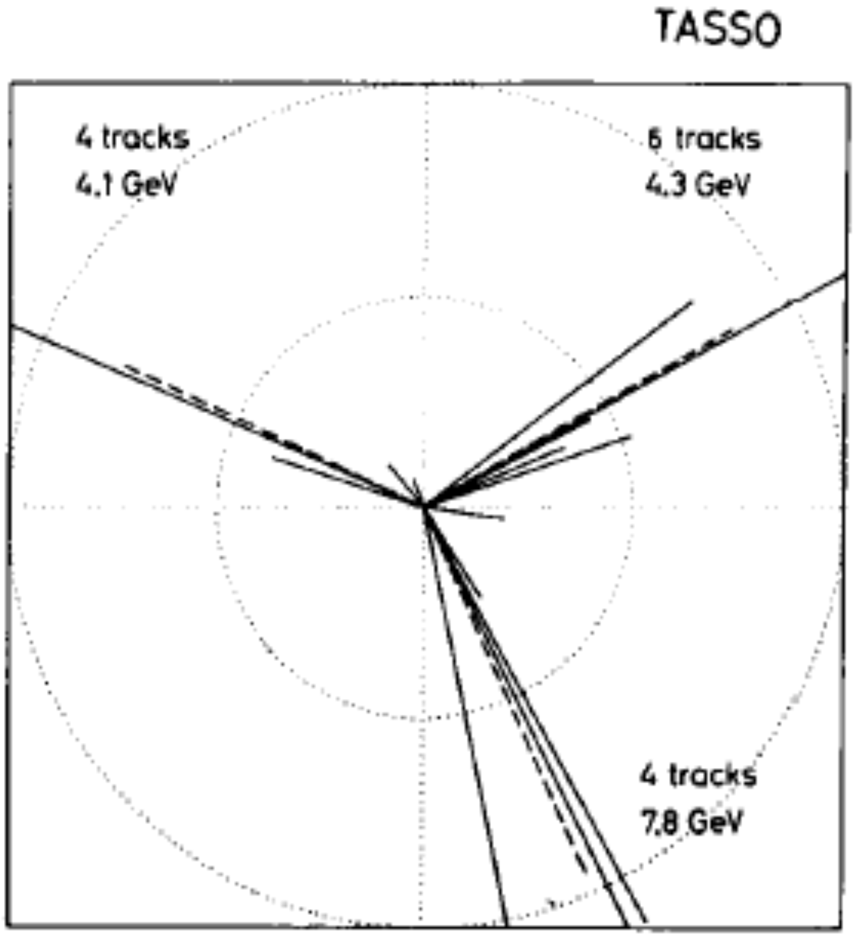
First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



Reputed to be the first three-jet event from TASSO

PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



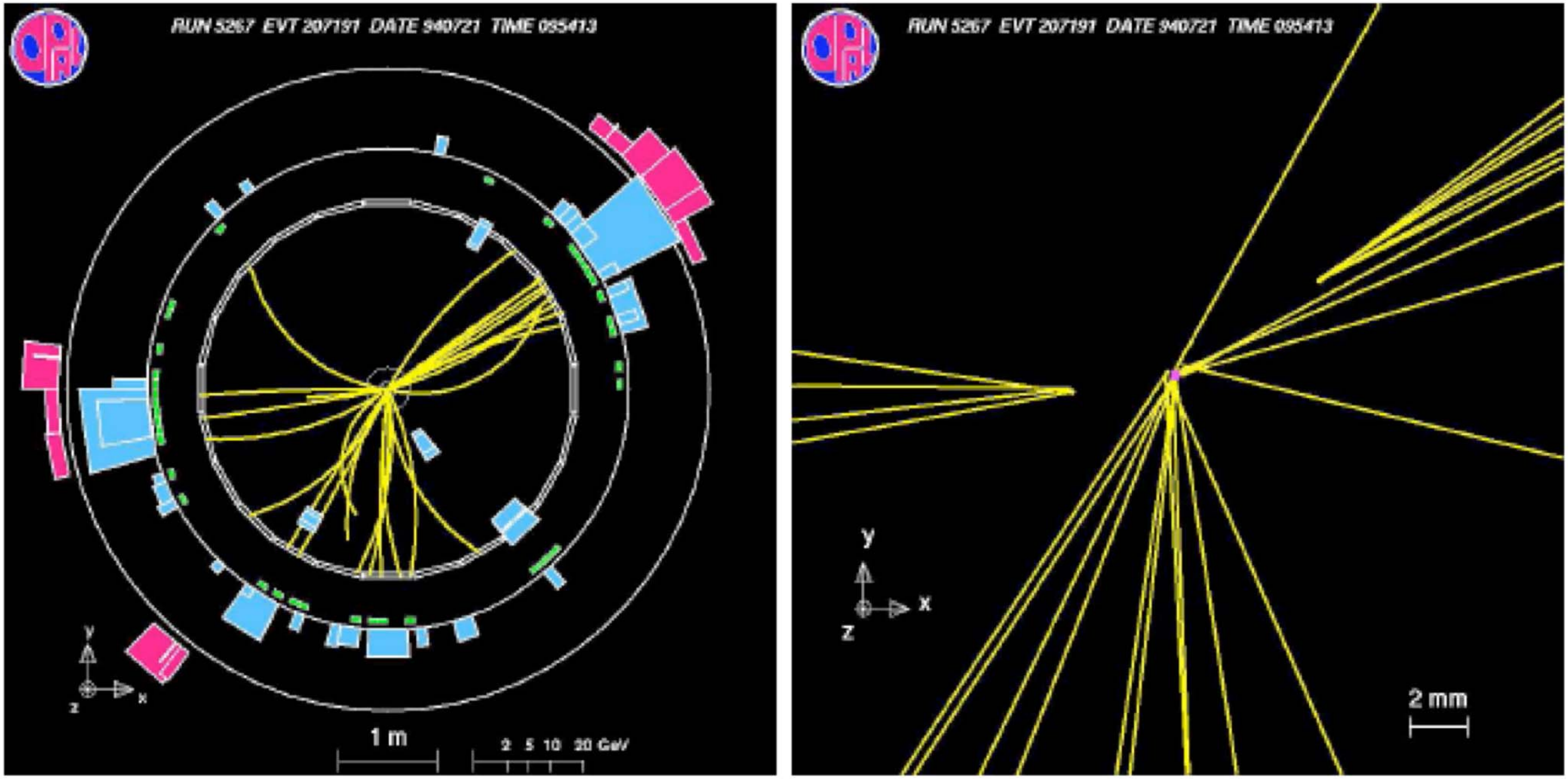
TASSO Collab., Phys. Lett. B86 (1979) 243

MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

Tagged Three-jet Event from LEP



Gluon Jet

The harder Question

❑ Question:

How to test QCD in a reaction with identified hadron(s)?
– to probe the quark-gluon structure of the hadron

❑ Facts:

Hadronic scale ~ 1/fm ~ Λ_{QCD} is non-perturbative

Cross section involving identified hadron(s) is not IR safe and is not perturbatively calculable!

❑ Solution – Factorization:

- ✧ Isolate the calculable dynamics of quarks and gluons
- ✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
  – provide information on the partonic structure of the hadron

Connecting the partons to the hadrons

❑ **Effective field theories + models:**

- ✧ Integrate out some degrees of freedom, express QCD in some effective degrees of freedom:

  HQEF, SCEF, …

  – approximation in field operators, still need the matrix elements to connect to the hadron states

- ✧ effective theory in hadron degrees of freedom, …
- ✧ models – Quark Models, …

❑ **PQCD factorization:**

- ✧ Connect partons to hadrons via matrix elements (PDFs, FFs, …)

$$\langle H(p,s)|\mathcal{O}(\phi,F_{\mu\nu})|H(p,s)\rangle$$

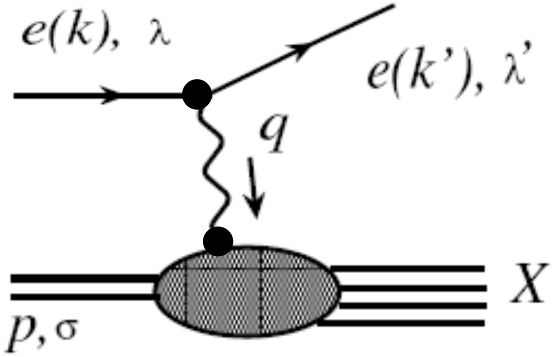
❑ **Lattice QCD – cannot calculate hadronic cross sections**

- ✧ can calculate matrix elements and partonic properties, …

Inclusive lepton-hadron DIS – one hadron

- ❑ **Cross section:**

$$E'\frac{d\sigma^{\text{DIS}}}{d^3k'}=\frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$



- ❑ **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S)=\frac{1}{4\pi}\int d^4z\ \mathrm{e}^{iq\cdot z}\ \left\langle p,S\left|J_\mu^\dagger(z)J_\nu(0)\right|p,S\right\rangle$$

$$W_{\mu\nu}=-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right)+\frac{1}{p\cdot q}\left(p_\mu-q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu-q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

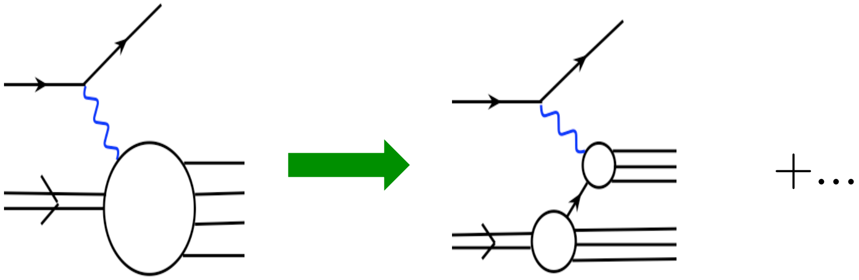
$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right)+\frac{(p.q)S_\sigma-(S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

- ❑ **Structure functions – infrared sensitive:**

$$F_1\left(x_B,Q^2\right),F_2\left(x_B,Q^2\right),g_1\left(x_B,Q^2\right),g_2\left(x_B,Q^2\right)$$

Perturbative QCD Factorization

❑ **Factorization – an approximation:**



(Diagrams with more active partons from each hadron!)

❑ **Leading Power:**

Single active parton from each hadron!

Short-distance

Power corrections

$$\sigma_{\text{phy}}(Q,1/R) \sim \widehat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

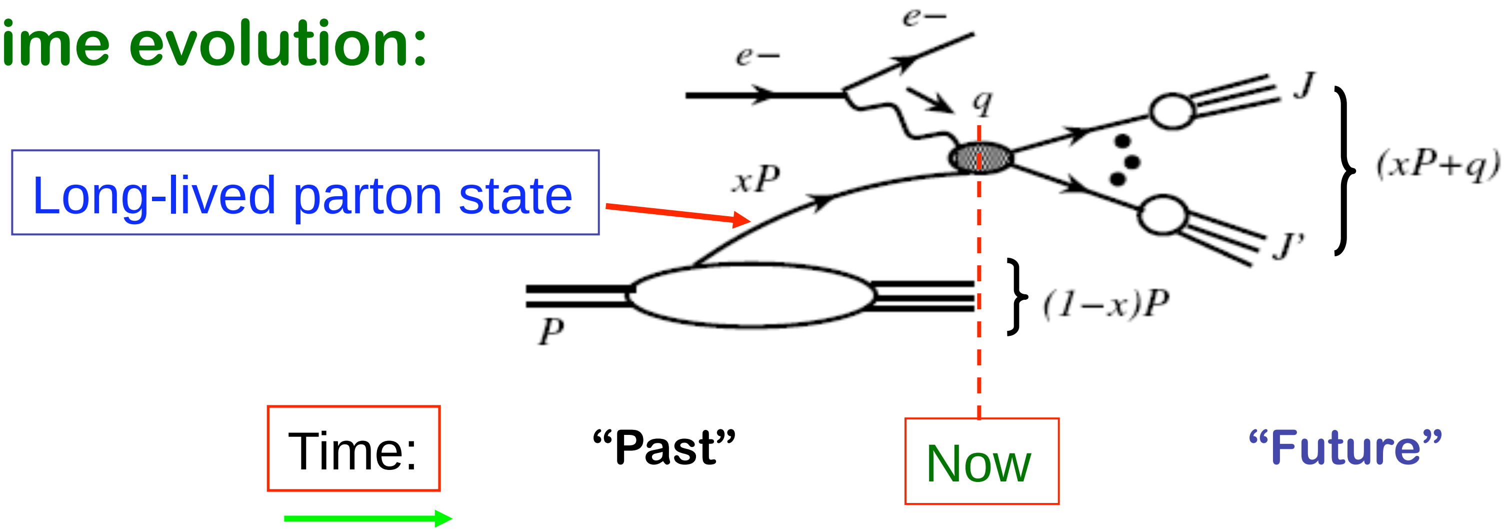
Measured

Long-distance

Factorization ⟷ needs a “long-lived” parton state

Picture of factorization for DIS

- **Time evolution:**



Long-lived parton state

Time:

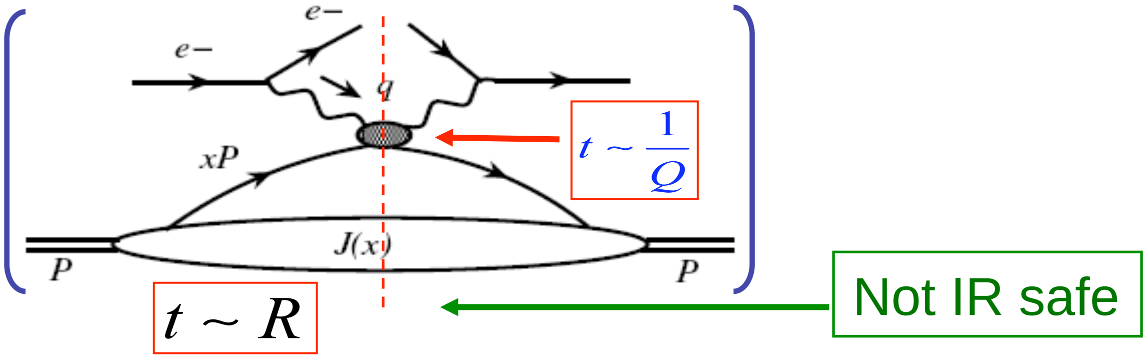
"Past"

Now

"Future"

- **Unitarity – summing over all hard jets:**

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}$$



$t \sim \frac{1}{Q}$

$t \sim R$

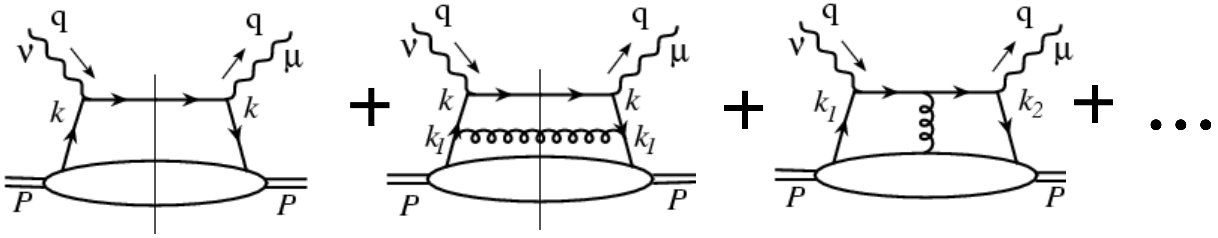
Not IR safe

Interaction between the "past" and "now" are suppressed!

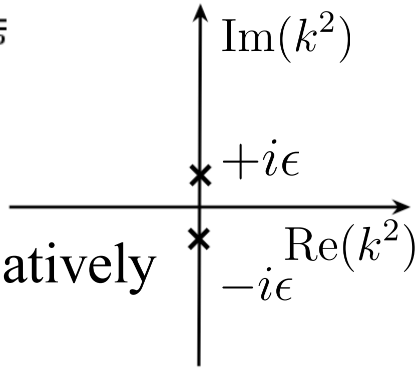
Long-lived Parton States

- ☐ **Feynman diagram representation:**

$$W^{\mu\nu} \propto$$



- ☐ **Perturbative pinched poles:**



$$\int d^4k\; \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \;\Rightarrow\; \infty \;\text{ perturbatively}$$

- ☐ **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

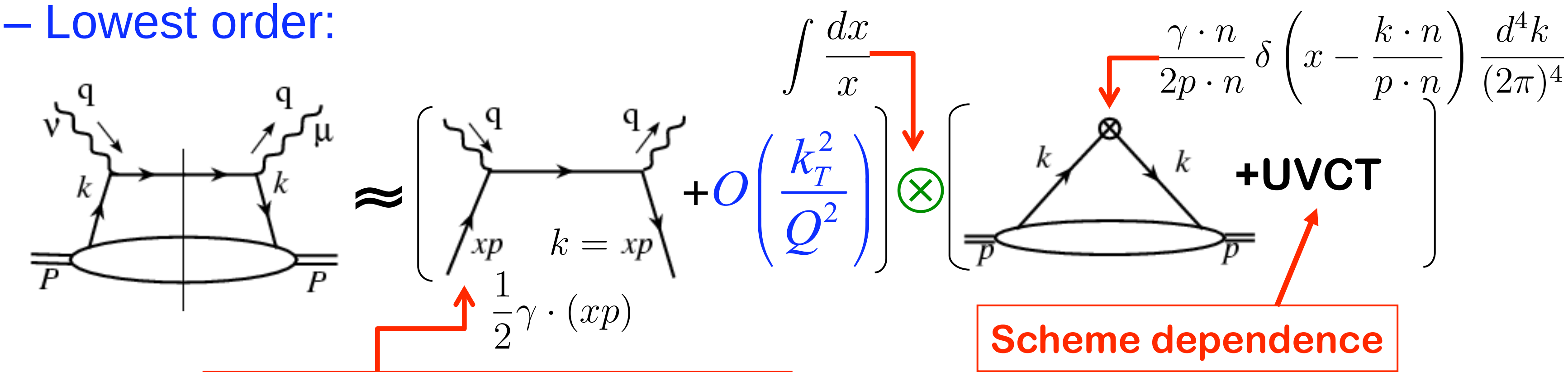
Nonperturbative matrix element

$$\int\frac{dx}{x}d^2k_T\;\mathrm{H}(Q,k^2=0)\;\boxed{\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0})}$$

Short-distance

Collinear factorization

- **Collinear approximation, if** $Q \sim xp\cdot n \gg k_T,\ \sqrt{k^2}$
  - – Lowest order:



$$\approx \left[\ \right] + O\left(\frac{k_T^2}{Q^2}\right) \otimes \left[\ + \text{UVCT}\ \right]$$

$\int \frac{dx}{x}$

$\frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x-\frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$

$\frac{1}{2}\gamma\cdot(xp)$

Same as elastic x-section

Scheme dependence

Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \to \infty,\ \text{while } x_B \text{ fixed}$

⇨ Feynman's parton model and Bjorken scaling

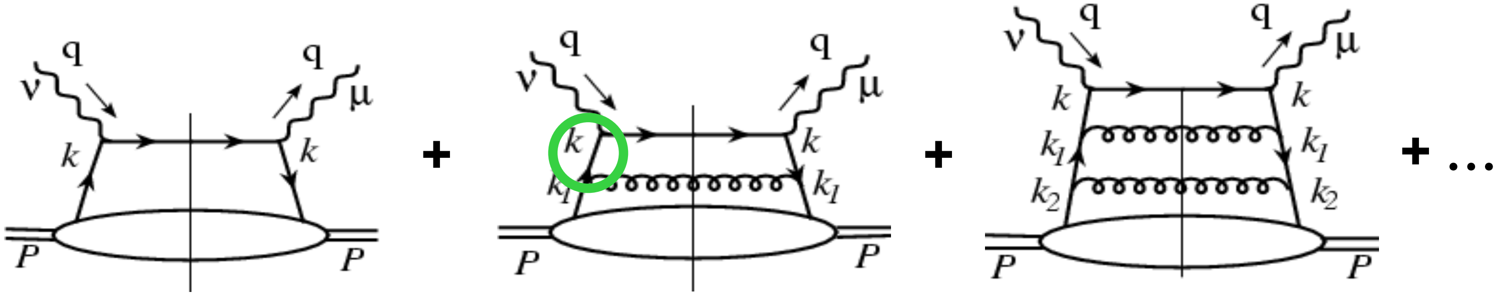
$$F_2(x_B,Q^2) = x_B \sum_f e_f^2\,\phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

Spin-½ parton!

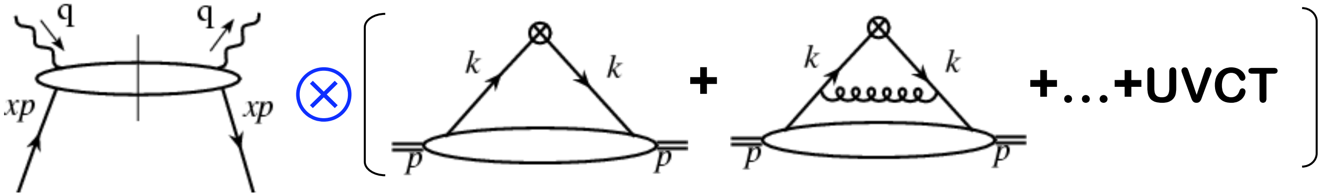
- **Corrections:** $\mathcal{O}(\alpha_s) + \mathcal{O}\left(\langle k^2\rangle/Q^2\right)$

Leading Power QCD Formalism

- **QCD corrections: pinch singularities in** $\int d^4 k_i$



- **Logarithmic contributions into parton distributions:**



$$\Longrightarrow \quad F_2(x_B, Q^2) = \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_f\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

- **Factorization scale:** μ_F^2

→ **To separate collinear from non-collinear contribution**

Recall: renormalization scale to separate local from non-local contribution

Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

⟶ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\text{QCD}}^2\right)$

Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

⟶ DGLAP evolution equation:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

- NLO is necessary for testing QCD calculation

Global QCD analysis of PDFs

❑ PDFs are extracted by using:

- ❖ **DGLAP**

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2}\varphi_i(x,\mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'},\alpha_s\right)\otimes\varphi_j(x',\mu_F^2)$$

- ❖ **Factorized hard cross sections, e.g.**

$$F_{2h}(x_B,Q^2) = \sum_q C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu_F^2\right)+O\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

- ❖ **Data:** to fix the boundary condition of DGLAP

❑ The order and scheme dependence of PDFs:

- ❖ **Leading order (tree-level) C_q**
- ❖ **Next-to-Leading order C_q**

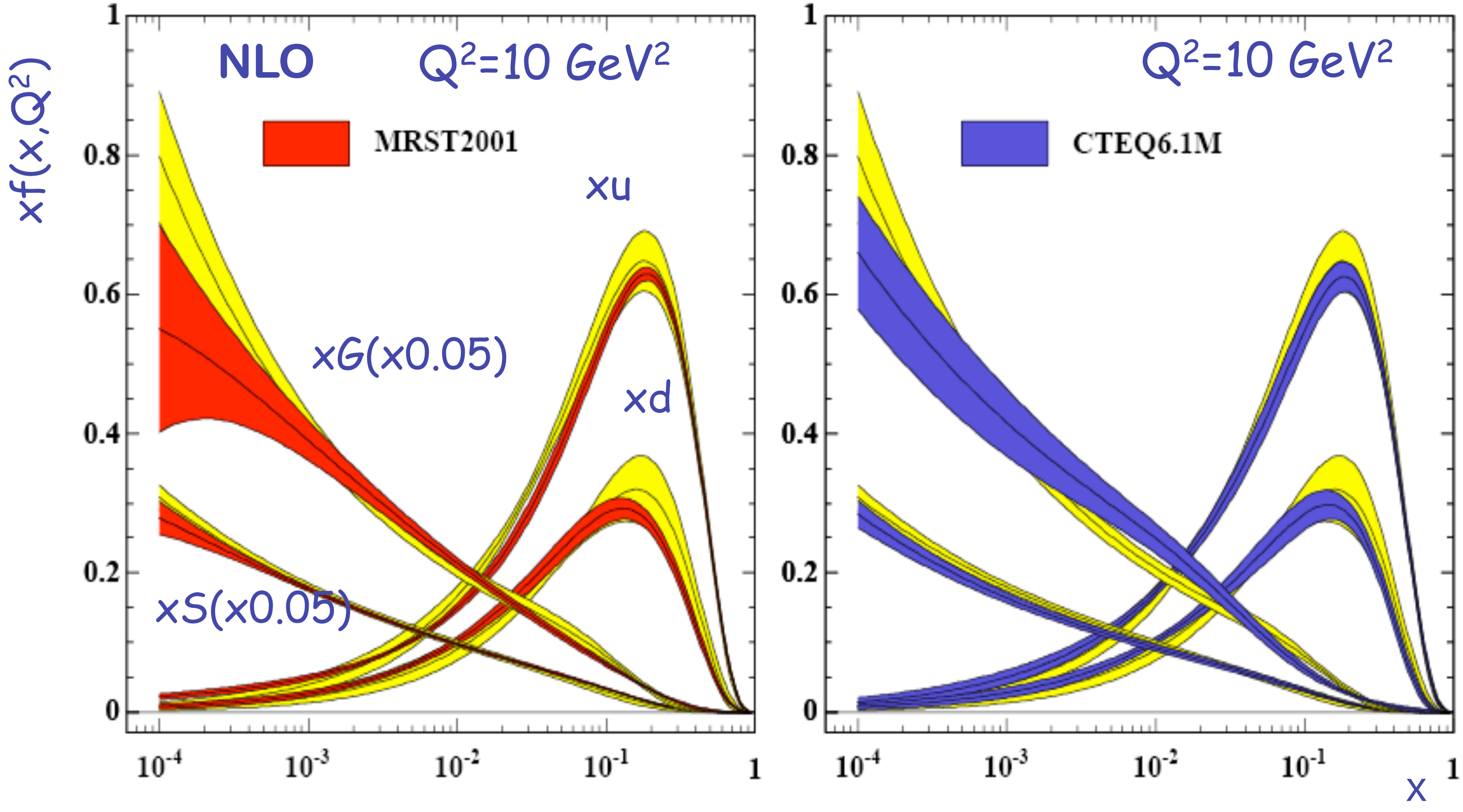
$\}$ ⟷ $\{$ **LO PDF's** / **NLO PDF's**

- ❖ **Calculation of C_q at NLO and beyond depends on the UVCT** → **the scheme dependence of C_q**

  → the scheme dependence of PDFs

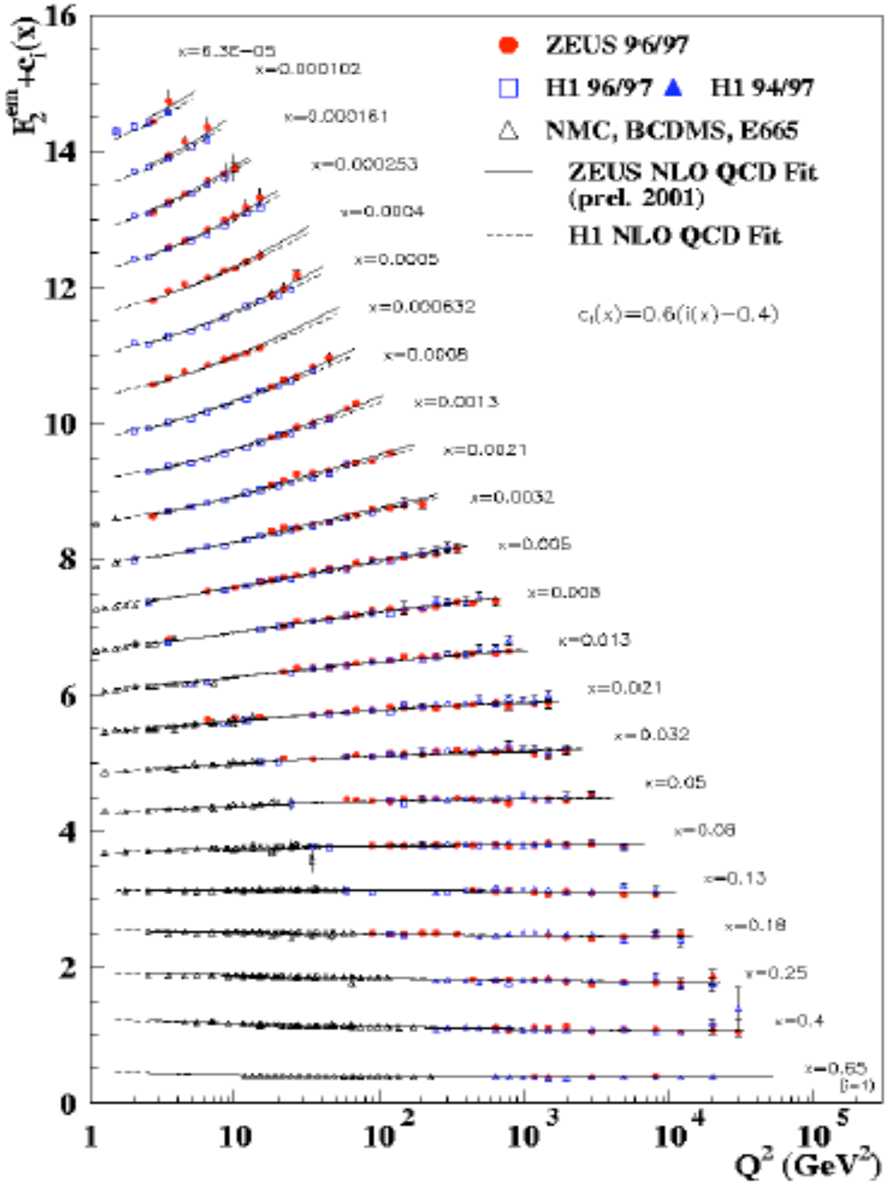
PDFs of a spin-averaged proton

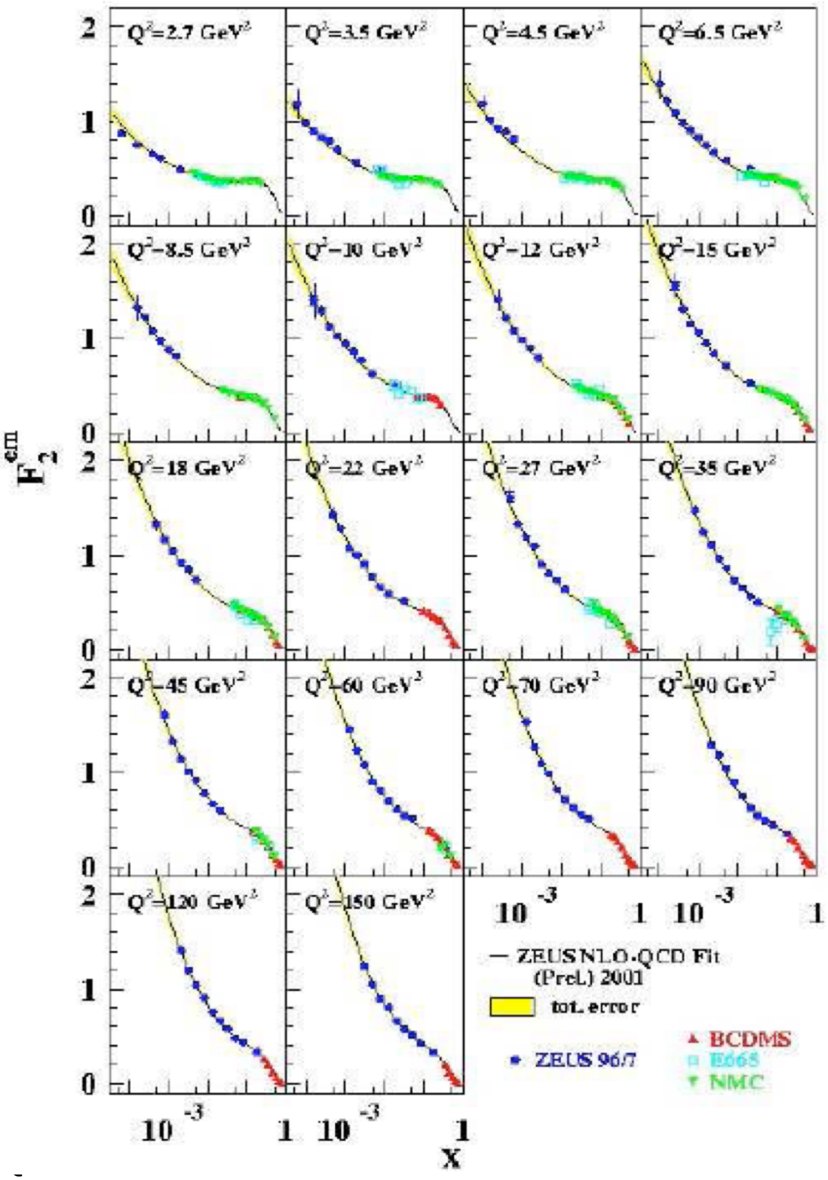
- ☐ **Modern sets of PDFs with uncertainties:**



Consistently fit almost all data with Q > 2GeV

Comparison with DIS Data

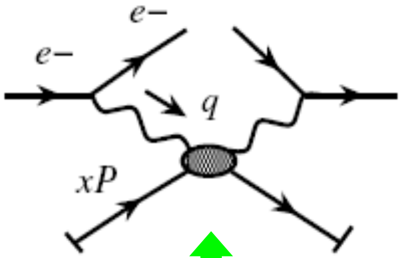




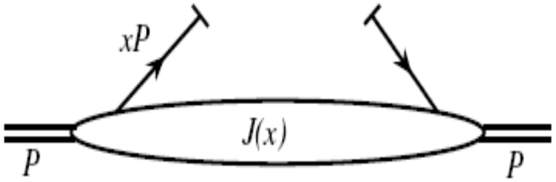
Cross Section with TWO Identified Hadrons

- **One hadron:**

$$\sigma_{\rm tot}^{\rm DIS} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

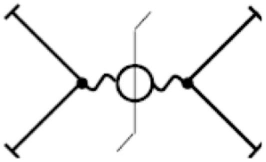
Now
Hard-part

Past
Parton-distribution

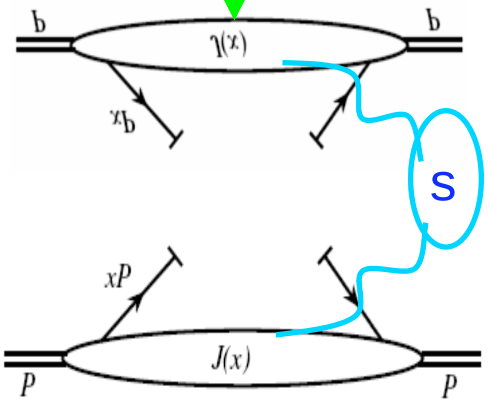
Connection
Power corrections

- **Two hadrons:**

$$\sigma_{\rm tot}^{\rm DY} \sim$$



$$\otimes$$



$$+\, O\left(\frac{1}{QR}\right)$$

Soft interactions between incoming hadrons break the universality of PDFs

"Drell-Yan" Cross Section

- ☐ **Drell-Yan process:**

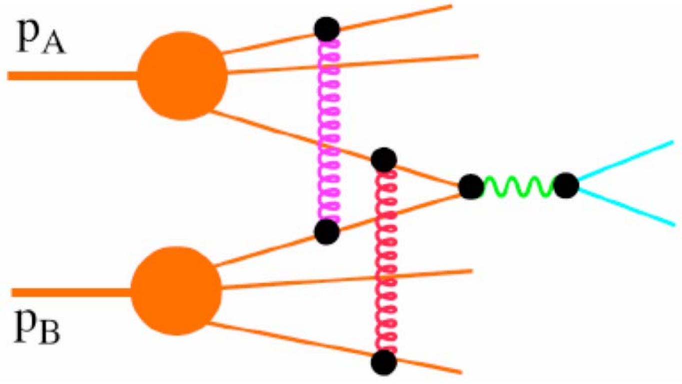
via a heavy colorless particle

$$h(p_A)+h'(p_B)\to \ell^+\ell^-(q)+X \qquad \text{with } Q^2=q^2$$

- ☐ **Parton model formula:**

$$\frac{d\sigma^{\mathrm{DY}}_{hh'}(p_A,p_B,q)}{dQ^2}=\sum_{f,f'}\int_0^1 dx\int_0^1 dx'\;\phi_f(x)\;\frac{d\hat{\sigma}^{\mathrm{el}}_{ff}(xp_A,x'p_B,q)}{dQ^2}\;\phi_{f'}(x')$$

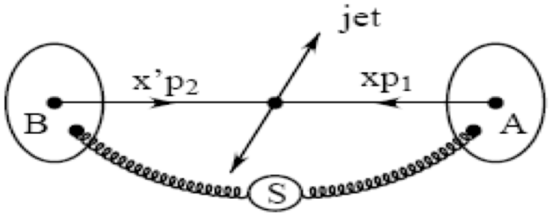
- ☐ **Long-range soft interactions before the hard collision could break PDF's universality – loss of predictive power**



$$K_{\text{factor}}=\frac{\sigma^{\mathrm{DY}}_{\mathrm{Exp}}}{\sigma^{\mathrm{DY}}_{\mathrm{PM}}}\sim 2$$

Long-range Soft Gluon Interactions

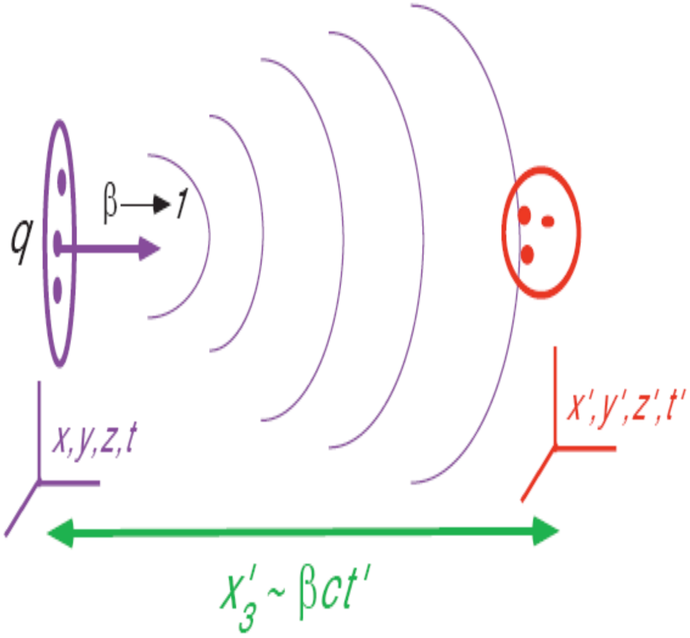
☐ **Soft-gluon interaction takes place all the time:**



Question:

What is its effect on a physical observable?

☐ **Factorization = soft-gluon interactions are suppressed:**



Field	x-Frame	x'-Frame
Scalar	$V(x) = \frac{e}{\lvert\vec{x}\rvert}$	$V'(x') = \frac{e}{(x_T^2 + \gamma^2\Delta^2)^{1/2}}$
		$\Rightarrow \frac{1}{\gamma}$ "contracted like a ruler"
Gauge	$A^-(x) = \frac{e}{\lvert\vec{x}\rvert}$	$A'^-(x') = \frac{e\gamma(1+\beta)}{(x_T^2 + \gamma^2\Delta^2)^{1/2}}$
		$\Rightarrow 1$ "not contracted!"

Field Strength is Strongly Contracted

Field	x-Frame	x'-Frame
Field Strength	$E_3(x) = \frac{e}{\lvert\vec{x}\rvert^2}$	$E_3(x') = \frac{-e\gamma\Delta}{(x_T^2+\gamma^2\Delta^2)^{3/2}}$
		$\Longrightarrow \frac{1}{\gamma^2}$ "strongly contracted!"

➡ *Lorentz contracted fields of incident particles do not overlap until the moment of the scattering!*

the $1/\gamma^2$ translates into a suppression factor of $1/Q^4$

➡ *Initial-state interaction disappear at high enough energies!*

$$\sigma(Q) = \sigma_0(Q) + \sigma_2(Q)\frac{1}{Q^2} + \sigma_4(Q)\frac{1}{Q^4} + \ldots$$

➡ the factorization should be valid at the order of $1/Q^2$

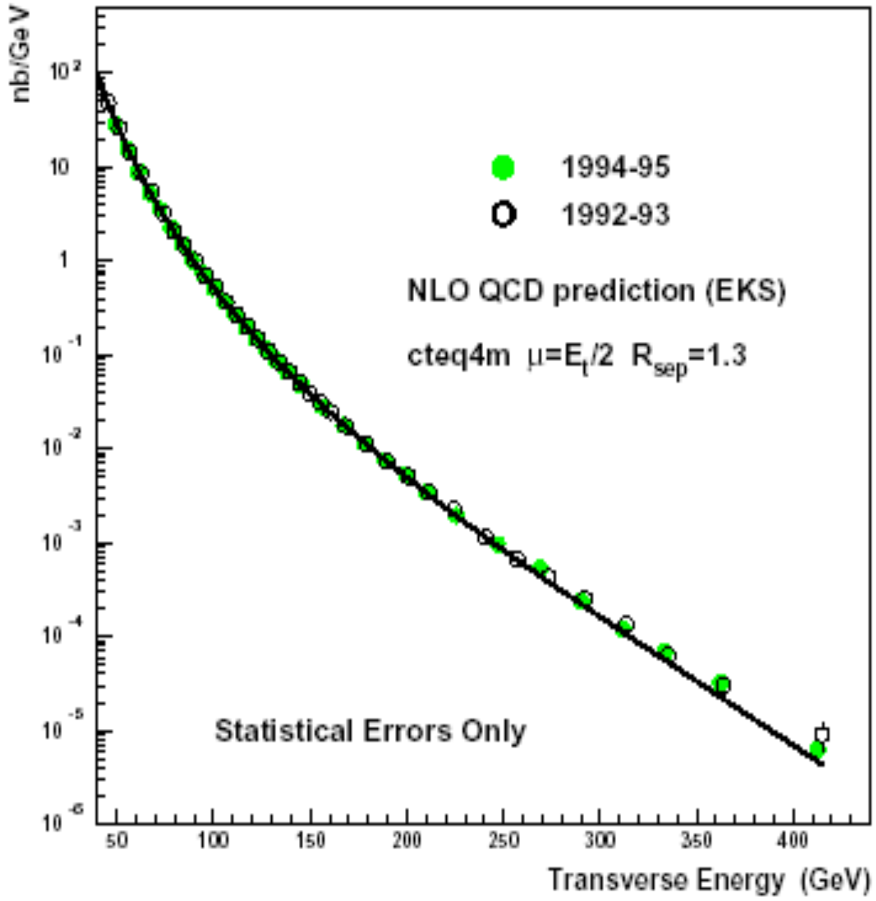
Leading power (twist): Collins, Soper, and Sterman; Bodwin

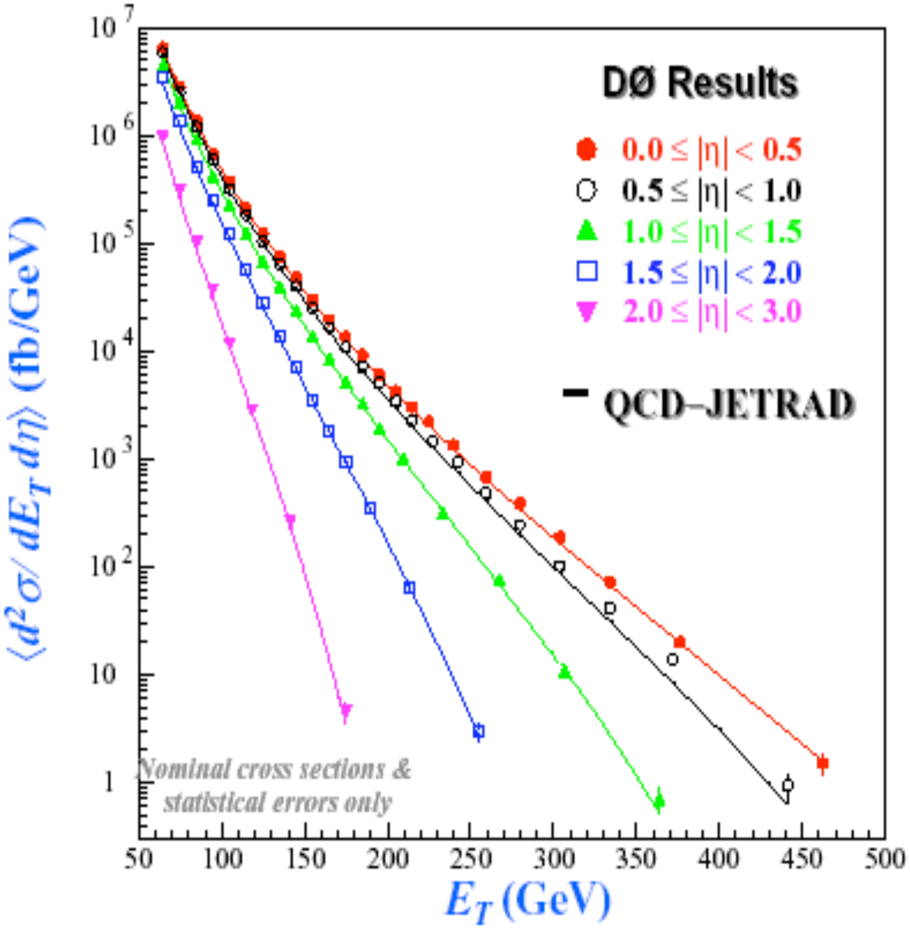
Next leading power: Qiu and Sterman

Factorization is violated at $1/Q^4$ via explicit calculation: Taylor et al.

Inclusive Jet Cross Section at Tevatron

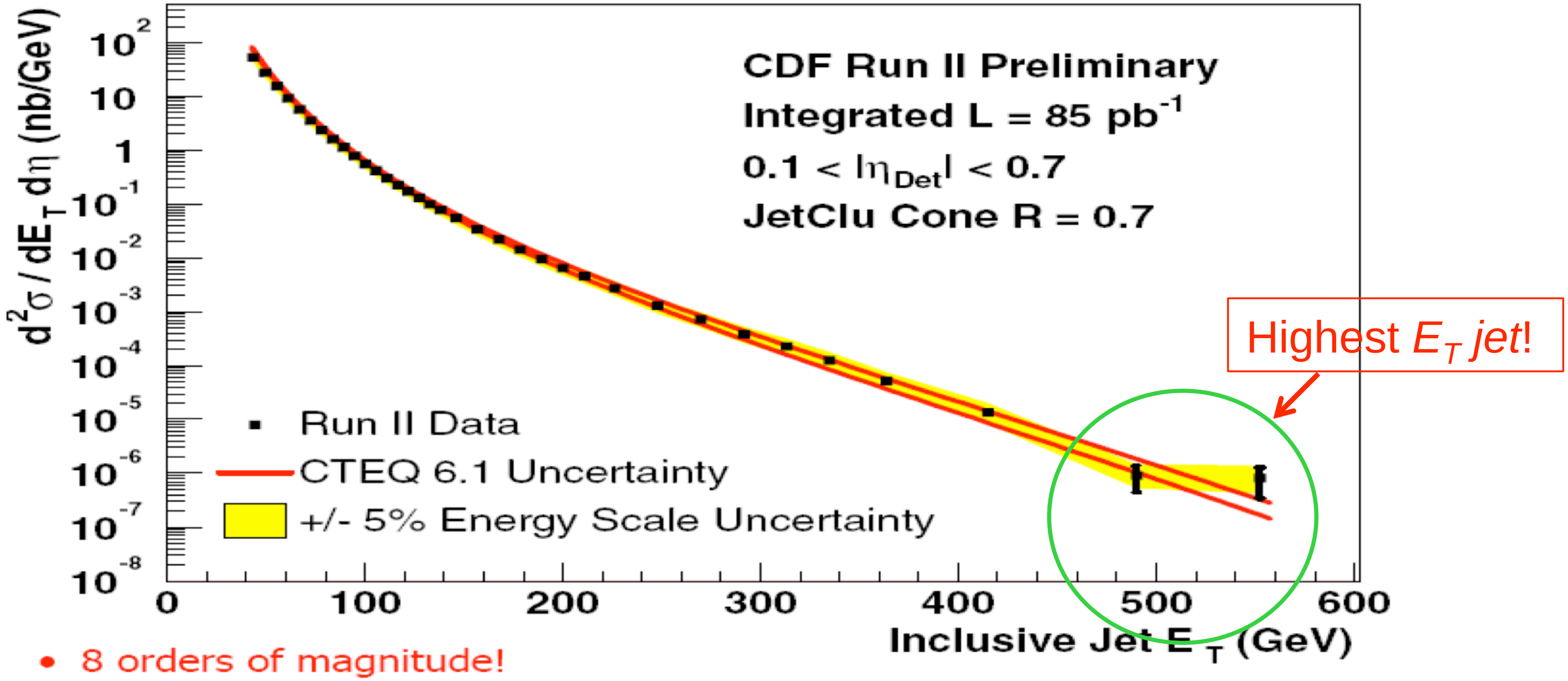
Run – 1b results





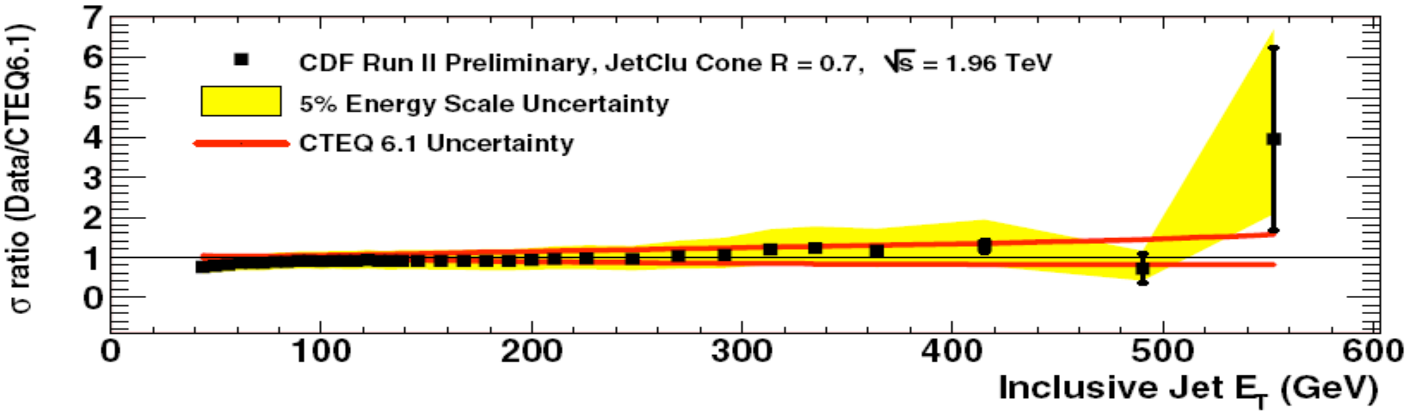
Data and Predictions span 7 orders of magnitude!

Prediction vs CDF RUN-II Data



Highest E_T *jet*!

- 8 orders of magnitude!



Are there anything left to do?

- ☐ **QCD has only been tested for the dynamics at a distance scale less than 1/10 fm!**

- ☐ **Connection between parton dynamics and the hadrons:**

Parton distribution functions grow fast as x→0

→ **Large phase space for gluon radiation** $\frac{Q^2}{s} \ll 0$

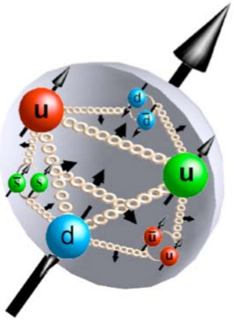
BFKL evolution → **violation of unitarity**

Large parton density – system is no longer dilute

→ **Parton recombination – saturation – CGC**

Active parton's virtuality → Q

→ **Parton k_T is important – power correction: $\langle k_T\rangle/Q$**

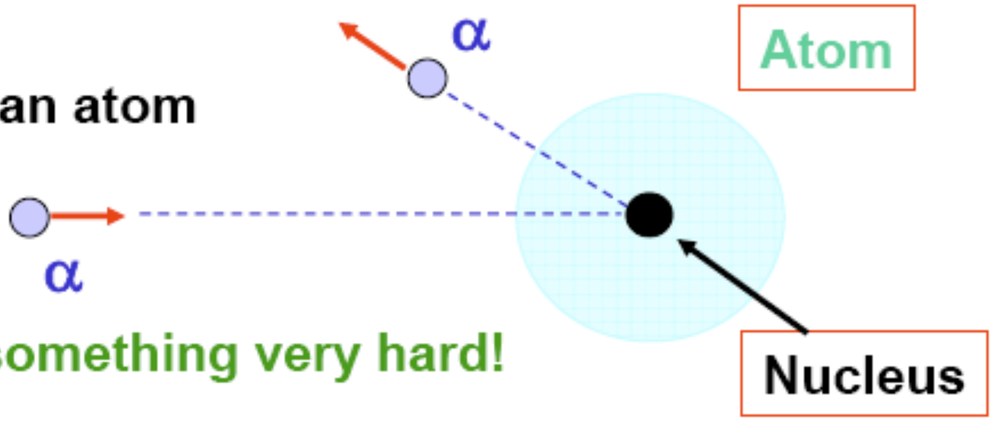


- ☐ **Novel phenomena in spin asymmtries:**

Single transverse spin asymmetries – parton transverse motion, …

“See” the substructure of a Nucleon?

- ☐ **Rutherford experiment:**
  - – to see the substructure of an atom

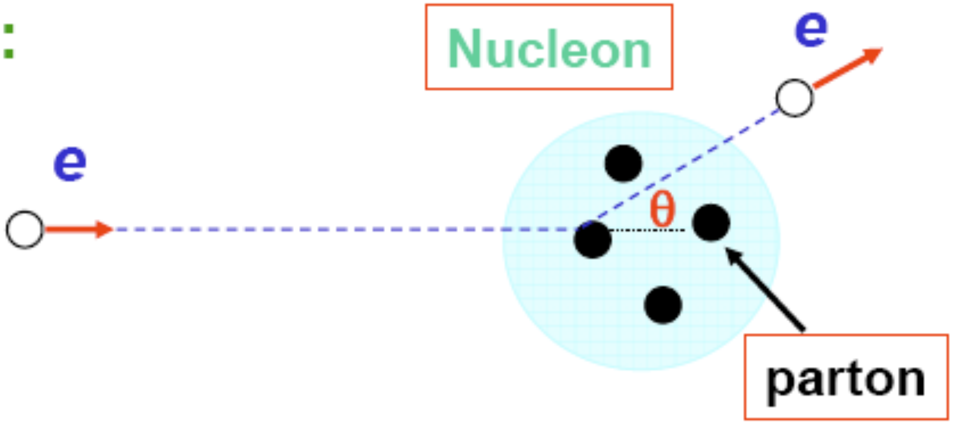


High energy α bounce off something very hard!

→ Discovery of nucleus inside an atom

- ☐ **SLAC experiment (1969):**

  Lepton-nucleon deeply inelastic scattering (DIS)



Scattering information on the θ-distribution

→ Discovery of the point-like spin-1/2 “partons”

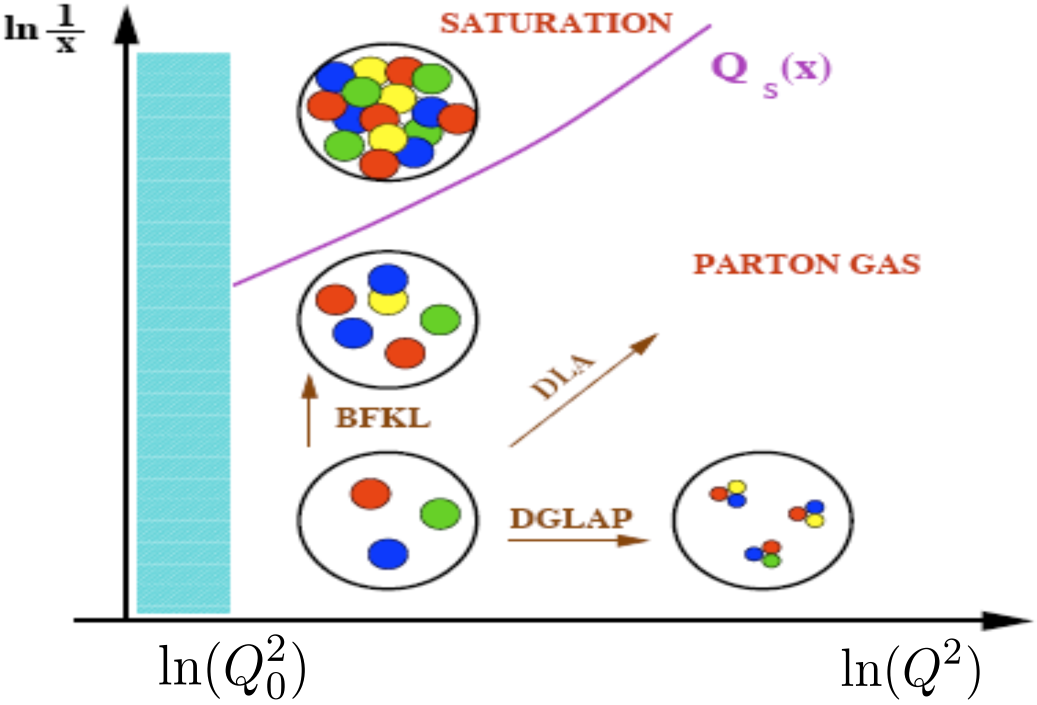
Electro-Ion Collider

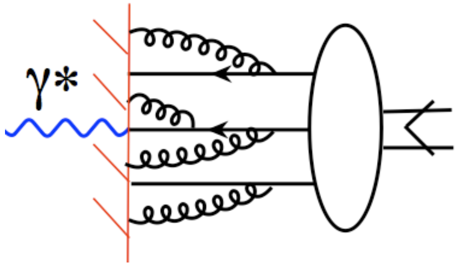
- ☐ **QCD in transition – from a dilute to a dense regime:**

  Parton k_T, parton correlation, QCD power expansion, …

- ☐ **QCD at high parton density:**

  Parton recombination – saturation – CGC





Parton density grows at small x

- **DGLAP evolution when** $x \to 0$
  - Gluon to gluon splitting function is singular as $x \to 0$

$$P_{gg}(x) \to 2N_c\left(\frac{\alpha_s}{2\pi}\right)\frac{1}{x} + \ldots$$

  - Corresponding moment:

$$\gamma(n) \equiv \int_0^1 dx\, x^{n-1}\; P(x)$$

$$\gamma_{gg}(n) \to 2N_c\left(\frac{\alpha_s}{2\pi}\right)\frac{1}{n-1} + \ldots \qquad \text{with pole at } \; \text{n=1}$$

  - Moments of gluon distribution:

$$\boxed{G(n,\mu^2) = G(n,\mu_0^2)\,\exp\left[\left(\frac{C}{n-1}\right)\ell n\left(\frac{\ell n(\mu^2/\Lambda_{\text{QCD}}^2)}{\ell n(\mu_0^2/\Lambda_{\text{QCD}}^2)}\right)\right]}$$

$$C = 4N_c/(-\beta_1) > 0$$

  - gluon at small x:

$$\boxed{xg(x,\mu^2) \approx \frac{G(n_0,\mu_0^2)}{\sqrt{2\pi a \ell n(1/x)}}\; \mathrm{e}^{2\sqrt{C\ell n(t)\ell n(1/x)}}}$$

Resummation of $\ln^n(1/x)$

BFKL Kinematics

- ☐ **Wthout k_T ordering, $\alpha_s^n \ell n^n (1/x)$, leading log approximation**

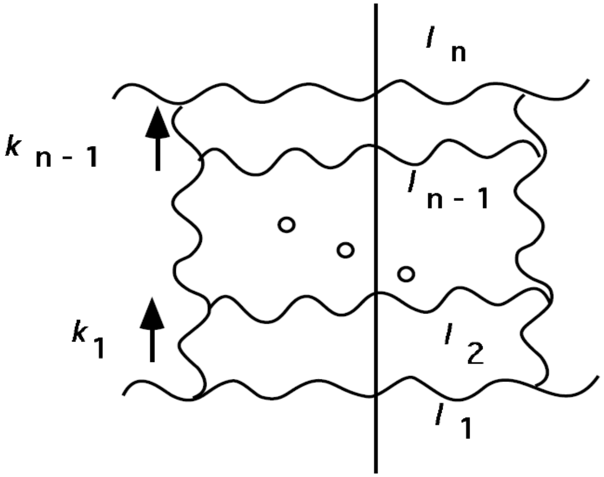
- ☐ **Consider n-gluon ladder at α_s^n**

  $x \ll 1, \quad Q^2 \ll W^2 = (1-x)Q^2/x,$

  so approximate

  $q^\mu \sim q^- \delta^{\mu -} + q^+ \delta^{\mu +}, q^2 \sim 2q^+ q^-$

  with $q^+ \ll q^-$



- ☐ **"Sudakov" parameterization:** $k_i = \alpha_i p + \beta_i q + k_{iT}$

- ☐ **Strong ordering:**

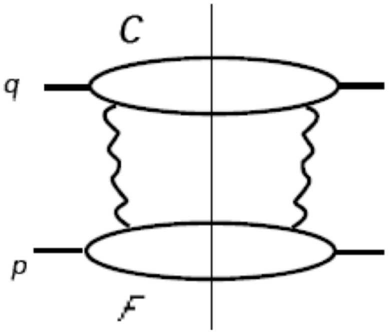
$$\boxed{\begin{array}{l} \alpha_1 \gg \alpha_2 \quad \gg \quad \cdots \gg \alpha_{n-1} \\ \beta_1 \ll \beta_2 \quad \ll \quad \cdots \ll \beta_{n-1} \end{array}} \qquad \sum_{i=1}^{j} \beta_i \sim \beta_j$$

$$\boxed{k_{iT} \quad \sim \quad k_{jT}} \qquad k_i^2 \sim -k_{i,T}^2 \qquad \sum_{i=j}^{n-1} \alpha_i \sim \alpha_j$$

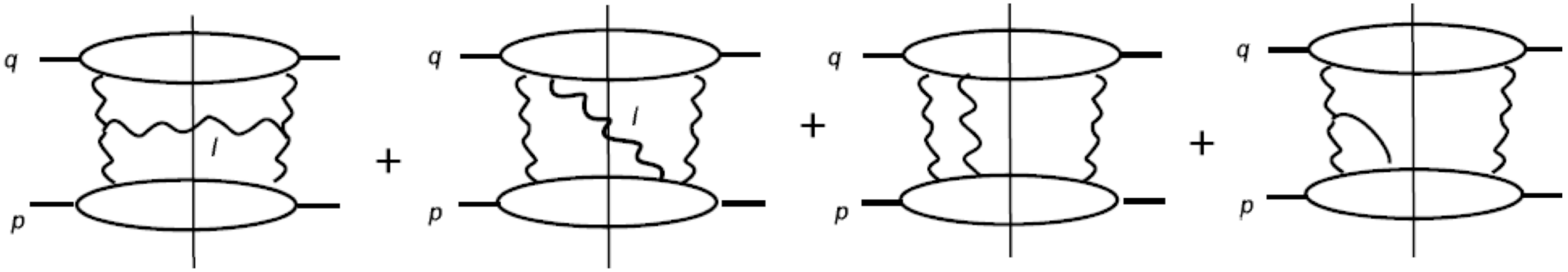
Generalized ladder diagrams

❑ **K_T factorization:**

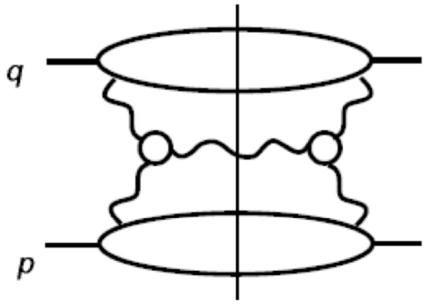
$$F(x,Q^2) = \int d^2k_T \int_x^1 \frac{d\xi}{\xi} C\left(\frac{x}{\xi},Q,k_T\right)\mathcal{F}(\xi,k_T)$$



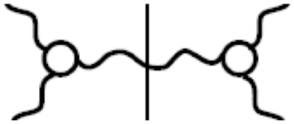
❑ **Add a gluon:**



❑ **Strong ordering leads to a generalized ladder:**



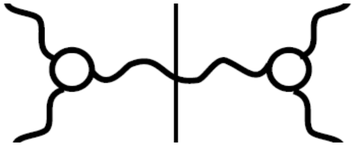
BFKL kennel:

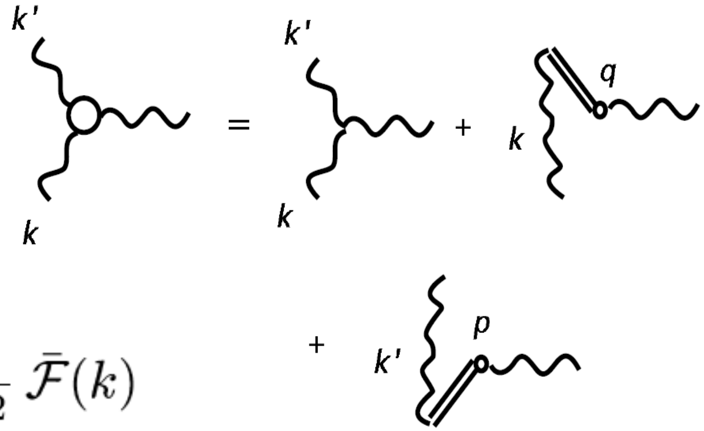


BFKL equation in DIS

❑ Calculation of the kernel:

$$\bar{K}(k'_T, k_T) =$$





$$\bar{\mathcal{F}}_{\rm real}(k') = \frac{1}{2}\frac{\alpha_s N}{\pi}\int_{k'^+}^{p^+}\frac{dk^+}{k^+}\int\frac{d^2k_T}{(k_T - k'_T)^2}\,\bar{\mathcal{F}}(k)$$

$$\bar{\mathcal{F}}_{\rm virtual}(k') = -\frac{1}{2}\frac{\alpha_s N}{\pi}\int_{k'^+}^{p^+}\frac{dk^+}{k^+}\int\frac{d^2k_T\,{k'_T}^2}{(k_T - k'_T)^2 k_T^2}\,\bar{\mathcal{F}}(k)$$

❑ BFKL equation:

$$\alpha\frac{\partial}{\partial\alpha}\bar{\mathcal{F}}(\alpha, k'_T) = -\frac{\alpha_s N}{\pi^2}\int\frac{d^2k_T}{(k_T - k'_T)^2}\left\{\bar{\mathcal{F}}(\alpha, k_T) - \frac{{k'_T}^2}{2k_T^2}\bar{\mathcal{F}}(\alpha, k'_T)\right\}$$

❑ Solution of BFKL equation:

$$\boxed{\mathcal{F}(x, q_T) \sim x^{-4N\ln 2(\alpha_s/\pi)}\,(q_T^2)^{-1/2}}$$

Much more singular than finite-order perturbative calculation!

Novel spin phenomena at EIC

- **Precision quark and gluon helicity distributions**

  Double spin asymmetries, … A_{LL}

  → Quark and gluon contribution to the proton's spin

- **Quark-gluon correlation inside a proton – inclusive DIS:**

  A_{LN} Twist-3 contribution to g2 structure funciton

  $A_N = 0$ One-photon exchange collision – precision tests

- **SIDIS:** $A_N \neq 0$ – Consequence of parton's transverse motion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

Three-parton correlation

$$\Delta\sigma(Q, s_T) \equiv [\sigma(Q, s_T) - \sigma(Q, -s_T)]/2$$

$$= (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}(1/Q^2)$$

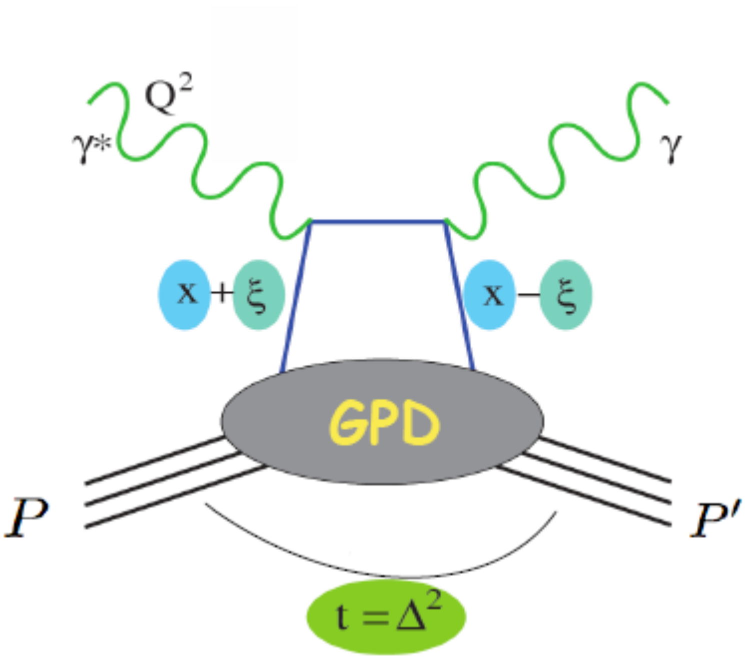
Direct measurement of QCD quantum interference

GPD – 3D parton distribution

Over the last decade, theory has understood that parton distributions and form factors are special cases of a much more powerful representation of nucleon structure:

“Generalized Parton Distributions”

Müller,Robaschik; Ji; Radyushkin



- x: average quark momentum frac^n
- ξ: “skewing parameter” $= x_1 - x_2$
- t: 4-momentum transfer^2

Generalized quark distribution

☐ Definition:

$$F_q(x,\xi,t,\mu^2) = \int \frac{d\lambda}{2\pi} \mathrm{e}^{-ix\lambda} \langle P'|\bar{\psi}_q(\lambda/2) \frac{\gamma\cdot n}{2P\cdot n} \psi_q(-\lambda/2)|P\rangle$$

$$\equiv \; H_q(x,\xi,t,\mu^2)\,[\bar{\mathcal{U}}(P')\gamma^\mu \mathcal{U}(P)]\,\frac{n_\mu}{2P\cdot n}$$

$$+ \; E_q(x,\xi,t,\mu^2)\left[\bar{\mathcal{U}}(P')\frac{i\sigma^{\mu\nu}(P'-P)_\nu}{2M}\mathcal{U}(P)\right]\frac{n_\mu}{2P\cdot n}$$

with $\xi = (P'-P)\cdot n/2 \text{ and } t = (P'-P)^2$

☐ Connection to normal quark distribution:

$$H_q(x,0,0,\mu^2) = q(x,\mu^2)$$

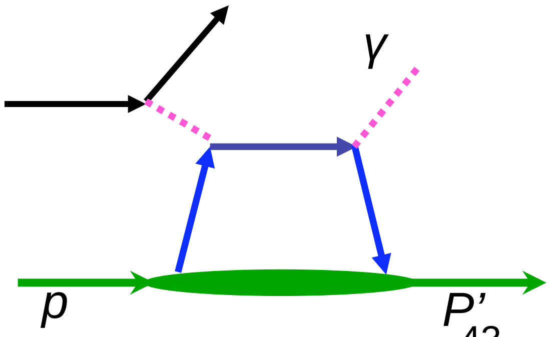
☐ Parton's orbital motion:

$$J_q = \frac{1}{2} \lim_{t\to 0} \int dx\, x\, [H_q(x,\xi,t) + E_q(x,\xi,t)]$$

$$= \frac{1}{2}\Delta q + L_q$$

Ji, PRL78, 1997

DVCS



Summary

- QCD is very rich in dynamics, much more than QED, while QED is the underline theory of all excitements of CMP, …

- After 35 years, we have learned only a very small part of QCD dynamics: less than 0.1 fm, although we have been successful

- EIC provides an unique opportunity to probe the partonic structure of hadron in many different kinematic regimes

- Let’s work and hope that we will have this QCD machine!

Thank you!