Introduction to QCD

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Outline

- Fundamentals of Quantum Chromodynamics (QCD)
- Why should we believe QCD?
- EIC as the next QCD machine
Quantum Chromodynamics (QCD) = A quantum field theory of quarks and gluons =

- **Fields:**
  - Quark fields: spin-$\frac{1}{2}$ Dirac fermion (like electron)
  - Color triplet: $i = 1, 2, 3 = N_c$
  - Flavor: $f = u, d, s, c, b, t$
  - Gluon fields: spin-1 vector field (like photon)
  - Color octet: $a = 1, 2, \ldots, 8 = N_c^2 - 1$

- **QCD Lagrangian density:**

\[
\mathcal{L}_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[ (i\partial_\mu \delta_{ij} - gA_{\mu,a}(t_a)_{ij})\gamma^\mu - m_f \delta_{ij} \right] \psi_j^f
\]
\[
- \frac{1}{4} \left[ \partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c} \right]^2
\]

+ gauge fixing + ghost terms

- **Color matrices:**

\[
[t_a, t_b] = i C_{abc} t_c
\]

Generators for the fundamental representation of SU3 color
Physical observables

- **Cross section:**
  
  Scattering amplitude square – Probability – Positive definite
  
  A function of in-state and out-state variables: momentum, spin, ...

- **Spin-averaged cross section:**
  
  \[ \sigma = \frac{1}{2} \left[ \sigma(\vec{s}) + \sigma(-\vec{s}) \right] \quad \text{– Positive definite} \]

- **Asymmetries or difference of cross sections:**
  
  \[ A(\vec{s}) = \frac{\Delta \sigma(\vec{s})}{\sigma} = \frac{\sigma(\vec{s}) - \sigma(-\vec{s})}{\sigma(\vec{s}) + \sigma(-\vec{s})} \quad \Delta \sigma(\vec{s}) = \frac{1}{2} \left[ \sigma(\vec{s}) - \sigma(-\vec{s}) \right] \]

  Not necessary positive!
  
  Chance to see quantum interference directly
Theorists: Lagrangian = “complete” theory

Experimentalists: Cross Section \(\rightarrow\) Observables

A road map – from Lagrangian to Cross Section:

- Particles \(\rightarrow\) Fields
- Interactions \(\rightarrow\) Symmetries
- Perturbation
- Lagrangian
  - Hard to solve exactly
  - Green Functions
    - Correlation between fields
    - Solution to the theory
      - find all correlations among any # of fields
- S-Matrix
- Cross Sections
- Observables
The Question

- **We measure:**
  
  Cross sections of hadrons and leptons – Observables

- **We believe:**
  
  Hadrons are bound states of quarks and gluons

- **But,**
  
  We have not been able to solve QCD analytically to understand the confinement

- **Question:**
  
  How to test QCD – a theory of quarks and gluons without seeing the quarks and gluons?
If there is no quantum interference between partons and hadrons,

\[
\sigma_{e^+e^- \to \text{hadrons}}^{\text{tot}} \propto \sum_n P_{e^+e^- \to n} = \sum_n \sum_m P_{e^+e^- \to m} P_{m \to n} = \sum_m P_{e^+e^- \to m} \sum_n P_{m \to n} = 1
\]

Finite in perturbation theory – KLN theorem

Test of QCD dynamics, color = 3, heavy quark mass threshold, …

Inclusive

\[ R = \frac{\sigma_{e^+e^- \to \text{Hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \]
QCD Asymptotic Freedom

- QCD is a renormalizable theory

- Running coupling: \[ \alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi} \alpha_s(\mu_1) \ln \left( \frac{\mu_2^2}{\mu_1^2} \right)} \equiv \frac{4\pi}{-\beta_1 \ln \left( \frac{\mu_2^2}{\Lambda_{QCD}^2} \right)} \]

\[ \mu_2 \text{ and } \mu_1 \text{ not independent} \]

Asymptotic Freedom \( \Leftrightarrow \) antiscreening

QCD: \( \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0 \)

Compare

QED: \( \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0 \)


2004 Nobel Prize in Physics
Effective Quark Mass

- **Running quark mass:**
  \[ m(\mu_2) = m(\mu_1) \exp \left[ - \int_{\mu_1}^{\mu_2} \frac{d\lambda}{\lambda} \left( 1 + \gamma_m(g(\lambda)) \right) \right] \]
  Quark mass depend on the renormalization scale!

- **QCD running quark mass:**
  \[ m(\mu_2) \Rightarrow 0 \quad as \quad \mu_2 \rightarrow \infty \quad since \quad \gamma_m(g(\lambda)) > 0 \]

- **Choice of renormalization scale:**
  \( \mu \sim Q \) for small logarithms in the perturbative coefficients

- **Light quark mass:** \( m_f(\mu) \ll \Lambda_{QCD} \) for \( f = u, d, \) even \( s \)

QCD perturbation theory \((Q >> \Lambda_{QCD})\) is effectively a massless theory
Consider a general diagram:

\[ p^2 = 0, \quad k^2 = 0 \quad \text{for a massless theory} \]

\[ k^\mu \to 0 \quad \Rightarrow \quad (p - k)^2 \to p^2 = 0 \]

\[ k^\mu \parallel p^\mu \quad \Rightarrow \quad k^\mu = \lambda p^\mu \quad \text{with} \quad 0 < \lambda < 1 \]
\[ \Rightarrow \quad (p - k)^2 \to (1 - \lambda)^2 p^2 = 0 \]

IR and CO divergences are generic problems of a massless perturbation theory
Infrared (IR) Safety

- Infrared safety:

\[ \sigma_{\text{Phy}} \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \frac{m^2(\mu^2)}{\mu^2} \right) \Rightarrow \hat{\sigma} \left( \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) + \mathcal{O} \left[ \left( \frac{m^2(\mu^2)}{\mu^2} \right)^\kappa \right] \]

Infrared safe = \( \kappa > 0 \)

Asymptotic freedom is useful only for quantities that are infrared safe

\[ \sigma_{\text{\text{Total}} {e^+e^-\rightarrow\text{Partons}}} (s = Q^2) = \sigma_0 \left[ 1 + \frac{\alpha_s(Q^2)}{\pi} + \ldots \right] \]

is IR safe

- Go beyond the inclusive total cross section?
Jets in e⁺e⁻ - Trace of Partons

- **Jets** – Inclusive x-section with a limited phase-space
- **Q:** will IR cancellation be completed?
  - Leading partons are moving away from each other
  - Soft gluon interactions should not change the direction of an energetic parton → a “jet” – “trace” of a parton
- **Many Jet algorithms**

Sterman-Weinberg Jet
A clean trace of two partons – a pair of quark and antiquark
Discovery of a Gluon Jet

First order in QCD ($\mathcal{O}(\alpha^2 \alpha_s^1)$):

PETRA $e^+e^-$ storage ring at DESY:

$E_{\text{c.m.}} \gtrsim 15$ GeV

Reputed to be the first three-jet event from TASSO

Tagged Three-jet Event from LEP
The harder Question

**Question:**

How to test QCD in a reaction with identified hadron(s)?
– to probe the quark-gluon structure of the hadron

**Facts:**

Hadronic scale $\sim 1/fm \sim \Lambda_{QCD}$ is non-perturbative

Cross section involving identified hadron(s) is not IR safe and is not perturbatively calculable!

**Solution – Factorization:**

✧ Isolate the calculable dynamics of quarks and gluons

✧ Connect quarks and gluons to hadrons via non-perturbative but universal distribution functions
  – provide information on the partonic structure of the hadron
Connecting the partons to the hadrons

- Effective field theories + models:
  - Integrate out some degrees of freedom, express QCD in some effective degrees of freedom:
    HQEF, SCEF, ...
  - approximation in field operators, still need the matrix elements to connect to the hadron states
  - effective theory in hadron degrees of freedom, ...
  - models – Quark Models, ...

- PQCD factorization:
  - Connect partons to hadrons via matrix elements (PDFs, FFs, ...)
  \[ \langle H(p, s) | \mathcal{O}(\phi, F_{\mu\nu}) | H(p, s) \rangle \]

- Lattice QCD – cannot calculate hadronic cross sections
  - can calculate matrix elements and partonic properties, ...
Inclusive lepton-hadron DIS – one hadron

- **Cross section:**

\[
E' \frac{d\sigma_{\text{DIS}}^{\mu\nu}}{d^3k'} = \frac{1}{2s} \left( \frac{1}{Q^2} \right)^2 L^{\mu\nu}(k,k') W_{\mu\nu}(q,p)
\]

- **Hadronic tensor:**

\[
W_{\mu\nu}(q,p,S) = \frac{1}{4\pi} \int d^4z \ e^{iq\cdot z} \ \langle p,S | J_\mu^+(z) J_\nu(0) | p,S \rangle
\]

\[
W_{\mu\nu} = -\left( g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2} \right) F_1(x_B,Q^2) + \frac{1}{p\cdot q} \left( p_{\mu} - q_{\mu} \frac{p\cdot q}{q^2} \right) \left( p_{\nu} - q_{\nu} \frac{p\cdot q}{q^2} \right) F_2(x_B,Q^2)
\]

\[+ iM_p \varepsilon^{\mu\nu\rho\sigma} q_\rho \left[ \frac{S_\sigma}{p\cdot q} g_1(x_B,Q^2) + \frac{(p\cdot q) S_\sigma - (S\cdot q) p_\sigma}{(p\cdot q)^2} g_2(x_B,Q^2) \right]\]

- **Structure functions – infrared sensitive:**

\[
F_1(x_B,Q^2), F_2(x_B,Q^2), g_1(x_B,Q^2), g_2(x_B,Q^2)
\]
Perturbative QCD Factorization

- Factorization – an approximation:
  - Short-distance Power corrections
  - Long-distance Measured

- Leading Power:
  - Single active parton from each hadron!

\[ \sigma_{\text{phy}}(Q, 1/R) \sim \hat{\sigma}(Q) \otimes \varphi(1/R) + O(1/Q R) \]
Picture of factorization for DIS

- **Time evolution:**
  - Long-lived parton state

- **Unitarity – summing over all hard jets:**
  - $\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im} \left[ \begin{array}{c} e^- \\
  xP \\
  q \\
  (1-x)P \\
  (xP+q) \\
  \end{array} \right]$

  Interaction between the “past” and “now” are suppressed!

- Not IR safe
Long-lived Parton States

- Feynman diagram representation:

\[ W^{\mu\nu} \propto \quad + \quad + \quad + \quad \ldots \]

- Perturbative pinched poles:

\[
\int d^4k \ H(Q,k) \left( \frac{1}{k^2 + i\epsilon} \right) \left( \frac{1}{k^2 - i\epsilon} \right) T(k, \frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}
\]

- Perturbative factorization:

\[
k^\mu = x p^\mu + \frac{k^2 + k_T^2}{2 x p \cdot n} n^\mu + k_T^\mu
\]

\[
\int \frac{dx}{x} d^2k_T \ H(Q, k^2 = 0) \quad \int dk^2 \left( \frac{1}{k^2 + i\epsilon} \right) \left( \frac{1}{k^2 - i\epsilon} \right) T(k, \frac{1}{r_0})
\]

Nonperturbative matrix element

Short-distance
**Collinear factorization**

- **Collinear approximation, if**
  \[ Q \sim x p \cdot n \gg k_T, \sqrt{k^2} \]

  - Lowest order:

    \[ \int \frac{dx}{x} + O\left( \frac{k_T^2}{Q^2} \right) \]

      \[ \gamma \cdot n \left( x - \frac{k \cdot n}{p \cdot n} \right) \frac{d^4k}{(2\pi)^4} \]

      \[ + \text{UVCT} \]

    **Scheme dependence**

    **Same as elastic x-section**

Parton’s transverse momentum is integrated into parton distributions, and provides a scale of power corrections.

- **DIS limit:** \( \nu, Q^2 \to \infty, \) while \( x_B \) fixed

  \[ F_2(x_B, Q^2) = x_B \sum_f e_f^2 \phi_f(x_B) = 2x_B F_1(x_B, Q^2) \]

  Spin-\( \frac{1}{2} \) parton!

- **Corrections:** \( \mathcal{O}(\alpha_s) + \mathcal{O}\left( \langle k^2 \rangle / Q^2 \right) \)

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Leading Power QCD Formalism

- **QCD corrections**: pinch singularities in \( \int d^4 k_i \)

- **Logarithmic contributions into parton distributions**:

- **Factorization scale**: \( \mu_F^2 \)
  - To separate collinear from non-collinear contribution
  - Recall: renormalization scale to separate local from non-local contribution
Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale
  \[ \mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0 \]

- Evolution (differential-integral) equation for PDFs
  \[ \sum_f \mu_F^2 \frac{d}{d\mu_F^2} C_f \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \otimes \varphi_f(x, \mu_F^2) + \sum_f C_f \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f(x, \mu_F^2) = 0 \]

- PDFs and coefficient functions share the same logarithms
  - PDFs: \( \log \left( \frac{\mu_F^2}{\mu_0^2} \right) \) or \( \log \left( \frac{\mu_F^2}{\Lambda_{QCD}^2} \right) \)
  - Coefficient functions: \( \log \left( \frac{Q^2}{\mu_F^2} \right) \) or \( \log \left( \frac{Q^2}{\mu^2} \right) \)

- DGLAP evolution equation:
  \[ \mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j} \left( \frac{x}{x'}, \alpha_s \right) \otimes \varphi_j(x', \mu_F^2) \]

- NLO is necessary for testing QCD calculation
Global QCD analysis of PDFs

PDFs are extracted by using:

- **DGLAP**
  \[
  \mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j} \left( \frac{x}{x'}, \alpha_s \right) \otimes \varphi_j(x', \mu_F^2)
  \]

- Factorized hard cross sections, e.g.
  \[
  F_{2h}(x_B, Q^2) = \sum_q C_{q/f} \left( \frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s \right) \otimes \varphi_{f/h}(x, \mu_F^2) + O\left( \frac{\Lambda_{QCD}^2}{Q^2} \right)
  \]

- **Data:** to fix the boundary condition of DGLAP

The order and scheme dependence of PDFs:

- Leading order (tree-level) **$C_q$**
- Next-to-Leading order **$C_q$**

Calculation of **$C_q$** at NLO and beyond depends on the UVCT and the scheme dependence of **$C_q$**
Modern sets of PDFs with uncertainties:

Consistently fit almost all data with $Q > 2\text{GeV}$
Comparison with DIS Data
Cross Section with TWO Identified Hadrons

- **One hadron:**

  \[ \sigma_{\text{DIS}}^{\text{tot}} \sim \]

  \[ xP \rightarrow J(x) + O\left(\frac{1}{QR}\right) \]

  Now

  Hard-part

  Past

  Parton-distribution

  Connection

  Power corrections

- **Two hadrons:**

  \[ \sigma_{\text{DY}}^{\text{tot}} \sim \]

  \[ xP \rightarrow J(x) + O\left(\frac{1}{QR}\right) \]

  Soft interactions between incoming hadrons break the universality of PDFs
“Drell-Yan” Cross Section

Drell-Yan process:

\[ h(p_A) + h'(p_B) \rightarrow \ell^+ \ell^- (q) + X \quad \text{with} \quad Q^2 = q^2 \]

Parton model formula:

\[
\frac{d\sigma^{\text{DY}}_{hh} (p_A, p_B, q)}{dQ^2} = \sum_{f,f'} \int_{x_0}^{1} dx \int_{x_0'}^{1} dx' \, \phi_f (x) \frac{d\hat{\sigma}^{\text{el}}_{ff} (xp_A, x' p_B, q)}{dQ^2} \phi_{f'} (x')
\]

Long-range soft interactions before the hard collision could break PDF’s universality – loss of predictive power

\[ K_{\text{factor}} = \frac{\sigma^{\text{DY}}_{\text{Exp}}}{\sigma^{\text{DY}}_{\text{PM}}} \sim 2 \]
Long-range Soft Gluon Interactions

- Soft-gluon interaction takes place all the time:

  Question:
  What is its effect on a physical observable?

- Factorization = soft-gluon interactions are suppressed:

\[
\begin{align*}
\text{Field} & \quad \text{x-Frame} & \quad \text{x'-Frame} \\
\text{Scalar} & \quad V(x) = \frac{e}{|x|} & \quad V'(x') = \frac{e}{(x_T^2 + \gamma^2 \Delta^2)^{1/2}} \\
& \quad \Rightarrow \frac{1}{\gamma} \quad \text{“contracted like a ruler”} & \quad \Rightarrow 1 \quad \text{“not contracted!”}
\end{align*}
\]
Field Strength is Strongly Contracted

Field

\[ E_3(x) = \frac{e}{|x|^2} \]

\[ E_3(x') = \frac{-e\gamma \Delta}{(x_T^2 + \gamma^2 \Delta^2)^{3/2}} \]

\[ \Rightarrow \frac{1}{\gamma^2} \quad \text{“strongly contracted!”} \]

Lorentz contracted fields of incident particles do not overlap until the moment of the scattering!

the \(1/\gamma^2\) translates into a suppression factor of \(1/Q^4\)

Initial-state interaction disappear at high enough energies!

\[ \sigma(Q) = \sigma_0(Q) + \sigma_2(Q) \frac{1}{Q^2} + \sigma_4(Q) \frac{1}{Q^4} + \ldots \]

the factorization should be valid at the order of \(1/Q^2\)

Leading power (twist): Collins, Soper, and Sterman; Bodwin
Next leading power: Qiu and Sterman
Factorization is violated at \(1/Q^4\) via explicit calculation: Taylor et al.
Inclusive Jet Cross Section at Tevatron

Run – 1b results

Data and Predictions span 7 orders of magnitude!
Prediction vs CDF RUN-II Data

- 8 orders of magnitude!
Are there anything left to do?

- QCD has only been tested for the dynamics at a distance scale less than 1/10 fm!

- Connection between parton dynamics and the hadrons:
  - Parton distribution functions grow fast as \( x \to 0 \)
  - Large phase space for gluon radiation
  - BFKL evolution
  - Violation of unitarity
  - Large parton density – system is no longer dilute
  - Parton recombination – saturation – CGC
  - Active parton’s virtuality \( \to Q \)
  - Parton \( k_T \) is important – power correction: \( \frac{<k_T>}{Q} \)

- Novel phenomena in spin asymmetries:
  - Single transverse spin asymmetries – parton transverse motion, …
“See” the substructure of a Nucleon?

- **Rutherford experiment:**
  - to see the substructure of an atom
  - High energy $\alpha$ bounce off something very hard!
  - Discovery of nucleus inside an atom

- **SLAC experiment (1969):**
  - Lepton-nucleon deeply inelastic scattering (DIS)
  - Scattering information on the $\theta$-distribution
  - Discovery of the point-like spin-1/2 “partons”
Electro-Ion Collider

- QCD in transition – from a dilute to a dense regime:
  Parton $k_T$, parton correlation, QCD power expansion, ...

- QCD at high parton density:

  Parton recombination – saturation – CGC
Parton density grows at small x

- **DGLAP evolution when** $x \to 0$
  - Gluon to gluon splitting function is singular as $x \to 0$
    \[ P_{gg}(x) \to 2N_c \left( \frac{\alpha_s}{2\pi} \right) \frac{1}{x} + \ldots \]
  - Corresponding moment:
    \[ \gamma(n) \equiv \int_0^1 dx \ x^{n-1} \ P(x) \]
    \[ \gamma_{gg}(n) \to 2N_c \left( \frac{\alpha_s}{2\pi} \right) \frac{1}{n-1} + \ldots \quad \text{with pole at } n=1 \]

- **Moments of gluon distribution:**
  \[ G(n, \mu^2) = G(n, \mu_0^2) \ \exp \left[ \left( \frac{C}{n-1} \right) \ln \left( \frac{\ln(\mu^2/\Lambda_{QCD}^2)}{\ln(\mu_0^2/\Lambda_{QCD}^2)} \right) \right] \]
  \[ C = 4N_c / (-\beta_0) > 0 \]

- **Gluon at small** $x$:
  \[ xg(x, \mu^2) \approx \frac{G(n_0, \mu_0^2)}{\sqrt{2\pi a \ell n(1/x)}} \ e^{2\sqrt{C\ell n(t)\ell n(1/x)}} \]
  | Resummation of $\ln^n(1/x)$ |
Without $k_T$ ordering, $\alpha_s^n \ln^n (1/x)$, leading log approximation

Consider $n$-gluon ladder at $\alpha_s^n$

$x \ll 1$, $Q^2 \ll W^2 = (1-x)Q^2 / x$,

so approximate

$q^\mu \sim q^- \delta_\mu^- + q^+ \delta_\mu^+ \quad q^2 \sim 2q^+ q^-$

with $q^+ \ll q^-$

"Sudakov" parameterization: $k_i = \alpha_i p + \beta_i q + k_{iT}$

Strong ordering:

$\alpha_1 \gg \alpha_2 \gg \cdots \gg \alpha_{n-1}$

$\beta_1 \ll \beta_2 \ll \cdots \ll \beta_{n-1}$

$k_{iT} \sim k_{jT} \quad k_i^2 \sim -k_{i,T}^2$

$\sum_{i=1}^{j} \beta_i \sim \beta_j$

$\sum_{i=1}^{n-1} \alpha_i \sim \alpha_j$
Generalized ladder diagrams

- **Kₜ factorization:**

\[ F(x, Q^2) = \int d^2k_T \int_x^1 \frac{d\xi}{\xi} C \left( \frac{x}{\xi}, Q, k_T \right) F(\xi, k_T) \]

- **Add a gluon:**

- **Strong ordering leads to a generalized ladder:**

BFKL kernel:
BFKL equation in DIS

- **Calculation of the kernel:**

\[
\bar{K}(k_T', k_T) = \int \bar{F}(k') \frac{d^2 k_T}{(k_T - k_T')^2 k_T^2} \bar{F}(k)
\]

- **BFKL equation:**

\[
\alpha \frac{\partial}{\partial \alpha} \bar{F}(\alpha, k_T') = - \frac{\alpha_s N}{\pi^2} \int \frac{d^2 k_T}{(k_T - k_T')^2} \left\{ \bar{F}(\alpha, k_T) - \frac{k_T'^2}{2k_T^2} \bar{F}(\alpha, k_T') \right\}
\]

- **Solution of BFKL equation:**

\[
\bar{F}(x, q_T) \sim x^{-4N \ln 2(\alpha_s/\pi)} (q_T^2)^{-1/2}
\]

**Much more singular than finite-order perturbative calculation!**
Novel spin phenomena at EIC

- **Precision quark and gluon helicity distributions**
  
  Double spin asymmetries, … \( A_{LL} \)
  
  Quark and gluon contribution to the proton’s spin

- **Quark-gluon correlation inside a proton – inclusive DIS:**
  
  \( A_{LN} \)  
  Twist-3 contribution to \( g_2 \) structure function

  \( A_N = 0 \)  
  One-photon exchange collision – precision tests

- **SIDIS:**  
  \( A_N \neq 0 \)  
  – Consequence of parton’s transverse motion

\[
\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + \frac{1}{Q} H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)
\]

Too large to compete!  
Three-parton correlation

\[
\Delta \sigma(Q, s_T) \equiv \left[ \sigma(Q, s_T) - \sigma(Q, -s_T) \right]/2
\]

\[
= \frac{1}{Q} H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}(1/Q^2)
\]

Direct measurement of QCD quantum interference
GPD – 3D parton distribution

Over the last decade, theory has understood that parton distributions and form factors are special cases of a much more powerful representation of nucleon structure: “Generalized Parton Distributions”

Müller, Robaschik; Ji; Radyushkin

\[ \begin{align*} 
Q^2 & \quad \gamma' \quad \gamma \\
\mathbf{t} = \Delta^2 \quad P \quad P' \\
\mathbf{x} & \text{: average quark momentum fraction} \\
\xi & \text{: “skewing parameter” } = x_1 - x_2 \\
\mathbf{t} & \text{: 4-momentum transfer}^2
\end{align*} \]
Generalized quark distribution

Definition:

\[ F_q(x, \xi, t, \mu^2) = \int \frac{d\lambda}{2\pi} e^{-ix\lambda} \langle P' \mid \bar{\psi}_q(\lambda/2) \gamma \cdot n \psi_q(-\lambda/2) \mid P \rangle \]

\[ \equiv H_q(x, \xi, t, \mu^2) \left[ \bar{\mathcal{U}}(P') \gamma^\mu \mathcal{U}(P) \right] \frac{n_\mu}{2P \cdot n} \]

\[ + E_q(x, \xi, t, \mu^2) \left[ \bar{\mathcal{U}}(P') \frac{i\sigma^{\mu\nu}(P' - P)_\nu}{2M} \mathcal{U}(P) \right] \frac{n_\mu}{2P \cdot n} \]

with \( \xi = (P' - P) \cdot n/2 \) and \( t = (P' - P)^2 \)

Connection to normal quark distribution:

\[ H_q(x, 0, 0, \mu^2) = q(x, \mu^2) \]

Parton’s orbital motion:

\[ J_q = \frac{1}{2} \lim_{t \to 0} \int dx \, x \left[ H_q(x, \xi, t) + E_q(x, \xi, t) \right] \]

\[ = \frac{1}{2} \Delta q + L_q \]

Ji, PRL78, 1997

Nov 5, 2007   Jianwei Qiu, ISU
Summary

- QCD is very rich in dynamics, much more than QED, while QED is the underline theory of all excitements of CMP, ...

- After 35 years, we have learned only a very small part of QCD dynamics: less than 0.1 fm, although we have been successful

- EIC provides an unique opportunity to probe the partonic structure of hadron in many different kinematic regimes

- Let’s work and hope that we will have this QCD machine!

Thank you!