Polarimetry with $e^+e^-$ pair conversions

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http://llr.in2p3.fr/~dbernard/polar/harpo-t-p.html
Polarimetry

- Modulation of azimuthal angle distribution

\[
\frac{d\Gamma}{d\phi} \propto (1 + \mathcal{A}P \cos [2(\phi - \phi_0)]),
\]

\[
\sigma_P \approx \frac{1}{\mathcal{A}} \sqrt{\frac{2}{N}},
\]

- \( P \) source linear polarisation fraction
- \( \mathcal{A} \) Polarization asymmetry
- \( \phi \) azimuthal angle
The enemy: multiple scattering

- Data
- MC simulation

The Glycine effect
Conversion in a Slab and Multiple Scattering: Dilution of the Polarisation Asymmetry

- \((1 + AP \cos[2(\phi)]) \otimes e^{-\phi^2/2\sigma^2_\phi} = (1 + A e^{-2\sigma^2_\phi} P \cos[2(\phi)])\)

\[\Rightarrow A_{\text{eff}} = A e^{-2\sigma^2_\phi}\]

- azimuthal angle RMS \(\sigma_\phi = \theta_{0,e^+} \oplus \theta_{0,e^-} / \hat{\theta}_{+-}\),

- \(\theta_0 \approx \frac{13.6 \text{ MeV}/c}{\beta_p} \sqrt{x/X_0}\),

- most probable opening angle \(\hat{\theta}_{+-} = 1.6 \text{ MeV}/E\)

\[\Rightarrow \sigma_\phi \approx 24 \text{ rad} \sqrt{x/X_0}\]  

(e.g. \(A_{\text{eff}}/A = 1/2\) for 110 \(\mu\)m of Si, 4 \(\mu\)m of W)

- This dilution is energy-independent.

Yu. D. Kotov, Space Science Reviews 49 (1988) 185,

**Developed, Validated, Event Generator**

- Development of a full (5D) exact (down to threshold) polarized evt generator
- Variables: azimuthal ($\phi_+$, $\phi_-$) and polar ($\theta_+$, $\theta_-$) angles of $e^+$ and $e^-$, and $x_+ \equiv E_+/E$

- Uses:
  - HELAS amplitude computation
  - SPRING event generator
  - Validation against published 1D distributions (nuclear and triplet conversions)


Event Generators: beware!

- Simulation of polarized \( \gamma \) conversion to pairs by G4:livermorepola surprising (polarisation asymmetry and polarisation angle)

@ 100 MeV, \[
\frac{A_{(\text{HELAS or BH})}}{A_{\text{G4:livermorepola}}} = \frac{(19.1 \pm 0.4)\%}{(8.7 \pm 0.6)\%} \approx 2.2
\]

Polarimetry with slab detector: with Evt Generator

- Full slab thickness 400 µm for all events
  
  Yes I know, 400 µm is obsolete!

- Thin line is Kotov’s $E$-independent, $\hat{\theta}_{+-}$-based approximation.

\[ t = \frac{x}{X_0} \]

\[ D \]

- Dilution improves with full generator, contribution of large opening angle events.

- Typical $D \approx 0.35$ for 400 µm Si (full thickness), fairly $E$-independent

\[ 10 \text{ MeV} \]
\[ 100 \text{ MeV} \]

- Attention: multiple scattering in conversion wafer only here

D. Bernard, 2nd Astrogam Workshop, Paris, mars 2015
Crab with slab detector: full simulation

- **Uniform conversion probability per unit depth inside wafer**

- With multiple scattering: kinematic cuts applied on the reco’ed angles.

- \( N_{\text{evt}}, A, \sigma_P : \)

<table>
<thead>
<tr>
<th></th>
<th>no multiple scattering</th>
<th>with multiple scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>no kinematic cuts</td>
<td>66211, 0.21, 0.026</td>
<td>66211, 0.11, 0.050</td>
</tr>
<tr>
<td>with kinematic cuts</td>
<td>6480, 0.19, 0.092</td>
<td>7473, 0.11, 0.149</td>
</tr>
</tbody>
</table>

- Effective dilution on whole spectrum \( \approx 0.5 \)

- So, precision on the Crab, \( 10^6 \text{ s} \) pointing mode, low background approximation
  \( \sigma_P = 15\% \)

- MDP (\( @ 3 \sigma \)) = 45%.

D. Bernard, 2nd Astrogam Workshop, Paris, mars 2015
**Polarimetry: Defining the Azimuthal Angle?**

\[ P = 1 \]

\[ P = 0 \]

Azimuthal angle distributions of 1.2 MeV photon conversions to pairs

- \( \omega \)
- \( \varphi_r \) recoil angle, \( \varphi_r = \varphi_{\text{pair}} \pm \pi \)
- \( \phi = (\varphi_+ + \varphi_-)/2 \), bisector of \( e^+ \) and \( e^- \) direction

We checked that on a $P = 0$ MC sample, the measured value is found to be $A \times P \approx 0$

We checked that form factors do not affect the polarization asymmetry

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**Polarimetry: Defining the Azimuthal Angle?**

- $\omega$
- $\varphi_r$ recoil angle, $\varphi_r = \varphi_{\text{pair}} \pm \pi$
- $\phi = (\varphi_+ + \varphi_-)/2$, bisector of $e^+$ and $e^-$ direction

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Polarimetry: Optimal Measurement

- Optimal measurement; $\Omega$
  - Let’s define $p(\Omega)$ the pdf of set of (here 5) variables $\Omega$
  - Search for weight $w(\Omega)$, $E(w)$ function of $P$, and variance $\sigma_P^2$ minimal;
  - A solution is $w_{\text{opt}} = \frac{\partial \ln p(\Omega)}{\partial P}$
  - Polarimetry: $p(\Omega) \equiv f(\Omega) + P \times g(\Omega)$, $w_{\text{opt}} = \frac{g(\Omega)}{f(\Omega) + P \times g(\Omega)}$.
    - If $A \ll 1$, $w_0 \equiv 2\frac{g(\Omega)}{f(\Omega)}$, and
    - for the 1D “projection” $p(\Omega) = (1 + AP \cos [2(\phi)])$:
      $w_1 = 2 \cos 2\phi$, $E(w_1) = AP$, $\sigma_P = \frac{1}{A\sqrt{N}}\sqrt{2 - (AP)^2}$.

D. Bernard, NIM A 729 (2013) 765
Optimal Measurement: 1D: fraction + angle

- polarisation fraction

\[ A \times P = 2 \sqrt{\langle \cos 2\phi \rangle^2 + \langle \sin 2\phi \rangle^2}, \]

- polarisation angle

\[ \phi_0 = \frac{1}{2} \arctan \left( \frac{\langle \sin 2\phi \rangle}{\langle \cos 2\phi \rangle} \right), \]

Polarimetry: Track matching issue

- Many foreseen project use $2 \times 2$D projections, not true 3D imaging (gas TPC, silicon strip detectors)

- Ambiguity:

  \[(\text{track}_1, x, \text{track}_1, y)(\text{track}_2, x, \text{track}_2, y) \leftrightarrow (\text{track}_1, x, \text{track}_2, y)(\text{track}_1, x, \text{track}_2, y)\]

- Ruins the azimutal angle information

- Assignment must be performed before multiple scattering blurs the picture
- Observed angular distribution

- for fully polarised pairs ($P_{\text{eff}} = 1$),
- in a fixed direction wrt the detector ($\omega_{\text{pola}} - \omega_{\text{det}}$)
- for different values of the matching efficiency $\varepsilon$. 

**Track matching issue: results**

- (time averaged) $\omega_{\text{pola}} - \omega_{\text{det}}$ distributions

Azimuthal distribution for different matching efficiencies

Dilution as a function of matching efficiency.

- A factor of 2 is at stake!

Recommendations

- All downstream (3rd wafer and the rest) information has azimuthal information screwed up.
- Don’t rely on downstream information for track matching
  - Do it yourself from the two hits in the 2nd wafer; Charge matching?
- In case two hits overlap (say, in $x$ strips): don’t throw the event away!
  - $(2, 1)$ and $(1, 2)$ events carry as much (more) azimuthal information as $(2, 2)$ events.
  - Examine two-bumps clusters?
  - Examine too large clusters (Ref: “merged” $\pi^0$ EM showers in HEP)
- Whatever smart event reco you make, the $\phi$ distribution will be rocky. (see Valentina et al.)
  - Don’t be afraid, the azimuthal information is there.
  - It can be extracted in an optimal way using moments’ method
- $\phi$-dependent acceptance / efficiency correction and background subtraction can be performed in a simple way using moments too
  
- May the force be with you!
Back-up slides
Polarisation asymmetry: asymptotic expressions

- Low energy
  \[ A = \frac{\pi}{4}. \]

- High energy
  \[ A \approx \frac{4}{9} \frac{\ln 2E}{\ln 2E} - \frac{20}{28} - \frac{218}{27}. \]
