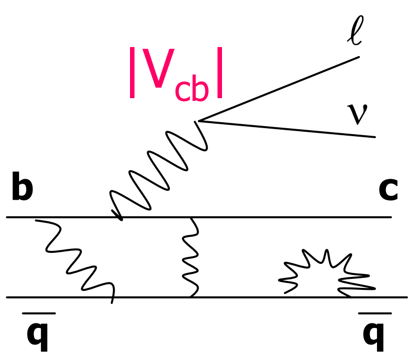
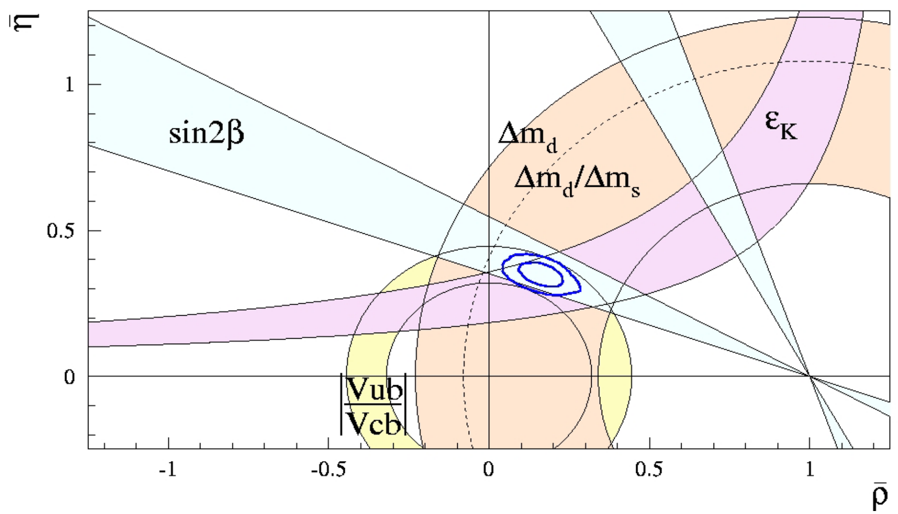
Inclusive and Exclusive V_{cb} measurements





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FPCP 2003 Flavor Physics and CP Violation Paris, June 3-6, 2003

V_{cb} from exclusive decays $\overline{B_d^0} \to D^{(*)} \ell^- \nu$

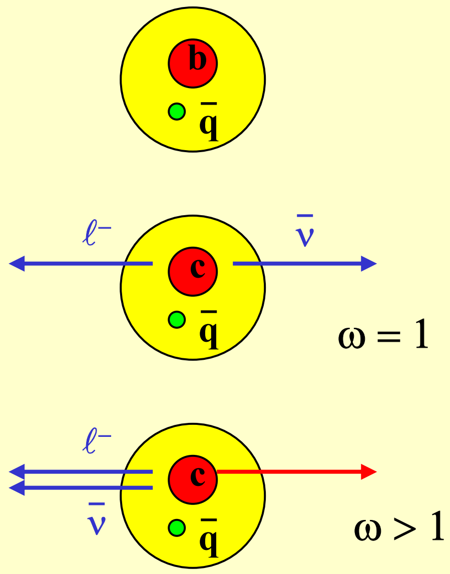
$$\frac{d\Gamma}{d\omega} = \frac{G_F^2}{48\pi^3} K(\omega) \mathcal{F}^2(\omega) |V_{cb}|^2$$

Kinematic factor known

Hadronic form factor

3 f.f. in $\overline{B_d^0} \to D^* \ell^- \nu$

1 f.f. in $\overline{B_d^0} \to D^+ \ell^- \nu$



$$\omega = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2 m_B m_D}$$

D* boost in B^0 rest frame

HQS:

in the heavy quark limit ($m_Q \to \infty$), at zero recoil (ω=1)

only one form factor $\mathcal{F}(1) \to \xi(1) = 1$

Strategy: Mesaure $d\Gamma/d\omega$ and extrapolate to ω=1 to extract

- Need form factor shape to extrapolate
- Need to calculate corrections to $\mathcal{F}(1)$ to extract $|V_{cb}|$

$\overline{B}_d^0 \to D^{*+}\ell^-\nu$ decays

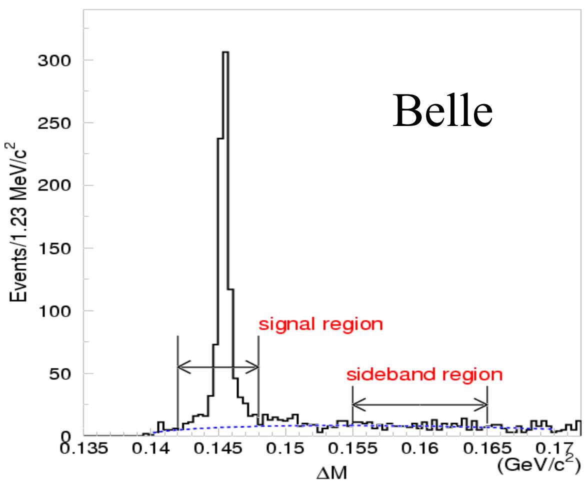
Υ (4S)

- ✓Good ω resolution
- ✓Kinematical constraints help to reduce background
- ✓Poor efficiency for low energy π_{soft}

$Z^0 \to b\overline{b}$

- ✓ Lower ω resolution
- ✓Background suppression more difficult
- ✓Fairly flat efficiency in ω (boosted π_{soft} p_π~1 GeV)

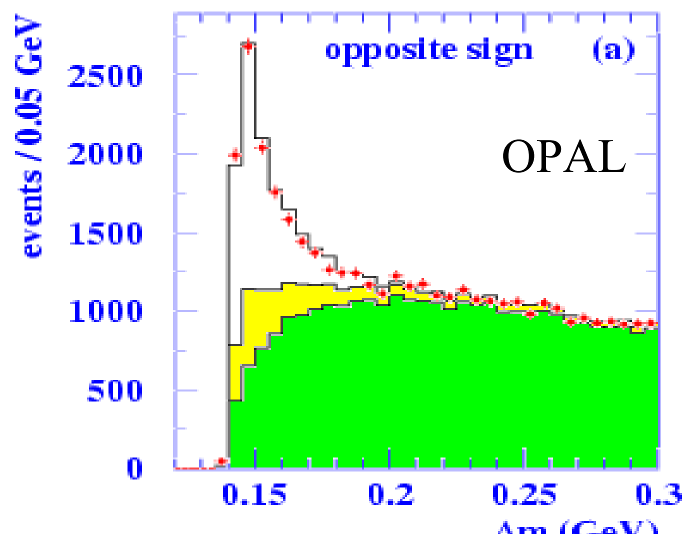
Exclusive D^* decays reconstruction
$D^* \to D\pi$, $D \to K\pi(\pi)$



Use:

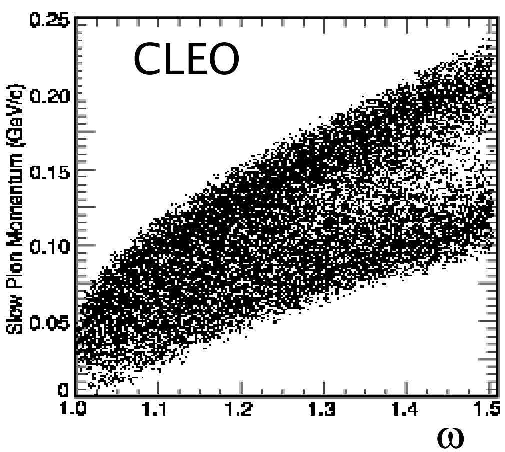
$\Delta m = m(D\pi) - m(D)$

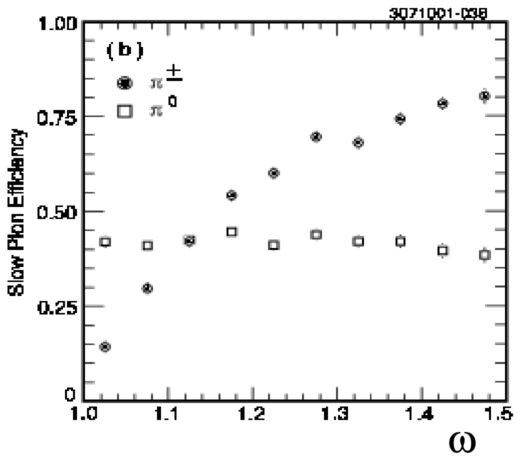
Exclusive and inclusive D^* decays reconstruction



ω reconstruction

Efficiency

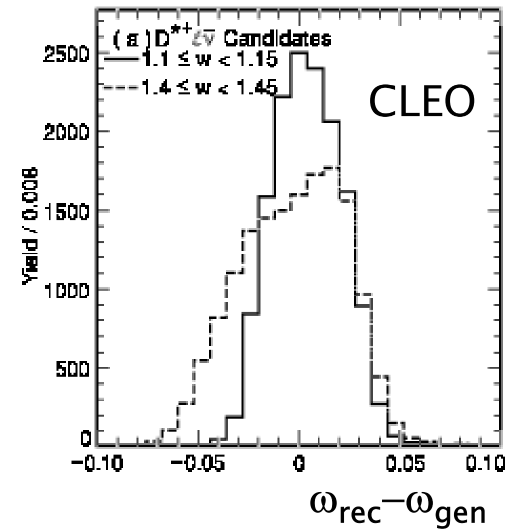




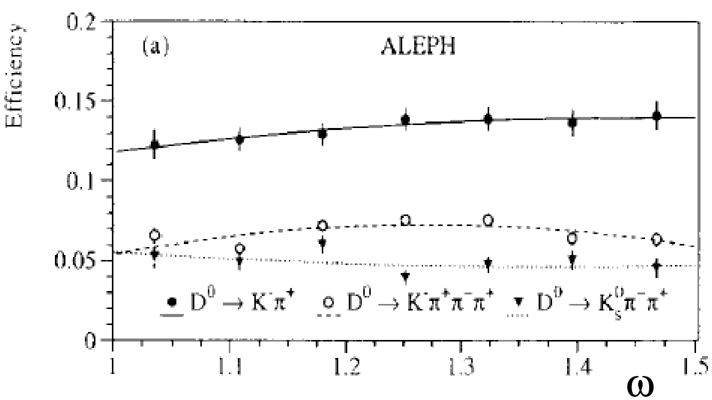
$\overline{B}_d^0 \rightarrow D^{*+}\ell^-\nu$

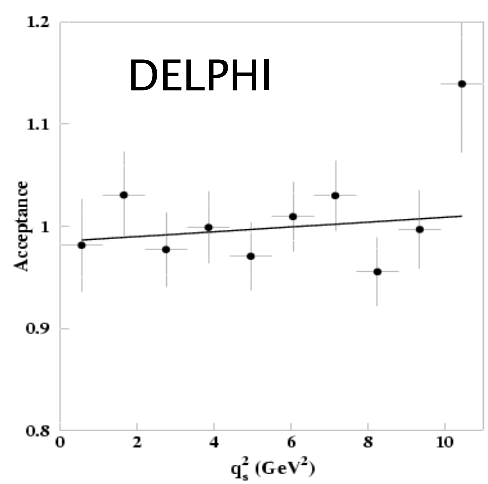
$B^- \rightarrow D^{*0}\ell^-\nu$

Resolution



Efficiency





q^2 is reconstructed as

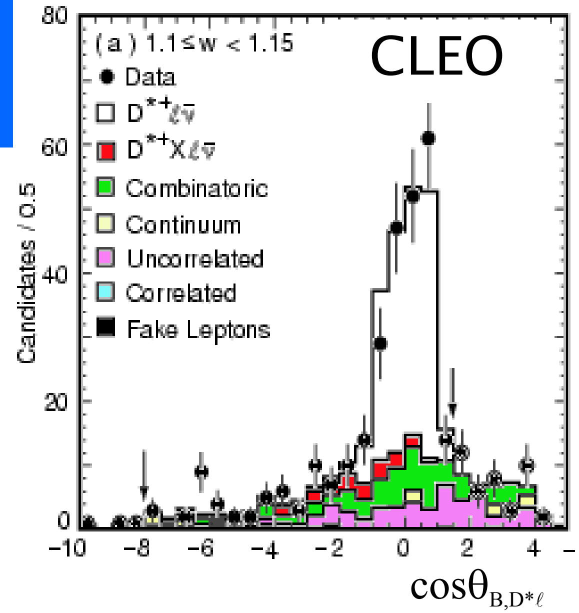
$q^2 = (p_{B^0} - p_{D^*})^2$

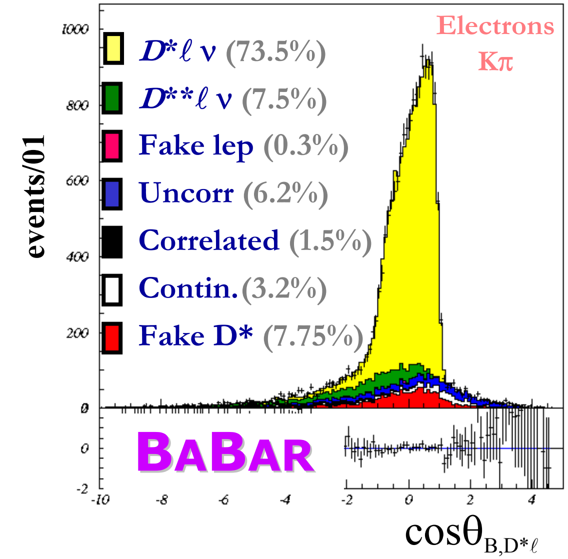
with ~0.3 GeV^2 resolution

Background from high mass D** states

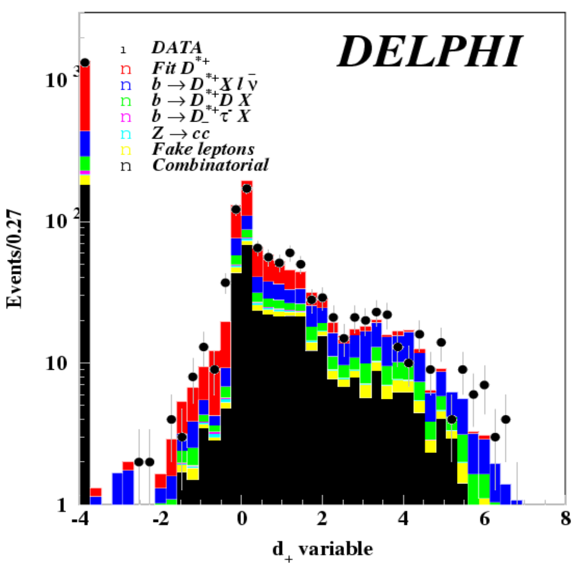
At Υ(4S) using:

$$\cos(\theta_{B-D^*\ell}) = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|p_B||p_{D^*\ell}|}$$





Delphi uses topological informations



and leave $\bar{B}_d^0 \to D^{*+} X \ell^- \nu$ free in the q^2 fit, obtaining:

$$BR(\bar{B}_d^0 \to D^{*+} X \ell^- \nu) = (0.64 \pm 0.08 \pm 0.09)\%$$

compatible with previous results

At LEP the composition of D** states is studied using *Leibovich, Ligeti, Stewart, Wise* model and the maximum variation of parameters compatible with available BR's measurements

Extrapolation: form factor shape

- Expansion around $\omega=1$ up to second order:

$$\mathcal{F}(\omega) = \mathcal{F}(1)[1 - \rho_F^{\,2}(\omega - 1) + c_F(\omega - 1)^2 + \ldots]$$

- Use dispersive relations to constraint the shape

Relate $\mathcal{F}(\omega)$ to the axial vector form factor $h_{A_1}(\omega)$ and ratios of HQET form factors $R_1(\omega)$, $R_2(\omega)$

Expand: $$h_{A_1}(\omega) \approx h_{A_1}(1)[1 - 8\rho_A^{\,2} z + (53\rho_A^{\,2} - 15)z^2 - (231\rho_A^{\,2} - 91)z^3]$$

with: $z = (\sqrt{\omega+1} - \sqrt{2})/(\sqrt{\omega+1} + \sqrt{2})$

Caprini, Lellouch, Neubert NP B530(98)153 and Boyd, Grinstein, Lebed PRD56(97)6895

Fit for $\mathcal{F}(1)|V_{cb}|$ and ρ_A^2

R_1, R_2 calculated using QCD sum rules

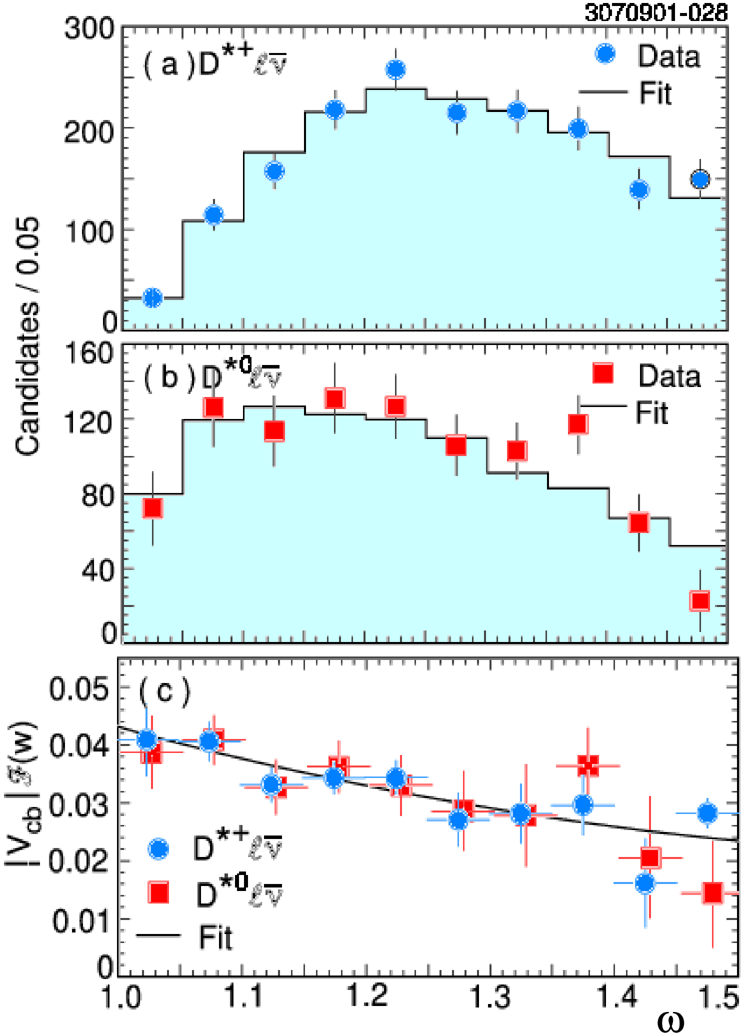
$R_1(\omega) \approx 1.27 - 0.12(\omega - 1) + 0.05(\omega - 1)^2$ $\qquad$ $R_2(\omega) \approx 0.80 + 0.11(\omega - 1) - 0.06(\omega - 1)^2$

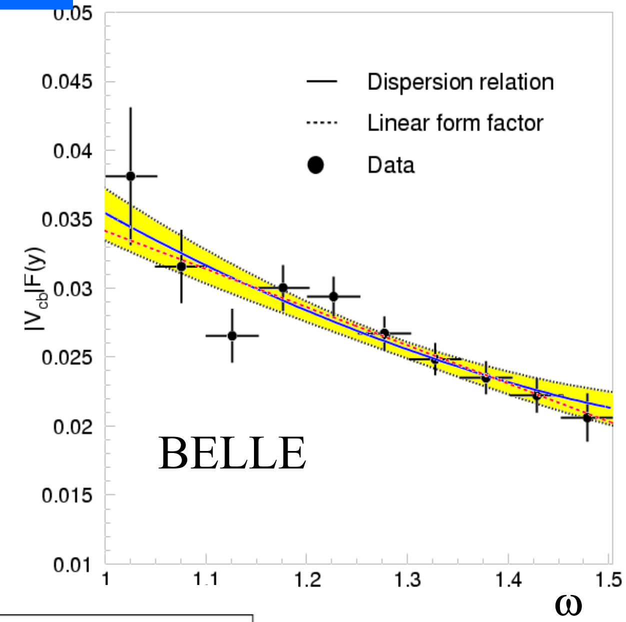
or measured by CLEO: $R_1(1) = 1.18 \pm 0.30 \pm 0.12$ $\qquad$ $R_2(1) = 0.71 \pm 0.22 \pm 0.07$

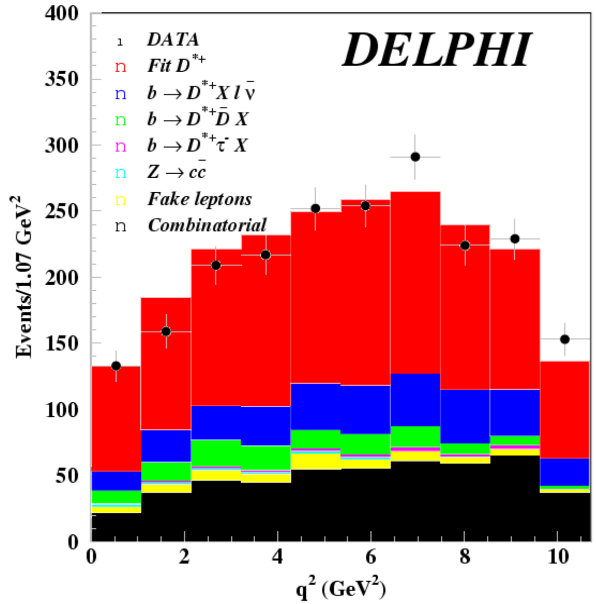
R_1, R_2 uncertainty is the major source of systematics on ρ_A^2, which should improve with future new measurements

$d\Gamma/d\omega$ fits

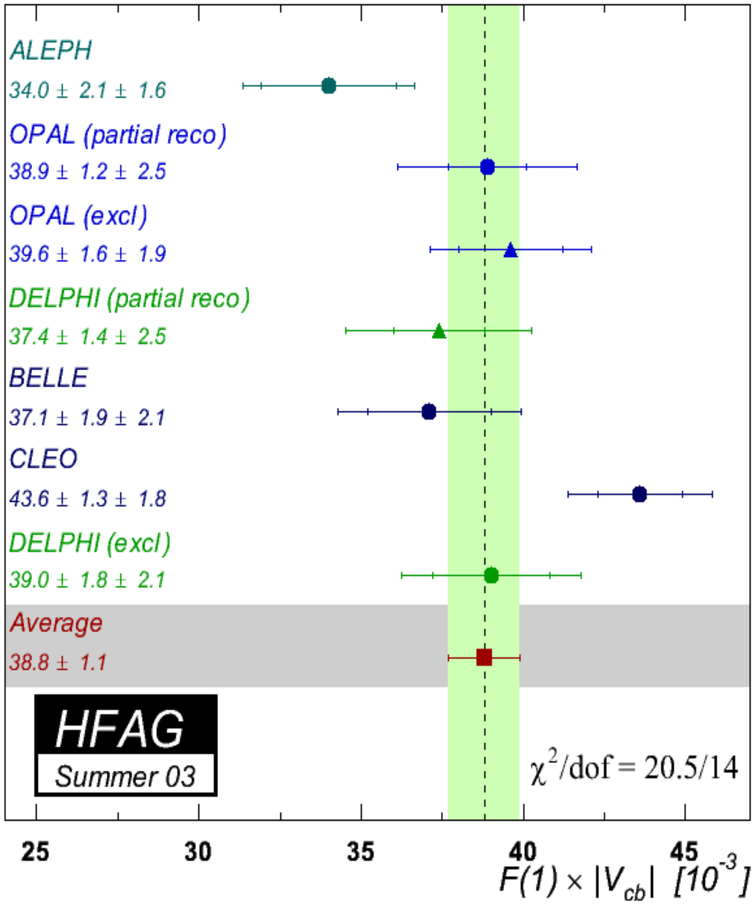
CLEO combined fit to
$\bar{B}_d^0 \to D^* \ell^- \nu$ and $B^- \to D^{*0} \ell^- \nu$

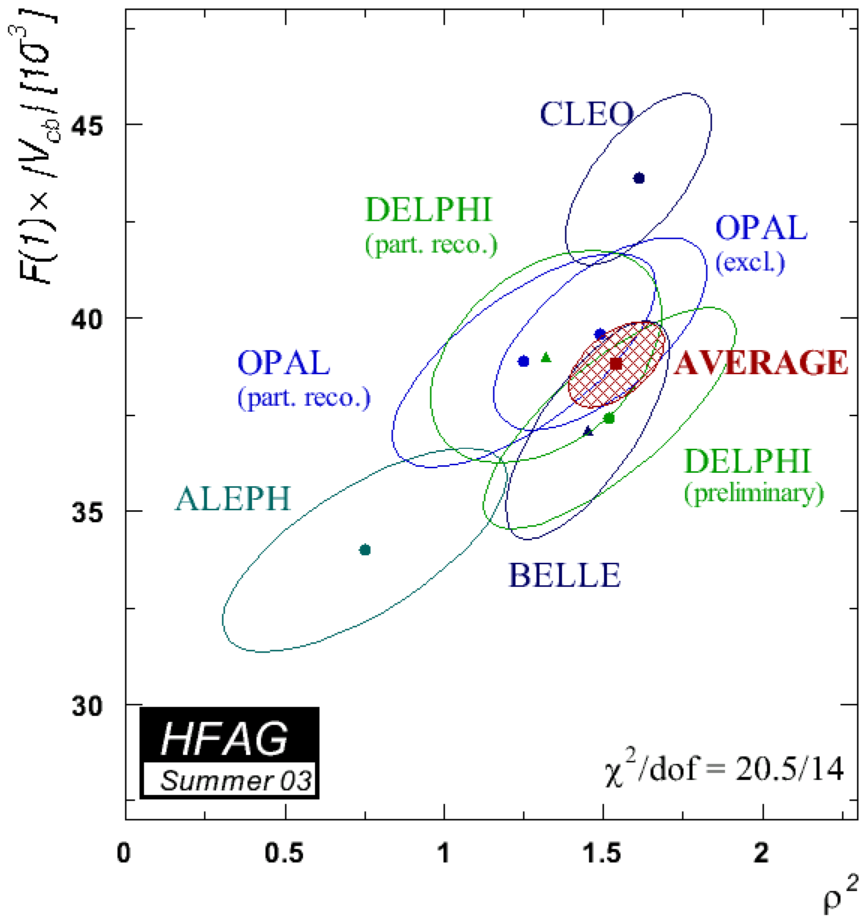






$\mathcal{F}(1)|V_{cb}|$ word average



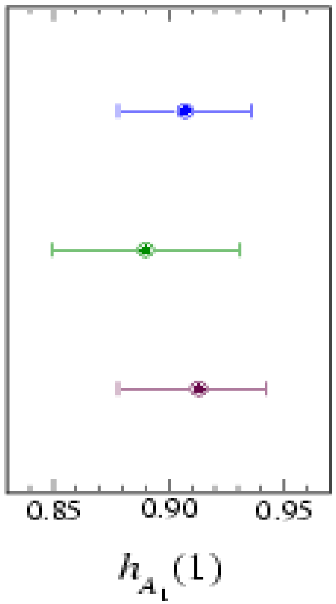


$\mathcal{F}(1)|V_{cb}|=(38.8\pm0.5_{stat}\pm0.9_{syst})\times10^{-3}$ $\rho_A^2=1.54\pm0.05_{stat}\pm0.13_{syst}$

corr($\mathcal{F}(1)|V_{cb}|$, ρ^2)=0.52

$\mathcal{F}(1)=?$

Need non-perturbative QCD calculations to correct $\mathcal{F}(1)=1 \quad (m_Q \to \infty)$

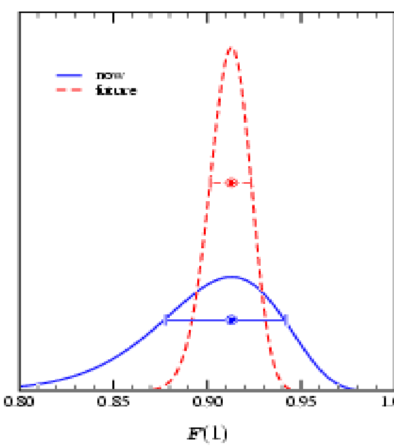


quark model $\mathcal{F}(1)=0.907 \pm 0.007 \pm 0.025 \pm 0.017$

sum rule $\mathcal{F}(1)=0.900 \pm 0.015 \pm 0.025 \pm 0.025$

Lattice QCD $\mathcal{F}(1)=0.913\,^{+0.024}_{-0.017}\,^{+0.017}_{-0.030}$

Future error reduction from unquenched calculations, more statistics, …



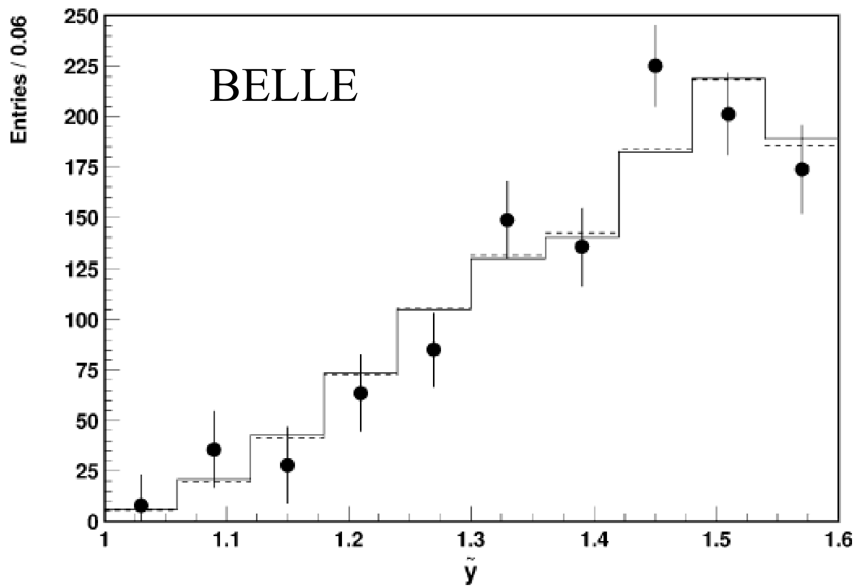
good agreement among different approaches

$$\mathcal{F}(1)=0.91 \pm 0.04$$

$$|V_{cb}|^{excl}=(42.6 \pm 1.2_{exp} \pm 1.9_{theo}) \times 10^{-3}$$

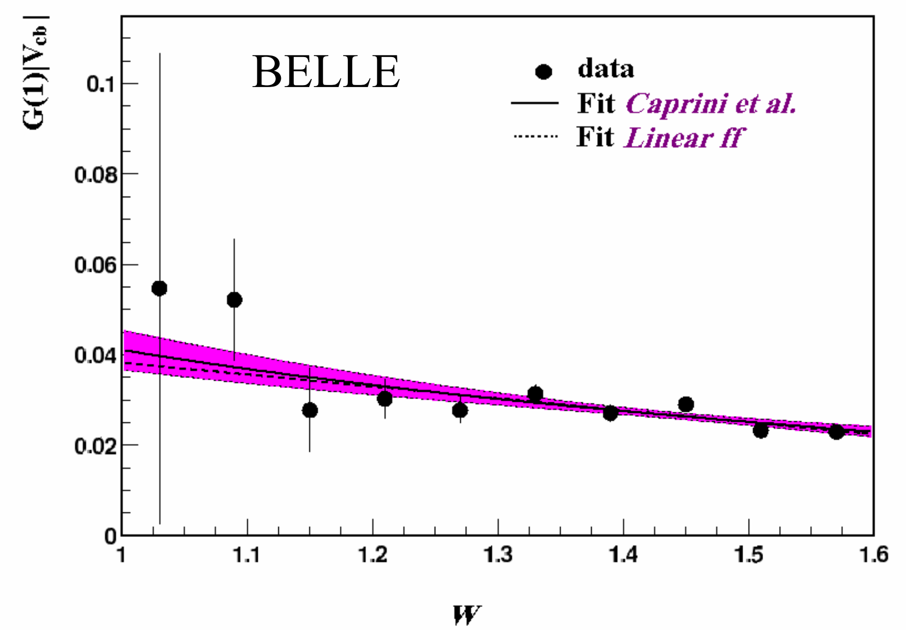
V_{cb} from $\overline{B_d^0} \to D^+ \ell^- \nu$ decays

D$^+$ vs D*

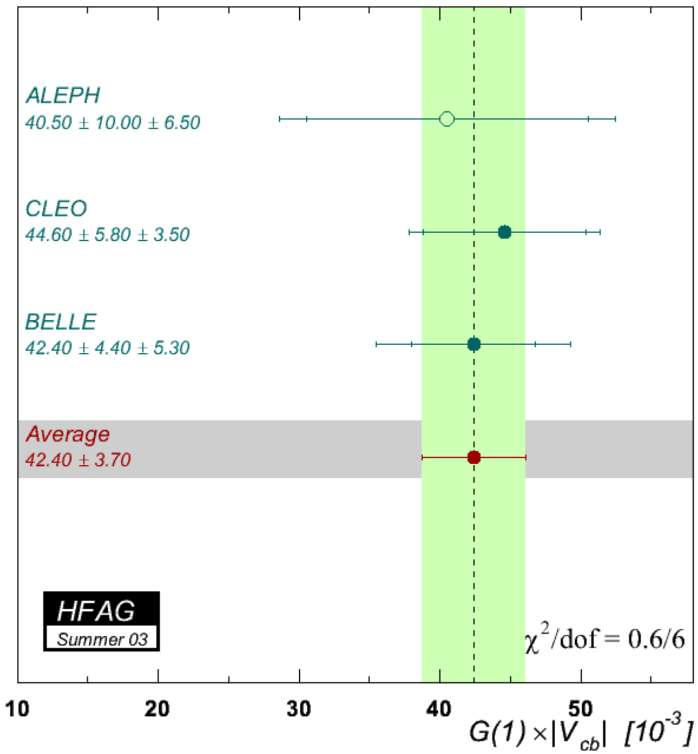


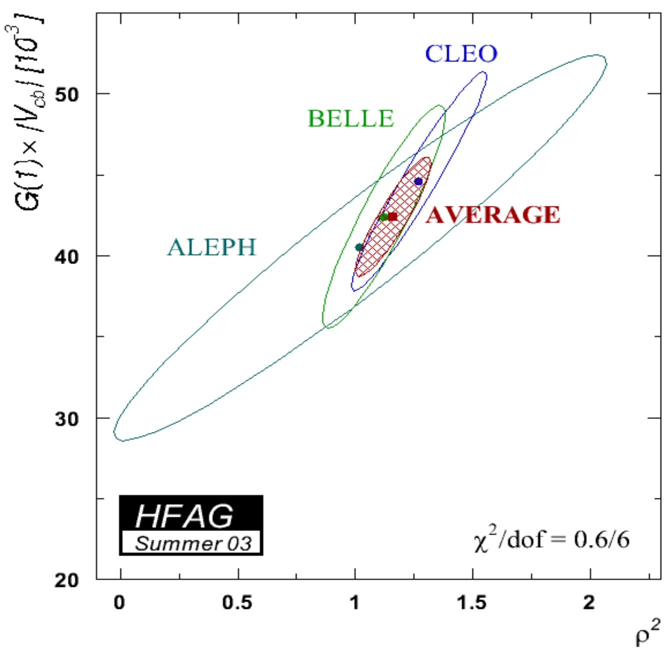
- BR($B_d^0 \to D \ell^- \nu$) ~ ¼ BR($B_d^0 \to D^* \ell^- \nu$)
- $d\Gamma_D/d\omega \sim (1-\omega)^{3/2}$; $d\Gamma_{D^*}/d\omega \sim (1-\omega)^{1/2}$
- Larger combinatoric backgound but no slow pion involved in the decay
- $1/m_Q$ corrections to $\mathcal{G}(1)$ are not zero

- However it provides a consistency check and allows test of theory



V_{cb} from $\overline{B_d^0} \to D^+ \ell^- \nu$





$$\mathcal{G}(1)|V_{cb}| = (42.4 \pm 3.7) \times 10^{-3}$$

$$\rho_{\mathcal{G}}^2 = 1.16 \pm 0.16$$

Comparing Belle D* and D+ results obtained with *Caprini et al.*

$$\rho^2_D - \rho^2_{D*} = -0.23 \pm 0.29 \pm 0.20$$

$$\mathcal{G}(1)/\mathcal{F}(1) = 1.16 \pm 0.14 \pm 0.12$$

Compatible with expectations

With $\mathcal{G}(1) = 1.04 \pm 0.6$
(CKM workshop average)

$$|V_{cb}| = (40.8 \pm 3.6 \pm 2.4) \times 10^{-3}$$

V_{cb} from inclusive decays

$$\Gamma_{sl}(b \to c\ell^-\overline{\nu}) = \gamma_{th}\left|V_{cb}\right|^2 = \frac{BR(b \to c\ell^-\overline{\nu})}{\tau_b}$$

Theory with perturbative and non-pert. corrections PDG2002 $\Delta|V_{cb}|$~5%

Y(4S),LEP,SLD,CDF measurements
Lot of data --> Experimental $\Delta|V_{cb}|$~1%

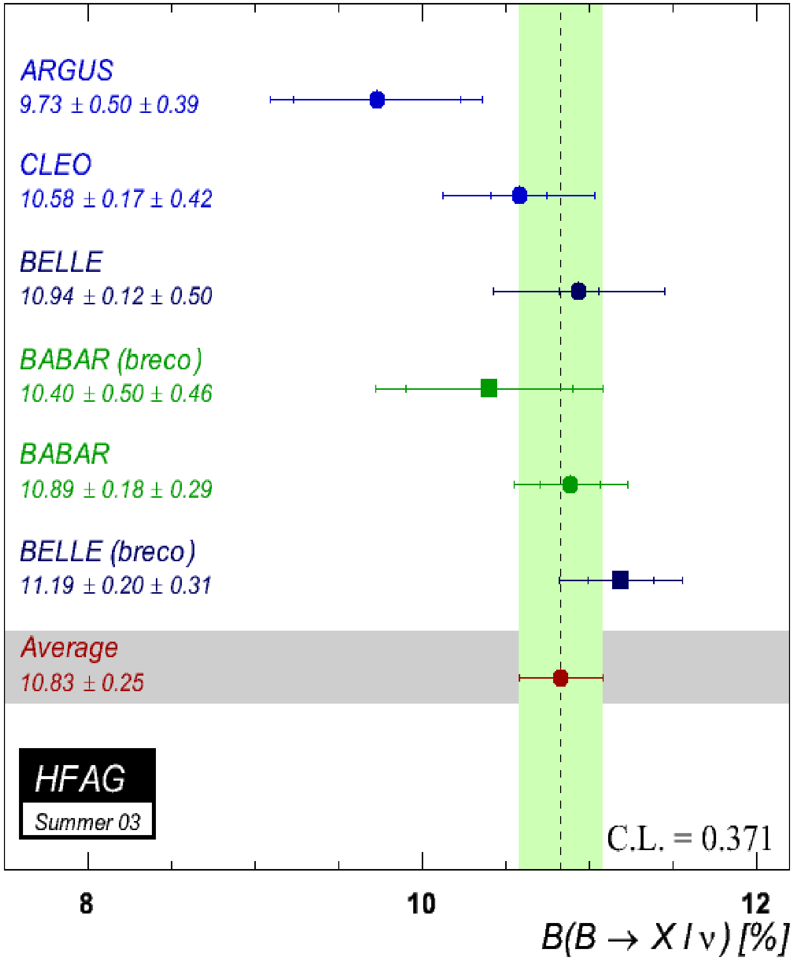
γ_{th} evaluated using OPE expansion in α_s and $1/m_b$ powers:

$O(1/m_b) \to$ 1 parameter: $\overline{\Lambda}$
$O(1/m_b^2) \to$ 2 parameters: λ_1 (μ_π^2), λ_2 (μ_G^2)
$O(1/m_b^3) \to$ 6 more parameters: $\rho_1, \rho_2, \mathcal{T}_{1-4}$ (ρ_D^3, ρ_{LS}^3)

Use other measurements of inclusive variables ⟹ spectral moments E_γ, M_X^2, E_ℓ to

- Test OPE predictions and underlying assumptions (quark-hadron duality)
- Constrain the unknown non-perturbative parameters and reduce $|V_{cb}|$ uncertainty

Inclusive $BR(B \to X_c \ell^- \nu)$ and τ_B



Υ(4S)

$BR(B \to X_c \ell^- \nu) = (10.83 \pm 0.25) \times 10^{-2}$

$\tau_B = (1.598 \pm 0.01)$ ps

$\Gamma_{B \to Xc\, \ell - \nu} = 0.446\ (1 \pm 0.023 \pm 0.007) \times 10^{-10}$ MeV

LEP

$BR(B \to X \ell^- \nu) = (10.59 \pm 0.22) \times 10^{-2}$

$BR(B \to X_u \ell^- \nu) = (\ 0.17 \pm 0.05) \times 10^{-2}$

$BR(B \to X_c \ell^- \nu) = (10.42 \pm 0.26) \times 10^{-2}$

$\tau_b = (1.573 \pm 0.01)$ ps

$\Gamma_{B \to Xc\, \ell - \nu} = 0.436\ (1 \pm 0.022 \pm 0.014) \times 10^{-10}$ MeV

Word average $\Gamma_{B \to X_c \ell^- \nu} = 0.441\ (1 \pm 0.018) \times 10^{-10}$ MeV

Parameter Extraction from CLEO data and V_{cb}

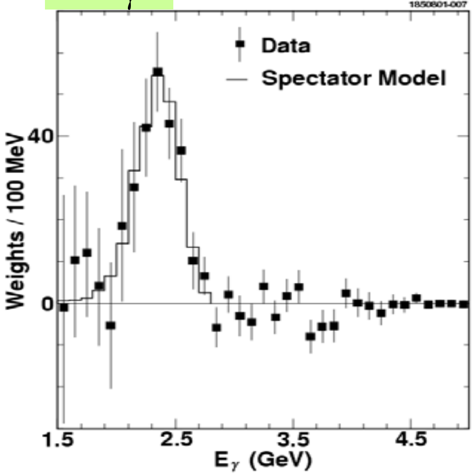
First derivations of OPE parameters from spectral moments used the pole mass expansion

$$\Gamma_{SL} = \frac{G_F^{\,2}\,|V_{cb}|^2\, m_B^{\,5}}{192\pi^3}\left(c_0 + \frac{1}{m_B} c_1(\overline{\Lambda}) + \frac{1}{m_B^{\,2}} c_2(\overline{\Lambda},\lambda_1,\lambda_2) + \frac{1}{m_B^{\,3}} c_3(\overline{\Lambda},\lambda_1,\lambda_2,\rho_1,\rho_2,\mathcal{T}_1,\mathcal{T}_2,\mathcal{T}_3,\mathcal{T}_4) + O(\frac{1}{m_B^{\,4}}) \right)$$

And photon energy spectrum in $B\to X_s\gamma$, hadronic mass spectrum and lepton energy spectrum in $B\to X_c\ell\,\nu$ measured by CLEO with a minimum energy cut (E_γ>2. GeV, p_ℓ>1.5 GeV/c)

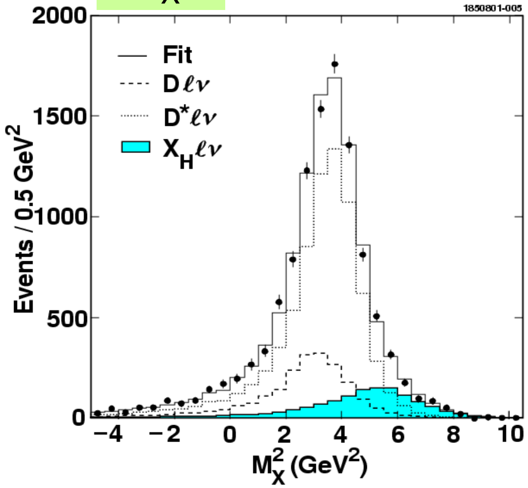
E_γ

PRL87 251807 '01



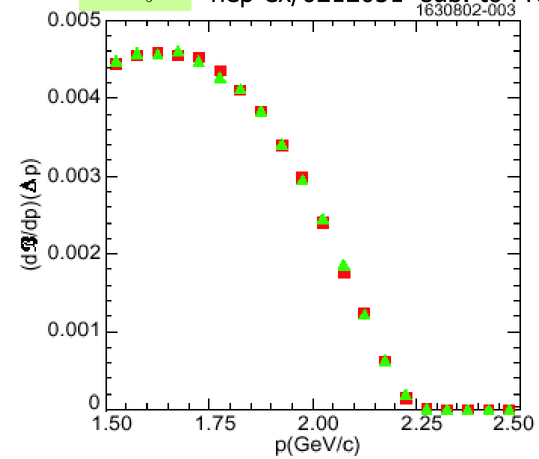
M_X^2

PRL87 251808 '01



E_ℓ

hep-ex/0212051 sub. to PRD



$$E_\gamma = \frac{m_B - \overline{\Lambda}}{2} + \ldots \qquad \frac{1}{m_B^{\,2}}\left\langle (M_X^{\,2} - \overline{m}_D^{\,2}) \right\rangle = \mathcal{M}_0 + \frac{1}{m_B}\mathcal{M}_1(\overline{\Lambda},\lambda_1,\lambda_2) + \ldots$$

Parameter Extraction from CLEO data and V_{cb}

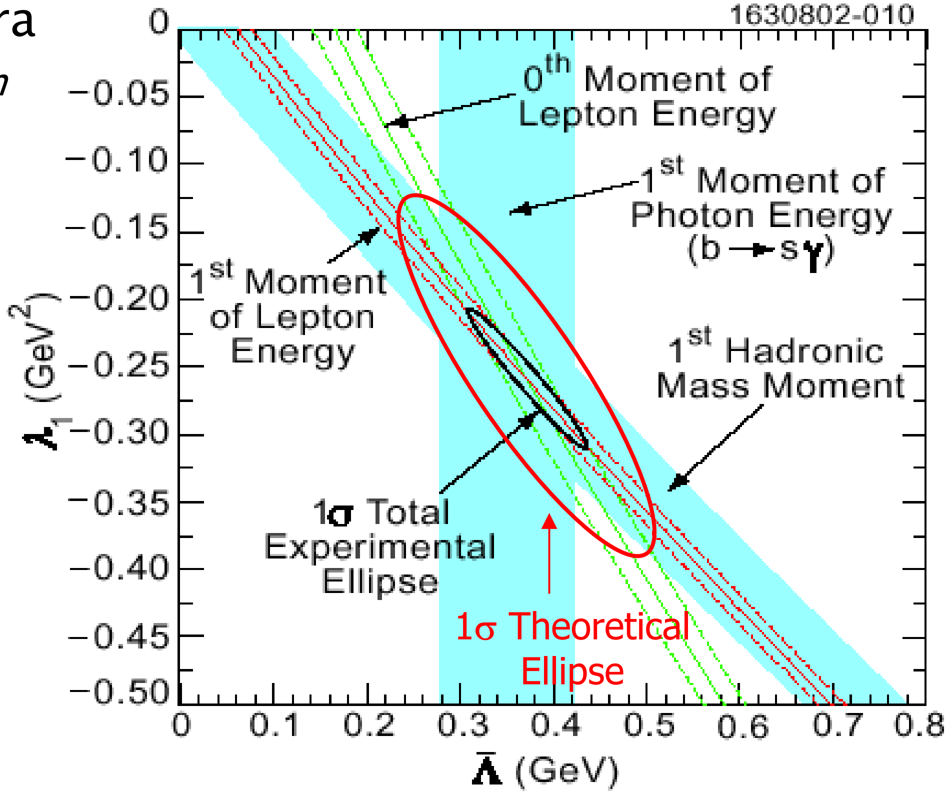
Using ratios of truncated lepton spectra

Gremm,Kapustin

$$R_0 = \frac{\int_{1.7GeV} \frac{d\Gamma_{sl}}{dE_l} dE_l}{\int_{1.5GeV} \frac{d\Gamma_{sl}}{dE_l} dE_l} \qquad R_1 = \frac{\int_{1.5GeV} E_l \frac{d\Gamma_{sl}}{dE_l} dE_l}{\int_{1.5GeV} \frac{d\Gamma_{sl}}{dE_l} dE_l}$$

$\bar{\Lambda} = 0.39 \pm 0.03_{stat} \pm 0.06_{sys} \pm 0.12_{th}$ GeV

$\lambda_1 = -0.25 \pm 0.02_{stat} \pm 0.05_{sys} \pm 0.14_{th}$ GeV^2



Parameters extracted from lepton spectra are in agreement with those extracted from M_X^2 and E_γ

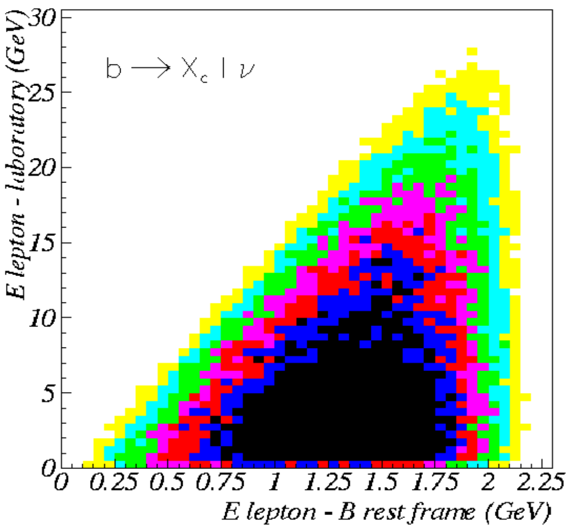
$$|V_{cb}| = 41.1 \times [1 \pm 0.009_{\Gamma_s} \pm 0.010_{\Lambda,\lambda_1} \pm 0.022_{theo}] \times 10^{-3}$$

Using $\Gamma_{sl} = (0.441 \pm 0.008) \times 10^{-10}$ MeV

Variation of input parameters: α_s, λ_2, ρ_1, ρ_2, $\mathcal{T}_{1\text{-}4}$ uncertainty due to higher order terms in the perturbative and non-perturbative terms not included

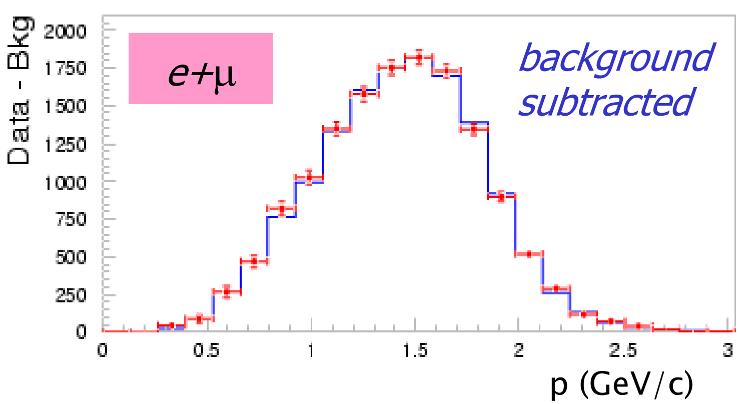
Moments of lepton energy spectrum and hadronic mass spectrum in $Z \to b\bar{b}$ events

First measurements of moments performed at LEP exploiting advantage of Z^0 kinematics:



- Large momentum of b-hadrons ($E_B \sim 30$ GeV) gives sensitivity to full lepton energy spectrum in B rest frame
- However: less statistics
  need to reconstruct the B system

- Measure first, second and third moment



Lepton spectrum in $B \to X_c \ell^- \nu$

DELPHI'02-071, ICHEP02

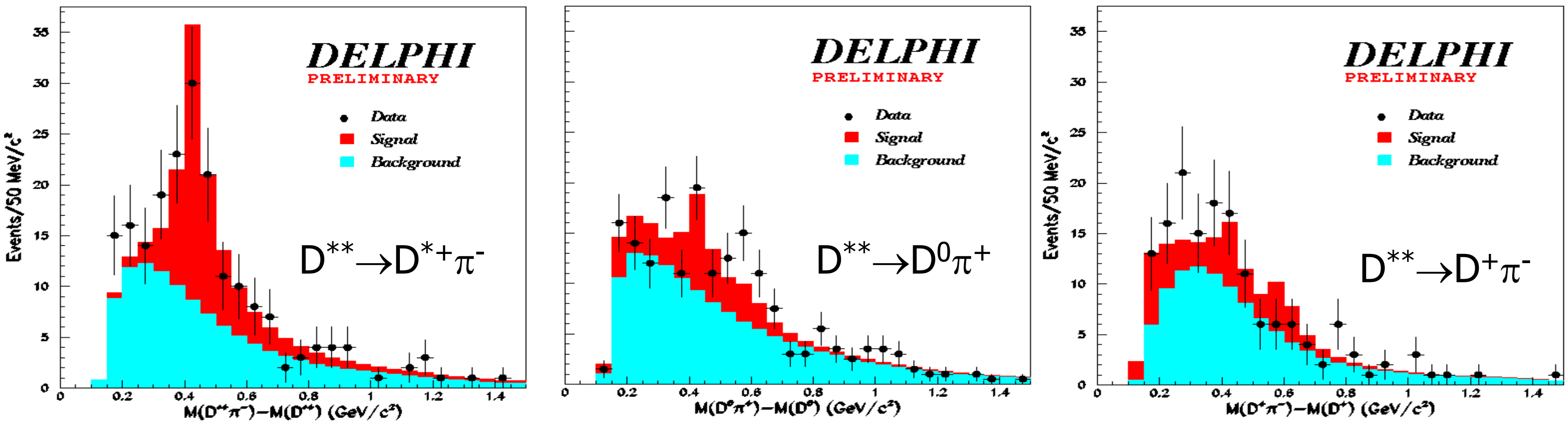
B system reconstructed from lepton+neutrino+charm vertex

lepton boosted in B rest frame

Hadronic mass spectrum

DELPHI'02-070, ICHEP02

$B_d^0 \rightarrow D^{**} \ell^- \nu$ decays exclusively reconstructed



$\Delta_M = M(D^{(*)}\pi) - M(D^{(*)})$ distributions fitted including contributions of resonant (narrow, broad) and non resonant states, $D^*X \ell^- \nu$ floating

$$BR(B^0 \rightarrow D^{**} \ell^- \nu) = (2.6 \pm 0.5 \pm 0.4)\ \%$$

From measured D** mass derive $\langle M^n_X \rangle = p_D M^n_D + p_{D^*} M^n_{D^*} + (1 - p_D - p_{D^*}) \langle M^n_{D^{**}} \rangle$

First three moments:

$$\langle M_X^2 - \overline{m_D}^2 \rangle = 0.534 \pm 0.041 \pm 0.074 \text{ GeV}$$

$$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle = 1.23 \pm 0.16 \pm 0.15 \text{ GeV}^2$$

$$\langle (M_X^2 - \langle M_X^2 \rangle)^3 \rangle = 2.97 \pm 0.67 \pm 0.48 \text{ GeV}^3$$

Non-perturbative parameter extraction with low scale running quark masses approach

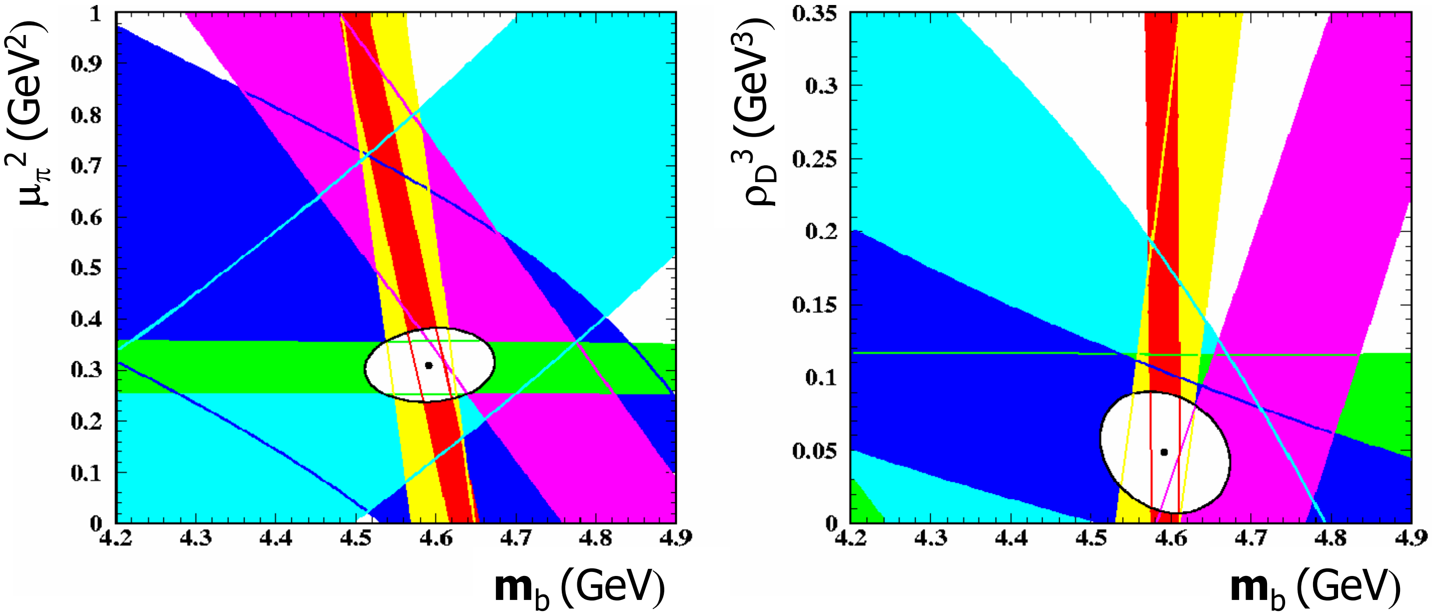
Another OPE formalism makes use of low scale running quark masses and does not rely on a $1/m_c$ expansion *(Bigi,Shifman,Uraltsev and Vainshtein)*

$$M_n(E_l) = \left(\frac{m_b}{2}\right)^n \left(\varphi_n(r) + a_n(r)\frac{\alpha_s}{\pi} + b_n(r)\frac{\mu_\pi^2}{m_b^2} + c_n(r)\frac{\mu_G^2}{m_b^2} + d_n(r)\frac{\rho_D^3}{m_b^3} + s_n(r)\frac{\rho_{LS}^3}{m_b^3} + \ldots \right) \qquad r = \frac{m_c^2(\mu)}{m_b^2(\mu)}$$

$m_b(\mu)$, $m_c(\mu)$ are independent parameters and two operators only contribute to $1/m_b^3$ corrections : ρ_D^3, ρ_{LS}^3

First applied to fit preliminary Delphi data   Phy.Lett B556(2003)41

- Multi-parameter χ^2 fit to first three moments of lepton energy spectra and hadronic mass spectra
  Higher moments used to get sensitivity to $1/m_b^3$ parameters
- Use expressions for non-truncated lepton spectra
- Simultaneous use of leptonic and hadronic moments in order to leave enough free parameters in the fit



Input constraints:

$\mu_G^2 = 0.35 \pm 0.05$ GeV^2
$\rho_{LS}^3 = -0.15 \pm 0.15$ GeV^3
$m_c = 1.05 \pm 0.30$ GeV
$m_b = 4.57 \pm 0.10$ GeV

equivalent to that derived from E_γ in $B \to X_s\gamma$

$m_{b,kin}$ (1GeV) $= 4.59 \pm 0.08_{fit} \pm 0.01_{sys}$ GeV
$m_{c,kin}$ (1GeV) $= 1.13 \pm 0.13_{fit} \pm 0.03_{sys}$ GeV
μ_π^2 (1GeV) $= 0.31 \pm 0.07_{fit} \pm 0.02_{sys}$ GeV^2
ρ_D^3 (1GeV) $= 0.05 \pm 0.04_{fit} \pm 0.01_{sys}$ GeV^3

$m_b(m_b) = 4.233$ GeV
$m_c(m_c) = 1.245$ GeV

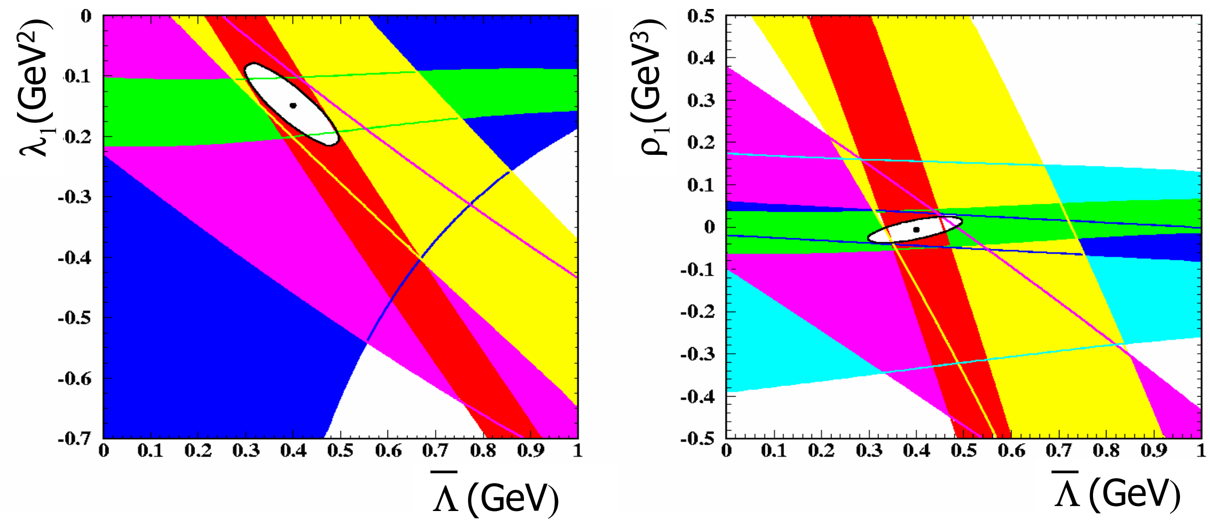
Good consistency of all measurements (χ^2/d.o.f.=0.96)

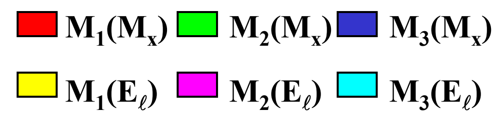
Within present accuracy no need to introduce higher order terms to establish agreement with data

❖The mass expansion: $$M_{H_Q} = m_Q + \overline{\Lambda} + \frac{\mu_\pi^{\;2} - \mu_G^{\;2}}{2m_Q} + \frac{\rho_D^{\;3} + \rho_{LS}^{\;3} - \rho_{nl}^{\;3}}{4m_Q^{\;2}} + O(\frac{1}{m_Q^{\;3}})$$

is not used in the fit. Provide a posteriori test: $\overline{\Lambda}(B)-\overline{\Lambda}(D)=-0.086\pm0.092$

❖ Repeating the multi-parameter fit in the pole mass expansion:





$\overline{\Lambda} = \; 0.40 \pm 0.10_{fit} \pm 0.02_{sys}$ GeV
$\lambda_1 = -\,0.15 \pm 0.07_{fit} \pm 0.03_{sys}$ GeV^2
$\rho_1 = -0.01 \pm 0.03_{fit} \pm 0.03_{sys}$ GeV^3
$\rho_2 = \; 0.03 \pm 0.03_{fit} \pm 0.01_{sys}$ GeV^3

❖ Good consistency of all measurements. Results compatible with CLEO.

❖ Similar results with m_b^{1S}-λ_1 formalism applied to CELO and DELPHI data
C.W.Bauer, Z.Ligeti, M.Luke, A.V.Manohar hep-ph/0210027.

Derivation of inclusive V_{cb}

- V_{cb} dependence on non-perturbative parameters in the running quark mass scheme: *N.Uraltsev* hep-ph/0302262

$$
\begin{aligned}
|V_{cb}| = |V_{cb}|_0 \{1 &- 0.65[\, m_b(1) - 4.6\ \mathrm{GeV}] + 0.40\, [m_c(1) - 1.15\ \mathrm{GeV}] \\
&+ 0.01[\, \mu_\pi^2 - 0.4\ \mathrm{GeV}^2] + 0.10\, [\, \rho_D^3 \ -0.12\ \mathrm{GeV}^3] \\
&+ 0.05[\, \mu_G^2 - 0.35\mathrm{GeV}^2] - 0.01\, [\, \rho_{LS}^3 + 0.15\ \mathrm{GeV}^3] \}
\end{aligned}
$$

- First moments $M_1(M_X)$, $M_1(E_\ell)$ are both sensitive to the same combination of masses ~ $(m_b - 0.65\, m_c)$ present in V_{cb}

- Using the word average Γ_{sl} :

$$|V_{cb}| = 41.9 \times [1 \pm 0.009_{\Gamma_{sl}} \pm 0.015_{fit} \pm 0.010_{pert} \pm 0.010_{theo}] \times 10^{-3}$$

- 0.015_{fit}: $m_b, m_c, \mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$ uncertainties
- 0.010_{pert}: α_s scale
- 0.010_{theo}: $O(1/m_b^4)$ contributions

Conclusions: Exclusive / Inclusive

$|V_{cb}|$ from exclusive B decays

- Large statistics on $B_d^0 \rightarrow D^{(*)}\ell^-\nu$ available and new measurements are coming
- Present precision (5%) is systematics limited:

  Experiments: D** states, D's BR

  Theory: form factor extrapolation, corrections to $\mathcal{F}(1)=1$ can be reduced in the future

$|V_{cb}|$ from inclusive B decays

- Experiment: large statistics on BR($B \rightarrow X_c\ell^-\nu$) and τ_B and small systematics
- Major limit from possible quark-hadron duality violation?

  From measurements on spectral moments no evidence of violation effects, at the level of present sensitivity. Derived constraints on non-perturbative parameters reduce the uncertainty on $|V_{cb}|$ to ~2.2%

- Future results on moments from B-factories can farther improve the picture.

Conclusions: results

Good precision measurements on V_{cb} are available, both from exclusive and inclusive semileptonic B decays.

They are a fundamental input to CKM matrix determinations:

$$|V_{cb}|^{excl} = (42.6 \pm 1.2_{exp} \pm 1.9_{theo}) \times 10^{-3}$$

$$|V_{cb}|^{incl} = (41.9 \pm 0.7_{exp} \pm 0.6_{theo}) \times 10^{-3}$$

Systematics

BELLE

CLEO

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Source	Combined fit $\lvert V_{cb}\rvert \mathcal{F}(1)$	ρ^2	Γ
Backgrounds	1.8	3.1	1.7
Reconstruction efficiency	2.9	3.2	4.6
B momentum & mass	0.1	0.1	0.2
$\bar{B}\to D^* X \ell \bar{\nu}$ model	0.3	1.6	0.9
Final-state radiation	0.7	0.3	1.1
Number of $B\bar{B}$ events	0.9	0.0	1.8
τ_B and branching fractions	1.8	0.0	3.5
$R_1(1)$ and $R_2(1)$	1.4	12.0	1.8
Total	4.3	13.0	6.6

Error sources	$\lvert V_{cb}\rvert F(1)$ (%)	$\rho^2_{A_1}$ (%)
$N_{B\bar{B}}$	0.5	–
$\tau_{\bar{B}^0}$	1.0	–
$\mathcal{B}(D^{*+} \to D^0\pi^+)$	0.4	–
$\mathcal{B}(D^0 \to K^-\pi^+)$	1.2	–
Tracking efficiency	2.5	–
Electron ID efficiency	1.0	–
D^0 reconstruction efficiency	1.4	–
Slow pion efficiency	2.6	1.2
Subtotal	4.3	1.2
y resolution	1.1	2.8
Combinatorial BG	1.0	0.1
Correlated BG (D^{**})	1.4	4.0
Uncorrelated BG	0.4	0.5
Fake electron BG	–	0.2
Continuum BG	0.5	2.9
MC statistics	1.7	4.4
$R_1(1)$, $R_2(1)$	1.1	11.8
Subtotal	2.9	13.7
Total	**5.2**	**13.8**

Systematics

LEP

	$\frac{\Delta\mathcal{F}(1)\lvert V_{cb}\rvert}{\mathcal{F}(1)\lvert V_{cb}\rvert}$				$\Delta\rho^2_{A_1}$			
Source	A_{exc}	D_{inc}	O_{inc}	O_{exc}	A_{exc}	D_{inc}	O_{inc}	O_{exc}
Correlated errors								
$\Gamma_{b\bar{b}}/\Gamma_{had}$	0.2	0.3	0.2	0.2	-	-	-	-
$BR(b \to \overline{B^0_d})$	1.1	1.8	1.3	1.3	-	0.02	-	-
$BR(D^{*+} \to D^0\pi^+)$	0.4	0.4	0.4	0.4	-	0.01	-	-
$BR(D^0 \to K^+\pi^-)$	0.6	-	-	0.3	-	-	-	0.01
$BR(D^0 \to K3\pi)$	1.3	-	-	-	-	-	-	-
$BR(D^0 \to K^0 2\pi)$	0.6	-	-	-	-	-	-	-
$BR(D^0 \to K^+\pi^-\pi^0)$	-	-	-	2.6	-	-	-	0.02
$BR(D \to Kn\pi)$	-	0.2	-	-	-	-	-	-
D^{**} rate	0.9	1.5	0.4	0.7	0.02	0.07	0.03	0.05
D^{**} shape	1.0	5.3	4.1	1.0	0.06	0.19	0.15	0.13
$B^- \to D^* X_c$	0.3	0.2	0.3	0.2	-	0.01	0.01	-
$B^- \to D^*\tau\bar{\nu}$	0.1	0.2	0.1	0.1	-	-	-	-
Fragmentation	0.9	1.0	1.0	0.5	0.01	-	0.11	-
τ_b lifetime	0.9	0.9	0.7	0.6	-	-	-	-
R_1 and R_2	2.4	1.1	1.0	1.0	0.4	0.3	0.2	0.2
Uncorrelated errors								
Combinatorial and fake D^0	1.1	0.5	-	1.2	-	0.07	-	0.01
Fake lepton	0.7	-	1.2	0.2	-	-	-	-
ℓ efficiency/modelling	0.7	1.1	-	1.2	-	0.03	-	-
Selection efficiency/modelling	1.2	2.6	2.9	3.6	0.02	0.08	0.12	0.01
MC statistics	1.6	0.2	-	-	0.05	-	-	-
w resolution	1.5	2.1	2.2	1.4	0.07	0.07	0.12	0.035
Total Systematic	4.0	7.2	6.0	4.8	0.41	0.36	0.35	0.24
Statistical	6.5	4.0	3.5	4.5	0.25	0.14	0.15	0.21