# Inclusive and Exclusive $|V_{ub}|$ Measurements

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# Methods for Determining $|V_{ub}|$

**Inclusive:**

Lepton Endpoint

$M_X < M_D$, ($q^2 > 7$ GeV$^2$)

Inclusive measurements are over 3D space,
e.g., $E_{lep}$, $q^2$, $M_X$.

Sensitivity can be:

uniform, over sharply defined subspace (good)

varying, over ill-defined subspace (bad)

**Exclusive:**

B □ □l□

B □ □l□

- Systematic errors dominate - ALWAYS
- Typically they involve theoretical uncertainties.

If objective assessment of dominant systematic errors cannot be made don't use that measurement.

# Lepton Endpoint Analysis

- For B mesons at rest, the maximum lepton momentum in $B \to X_u l \nu$ is

  $(M_B^2 - M_\pi^2)/2M_B = 2.64$ GeV/c.

- For $B \to X_c l \nu$ it is

  $(M_B^2 - M_D^2)/2M_B = 2.31$ GeV/c.

- For B's in the $\Upsilon$(4s) rest frame, these cutoffs are smeared ± 150 MeV/c.
- Thus, the yield of leptons above ~ 2.2 GeV/c will be substantially enhanced in $b \to u l \nu$ leptons

# Lepton Endpoint Analysis - cont'd

a) Pick a lepton momentum cut. (e.g. 2.2 GeV, 2.4 GeV)

b) Measure yield of leptons above the cut, On   (4s), Below   (4s).

c) Subtract Below from On.

d) Determine contribution from b □ cl□, subtract that also.

e) Extrapolate from b □ ul□ yield above cut to total b □ ul□ yield.

f) Obtain $|V_{ub}|$ from total b □ ul□ yield.

Problem: Continuum background is huge, requires continuum suppression cuts. These can (in earlier measurements did) introduce $q^2$ dependence to efficiency

Problem: Extrapolation from yield above cut to total yield - used to be done with models.

# CLEO's Latest Endpoint Analysis

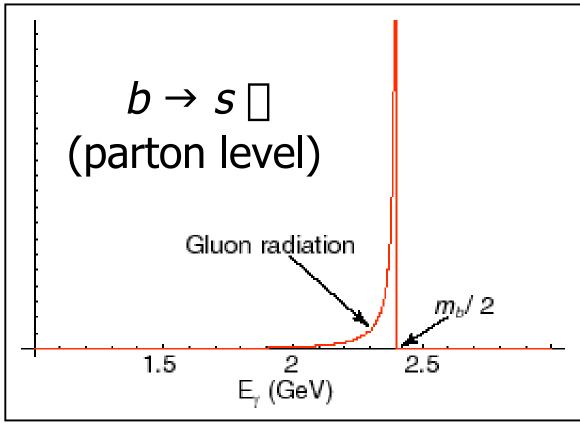
PRL 88, 231803, 2002

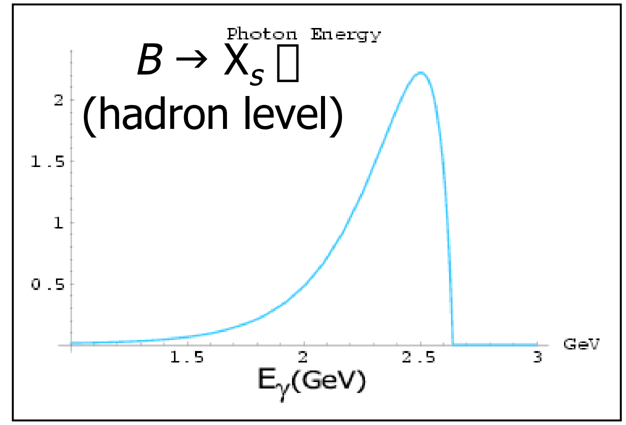
9.1 $fb^{-1}$ On $\Upsilon(4s)$, 4.35 $fb^{-1}$ Below $\Upsilon(4s)$

- Continuum suppression - energy distribution relative to lepton direction, excluding 25$^{o}$ cone away from lepton. Reduced dependence of efficiency on $q^2$, compared to previous analyses (factor of 3 less model dependence, now 30% fall-off in efficiency from high $q^2$ to low $q^2$)
- Extrapolation from yield above cut to total yield - Done using CLEO's photon energy spectrum for $b \to s\gamma$, [$b \to ul\nu$ and $b \to s\gamma$, both $b \to$ *light*, are governed by same light cone shape function]
  - Kagan, Neubert, deFazio, Leibovich, Low, Rothstein

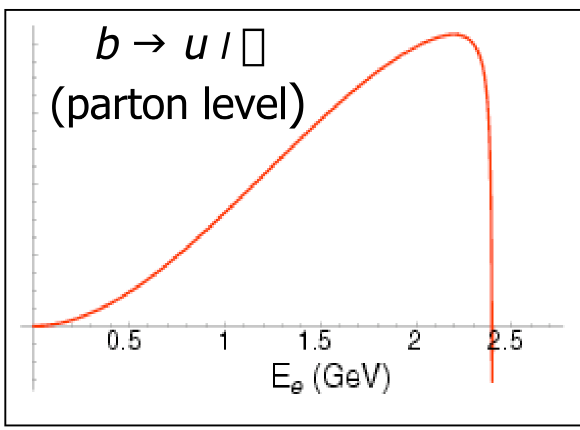
B → light quark shape function, SAME (to lowest order in $\Lambda_{QCD}/m_b$)

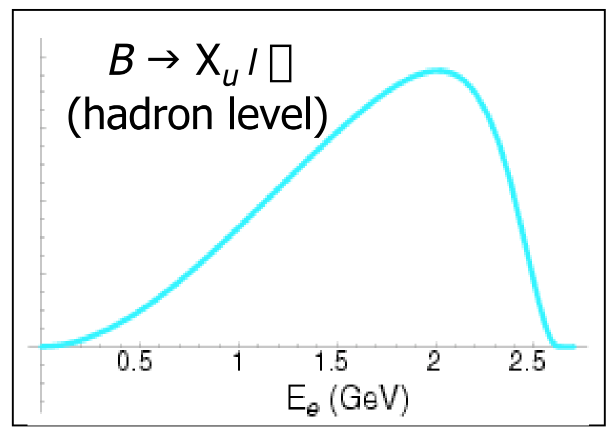
For $b \to s\gamma$ ⇨ $B \to X_s\gamma$ and $b \to u\, l\nu$ ⇨ $B \to X_u\, l\nu$.



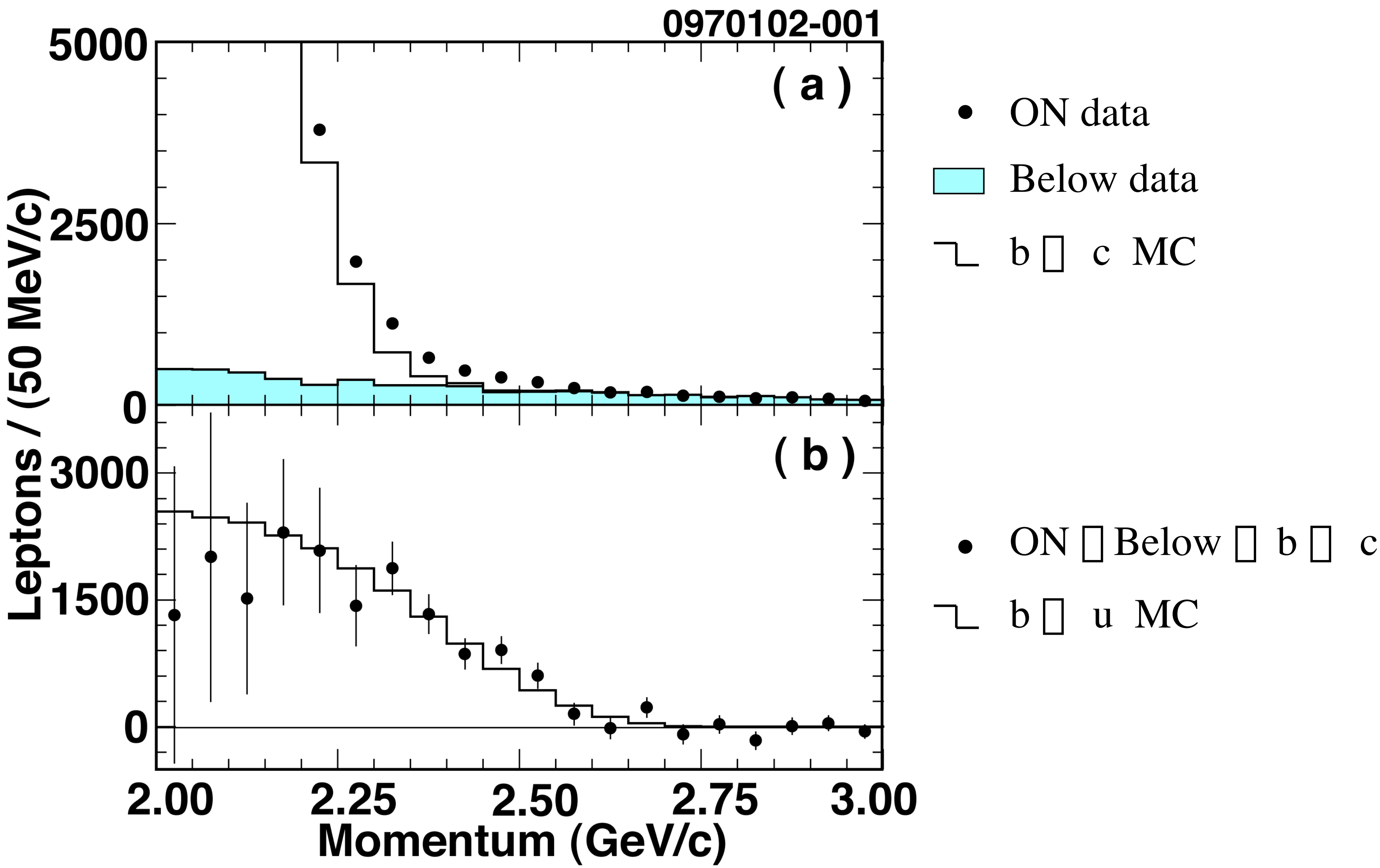




**Convolute with light cone shape function.**



# CLEO's Latest Endpoint Analysis



# Endpoint Results

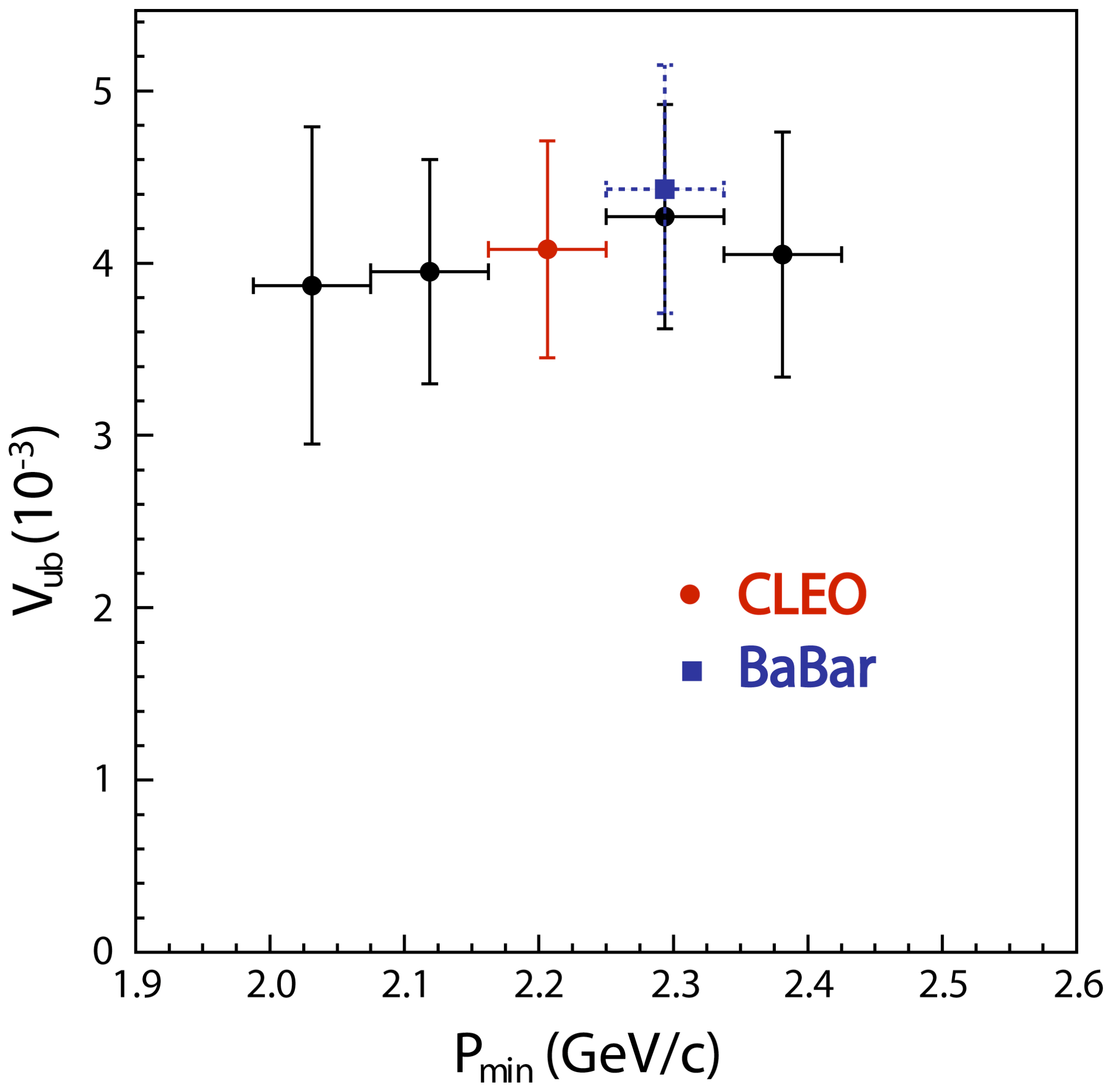
| $p$ (GeV/$c$) | $\|V_{ub}\|$ $(10^{-3})$ |
|---|---|
| 2.0 - 2.6 | 3.90 ± 0.84 ± 0.35 ± 0.22 ± 0.12 |
| 2.1 - 2.6 | 3.98 ± 0.46 ± 0.40 ± 0.22 ± 0.16 |
| 2.2 - 2.6 | 4.11 ± 0.34 ± 0.44 ± 0.23 ± 0.24 |
| 2.3 - 2.6 | 4.30 ± 0.24 ± 0.47 ± 0.24 ± 0.34 |
| 2.4 - 2.6 | 4.08 ± 0.28 ± 0.45 ± 0.23 ± 0.45 |

BaBar: 2.3 - 2.6 4.43 ± 0.29 ± 0.50 ± 0.25 ± 0.35

ICHEP 02

↑ yield; ↑ b □ s□ spectrum; ↑ BR to $|V_{ub}|$; ↑ b □ s□to b □ ul□

# Endpoint Results



# The LEP Analyses

| | |
|---|---|
| ALEPH | Eur. Phys. J. C 6, 555 (1999) |
| OPAL | Eur. Phys. J. C 21, 399 (2001) |
| DELPHI | Phys. Lett. B 478, 14 (2000) |
| L3 | Phys. Lett. B 436, 174 (1998) |

Approaches have similarities, differences:

- Each has a few million hadronic $Z^o$ decays
- Select $Z^o \to b\bar{b}$, with a lepton
- Find variables that distinguish between $b \to u l \nu$ and $b \to c$
- Feed into neural net and fit the distribution in the net output (ALEPH, OPAL)

OR

- Cut on variables, count yield; subtract $b \to c$ backgrounds (DELPHI, L3)

# The LEP Analyses - cont'd

LEP Average:

$$|V_{ub}| = (4.09 \pm 0.37 \pm 0.44 \pm 0.34) \times 10^{-3}$$

↑ Stat. + exp. sys

↑ $b \to c$ sys

↑ $b \to u l \nu$ sys ← (Needs to be reevaluated)

Comments:

- $b \to c$ backgrounds large (5-20x signal), so systematic error from $b \to c$ modelling important (re-evaluate using current knowledge?)
- Efficiency (DELPHI, L3), or sensitivity (ALEPH, OPAL) varies over $E_{lep}$, $q^2$, $M_X^2$ 3D phase space, so hard to evaluate determination of systematic error from $b \to u l \nu$ modelling.
- Systematic error from $b \to u l \nu$ modelling needs to be reevaluated using contemporary techniques, e.g. light-cone shape function, with parameter range as given by $b \to s \gamma$ photon energy spectrum.

# Belle

Belle has recently obtained Br(B□ $X_u$l□) via two novel approaches - "D$^{(*)}$l□ tag" and "Advanced □ Reconstruction"

85 million $B\bar{B}$ events used in each

- D$^{(*)}$l□ tag: $B_1$□ D$^{(*)}$$l_1$□$_1$, $B_2$ □ X$l_2$□$_2$
  - If one detects D$^{(*)}$ and $l_1$, then the constraint

$$m^2_{\nu_1} = \left(P^\mu_{B_1} - P^\mu_{D^{(*)}\ell_1}\right)^2 = 0 \Rightarrow cos\theta_{B_1 - D^{(*)}\ell_1}$$

  - If the other B decays $B_2$ □ X$l_2$□$_2$, then the constraint

$$m^2_{\nu_2} = \left(P^\mu_{B_2} - P^\mu_{X\ell_2}\right)^2 = 0 \Rightarrow cos\theta_{B_2 - X\ell_2}$$

  - If one has things right, the two cones will intersect. ($B_1$ and $B_2$ are back-to-back)

# Belle - $D^{(*)}l\nu$ tag, cont'd

- If one has things right, the two cones will intersect. ($B_1$ and $B_2$ are back-to-back)
- So: Require that cones intersect (or almost do)
  - Require Net Q = 0
  - Require X contain no $K^{\pm}$
  - Require $M_X < 1.5$ GeV, $P_l > 1.0$ GeV/c
- S/N ≈ 1:2
  - Subtract $b \to c$ contribution

    $\Rightarrow b \to ul\nu$ signal

$$|V_{ub}| = (5.00 \pm 0.60 \pm 0.23 \pm 0.05 \pm 0.39 \pm 0.36) \times 10^{-3} \text{ (Prelim)}$$

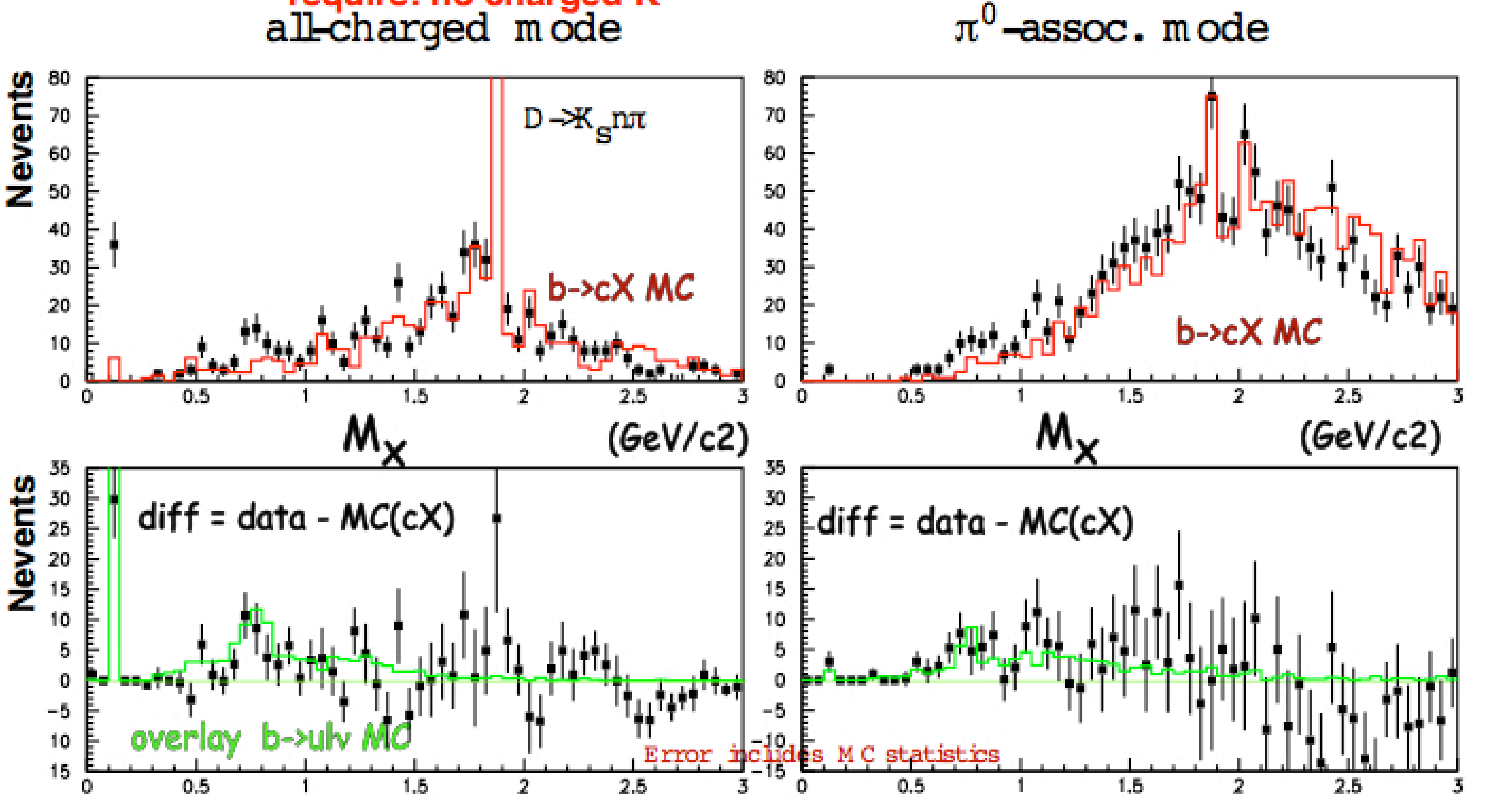
(errors, left to right: stat, sys, $b \to c$, $b \to u$, theor)

How does efficiency vary over $M_X$, $q^2$, $E_{lep}$?

# |Vub| through D(*) l ν tag: Mx distribution for Xulν signal

$\Upsilon(4S)$ → B ->D(*) l ν
→ B -> Xu l ν

require: no charged-K




82 ± 19 events in Mx < 1.5 GeV

92 ± 21 events in Mx < 1.5 GeV

# Belle - Advanced □ Reconstruction

- Select events with signal B decaying semileptonically, and tag B decaying hadronically
- By magic, sort all particles into those belonging to tag B, and those belonging to signal B
  - "Annealing", S. Kirkpatrick et al., Science 220, 4598 (1983)
  - Make cuts to throw out events that have annealed poorly.

# Belle - Advanced □ Recon, cont'd

- Now, knowing which particles belong to signal B, calculate $q^2$, $M_X$, with good resolution.
- Require $M_X < 1.5$ GeV, $q^2 > 7$ GeV$^2$
- Estimate b □ c background, subtract, count.

S/N = 0.27, 1148 signal events

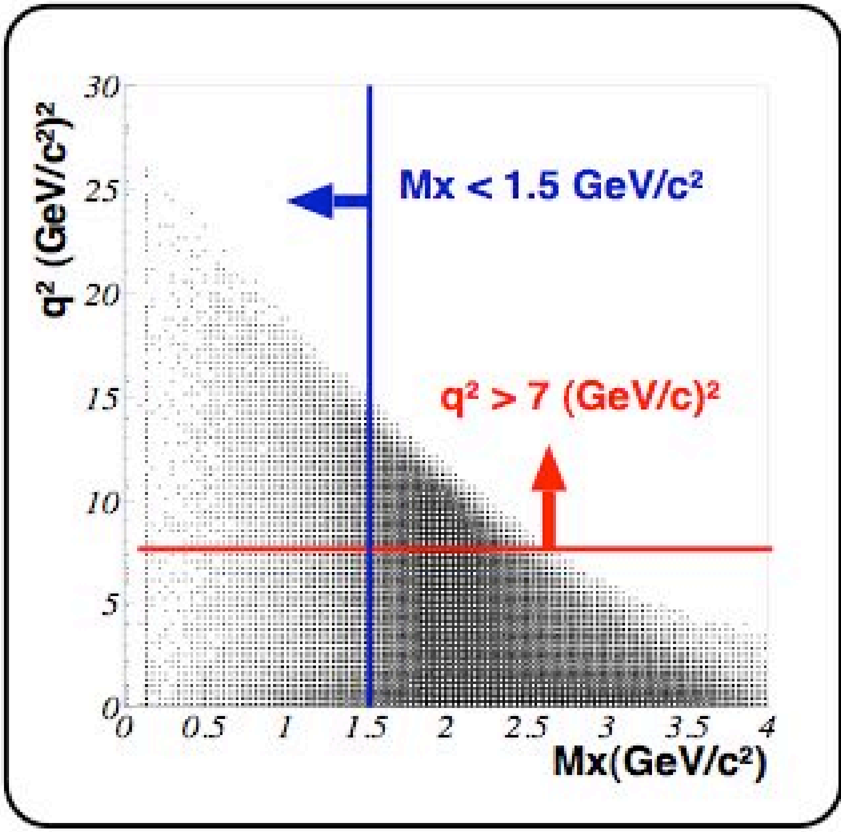
$|V_{ub}| = (3.96 \pm 0.17 \pm 0.44 \pm 0.34 \pm 0.26 \pm 0.29)$ □ $10^{-3}$

| 0.17 | 0.44 | 0.34 | 0.26 | 0.29 |
|---|---|---|---|---|
| stat | sys | b□ c | b□ u | theor. |

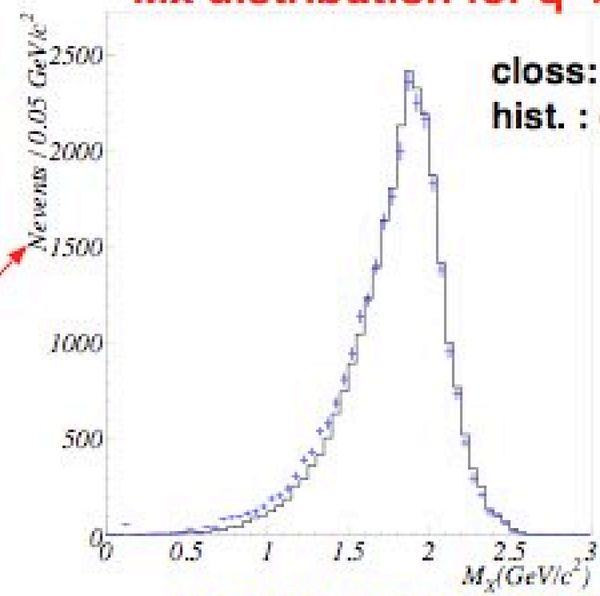
? How does efficiency vary over $M_X$, $q^2$, $E_{lep}$?

# |Vub| through Advanced ν recon: Mx,q² distribution

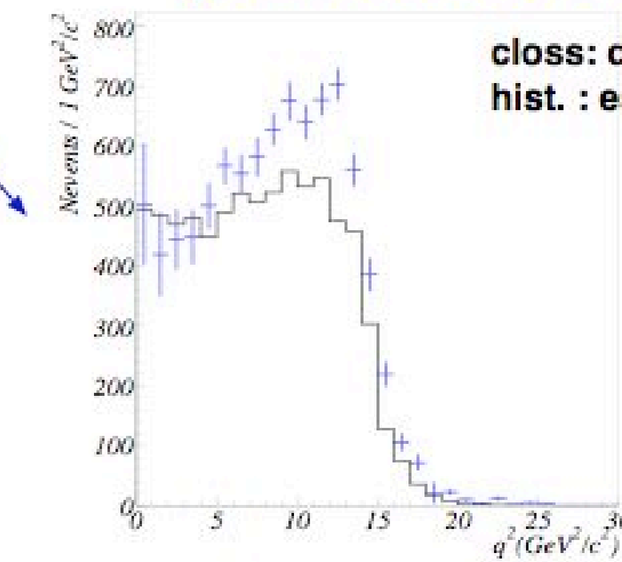
## Combined cuts on Mx & q²




Mx distribution for q² > 7GeV




q² distribution for Mx < 1.5GeV



# BaBar - Br($B \to X_u l \nu$), Inclusive

88 million $B\bar{B}$ pairs
Fully reconstruct one B.
Study semileptonic decay of other B.

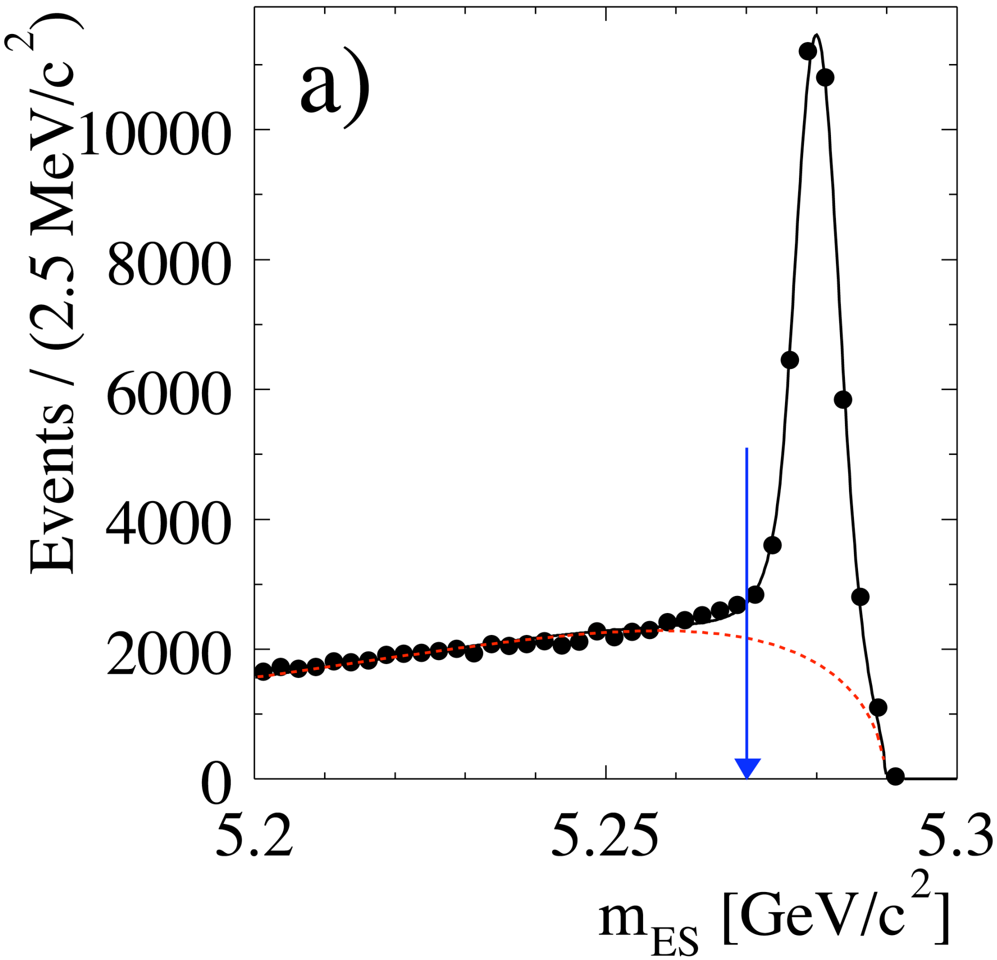
Try MANY B decay modes - over 1100!
Use the relatively clean modes (purity > 9% to 24%).
Keep 0.3% of $B^0\bar{B}^0$ events, 0.5% of $B^+B^-$ events
$\Rightarrow$ 350,000 events with one B reconstructed

# BaBar - Br($B \to X_u l \nu$), Inclusive, cont'd

Now require high momentum lepton (P > 1.0 GeV/c) from other B.
30K events, purity 67%

Interest is in $B \to Xl\nu$

- X - all particles not used for fully reconstructed B
- $\nu$ - from missing 4-momentum
- Kinematic fit → $M_X$ to ± 350 MeV.



# BaBar - Br(B  $X_u$l), Inclusive, cont'd

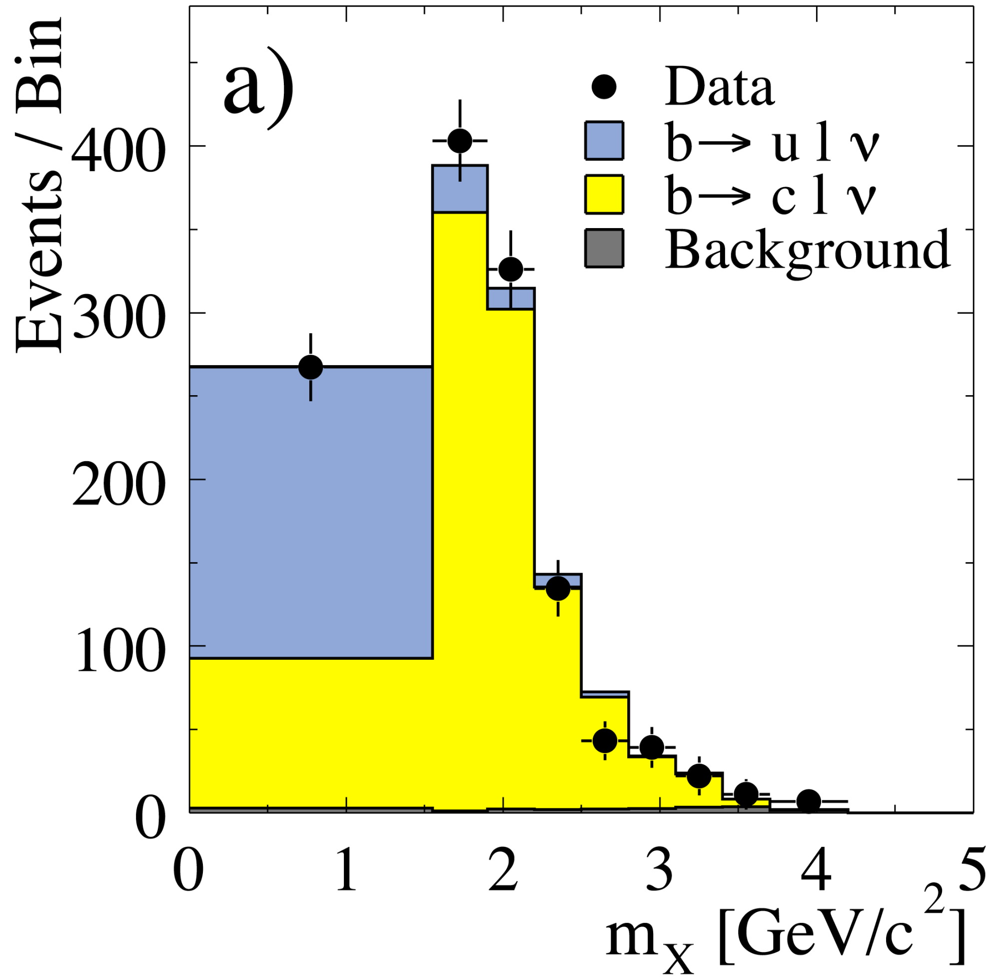
Clean up cuts

- Net charge = 0, only one lepton above 1.0 GeV/c, $m^2_{miss}$ small.

Improves $M_X$ resolution

- No $K^\pm$ or $K_s^0$ in X, D*l partial reconstruction veto,

Suppresses b  cl




For $M_X$ < 1.55 GeV, S/N = 2:1 - impressive!

# BaBar - Br($B \to X_u l \nu$), Inclusive, cont'd

Fit $M_X$ distribution

- Region for $M_X > 1.55$ fixes $b \to cl\nu$, allows extrapolation to $M_X < 1.55$
- Region for $M_X < 1.55$, with $b \to cl\nu$ subtracted, gives $b \to ul\nu$

- Because S/N is so good, systematic error from modelling $b \to c$ backgrounds is small.
- Systematic errors from modelling $b \to ul\nu$ are *not* small. They are evaluated by varying HQET parameters $\Lambda$ and $\lambda_1$ within their errors, as determined by CLEO.

# BaBar - Br($B \to X_u l \nu$), Inclusive

Results:

$$\mathcal{B}\left(B \to X_u \ell \nu\right) = (2.24 \pm 0.27 \pm 0.26 \pm 0.39) \times 10^{-3}$$

↑ stat ↑ other sys ↑ $b \to u$ sys

$$|V_{ub}| = (4.62 \pm 0.28 \pm 0.27 \pm 0.40 \pm 0.26) \times 10^{-3}$$

↑ stat ↑ other sys ↑ $b \to u$ sys ↑ $V_{ub}$ from $b \to ul\nu$

# Exclusive $b \to ul\nu$ Measurements

1. CLEO, $B \to \pi l\nu, \rho l\nu$ PRL 77, 5000 (1996)
   - $Br(B^0 \to \pi^- l^+ \nu) = (1.8 \pm 0.5) \times 10^{-4}$
   - $Br(B^0 \to \rho^- l^+ \nu) = (2.5 \pm 0.9) \times 10^{-4}$
2. CLEO, $B \to \rho l\nu$ PRD 61, 052001 (2000)
   - $Br(B^0 \to \rho^- l^+ \nu) = (2.69 \pm 0.75) \times 10^{-4}$
3. BaBar, $B \to \rho l\nu$ PRL 90, 181801 (2003)
   - $Br(B^0 \to \rho^- l^+ \nu) = (3.29 \pm 0.83) \times 10^{-4}$
4. CLEO $B \to \pi l\nu$, $\rho l\nu$, [illegible]$l\nu$ hep-ex/0304019
   - $Br(B^0 \to \pi^- l^+ \nu) = (1.33 \pm 0.22) \times 10^{-4}$
   - $Br(B^0 \to \rho^- l^+ \nu) = (2.17 \pm 0.74) \times 10^{-4}$

- All measurements “detect” the $\nu$ via the missing 4-momentum in the event.
- (2.) and (3.) suppress $b \to cl\nu$ background by (effectively) requiring $P_{lep} > 2.3$ GeV/c.
- (1.) is superseded by (4.)

# CLEO's Latest Exclusive Analysis

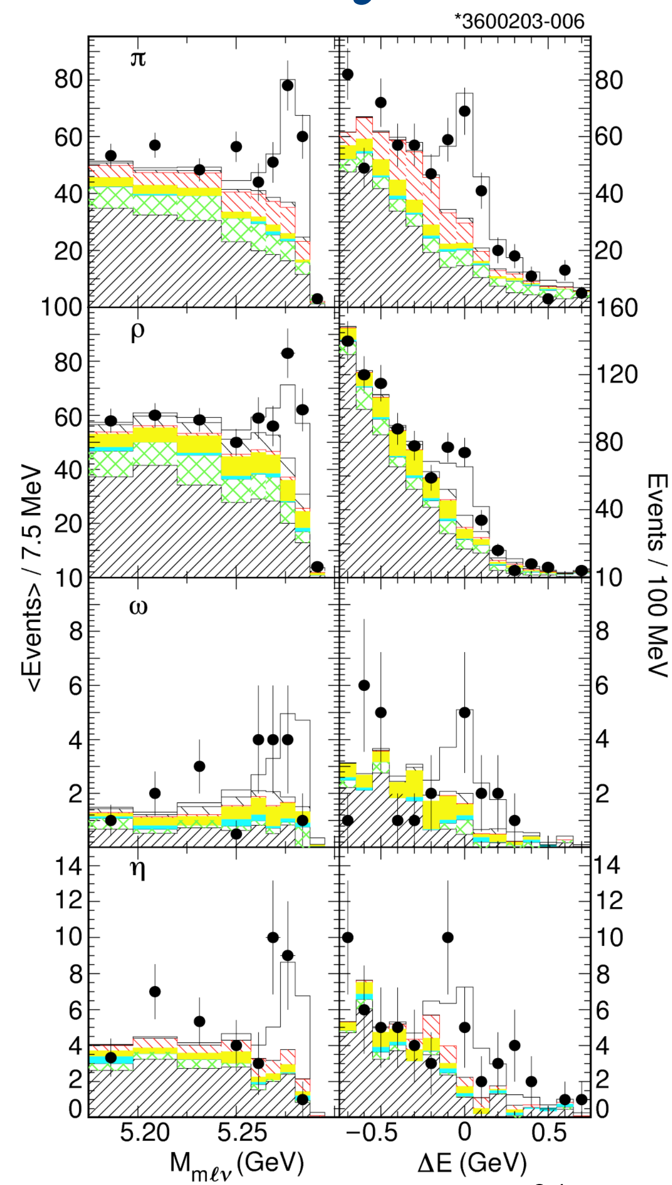
9.7 million $B\bar{B}$ pairs
Split data into 3 $q^2$ intervals
Reduces model dependence

Lower lepton momentum cut:
$P_{lep} > 1.0$ GeV/c for □
$P_{lep} > 1.5$ GeV/c for □
Reduces model dependence

$$\mathcal{B}\left(B^0 \rightarrow \pi^- \ell^+ \nu\right) =$$
$$(1.33 \pm 0.18 \pm 0.11 \pm 0.01 \pm 0.07) \times 10^{-4}$$

↑ stat ↑ exp sys ↑ □ FF shape ↑ □ FF shape

$$\mathcal{B}\left(B^0 \rightarrow \rho^- \ell^+ \nu\right) =$$
$$(2.17 \pm 0.34 \pm^{+0.47}_{-0.54} \pm 0.01 \pm 0.41) \times 10^{-4}$$



# CLEO'S Latest Exclusive Analysis, cont'd

## Obtaining $|V_{ub}|$

For $0 < q^2 < 16$ GeV$^2$, use light-cone sum rules

For 16 GeV$^2 < q^2 < q_{max}^2$, use lattice QCD

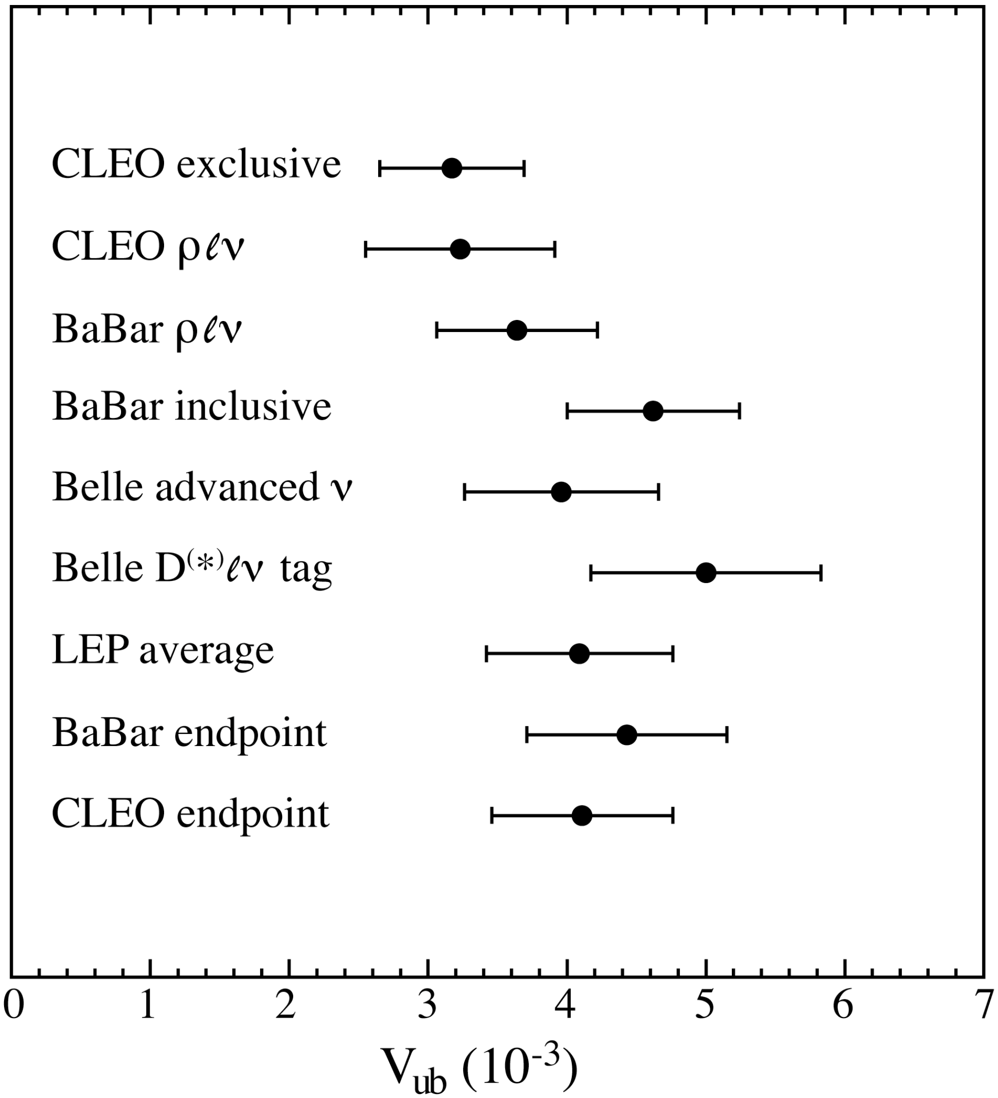
Combine the two results

$|V_{ub}|$ $(10^{-3})$

| | |
|---|---|
| □1□ | $3.24 \pm 0.22 \pm 0.13 \,^{+0.55}_{-0.39} \pm 0.09$ |
| □1□ | $3.00 \pm 0.21 \,^{+0.29}_{-0.35} \,^{+0.49}_{-0.38} \pm 0.28$ |
| Combined | $3.17 \pm 0.17 \pm 0.17 \,^{+0.53}_{-0.39} \pm 0.03$ |

↑ stat ↑ exp sys ↑ theo ↑ □ FF shape

# $|V_{ub}|$ Results



CLEO exclusive
CLEO ρℓν
BaBar ρℓν
BaBar inclusive
Belle advanced ν
Belle D(*)ℓν tag
LEP average
BaBar endpoint
CLEO endpoint
0
1
2
3
4
5
6
7
Vub (10-3)

# Averaging - ?!?

Simple averages of these measurements:

Means: 4.03

Exclusive: 3.35

Inclusive: 4.37

Errors: $\pm$ 0.66

r.m.s. spread of means $\pm$ 0.61

No evidence of any problem.

(Weak hint of exclusive/inclusive difference)

# Averaging - ?!?

Q. Can't one, shouldn't one, combine all the existing measurements of $|V_{ub}|$, taking into account correlated errors?

A. Maybe not. It looks like a nightmare to me.

- Partially correlated errors, that must be treated in a consistent fashion
  - Model of $b \to c$, for backgrounds
  - Model of $b \to u l \nu$, for signal
- The "unquantifiable errors"
  - Higher twist, power correction from non-local operators
  - Local quark-hadron duality

Maybe it would be better to include only those measurements that have uniform sensitivity over a well-defined region of phase space.

# Closing Comments

Exclusives

"Soon" there will be unquenched lattice QCD calculations for $B \to \pi l \nu$, over the full $q^2$ range, accurate to ~ ±10% in rate, therefore ± 5% in $|V_{ub}|$. Experiment (CLEO, 9.7M $B\bar{B}$) already gives $Br(B \to \pi l \nu)$ to ±17%, with statistical errors 1.4 times the sum of the systematic ones. A 12% measurement in rate is possible with data already taken by BaBar, Belle.

A very promising route to $|V_{ub}|$ to < ±10%.

Inclusives

The best way to get systematic error from modelling of $B \to X_u l \nu$ is from shape function, obtained from photon energy spectrum from $B \to X_s \gamma$.

$\Rightarrow$ Need a second measurement of the spectrum, down to 2.0 GeV at least, preferably 1.5 GeV.