Recent Advances in Semileptonic $\boldsymbol{B}$ Decays

Theory

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Extracting V_{cb} and V_{ub} is a challenge to strong interaction theory probing our control of QCD dynamics

- Golden way $\Gamma_{\rm sl}(B)$, inclusive decay distributions
- Gold-plated modes: $B \to D^*\,\ell\nu$ and $B \to D\,\ell\nu$ near zero recoil

$1/m_c^k$ corrections are not too small...

Inclusive decays provide a host of dynamic info vital for $B \to D^*$ and $B \to D$ decays

Recent inclusive data fuel advances in the old field

Theory progress:

New HQ sum rules (exact spin sum rules)

exact inequalities

D$'$Orsay sum rules

HQ relations for higher IW derivatives

BPS expansion

The $1/m_c$ corrections to HQ spin symmetry are too significant
A subgroup, HF symmetry for ground-state pseudoscalar mesons is good

'light charm' problems may even affect inclusive decays via $\langle B|\bar{b}c\,\bar{c}b|B\rangle$ BBMU 2003

Theoretical fidelity exceeds lattice accuracy, can be used to cross-check lattice simulations

Some problems may be emerging

Highlights:

♠ $B \to D\,\ell\nu$ amplitude may be known theoretically with a $1-2\%$ accuracy at small recoil

♣ $B \to D\,\ell\nu$ decay rate may be measuring the whole IW function without significant power corrections

Accuracy of the predictions strongly depends on the precise value of $\mu_\pi^2(1\,\mathrm{GeV})$ best for low, qualitative at upper end

will have been clarified in inclusive decays

♥ $B \to D^{(*)}\,\tau\nu$ and $B \to X_c\,\tau\nu$ offer interesting studies (m_c, local duality violation, ...)

Recent progress in 'inclusive' OPE:

- Perturbative resummation for $\Gamma_{\rm sl}(B)$

  hep-ph/0210413, hep-ph/0302262

- Higher orders in $1/m_b$ and a new class of nonperturbative effects from $\langle \bar{b}c\,\bar{c}b\rangle$ hep-ph/0302262

Present stage:

♠ Have an accurate and reliable determination of some HQ parameters from experiment

♣ Extracting $|V_{cb}|$ from $\Gamma_{\rm sl}(B)$ has good accuracy and solid grounds

♥ Have at least one precision check of the OPE at the nonperturbative level

Theoretical status

Can go down to a % level in $|V_{cb}|$ if relevant parameters are determined:

- $m_{b,c}(\mu)$, $\mu_\pi^2(\mu)$, $\mu_G^2(\mu)$, ... are completely defined and can (in principle) be determined from experiment with an unlimited accuracy
- Duality violation is very small in $\Gamma_{\rm sl}(B)$ BU 2001
- α_s corrections to Wilson coefficients are feasible Limiting factor
- Know how to analyze higher power corrections BBMU 2003

m_b, m_c, μ_π^2, ... (properly defined) can be determined from the semileptonic $(b \to s + \gamma)$ decay distributions themselves BSUV, 1993-1994

Nowadays is being implemented in a number of experiments

New strategy: formulated at CKM 2002 @ CERN

Comprehensive approach: measure many observables to extract the 'theoretical' input parameters

We can do without relying on $1/m_c$ expansion at all

Expansion in $1/m_c$ is questionable: $\frac{1}{m_c^2} > 14\,\frac{1}{m_b^2},\quad 8\,\frac{1}{(m_b - m_c)^2}$

Hadronic moments: if m_c were large enough first would yield $\overline{\Lambda}$, second μ_π^2, third ρ_D^3 more or less directly BSUV 1993-94

The comprehensive studies allow a robust analysis

- width is affected only by ρ_D^3 to order $1/m_b^3$
  moments also depend (weakly) on ρ_{LS}^3
- No non-local correlators ever enter
- Deviations from the HQ limit are driven by $1/m_b$
  actually, $\propto \mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$ in B BPS limit
- Exact sum rules and inequalities for properly defined parameters
  e.g., $\mu_\pi^2 > \mu_G^2 \simeq 0.35\,\mathrm{GeV}^2$

How this works: illustration

$$\langle E_\ell \rangle = 1.38\,\mathrm{GeV} + 0.38[(m_b - 4.6\,\mathrm{GeV}) - 0.71\,(m_c - 1.15\,\mathrm{GeV}) + 0.09\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.22\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)]$$

$$\mathcal{L}_2 = 0.18\,\mathrm{GeV}^2 + 0.1[(m_b - 4.6\,\mathrm{GeV}) - ...] + 0.01(m_c - 1.15\,\mathrm{GeV}) + 0.04\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.04\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

$$\mathcal{L}_3 = -0.033\,\mathrm{GeV}^3 - 0.03[(m_b - 4.6\,\mathrm{GeV}) - ...] + 0(m_c - 1.15\,\mathrm{GeV}) + 0.03\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.04\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

Practically the same combination $m_b - 0.7 m_c$

weak dependence on μ_π^2, ρ_D^3

$$\frac{|V_{cb}|}{0.042} = 1 - 0.65\,[(m_b - 4.6\,\mathrm{GeV}) - ...] - 0.06\,(m_c - 1.15\,\mathrm{GeV}) - 0.07\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.05\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

Precise value of m_c is irrelevant!

Need to know with some accuracy μ_π^2 and ρ_D^3

no hidden assumptions

Hadronic moments

$$\langle M_X^2 \rangle \simeq 4.54\,\mathrm{GeV}^2 - 5\,[(m_b - 4.6\,\mathrm{GeV}) - 0.62\,(m_c - 1.15\,\mathrm{GeV}) + 0.13\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.2\,(\tilde\rho_D^3 - 0.12\,)]$$

Nearly the same combination $m_b - 0.7 m_c - 0.1\mu_\pi^2 - 0.2\tilde\rho_D^3$ as in $\langle E_\ell \rangle$

Not very constraining ... – instead checks how HQ expansion works: Theory predicts $\langle E_\ell \rangle = 1.377\,\mathrm{GeV}$

Experiment: $\langle E_\ell \rangle = 1.383 \pm 0.015\,\mathrm{GeV}$

A highly nontrivial nonperturbative check of the OPE: the sum rule for $M_B - m_b \simeq 650\,\mathrm{MeV}$ verified at a $40\,\mathrm{MeV}$ level!

$$\langle (M_X^2 - \langle M_X^2 \rangle)^2 \rangle \simeq 1.2\,\mathrm{GeV}^4 + 0.02(m_b - 4.6\,\mathrm{GeV}) - 0.7\,(m_c - 1.15) + 4.5\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 5.3\,(\tilde\rho_D^3 - 0.12\,\mathrm{GeV}^3)$$

$$\langle (M_X^2 - \langle M_X^2 \rangle)^3 \rangle \simeq 4\,\mathrm{GeV}^6 - (m_b - 4.6\,\mathrm{GeV}) - 3\,(m_c - 1.15\,\mathrm{GeV}) + 5\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) + 12\,(\tilde\rho_D^3 - 0.12\,\mathrm{GeV}^3)$$

Ideally, these moments measure kinetic and Darwin expectation values. In practice, for ρ_D^3 only approximate evaluation and an informative upper bound

Current sensitivity to μ_π^2 is about $0.1\,\mathrm{GeV}^2$, $0.1\,\mathrm{GeV}^3$ to ρ_D^3

$$\frac{|V_{cb}|}{0.042} = 1 - 0.13\,[\langle M_X^2\rangle - 4.54\,\mathrm{GeV}^2] - 0.005\,(m_c - 1.15\,\mathrm{GeV}) + 0.10\,(\mu_\pi^2 - 0.4\,\mathrm{GeV}^2) - 0.03\,(\tilde{\rho}_D^3 - 0.12\,\mathrm{GeV}^3)$$

Measuring $\langle M_X^4\rangle$ and $\langle M_X^6\rangle$ is the real step in implementing the comprehensive program of extracting $|V_{cb}|$

further work is required

Example:

$$|V_{cb}| = 0.0421 \cdot (1 \pm 0.015_{\mathrm{SL\,width}} \pm 0.02_{\mathrm{HQ\ param}})$$

solely from DELPHI hadronic moments

Does not rely on expansion in $1/m_c$!

small uncertainties are not dominated by theory

(Recent theory review: hep-ph/0302262, ND – Karlsruhe – Milan)

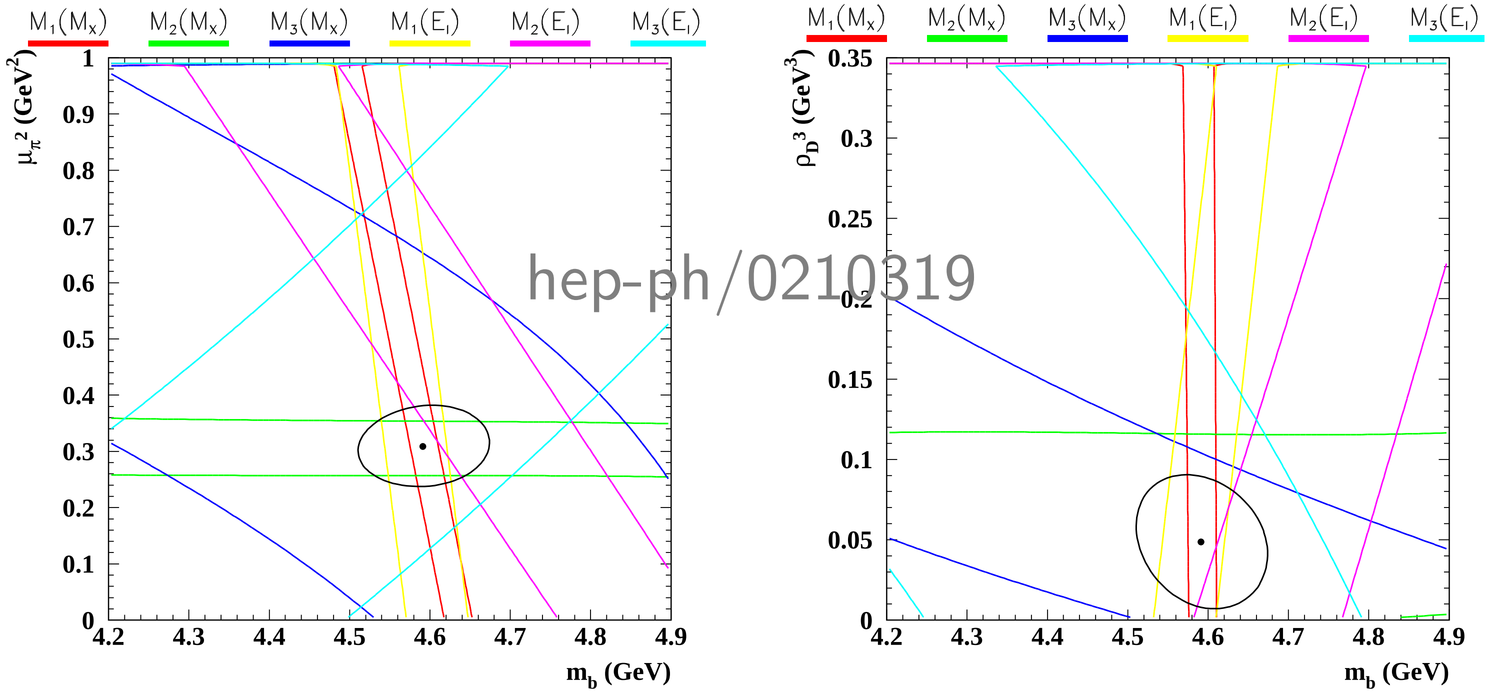
Moreover,

assuming $\bar{m}_c(m_c) = 1.23\,\mathrm{GeV}$ $\quad m_b(1\,\mathrm{GeV}) \simeq 4.58\,\mathrm{GeV}$

(too) good agreement with the theoretical expectations ?

Hadronic parameters

$$\overline{\Lambda}(1\,\mathrm{GeV}),\ \ \mu_\pi^2(1\,\mathrm{GeV}),\ \ \mu_G^2(1\,\mathrm{GeV}),\ \ \ldots$$



Using the same accurate regularized definition for kinetic $(\{j,k\})$ and chromomagnetic $([j,k])$ operators allows precision numerical evaluation

Product of covariant derivatives $\bar{Q}(x) iD_j \, P\exp \, iD_k Q(0)$ offset along t direction $it \sim 1/\mu$

$$M_{B^*} - M_B \simeq \frac{2}{3}\frac{\mu_G^2}{m_b}$$

$$\mu_G^2(1\,\mathrm{GeV}) = 0.35^{+.03}_{-.02}\,\mathrm{GeV}^2$$ N.U. 2001

$\mu_\pi^2(\mu) > \mu_G^2(\mu)$ at any μ rigorous inequality

BSUV, Voloshin 1993–1994

Physical observables, renormalon-free $\Rightarrow$ have definite values in Nature

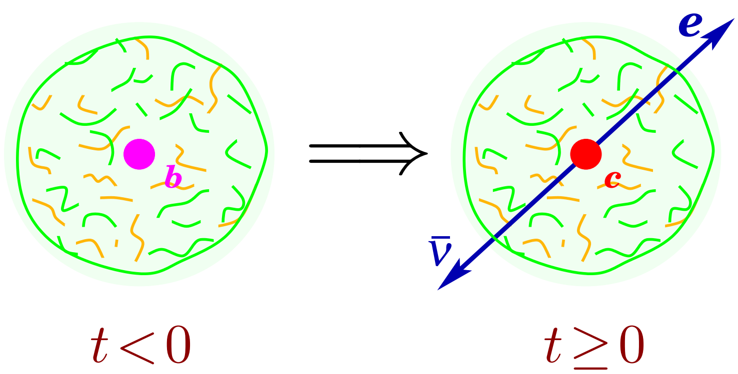
Experiment: typically $\mu_\pi^2(1\,\mathrm{GeV}) \simeq (0.37 \pm 0.1)\,\mathrm{GeV}^2$

$\mu_\pi^2(1\,\mathrm{GeV}) > 0.45\,\mathrm{GeV}^2$ excluded ?

Assume this for what follows

$B \to D^* + \ell\bar{\nu}$ at zero recoil

$$\mathrm{d}w\,(B \to D^* + \ell\bar{\nu}) \sim G_F^2 \cdot |V_{cb}|^2 \cdot |\vec{p}| \cdot \left|F_{B\to D^*}(\vec{p})\right|^2$$



$F_{B\to D^*}$ is determined by bound state dynamics

If $\vec{p}=0$ $(\vec{p}_e = -\vec{p}_{\bar{\nu}})$

almost nothing has changed!

$F(\vec{p}=0) = 1$ up to 'isotopic effects'

$$F_{\mathsf{n/p}}(0) = 1 + \frac{0}{m_{c,b}} + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{m^2_{c,b}}\right) + \mathcal{O}\left(\frac{\Lambda^3_{\mathrm{QCD}}}{m^3_{c,b}}\right) + \ldots$$

$1/m_{b,c}$ effects are absent

1986 Voloshin, Shifman
1990 Luke

Important to estimate δ_{1/m^2}

Before May 1994 : $\delta_{1/m^2} \simeq -0.02$

OPE $\Longrightarrow$ HQ Sum Rules

SUV, BSUV April 1994
Experiment June 1994

$$-\delta_{\mathsf{n/p}} > \frac{M^2_{B*} - M^2_B}{8m_c^2} \simeq -0.04$$

rigorous bound on $F(0)$

$F(0) \simeq 0.9$ actual estimate

Numerical estimates of F_{D^*}

$$F_{D^*}=\left[\xi_A(\mu)-\frac{\mu_G^2}{3m_c^2}-\frac{\mu_\pi^2-\mu_G^2}{4}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}+\frac{2}{3m_cm_b}\right)-\sum_{f\neq D^*}^{\epsilon<\mu}|F_{B\to f}|^2+\mathcal{O}\left(\frac{1}{m^3}\right)\right]^{\frac{1}{2}}$$

$$2\delta_{1/m^2}(\mu)$$

$\xi_A^{\frac{1}{2}}(\mu)$ is the short-distance renormalization factor 0.97 ± 0.01

$$\sum_{f\neq D^*}^{\epsilon<\mu}|F_{B\to f}|^2=\chi\left[\frac{\mu_G^2}{3m_c^2}+\frac{\mu_\pi^2-\mu_G^2}{4}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}+\frac{2}{3m_cm_b}\right)+\mathcal{O}\left(\tfrac{1}{m^3}\right)\right]$$

χ describes the w f overlap deficit guess: $0<\chi\le 1$ SUV 1994

$$F_{D^*}\simeq\xi_A^{\frac{1}{2}}-(1+\chi)\left[\frac{\mu_G^2}{6m_c^2}+\frac{\mu_\pi^2-\mu_G^2}{8}\left(\frac{1}{m_c^2}+\frac{1}{m_b^2}+\frac{2}{3m_cm_b}\right)+\Delta_{\frac{1}{m^3}}\right]$$

if $\chi=0.5\pm 0.5$ $\mu\approx 0.8\,\text{GeV}$

$$F_{D^*}\simeq 0.89-0.015\,\frac{\mu_\pi^2-0.4\,\text{GeV}^2}{0.1\,\text{GeV}^2}\pm 0.03_{\text{exc}}\pm 0.01_{\text{pert}}$$

$1/m_c^3$ correction is significant!

$F_{D^*}\lesssim 0.92$ for $\chi\simeq 0$

$\chi^{\text{pert}}=1$ @ $\mathcal{O}(\alpha_s^1)$

't Hooft model: $\chi=\frac{13}{21}+\frac{5}{21}\frac{m^2-\beta^2}{\overline{\Lambda}^2-m^2+\beta^2}-\frac{4}{21}\left(\varrho^2-\frac{3}{4}\right)\simeq 0.55$

Burkardt, N.U. 2000

$B \to D\,\ell\nu$ near zero recoil

$$\langle D(p_2)|\bar{c}\gamma_\nu b|B(p_1)\rangle = f_+(p_1+p_2)_\nu + f_-(p_1-p_2)_\nu$$

$$f_\pm \equiv f_\pm(\vec{q}^{\,2})$$

One amplitude $J_0 = (M_B+M_D)f_+(0) + (M_B-M_D)f_-(0)$ at $\vec{q}=0$

HQ limit: $f_+ = \frac{M_B+M_D}{2\sqrt{M_B M_D}}, \quad f_- = -\frac{M_B-M_D}{M_B+M_D}\, f_+$

$$\frac{J_0}{2\sqrt{M_B M_D}} = 1 - a_2\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2 - a_3\left(\frac{1}{m_c}-\frac{1}{m_b}\right)^2\left(\frac{1}{m_c}+\frac{1}{m_b}\right) + \ldots$$

Corrections are well under control and small$_{\text{quantify later}}$

Any amplitude with massless leptons depends, however solely on f_+, while only the combination of f_+ and f_- has no $1/m$ corrections

$F_+ \equiv \frac{2\sqrt{M_B M_D}}{M_B+M_D}\, f_+$ has $1/m_Q$ corrections since $\vec{J}$ has such a term...

Good news: we know it!

$$F_+ = 1 + \left(\frac{\overline{\Lambda}}{2} - \overline{\Sigma}\right)\left(\frac{1}{m_c} - \frac{1}{m_b}\right)\frac{M_B - M_D}{M_B + M_D} - \mathcal{O}\left(\frac{1}{m_Q^2}\right)$$

Thanks to inclusive decays and exact sum rules we know $\frac{\overline{\Lambda}}{2} - \overline{\Sigma}$ (positive, but very small $\propto \frac{\mu_\pi^2 - \mu_G^2}{3\mu_{\rm hadr}}$)

Moreover, we know `all` power corrections are small, the concern is rather exponential terms $\sim e^{-2m_c/\mu_{\rm hadr}}$

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D}\, f_+(0) = 1.03 \pm 0.025$$

All orders in $1/m$ in BPS, to $1/m \cdot 1/\mathrm{BPS}^2$, α_s^1

This formfactor is known better than for 'gold-plated' $B \to D^*$

differs from existing estimates

Obsolete evaluations of the perturbative effects to be refined

If this can be measured, nothing else `exclusive` may be required for $|V_{cb}|$

How do we know?

Sum Rules in the HQ Limit

$$\varrho^2 - \frac{1}{4} = 2\sum_m |\tau_{3/2}^{(m)}|^2 + \sum_n |\tau_{1/2}^{(n)}|^2$$

$$\frac{1}{2} = 2\sum_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n |\tau_{1/2}^{(n)}|^2$$

$$\frac{\overline{\Lambda}}{2} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2$$

$$\overline{\Sigma} = 2\sum_m \epsilon_m |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n |\tau_{1/2}^{(n)}|^2$$

$$\frac{\mu_\pi^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2$$

$$\frac{\mu_G^2}{3} = 2\sum_m \epsilon_m^2 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^2 |\tau_{1/2}^{(n)}|^2$$

$$\frac{\rho_D^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 + \sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2$$

$$-\frac{\rho_{LS}^3}{3} = 2\sum_m \epsilon_m^3 |\tau_{3/2}^{(m)}|^2 - 2\sum_n \epsilon_n^3 |\tau_{1/2}^{(n)}|^2$$

Second and Fourth sum rules are superconvergent

$$\epsilon_k = M_k - M_B$$

$$\langle B(v)|\, \bar{b}\,\gamma_0\, b\,|B(0)\rangle = 1 - \varrho^2 \frac{\vec{v}^{\,2}}{2} + \mathcal{O}(\vec{v}^{\,4})$$

$$\langle P^{(1/2)}(v_2)|\bar{b}\gamma_\mu\gamma_5 b|B(v_1)\rangle = -\tau_{1/2}\,(v_1 - v_2)_\mu$$

$$\langle P^{(3/2)}(v_2)|\bar{b}\gamma_\mu\gamma_5 b|B(v_1)\rangle = -\frac{1}{\sqrt{2}}\, i\, \tau_{3/2}\, \epsilon_{\mu\alpha\beta\gamma}\, \varepsilon^{*\alpha}\, v_2^\beta\, v_1^\gamma$$

spin of light cloud is $\begin{cases} \frac{1}{2} & \text{in } P^{(1/2)} \\ \frac{3}{2} & \text{in } P^{(3/2)} \end{cases}$

Remarkable extension of first sum rules to v^4 and higher orders:

D' Orsay Sum Rules

Le Yaouanc, Oliver, Raynal 10/2002

OPE for nonforward scattering amplitude

$$\varrho_L^2 = (2L+1) \sum_n \left| \tau^{(n)}_{L+\frac{1}{2}} \right|^2 \qquad \varrho_L^2 \equiv \frac{(-1)^L}{L!} \frac{\mathrm{d}^L \xi(w)}{(\mathrm{d}w)^L} \Big|_{\mathrm{w=1}}$$

$$L \sum_n \left| \tau^{(n)}_{L+\frac{1}{2}} \right|^2 - \sum_k \left| \tau^{(k)}_{L-\frac{1}{2}} \right|^2 = \frac{2L-1}{4} \sum_n \left| \tilde{\tau}^{(n)}_{(L-1)+\frac{1}{2}} \right|^2$$

Divergent – undergo renormalization...

Peculiar: only L-th orbital waves enter for L-th derivative!

For instance

$$\varrho_2^2 \geq \frac{5}{4} \varrho^2 \geq \frac{15}{16}$$

↑ IW curvature ↑ IW slope

$$\varrho_L^2 \geq \frac{(2L+1)!!}{2^{2L} L!} \varrho^2$$

'Extended BPS' limit: All $\tau^2_{L-\frac{1}{2}}$ suppressed ?!

all 'spin' inequalities are approximately saturated

$$\xi_{\mathrm{BPS}}(w) = \left(\frac{2}{w+1} \right)^{\frac{3}{2}}$$

$w \equiv v_0$

Can be directly measured in $B \to D\, \ell \nu$

Sum rules yield strict inequalities

$$\varrho^2 > \tfrac{3}{4}\,, \qquad \overline{\Lambda} > 2\overline{\Sigma}\,, \qquad \mu_\pi^2 > \mu_G^2\,, \qquad \rho_D^3 > -\rho_{LS}^3$$

$$\rho_D^3 > |\rho_{LS}^3|/2$$

Likewise

$$\mu_\pi^2 \ge \frac{3\overline{\Lambda}^2}{4\varrho^2 - 1}\,, \quad \rho_D^3 \ge \frac{3}{8}\,\frac{\overline{\Lambda}^3}{(\varrho^2 - \frac{1}{4})^2}\,, \quad \rho_D^3 \ge \frac{(\mu_\pi^2)^{3/2}}{\sqrt{3(\varrho^2 - \frac{1}{4})}}$$

Similarly for W_- moments

Positivity for many non-local correlators

Hold in our renormalization scheme

Maximal physical information – the case of ‘kinetic’ mass and other definitions based on the SV sum rules

Good example: $\varrho^2 > \frac{3}{4}$ N.U. 2000

Dynamic, much stronger than the Bjorken's $\varrho^2 > \frac{1}{4}$

Moreover

$$\mu_\pi^2(\mu) - \mu_G^2(\mu) = 3\tilde\varepsilon^2 \cdot \left(\varrho^2(\mu) - \tfrac{1}{4} - S(\mu)\right) \qquad 0.5\,\mathrm{GeV} < \tilde\varepsilon < \mu$$

$$S(\mu) = 2\sum_{\varepsilon<\mu} |\tau_{3/2}^{(m)}|^2 - |\tau_{1/2}^{(n)}|^2 \underset{\mu\to\infty}{\longrightarrow} \tfrac{1}{2} + 0$$

If the first spin sum rule is saturated at $\mu = 1\,\mathrm{GeV}$ then

$$\mu_\pi^2 - \mu_G^2 = 3\tilde\varepsilon^2 \cdot (\varrho^2 - \tfrac{3}{4})$$

Quite a constraint: $\left(\varrho^2 - \frac{3}{4}\right) = \frac{\mu_\pi^2 - \mu_G^2}{3\tilde\varepsilon^2} \lesssim 0.2 \ (0.3)$

at $\mu_\pi^2 = 0.43\ (0.5)\ \mathrm{GeV}^2$ since $\tilde\varepsilon > 0.4\,\mathrm{GeV}$

ϱ^2 is probed in experiment

important for V_{cb}

radically improves $B\to D^*$ extrapolation to zero recoil

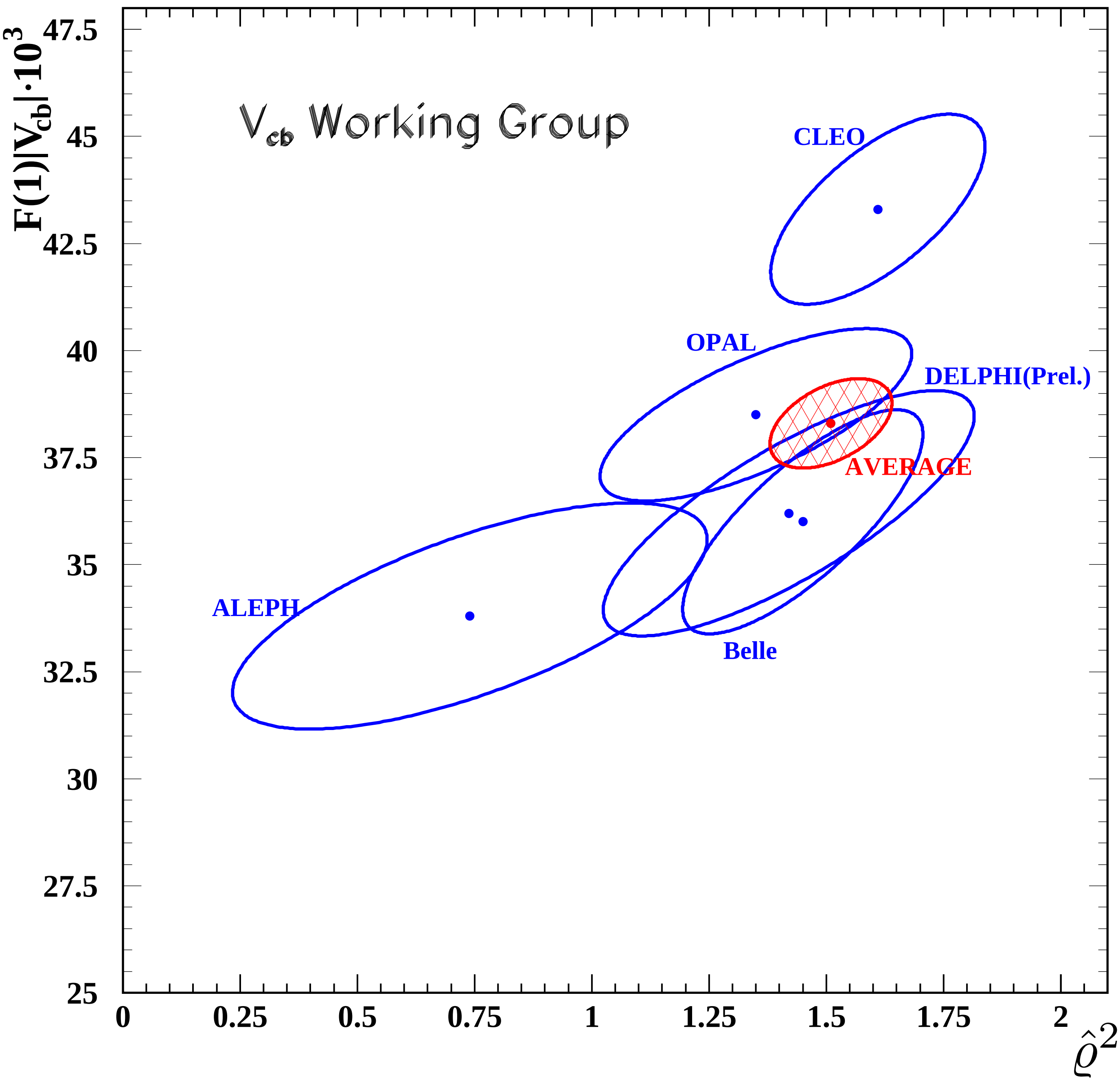
Neubert, 1993: $\hat\varrho^2 \simeq \varrho^2 - 0.09$

hardly correct

Excluded by experiment

Recent UKQCD lattice is quite compatible with the prediction:

$$\varrho^2 = 0.83^{+.15\,+\,.24}_{-.11\,-\,.01}$$ hep-lat/0202029



Question to experiment and fits:

What is the value for $F(1)\cdot|V_{cb}|$ with the constraint $\hat{\varrho}^2 < 1.2$?

The whole set of the sum rule constraints is even more interesting

their strength depends on the size of μ_π^2

If indeed $\mu_\pi^2 \lesssim 0.45\,\mathrm{GeV}^2$, i.e. $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2,\, \mu_G^2$

BPS expansion

N.U. 2001

Expand around $\mu_\pi^2 - \mu_G^2 = 0$

- $\varrho^2 - \frac{3}{4}$, $\overline{\Lambda} - 2\overline{\Sigma}$, $\mu_\pi^2 - \mu_G^2$, $\rho_D^3 + \rho_{LS}^3$, ... are all moments of one and the same HQ positive structure function which then must be suppressed

- At $\mu_\pi^2 = \mu_G^2$ there is a `functional` relation $\vec{\sigma}\vec{\pi}|B\rangle = 0$

Reminiscent to a "BPS"-saturated state not literally ?

$$\mathcal{H}_Q = A_0 + \frac{(\vec{\sigma}\vec{\pi})^2}{2m_Q}$$

Yet

$$\mathcal{P}_z|B_{\frac{1}{2}}\rangle = 0\,, \qquad \mathcal{P}_x - i\mathcal{P}_y\,|B_{\frac{1}{2}}\rangle = 0$$

Ultrarelativistic light cloud – antipode to NR quark models

Remarkable limit in many respects

Infracted by hard gluons rather a property of soft dynamics

Often extends Heavy Flavor (but not Spin) symmetry to all orders in $1/m_Q$

No formal power corrections to $m_b - m_c = M_B - M_D$

only exponential in $2m_c/\mu_{\rm hadr}$

A number of relations, among them

$$\varrho^2 = \frac{3}{4}, \qquad \overline{\Lambda} = 2\overline{\Sigma}, \qquad \rho_D^3 + \rho_{LS}^3 = 0, \; ...$$

$$\rho_{\pi G}^3 = -2\rho_{\pi\pi}^3, \qquad \rho_A^3 + \rho_{\pi G}^3 = -(\rho_{\pi\pi}^3 + \rho_S^3), \; ...$$

A chain of higher-order corrections is suppressed

Let us neglect perturbative corrections

Miracles of the BPS limit

- $\varrho^2 = \frac{3}{4}$

- No power corrections to $M = m_Q + \overline{\Lambda}$ for the ground state

$$M_B - M_D = m_b - m_c \text{ to all orders in } 1/m_Q$$

All $1/m_Q^k$ terms in the Hamiltonian annihilate the ground state

Foldy-Wouthuysen transformation is trivial on the ground state at rest; no lower component of the Q bispinor appears to any order

- For $B \to D$ amplitude

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2) \text{ to any order in } 1/m_Q$$

- Zero recoil $B \to D$ amplitude: $\delta_{1/m^k} = 0$

regardless of mass ratio

- In $B \to D$ at zero recoil

$$f_+ = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \qquad \text{to all orders in } 1/m_Q$$

- At arbitrary velocity power corrections in $B \to D$ vanish (or only kinematic)

$$f_+(q)^2 = \frac{M_B+M_D}{2\sqrt{M_B M_D}}\;\xi\left(\frac{M_B^2+M_D^2-q^2}{2M_B M_D}\right)$$

Decay rate directly gives the IW function

Experiment: $B \to D$ slope much closer to $\varrho^2 \simeq 0.9$

$B \to D^*$: $\delta_{1/m^2} = -(1+\chi)\left(\frac{\mu_G^2}{6m_c^2}+\ldots\right)$

usually assume $0<\chi<1$

$1-\frac{\mu_G^2}{6m_c^2}$ itself is just normalization, what about the overlap χ, maybe χ is also small?

No, even in the BPS limit spin symmetry receives large corrections, χ is significant in $B \to D^*$ and δ_{1/m^2}, δ_{1/m^3}, ... are not suppressed at all

N.U. 2001

$$\mathrm{BPS}: \qquad F_{D^*} \lesssim 0.9$$

Likewise corrections to the `shape` of the $B \to D^*$ formfactor are way too significant

Irreducible uncertainties:

$$e^{-\frac{2m_c}{\mu_{\rm hadr}}} \sim \text{ a few } \%$$

Quantifying Corrections to BPS

BPS limit is not exact in QCD. How significant are corrections to its relations? It depends

The deviation parameter ? The most natural would be

$\alpha = \|(\vec{\sigma}\vec{\pi})\,|B\rangle\| \equiv \sqrt{\mu_\pi^2 - \mu_G^2}$ – dimensionfull...

Dimensionless parameter is

$$\beta = \|\pi_0^{-1}(\vec{\sigma}\vec{\pi})\,|B\rangle\| \equiv \sqrt{3\left(\varrho^2 - \tfrac{3}{4}\right)} = 3\left(\sum_n |\tau_{1/2}^{(n)}|^2\right)^{\frac{1}{2}}$$

Numerically β is not a too small number, similar in size to generic $1/m_c$ expansion parameter

However, $\mu_\pi^2 - \mu_G^2 \propto \beta^2$ should be good

We can count together powers of $1/m_c$ and β to judge the real quality of the HQ relations

At which order in β the BPS relations can be violated?

Not difficult to answer to the leading orders in $1/m_Q$...

Need classification in β to **all orders** in $1/m_Q$

N.U. 2003

- Absence of corrections to $M_D = m_c + \overline{\Lambda}$,
  $M_B - M_D = m_b - m_c$ holds up to β^2

- $\overline{\Sigma} = \frac{\overline{\Lambda}}{2}$, $\rho_{LS}^3 = -\rho_D^3$, $\rho_A^3 + \rho_{\pi G}^3 = -(\rho_{\pi\pi}^3 + \rho_S^3)$ hold up to β^2,
  but $\rho_{\pi G}^3 = -2\rho_{\pi\pi}^3$ only to the leading order in BPS

- Zero recoil $B \to D$ amplitude is unity up to β^2

- At arbitrary velocity relation between f_+ and f_- in $B \to D$ holds only to the leading order

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D}\, f_+(q^2) + \mathcal{O}(\beta)$$

- At arbitrary velocity the relations between $f_\pm$ in $B \to D$ and the IW function may receive corrections $\propto \beta^1$

- f_+ near zero recoil receives only second order corrections in β to any order in $1/m_Q$:

$$f_+\left((M_B - M_D)^2\right) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} + \mathcal{O}(\beta^2)$$

Analogue of the Ademollo-Gatto theorem for the BPS expansion

the same applies to f_-

Must be quite accurate, f_-/f_+ can be checked in $B \to D\, \tau \nu_\tau$

Semileptonic B decays with $\tau\nu_\tau$

$B \to D\,\tau\nu_\tau$ amplitude does not vanish at $\vec{q} \to 0$ due to f_- although still suppressed

can check the BPS relation between f_+ and f_-

Velocity range is limited, $1 \le w \le 1.43$ and $\mathrm{BR}(B \to D\,\tau\nu_\tau)$ is well predicted in terms of $\varrho^2 - \frac{3}{4}$

$\mathrm{Inclusive}$ width $B \to X_c\,\tau\nu_\tau$ is very sensitive to $m_b - m_c$ (for light leptons $m_b - 0.6\,m_c$)

Lattices may be reliable for $m_b - m_c$ from M_Υ, M_{B_c} and $M_{J/\Psi}$

Energy release is limited, $m_b - m_c - m_\tau \simeq 1.6\,\mathrm{GeV}$

sensitive to higher power corrections
probe of duality violation
possible effects of nonperturbative charm operators

$\langle B|\,\bar{b}c\,\bar{c}b\,|B\rangle$ BBMU 2003

More possibilities if these decays are feasible!

Can we separately measure inclusive width for vector and axial? A good test of inclusive vs. exclusive HQE

Conclusions:

Inclusive studies yield crucial info for HQ physics, even for exclusive amplitudes Formerly viewed as antipodes

Power corrections to HQ symmetry are very significant in charm. There is a subset of relations which are stable, they are limited to the ground-state pseudoscalar B and D mesons, but exclude `spin` symmetry for charm

Experiment must verify the actual value of the kinetic expectation value, with higher accuracy and fidelity
in inclusive decays

$B \to D$ decays can be reliable theory-wise
in the BPS case

If $\mu_\pi^2 \lesssim 0.43\,\mathrm{GeV}^2$ is confirmed then

$\mathcal{F}_+(0) \simeq 1.03$ is an accurate prediction for $B \to D$

Many nontrivial consequences of the BPS regime

Slope ϱ^2 is close to 1-

Fits of $B \to D^*$ should incorporate constraints on $\hat{\varrho}^2$

$B \to D^{(*)}\,\tau\nu$ and $B \to X_c\,\tau\nu$ offer a number of interesting possibilities