PQCD Approach For B Decays

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The pQCD collaboration

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Outline

1. Introduction of The PQCD Approach

2. Some Results (CP asymmetry in $B \to \pi^+\pi^-$)

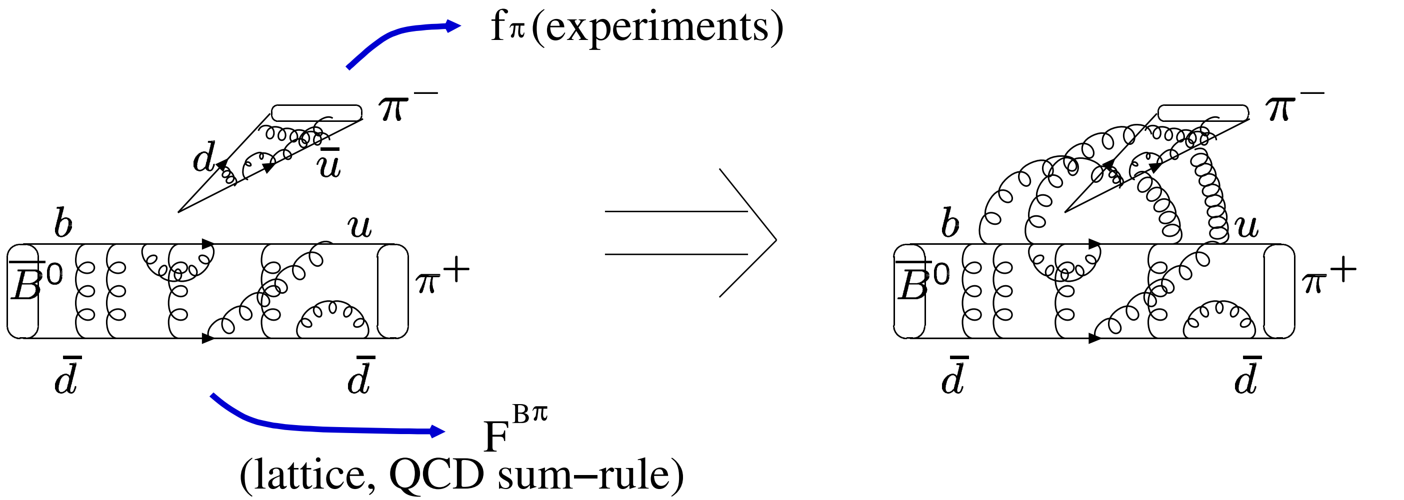
3. Theoretical Uncertainties (Inputs from the Light-Cone QCD Sum-Rule)

4. Future Prospects

5. Conclusions

Hadronic B Decays vs Theoretical Uncertainties

While there are plenty of interesting programs in hadronic two body decays, such as ϕ_2 determination in $B \to \pi\pi$, ϕ_3 determination in $B \to K\pi$, the strong interaction complicates our computation.



Naive factorisation

Non-factorisable contributions?!

$\longrightarrow$ **Attempts to go beyond the naive factorisation**

✍ **Perturbative QCD Approach** Keum, Li, Sanda, see e.g. PRD63 (2001)

✍ **QCD Factorisation** Beneke, Buchalla, Neubert, Sachrajda, see e.g. NPB606(2001)

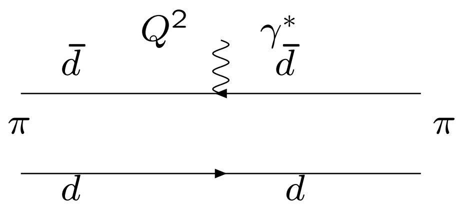
Inclusion of $\mathcal{O}(\alpha_s)$ corrections to the naive factorisation approximation. Factorisation has been shown neglecting $\mathcal{O}(1/m_b)$.

Perturbative QCD for Exclusive B decays

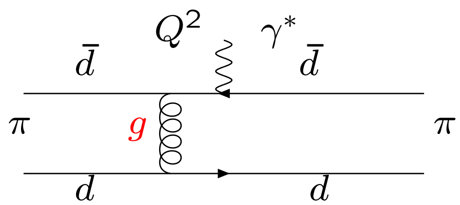
Based on the calculation of electromagnetic pion form factor at large Q^2.

P.Lepage, S.Brodsky, Phys. Rev. D22(1980)

H.-n.Li, G.Sterman, Nucl. Phys. B381, (1992)



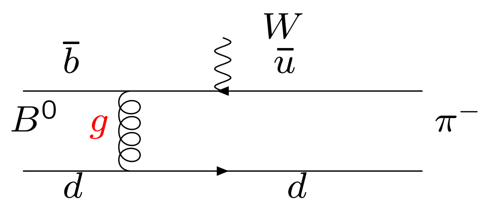
$\rightarrow$



$\longrightarrow$ applied to $B \longrightarrow \pi$ transition form factor

R.Akhoury, G.Sterman, Y.P.Yao, Phys. Rev. D 50(1994)

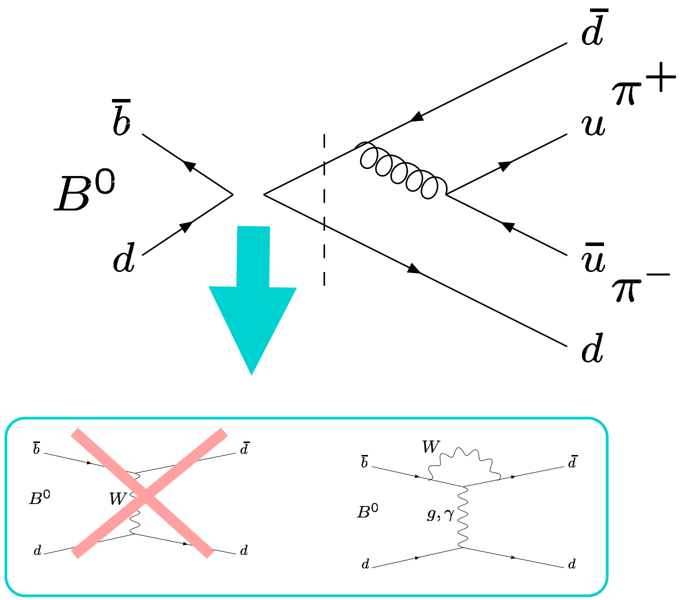
H.-n.Li, H.L.Yu, Phys. Rev. D53(1996)



Hard gluon exchange is crucial!

Importance of Annihilation Diagrams

Y.Y. Keum, H.-n. Li, A.I. Sanda, PLB504 (2001)



Annihilation diagrams had been neglected due to:

- α_s suppressed $\longrightarrow$ **Not in pQCD**
- $\frac{1}{m_b}$ suppressed comparing to the emission diagrams.
- Angular momentum conservation forbids the $V - A$ currents ($O_{1\sim4}$) by a factor of m_π^2 (as $\pi \longrightarrow e\bar{\nu}$).

However, $V + A$ currents ($O_{5,6}$) remain accompanied by the chiral enhancement factor $m_0^\pi = m_\pi^2/(m_u + m_d)$.

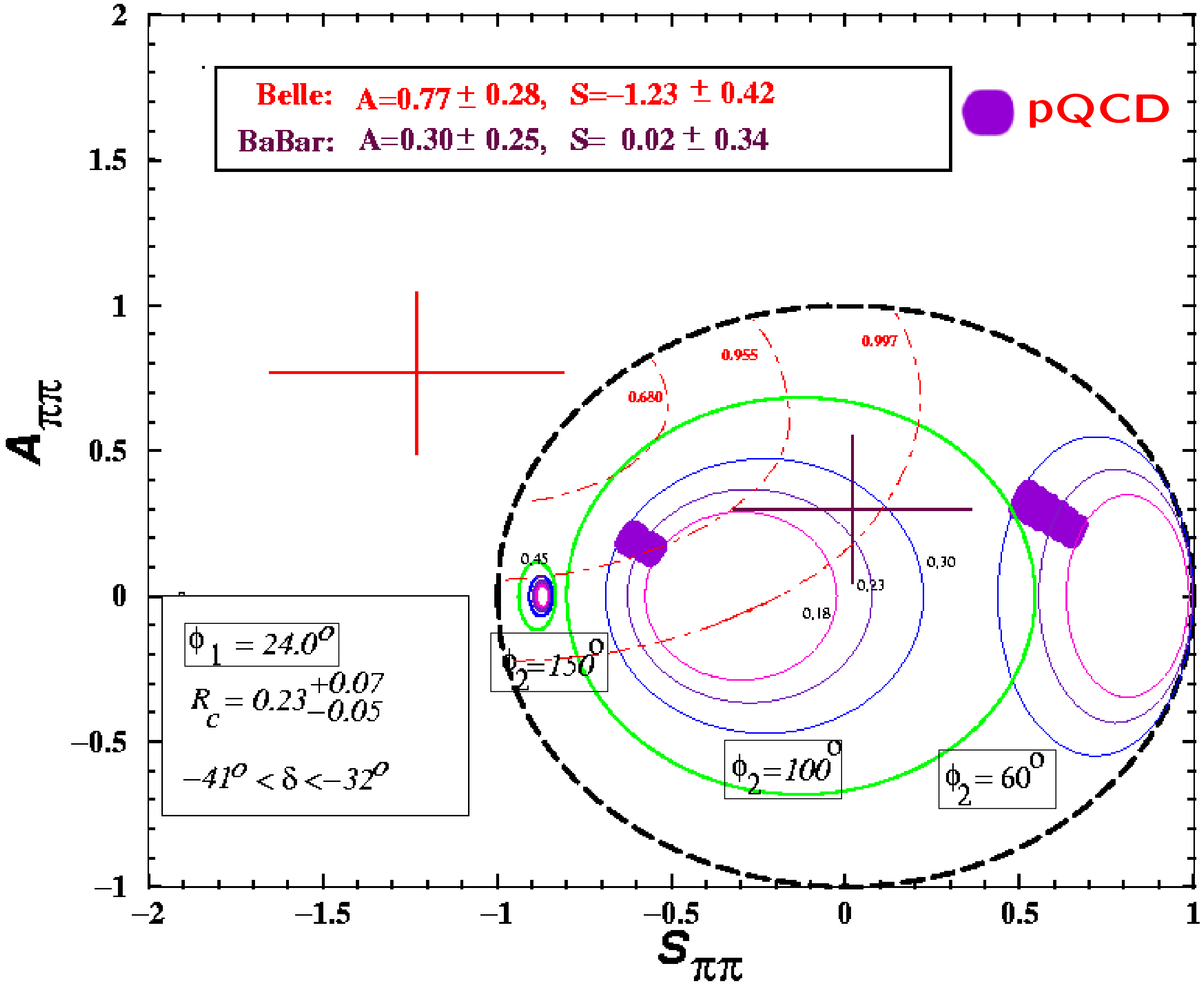
Furthermore, we found that:

☛ The large absorptive part arises from cuts on the intermediate state.

☛ The strong phase associated with $O_{5,6}$ annihilation diagrams is nearly 90° in $B \longrightarrow \pi\pi$ as well as $B \longrightarrow K\pi$.

CP Violation in $B \to \pi^+\pi^-$

Y.Y. Keum and A.I. Sanda, PRD67 (2003)



Thanks to Y.Y. Keum for the figure!

$$A_{CP} = \frac{\Gamma(\overline{B}^0 \to \pi^+\pi^-) - \Gamma(B^0 \to \pi^+\pi^-)}{\Gamma(\overline{B}^0 \to \pi^+\pi^-) + \Gamma(B^0 \to \pi^+\pi^-)} = S_{\pi\pi} \sin \Delta M_B \Delta t + A_{\pi\pi} \cos \Delta M_B \Delta t$$

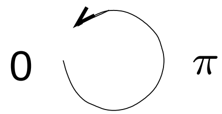
$$S_{\pi\pi} = \frac{2Im(\lambda_{\pi\pi})}{|\lambda_{\pi\pi}|^2 + 1}$$

$$A_{\pi\pi} = \frac{|\lambda_{\pi\pi}|^2 - 1}{|\lambda_{\pi\pi}|^2 + 1}$$

✔ $R_c = |P/T|$

0.18 0.23 0.30

✔ $\delta = \delta_P - \delta_T$



PQCD predictions: $|P/T| = 0.23^{+0.07}_{-0.05}$ and $\delta_P - \delta_T = -41^\circ \sim -32^\circ$

Form Factor Calculation in PQCD

see. e.g. Y.Y. Keum, H.-n. Li, A.I. Sanda, PRD63 (2001)

The form factor is written as a convolution of the distribution amplitude and the hard scattering amplitude:

$$\langle \pi(P_2)|\bar{b}j_\mu u|B(P_1)\rangle = \int_0^1 dx_1 dx_2 \int_0^\infty db_1 db_2$$
$$\mathcal{P}_\pi(x_2, b_2, P_2, \mu) T_H(x_1, x_2, b_1, b_2, Q, \mu) \mathcal{P}_B(x_1, b_1, P_1, \mu)$$

where x_i and b_i are momentum fraction and impact parameter of the quark inside meson, respectively. $Q^2 = -(P_2 - P_1)^2$.

- Distribution Amplitude

$$\mathcal{P}_M(x, b, P, \mu) =$$
$$\exp\left[-s(x, b, Q) - s(1 - x, b, Q) - 2\int_{1/b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu}))\right] \Psi_M(x, 1/b, P)$$

where $s(x, b, Q)$ is Sudakov exponent. Ψ_M denotes a wave function of meson M.

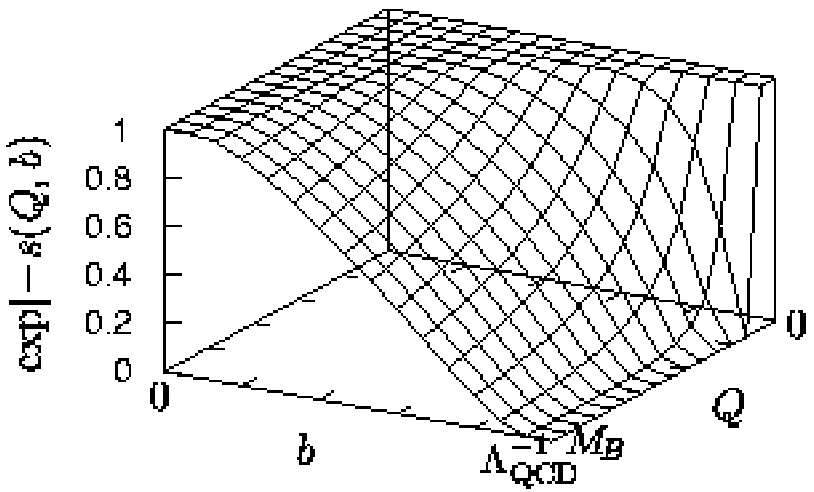
- Hard Scattering Amplitude

$$T_H(x_1, x_2, b_1, b_2, Q, \mu) \sim \int \frac{d^2\mathbf{k}_{\perp 1,2}}{(2\pi)^2} \exp[-i\mathbf{k}_{\perp 1,2} \cdot \mathbf{b}_{1,2}]$$
$$\frac{C_F}{x_1 x_2 Q^2 + (\mathbf{k}_{\perp 1} - \mathbf{k}_{\perp 2})^2} \frac{1}{(x_2 Q^2 + \mathbf{k}_{\perp 2}^2)} \exp\left[4\int_{\mu}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(g(\bar{\mu}))\right]$$

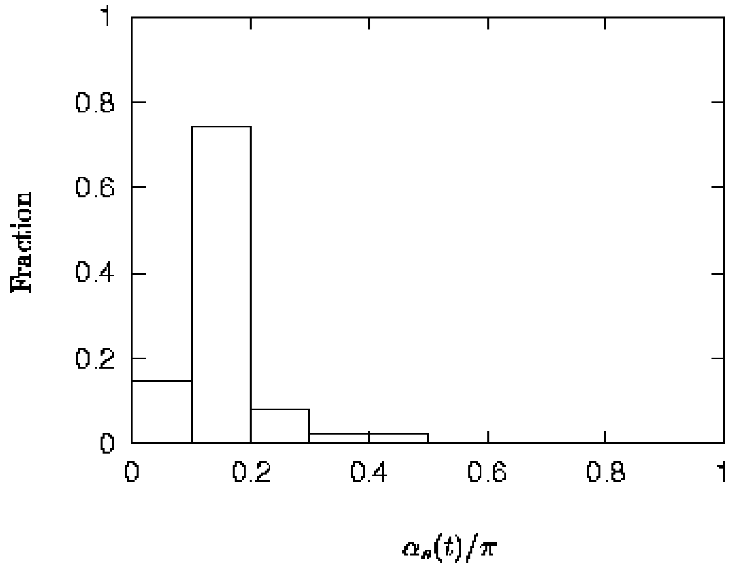
where t is the largest scale appearing in T_H, $t = max(\sqrt{x} M_B, 1/b)$.

Sudakov Suppression and Applicability of PQCD

Sudakov Suppression



Applicability of PQCD



Wave Functions For Light Mesons

$$\Psi_M(P,x,\zeta) \equiv \not{P}\phi_M^A(x) + m_0^M \phi_M^P(x) + \zeta m_0^M (\not{v}\not{n} - v\cdot n)\phi_M^{\sigma\prime}(x)$$

where P and x are the momentum and the momentum fraction of meson M, respectively.

see e.g. P. Ball JHEP 9809(1998)

$$\begin{aligned}
\langle \pi^-(P)|\bar{d}(z)\gamma_\mu\gamma_5 u(0)|0\rangle &\equiv -i\frac{f_\pi}{N_c}P_\mu \int_0^1 dx e^{ixP\cdot z}\phi_\pi^A \\
\langle \pi^-(P)|\bar{d}(z)\gamma_5 u(0)|0\rangle &\equiv -i\frac{f_\pi}{N_c}m_0^{\pi^-} \int_0^1 dx e^{ixP\cdot z}\phi_\pi^P \\
\langle \pi^-(P)|\bar{d}(z)\sigma_{\mu\nu}\gamma_5 u(0)|0\rangle &\equiv -i\frac{f_\pi}{6N_c}m_0^{\pi^-} \int_0^1 dx e^{ixP\cdot z}\phi_\pi^{\sigma\prime}
\end{aligned}$$

We include up to the second (first) terms of the Gegenbauer expansion of the distribution amplitudes ϕ_π^A and ϕ_π^P (ϕ_π^T) in our calculation.

A relatively large theoretical uncertainty occurs from the parameters m_0^i $(i = e.g. \pi, K)$:

$$m_0^\pi \equiv \frac{m_\pi^2}{(m_u+m_d)}, \qquad m_0^K \equiv \frac{M_K^2}{m_d+m_s}.$$

Wave Functions For B Mesons

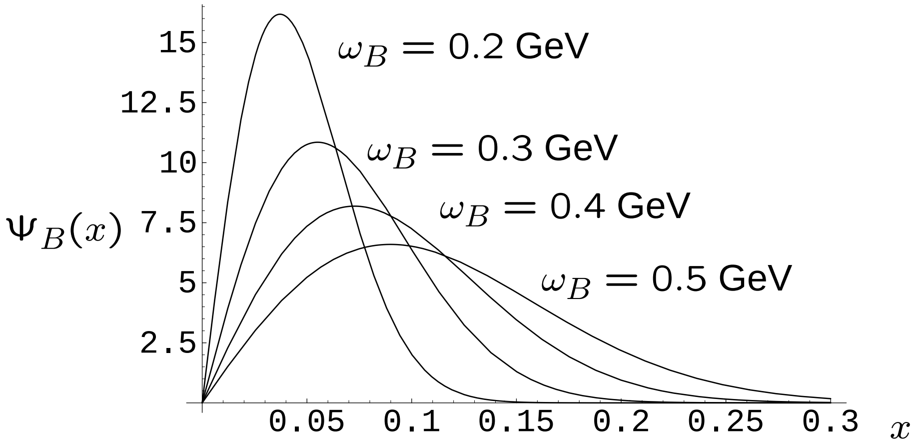
see e.g. M. Bauer and M. Wirbel, Z. Phys. C42(1989)

A.G.Grozin and M. Neubert, Phys. R ev. D55(1997)

$$\Psi_B(x,b) = N_B x^2(1-x)^2 exp[-\frac{1}{2}(\frac{xM_B}{\omega_B})^2 - \frac{\omega_B^2 b^2}{2}]$$

where x is the momentum fraction carried by the spectator. $\Psi_B(x,b)$ is normalised by:

$$\int_0^1 dx \Psi_B(x, b=0) = \frac{f_B}{2\sqrt{2N_C}}$$



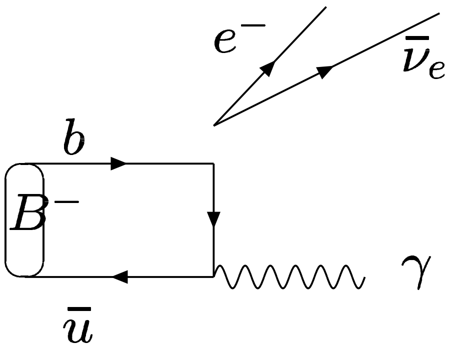
☛ Our results are sensitive to ω_B. On the other hand, since this wave function must be process independent, ω_B are constrained by our analysis for various modes.

Our present best fit value is:

$$\omega_B \simeq 0.4 \text{ GeV}$$

B Meson Properties and $B \to \gamma e \nu$ Process

$B \to \gamma e \nu$ Process



✍The only hadron in the decay is the B meson, making it easier to focus on the properties of Ψ_B.

G.P. Korchemsky, D. Pirjol and T.-M Yan, PRD61(2000)

S. Descotes-Genon and C.T. Sachrajda, NPB650(2003)

E. Lunghi, D. Pirjol and D. Wyler, NPB649(2003)

Factorisation is shown in the framework of the QCD factorisation (BBNS):

$$F^{\text{hard}}(E_\gamma) = \frac{f_B m_B Q_u}{2\sqrt{2}E_\gamma} \int_0^\infty dk_+ \frac{\Phi_+^B(k_+)}{k_+} \equiv \frac{f_B m_B Q_u}{2E_\gamma \lambda_B}$$

In general, one can argue that $\lambda_B \simeq \Lambda_{QCD}$ and use $\lambda_B \simeq 0.35$ GeV.

We can evaluate λ_B by using Light-Cone QCD Sum-Rule!

Evaluation of ω_B in Light-Cone QCD Sum-Rule

P. Ball and E.K. JHEP04(2003)

The Sum-Rule for the $B \to \gamma$ form factor is written as:

$$e^{-\overline{\Lambda}/\tau}\frac{f_B^2 m_B^2}{m_b E_\gamma}\frac{1}{\lambda_B} = \frac{3}{\pi^2 E_\gamma}\int_0^{\omega_0} d\omega\omega e^{-\omega/\tau}$$

where at the heavy quark limit, τ and ω_0 are related to the the Borel parameter and the continuum threshold as $M^2 \to 2m_b\tau$ and $s_0 \to m_b^2 + 2m_b\omega_0$.

Since the Sum-Rule for the statistic limit of the decay constant $f_{\text{stat}}^2 = f_B^2 m_B^2/m_b$ is also known, we can write λ_B as:

$$\lambda_B = \frac{\int_0^{\omega_0} d\omega\omega^2 e^{-\omega/\tau}}{\int_0^{\omega_0} d\omega\omega e^{-\omega/\tau}}$$

Using the optimised values of the continuum threshold and the Borel parameter, which depend on the $\overline{m_b} = (4.22 \pm 0.08)$ GeV, we obtain:

$$\lambda_B = 0.56 \sim 0.60 \text{ GeV} \to \boxed{\omega_B = 0.48 \sim 0.51 \text{ GeV}}$$

Future Prospect

✎ Theoretical Test

The NLO calculation is extremely important for pQCD approach. PQCD collaboration has already started climbing this high mountain.

✌The contributions from the chromomagnetic operator is now computable! S. Mishima and A.I. Sanda, hep-ph/0305073

✎ Phenomenological Test

✔ Use of pure annihilation processes such as $B \to D_s K^{0(*)}$

Li and C.-D. Lü, hep-ph/0305278

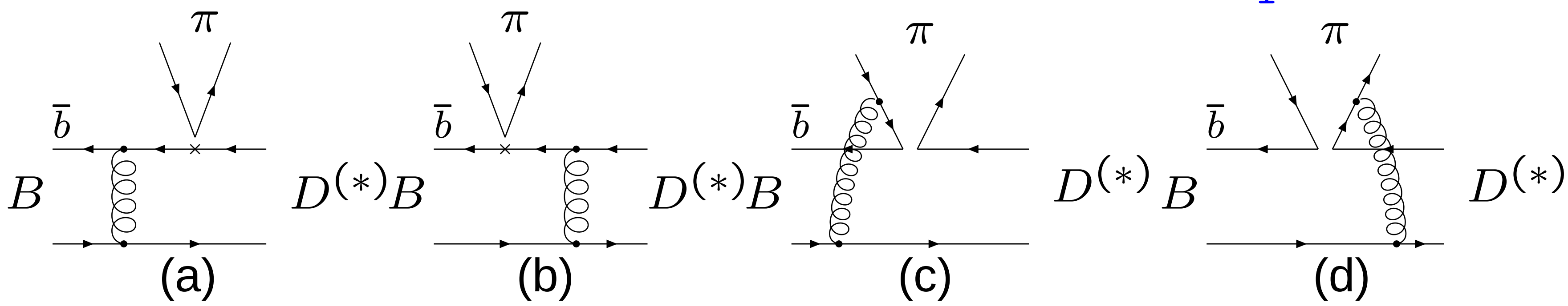
✔ Test of nonfactorisable contributions by $B \to D^{(*)}\pi$

Y.Y. Keum, T. Kurimoto, H.-N. Li, C.-D. Lü and A.I. Sanda, hep-ph/0305335

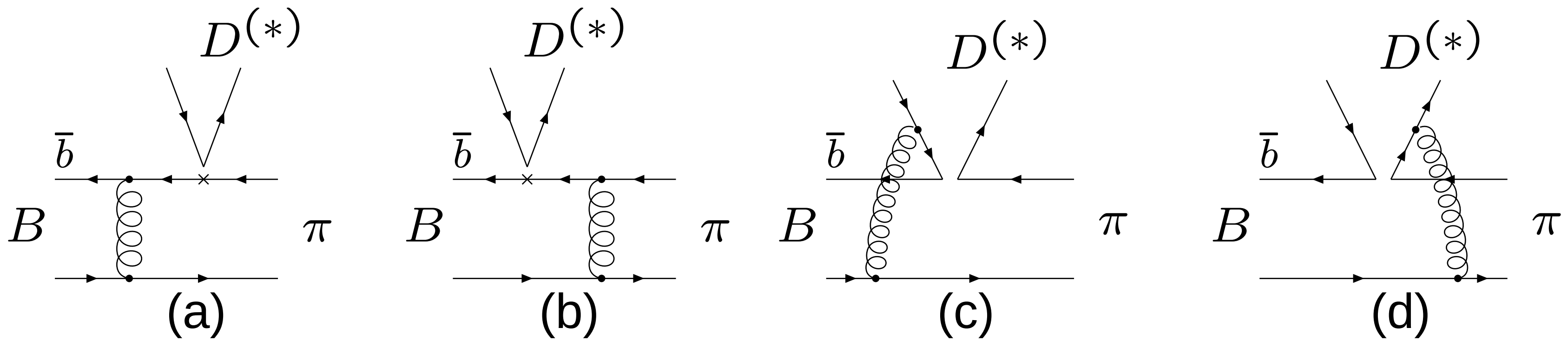
a_2/a_1 in PQCD Approach

Recent measurements indicate rather large value of a_2 and large imaginary part in a_2/a_1.

CLASS I: Color-Allowed Factorisable and Nonfactorisable $\longrightarrow a_1$



CLASS II: Color-Suppressed Factorisable and Nonfactorisable $\longrightarrow a_2$



Numerical Result on $B \to D\pi$ Mode

Amp	$C_D = 0.6$	$C_D = 0.8$	$C_D = 1.0$	
$f_\pi \xi_{\text{ext}}$	6.90	7.46	8.01	$\rightarrow$ CLASSI factorisable
$f_D \xi_{\text{int}}$	-1.44	-1.44	-1.44	$\rightarrow$ CLASSII factorisable
$f_B \xi_{\text{exc}}$	$-0.01 - 0.03i$	$-0.02 - 0.03i$	$-0.02 - 0.03i$	$\rightarrow$ Annihilation factorisable
$\mathcal{M}_{\text{ext}}$	$-0.24 + 0.57i$	$-0.25 + 0.60i$	$-0.27 + 0.65i$	$\rightarrow$ CLASSI non-factorisable
$\mathcal{M}_{\text{int}}$	$3.34 - 3.02i$	$3.22 - 3.07i$	$3.10 - 3.12i$	$\rightarrow$ CLASSII non-factorisable
$\mathcal{M}_{\text{exc}}$	$-0.26 - 0.89i$	$-0.31 - 0.95i$	$-0.37 - 1.02i$	$\rightarrow$ Annihilation non-factorisable

Amplitude in units of 10^{-2} GeV. C_D is a parameter entering to the wave function of D meson, which can be determined by the semileptonic $B \to Dl\nu$ process.

Numerical Results on $B \to D^*(\pi, \rho, \omega)$ Modes

Quantities	$C_D = 0.6$	$C_D = 0.8$	$C_D = 1.0$	Data
$B(\bar{B}^0 \to D^+\pi^-)$	2.37	2.74	3.13	3.0 ± 0.4
$B(\bar{B}^0 \to D^0\pi^0)$	0.26	0.25	0.24	0.29 ± 0.05
$B(B^- \to D^0\pi^-)$	4.96	5.43	5.91	5.3 ± 0.5
$\lvert a_2/a_1 \rvert$ (w/o anni.)	0.47(0.51)	0.43(0.46)	0.39(0.42)	
$Arg(a_2/a_1)$ (w/o anni.)	$-42.5^o(-61.5^o)$	$-41.6^o(-63.5^o)$	$-41.9^o(-65.3^o)$	

Quantities	$C_{D^*} = 0.5$	$C_{D^*} = 0.7$	$C_{D^*} = 0.9$	Data
$B(\bar{B}^0 \to D^{*+}\pi^-)$	2.16	2.51	2.88	2.76 ± 0.21
$B(\bar{B}^0 \to D^{*0}\pi^0)$	0.29	0.28	0.27	0.17 ± 0.05
$B(B^- \to D^{*0}\pi^-)$	4.79	5.26	5.75	4.60 ± 0.40
$\lvert a_2/a_1 \rvert$ (w/o anni.)	0.52 (0.55)	0.47 (0.50)	0.43 (0.47)	
$Arg(a_2/a_1)$ (w/o anni.)	$-40.5^o(-61.4^o)$	$-40.7^o(-63.1^o)$	$-40.8^o(-64.8^o)$	

Branching ratio is in units of 10^{-3}.

Branching ratios	$C_D = 0.6$	$C_D = 0.8$	$C_D = 1.0$	Data
$B(\bar{B}^0 \to D^+\rho^-)$	5.31	6.16	7.06	7.8 ± 1.4
$B(\bar{B}^0 \to D^0\rho^0)$	0.15	0.15	0.15	
$B(B^- \to D^0\rho^-)$	8.74	9.85	11.0	13.4 ± 1.8
$B(\bar{B}^0 \to D^0\omega)$	0.14	0.14	0.14	
Branching ratios	$C_{D^*} = 0.5$	$C_{D^*} = 0.7$	$C_{D^*} = 0.9$	Data
$B(\bar{B}^0 \to D^{*+}\rho^-)$	4.89	5.67	6.51	7.3 ± 1.5
$B(\bar{B}^0 \to D^{*0}\rho^0)$	0.41	0.41	0.42	< 0.56
$B(B^- \to D^{*0}\rho^-)$	10.53	11.72	13.02	15.5 ± 3.1
$B(\bar{B}^0 \to D^{*0}\omega)$	0.69	0.71	0.75	< 0.74

Conclusions

- PQCD approach is one of the most promising attempts to go beyond the naive factorisation approximation.

- We emphasised the importance of the annihilation diagrams, which produce a large strong phase through $O_{5,6}$.

- We showed our result for the CP asymmetry in $B \to \pi^+\pi^-$. Our predictions $\boxed{P/T = (0.23^{+0.07}_{-0.05}) \text{ and } \delta_P - \delta_T = -41^\circ \sim -32^\circ}$ accompanied by the Babar result determine $\phi_2 = 55^\circ \sim 100^\circ$.

- We discussed the theoretical errors in our calculation, which is mainly caused by the parameters in distribution amplitudes of mesons.

- We showed that our best fit value of the parameter characterising B meson, $\omega_B \simeq 0.4$ is comparable to the latest Light-Cone QCD Sum-Rule result.