

Theory introduction to charm physics



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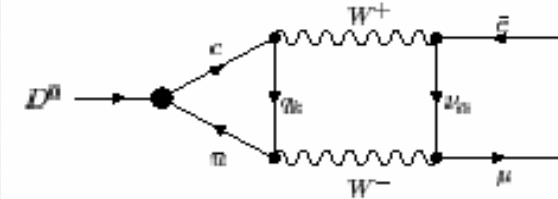
1. Introduction
2. Spectroscopy
3. Leptonic and semileptonic decays
4. Mixing and Rare decays
5. Conclusions and outlook

1. Introduction: role of charm

1. Charm transitions serve as excellent probes of New Physics

1. Processes **forbidden** in the Standard Model **to all orders** (or very rare)

Examples: $D^0 \rightarrow \mu^+ e^-$



2. Processes **forbidden** in the Standard Model **at tree level**

Examples: $D^0 - \bar{D}^0$ mixing, $D \rightarrow X\gamma$, $D \rightarrow X\nu\bar{\nu}$

3. Processes **allowed** in the Standard Model

Examples: relations, valid in the SM, but not necessarily in general

2. Provide unique QCD laboratory

Start from the bottom...

Introduction

Murphy's law:

Modern charm physics experiments acquire ample statistics; many decay rates are quite large.

THUS:

It is very difficult to provide model-independent theoretical description of charmed quark systems.

2. Charm Spectroscopy

- HQL: Charm spectroscopy is “simple”

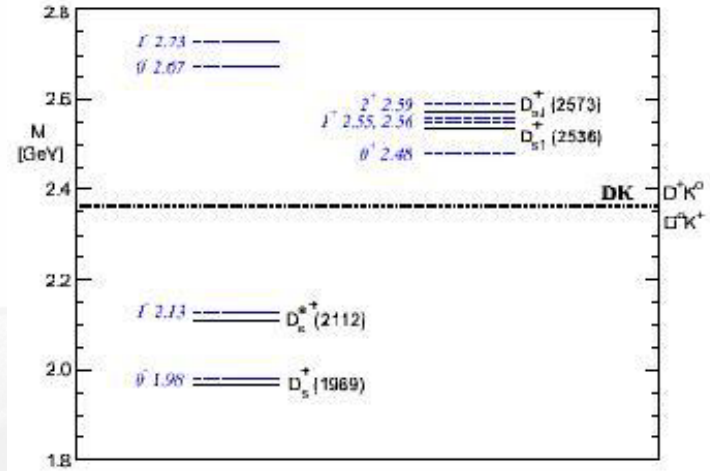
$$\vec{S} = \vec{J}_l + \vec{S}_Q, \quad \vec{J}_l = \vec{S}_l + \vec{L}_l$$

good quantum numbers decouples

- All states appear as doublets classified by parity and spin of light DoF:

$$S^P = J_l^P \pm \frac{1}{2}$$

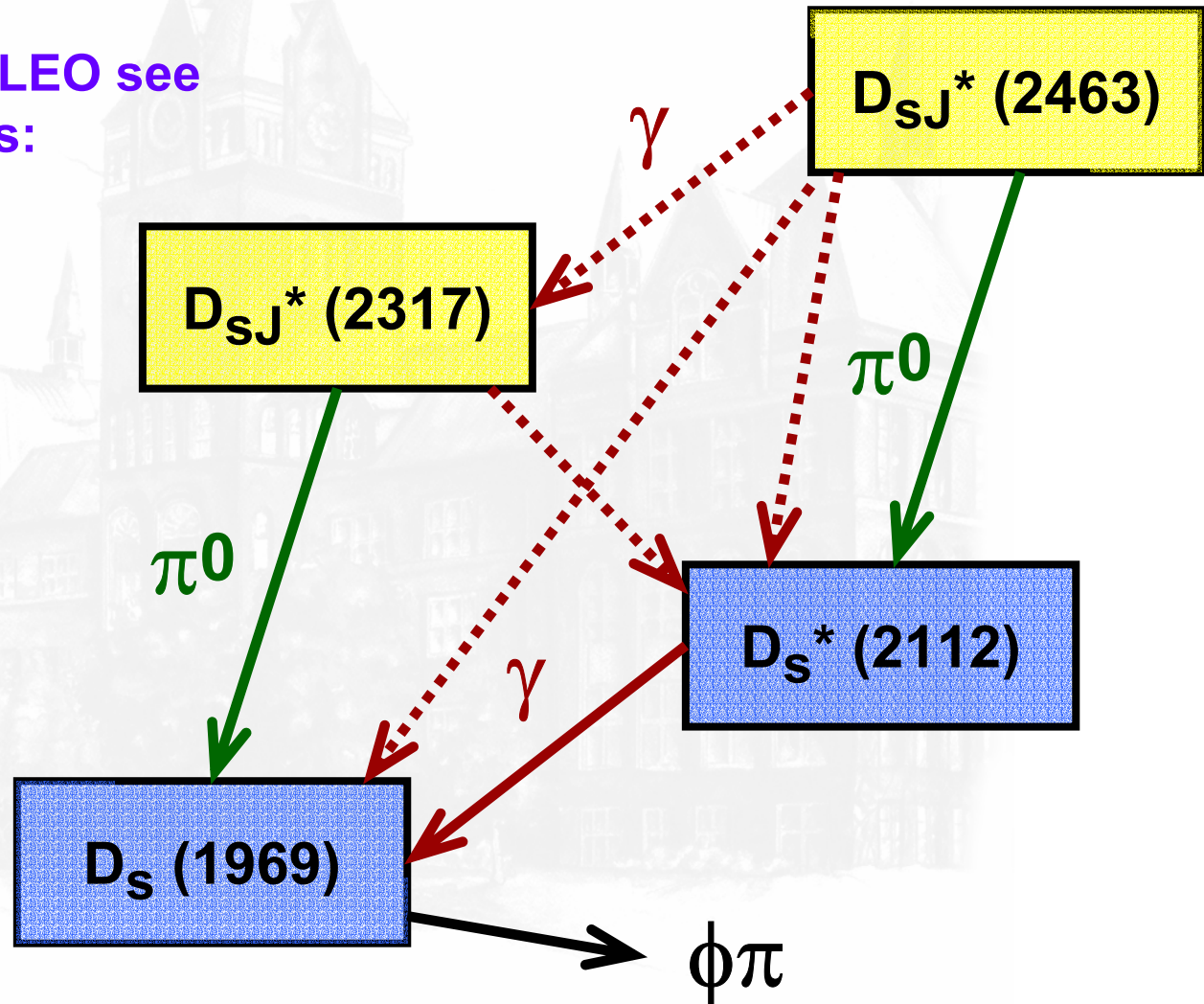
<i>L</i>	0	1	2
<i>J_l</i>	1/2	1/2, 3/2	3/2, 5/2
<i>S</i>	0,1	0,1	1,2



Spin		<i>state</i>	Exp. Data, MeV	
<i>J_l^P</i>	<i>S^P</i>		<i>M</i>	<i>Γ</i>
1/2 ⁻	0 ⁻	<i>D_s</i>	1969	0.49 ps
	1 ⁻	<i>D_s[*]</i>	2110	<1.9
1/2 ⁺	0 ⁺	<i>D₀[*]</i>		
	1 ⁺	<i>D₁[']</i>		
3/2 ⁺	1 ⁺	<i>D_{s1}</i>	2536	<2.3
	2 ⁺	<i>D_{s2}</i>	2572	15

Charm Spectroscopy: new states

- BaBar/Belle/CLEO see new D_{sJ}^* states:



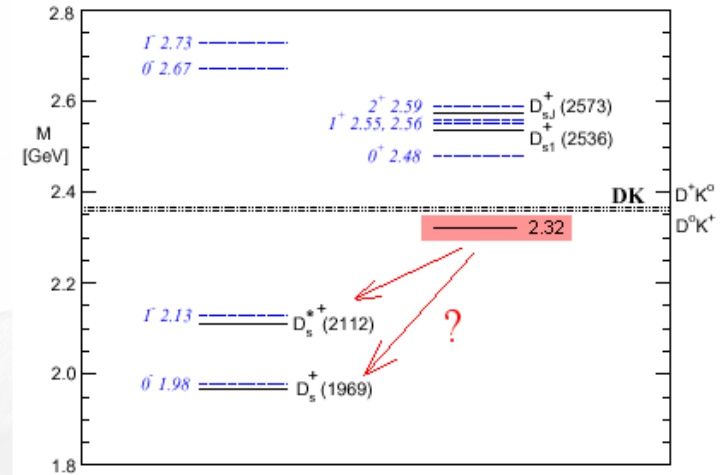
Charm Spectroscopy: problem?

➤ BaBar/Belle/CLEO report new D_{sJ} states

$$D_{sJ}^*(2317) \rightarrow D_s \pi^0, D_{sJ}(2463) \rightarrow D_s^* \pi^0$$

$$\not\rightarrow D_s^* \gamma, D_s^* \gamma \gamma, D_s \pi^+ \pi^-$$

➤ Interpretation? 0^+ and 1^+ p-wave Qq states?



Possible problems:

1. Mass is too low?
2. Width is too narrow?

non- Qq state?

Barnes, Close, Lipkin;
Szczepaniak, Bali

4-quark (“baryonium”) state

➤ DK or $D\pi$ molecule?

1. mass is naturally in the vicinity of DK threshold
2. since $M(D_{sJ}(2130)) < M(D+K)$ width is naturally small

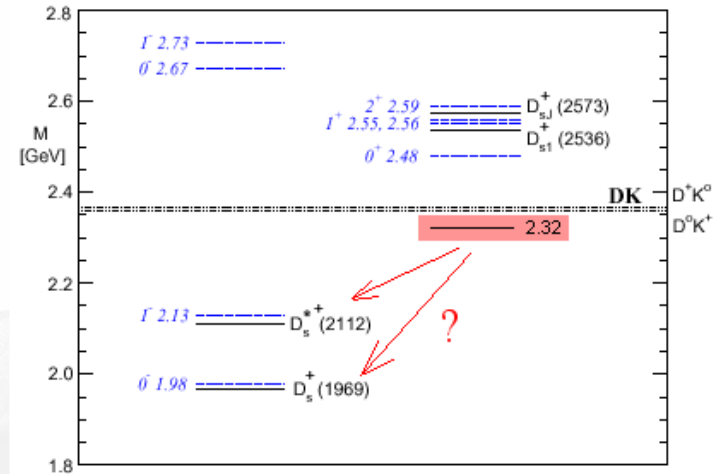
Charm Spectroscopy: problem?

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➤ Interpretation? 0^+ and 1^+ p-wave Qq states!



Possible problems:

1. Mass is too low?
2. Width is too narrow?

Broken chiral symmetry: positive parity-partners of $D_s D_s^*$

Bardeen, Eichten, Hill

Reference	Mass
Ebert et. al. (98)	2.51 GeV
Godfrey-Isgur (85)	2.48 GeV
DiPierro-Eichten (01)	2.49 GeV
Gupta-Johnson (95)	2.38 GeV
Zeng et. al. (95)	2.38 GeV

Charm Spectroscopy

New states:

1. Why is $M(D_{sJ}^*(2130)) < M(D+K)$ and $M(D_{sJ}^*(2130)) < M(D^*+K)$?

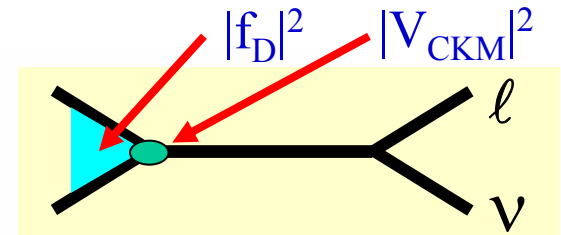
Van Beveren and Rupp

2. Interpretation? Radiative decays?

Godfrey; Colangelo and De Fazio

3. Similar states in D and B systems?

3. Leptonic and semileptonic decays



Form-factors and decay constants

- Heavy quark symmetry relates observables in B and D transitions

Example 1: decay constants

$$\langle 0 | A_\mu(0) | X(p) \rangle = f_X p_\mu$$

HQS requires:

$$\frac{f_B}{f_D} = \sqrt{\frac{M_D}{M_B}} + O(1/M)$$

Large!

HQS+Chiral symmetry:

$$\frac{f_{B_s}/f_B}{f_{D_s}/f_D} = 1 + O(m_s) \times O(1/m_b - 1/m_c)$$

$$\frac{\Delta M_d}{\Delta M_s} \propto \left[\frac{\sqrt{B_{B_d}} f_{B_d}}{\sqrt{B_{B_s}} f_{B_s}} \right]^2 \left[\frac{|V_{td}|}{|V_{ts}|} \right]^2$$

$$\frac{f_{D_s}}{f_D} = 1 - \frac{5}{6} (1 + 3g^2) \frac{M_K^2}{16\pi^2 f^2} \log[M_K^2/\mu^2] + \dots$$

$D^* D \pi$ coupling constant

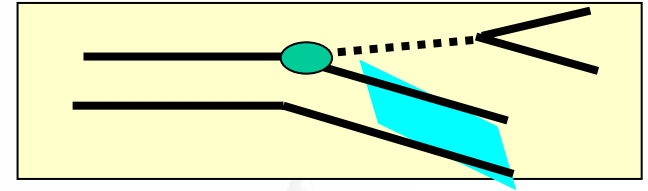
- CLEO-c is expected to provide accurate measurements

	Reaction	CLEO-c CM Energy (MeV)	L fb ⁻¹	PDG	CLEO-c
f_{D_s}	$D_s^+ \rightarrow \mu\nu$	4140	3	17%	1.7%
f_{D_s}	$D_s^+ \rightarrow \tau\nu$	4140	3	33%	1.6%
f_{D^+}	$D^+ \rightarrow \mu\nu$	3770	3	UL	2.3%

- If charm production data is used to obtain V_{cs} ($\delta V_{cs}/V_{cs} \sim 1.3\%$), the ratio gives information about decay constants
 - input for lattice calculations

$$|V_{CKM}|^2 |f_+(q^2)|^2$$

Form-factors and decay constants



- Heavy quark symmetry relates observables in B and D transitions

Example 2: decay form-factors

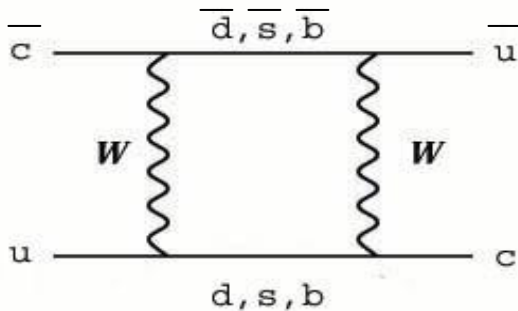
$$\langle P | j_\mu | X(p) \rangle = f_+^{XP}(q^2) (p_X + p_P)_\mu + f_-^{XP}(q^2) (p_X - p_P)_\mu$$

- If charm production data is used to obtain V_{cs} ($\delta V_{cs}/V_{cs} \sim 1.3\%$), the ratio gives information about decay form factors

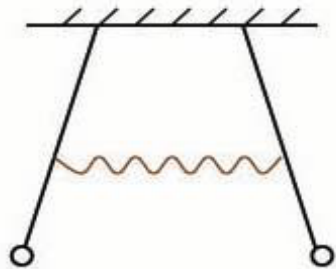
- q^2 shape can be measured
- input for lattice calculations

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{cs}|^2 p_K^3 |f_+(q^2)|^2$$

4. D^0 - \bar{D}^0 mixing



D-D mixing



Coupled oscillators

$\Delta Q=2$: only at one loop in the Standard Model:
possible **new physics** particles in the loop

$\Delta Q=2$ interaction couples dynamics of D^0 and \bar{D}^0

$$|D(t)\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = a(t) |D^0\rangle + b(t) |\bar{D}^0\rangle$$

- Time-dependence: coupled Schrödinger equations

$$i \frac{\partial}{\partial t} |D(t)\rangle = \left(M - \frac{i}{2} \Gamma \right) |D(t)\rangle = \begin{bmatrix} A & p^2 \\ q^2 & A \end{bmatrix} |D(t)\rangle$$

- Diagonalize: mass eigenstates \neq flavor eigenstates

$$|D_{1,2}\rangle = p |D^0\rangle \pm q |\bar{D}^0\rangle$$

Mass and lifetime differences of mass eigenstates: $x = \frac{M_2 - M_1}{\Gamma}$, $y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$

CP violation in charm

- Possible sources of CP violation in charm:

- CPV in **decay amplitudes** (“direct” CPV)

$$A(D \rightarrow f) \neq A(\bar{D} \rightarrow \bar{f})$$

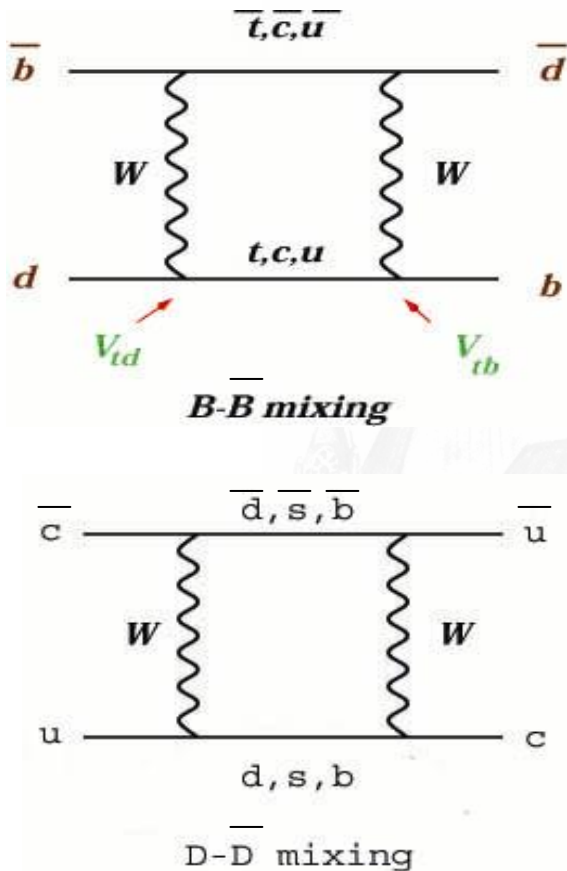
- CPV in $D^0 - \bar{D}^0$ **mixing matrix**

$$R_m^2 = \left| \frac{p}{q} \right|^2 = \frac{2M_{12} - i\Gamma_{12}}{2M_{12}^* - i\Gamma_{12}^*} \neq 1$$

- CPV in the **interference of decays with and without mixing**

$$\lambda_f = \frac{q}{p} \frac{\overline{A}_f}{A_f} = R_m e^{i(\phi+\delta)} \left| \frac{\overline{A}_f}{A_f} \right|$$

Mixing: why do we care?



$\bar{D}^0 - D^0$ mixing	$\bar{B}^0 - B^0$ mixing
<ul style="list-style-type: none"> intermediate down-type quarks SM: b-quark contribution is negligible due to V_{ub} rate $\propto f(m_s) - f(m_d)$ (zero in the SU(3) limit) <p>Falk, Grossman, Ligeti, A.A.P. (Phys.Rev. D65, 054034, (2002)): 2nd order in SU(3) breaking!!!</p>	<ul style="list-style-type: none"> intermediate up-type quarks SM: t-quark contribution is dominant rate $\propto m_t^2$ (expected to be large)
<ol style="list-style-type: none"> Sensitive to long distance QCD Small in the SM: New Physics! (must know SM x and y) 	<ol style="list-style-type: none"> Computable in QCD (*) Large in the SM: CKM!

(*) up to matrix elements of 4-quark operators

How would new physics affect mixing?

- Look again at time development:

$$i \frac{\partial}{\partial t} |D(t)\rangle = \left(M - \frac{i}{2} \Gamma \right) |D(t)\rangle = \begin{bmatrix} A & p^2 \\ q^2 & A \end{bmatrix} |D(t)\rangle$$

- Expand $\overline{D^0} - D^0$ mass matrix:

$$\left(M - \frac{i}{2} \Gamma \right)_{ij} = m_D^{(0)} \delta_{ij} + \frac{1}{2m_D} \langle D_i^0 | H_W^{\Delta C=2} | D_j^0 \rangle + \frac{1}{2m_D} \sum_I \frac{\langle D_i^0 | H_W^{\Delta C=1} | I \rangle \langle I | H_W^{\Delta C=1} | D_j^0 \rangle}{m_D^2 - m_I^2 + i\varepsilon}$$

Local operator, affects x,
possible **new physics**

Real intermediate states, affect
both x and y \Rightarrow **Standard Model**

1. $x \gg y$: signal for New Physics?

$x \approx y$: Standard Model?

2. CP violation in mixing/decay

new CP-violating phase ϕ

With b-quark contribution neglected:
only **2** generations contribute
 \Rightarrow **real 2x2 Cabibbo matrix**

Experimental constraints

1. Time-dependent $D^0(t) \rightarrow K^+\pi^-$ analysis

$$\Gamma[D^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |A_{K^+\pi^-}|^2 \left[R + \sqrt{R} R_m (y' \cos \phi - x' \sin \phi) \Gamma t + \frac{R_m^2}{4} (y^2 + x^2) (\Gamma t)^2 \right]$$

Sensitive to DCS/CF strong phase

2. Time-dependent $D^0(t) \rightarrow K^+K^-$ analysis (lifetime difference)

$$y_{CP} = \frac{\tau(D \rightarrow \pi^+ K^-)}{\tau(D \rightarrow K^+ K^-)} - 1 = y \cos \phi - x \sin \phi \frac{A_m}{2}$$

3. Semileptonic analysis $rate \propto x^2 + y^2$

Quadratic in x,y: not so sensitive

4. Time-independent analysis at tau-charm factory: (QM) entangled initial state

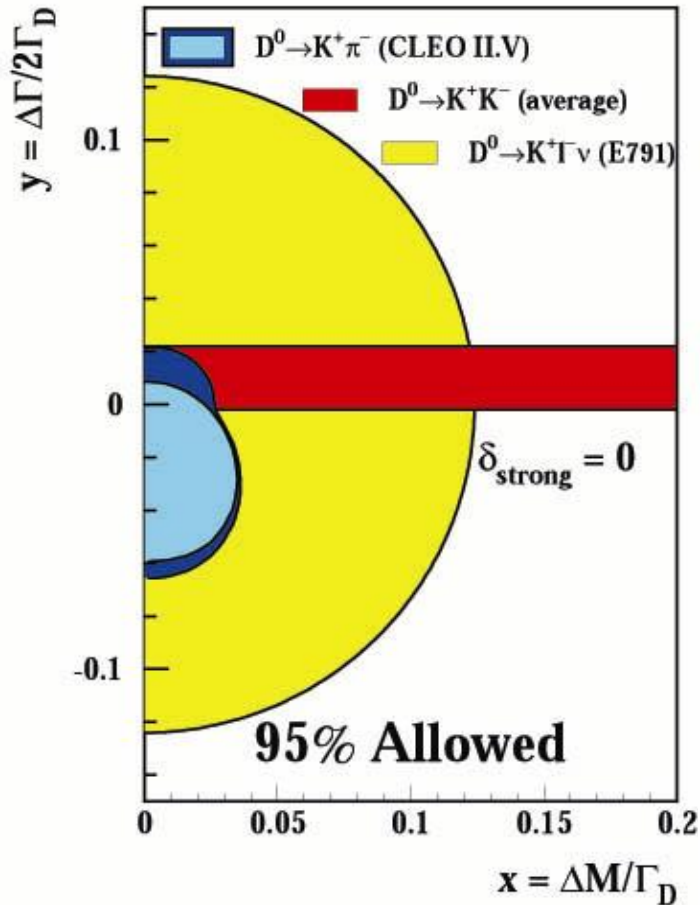
$$y \cos \phi = (-1)^L \sigma \frac{R_\sigma^L - 1}{R_\sigma^L}$$

$$R_\sigma^L = \frac{1}{Br(D^0 \rightarrow Xl\nu)} \frac{\Gamma[\psi_L \rightarrow (D \rightarrow [CP]_\sigma)(D \rightarrow Xl\nu)]}{\Gamma[\psi_L \rightarrow (D \rightarrow [CP]_\sigma)(D \rightarrow X)]}$$

D. Atwood and A.A.P., hep-ph/0207165

Experimental constraints 1

D^0 - \bar{D}^0 Mixing Limits



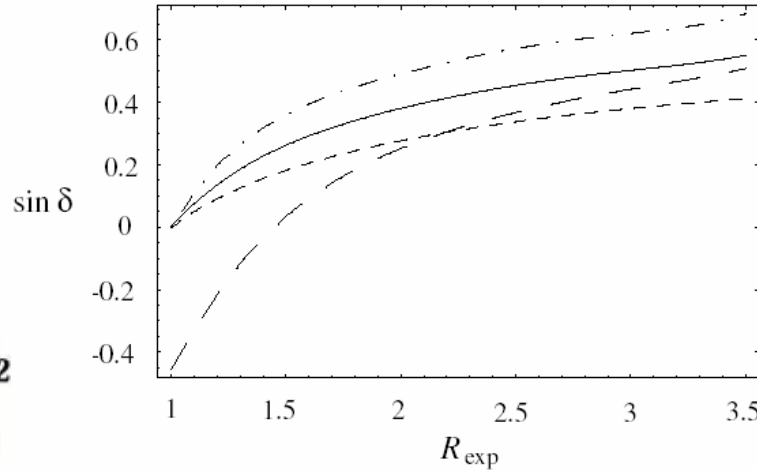
1. Time-dependent $D^0(t) \rightarrow K^+ \pi^-$ analysis

$$\Gamma[D^0(t) \rightarrow K^+ \pi^-] = e^{-\Gamma t} |A_{K^+ \pi^-}|^2 \times \left[R + \sqrt{R} R_m (y' \cos \phi - x' \sin \phi) \Gamma t + \frac{R_m^2}{4} (y^2 + x^2) (\Gamma t)^2 \right]$$

$$R = \left| \frac{A_{K^+ \pi^-}}{A_{\bar{K}^+ \pi^-}} \right|^2$$

$$\begin{aligned} x' &= x \cos \delta + y \sin \delta \\ y' &= y \cos \delta - x \sin \delta \end{aligned}$$

$$\begin{aligned} \text{for } D^0 \leftrightarrow \bar{D}^0: \\ R_m &\leftrightarrow R_m^{-1}, \quad x' \leftrightarrow -x' \end{aligned}$$



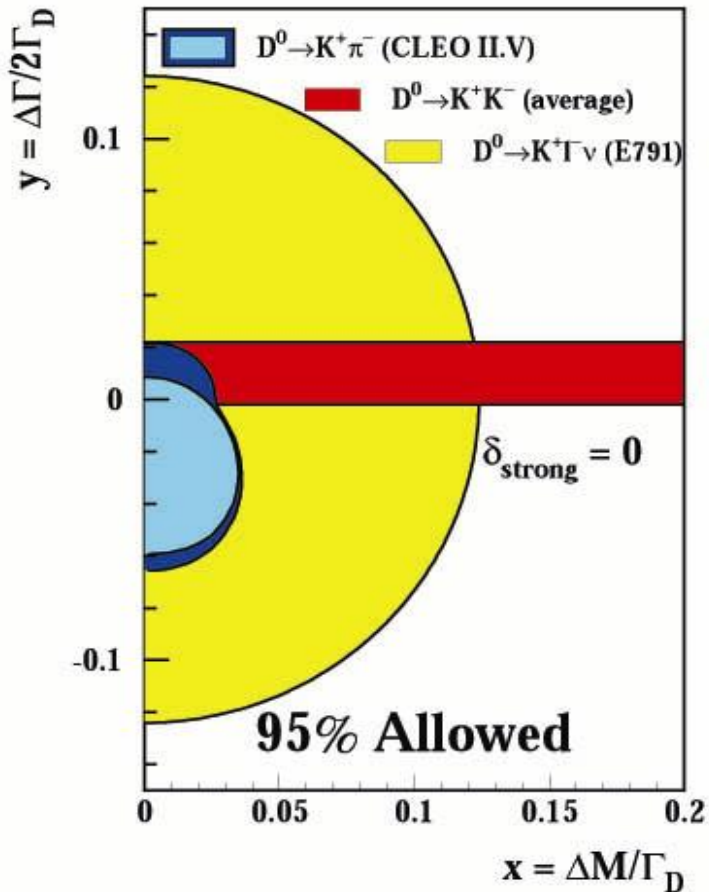
Strong phase is zero in the SU(3) limit

A. Falk, Y. Nir and A.A.P.,
JHEP 12 (1999) 019

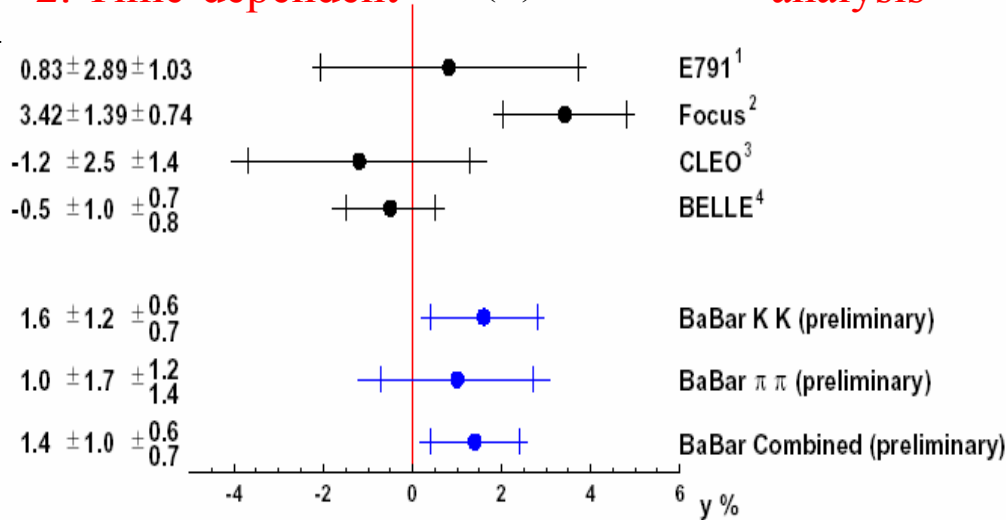
Can be measured at CLEO-c!

Experimental constraints 2

D^0 - \bar{D}^0 Mixing Limits



2. Time-dependent $D^0(t) \rightarrow K^+ K^-$ analysis

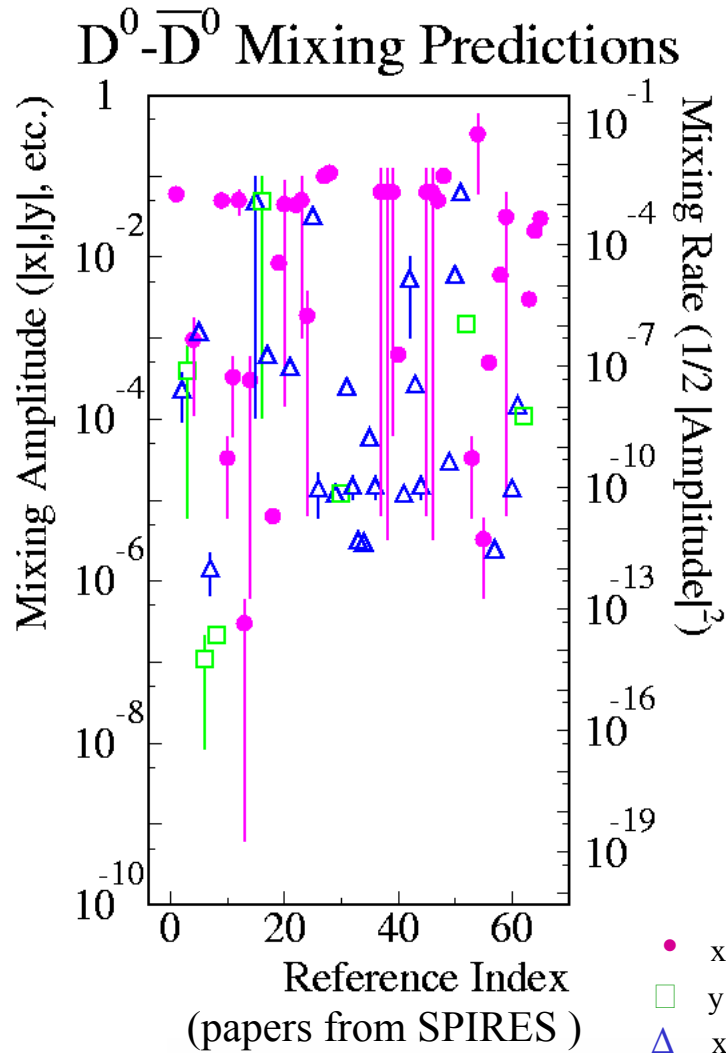


Experiment	Value
FOCUS (2000)	$(3.42 \pm 1.39 \pm 0.74)\%$
E791(2001)	$(0.8 \pm 2.9 \pm 1.0)\%$
CLEO (2002)	$(-1.2 \pm 2.5 \pm 1.4)\%$
Belle (2002)	$(-0.5 \pm 1.0^{+0.7}_{-0.8})\%$
BaBar (2002)	$(1.4 \pm 1.0^{+0.6}_{-0.7})\%$

World average: $y_{CP} = (1.0 \pm 0.7)\%$

What are the expectations for x and y?

Theoretical estimates



- Theoretical predictions are all over the board...
- Nevertheless, it can be that $y \sim 1\%$!

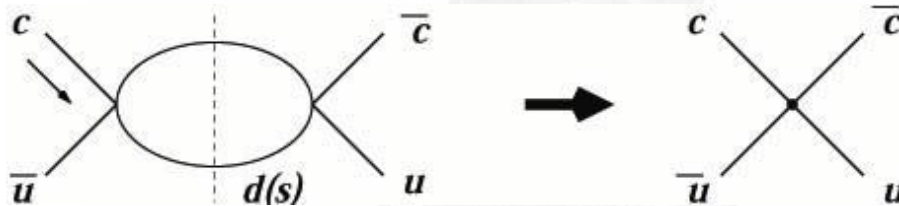
Falk, Grossman, Ligeti, A.A.P., Phys.Rev. D65, 054034, (2002)

- Seems like D-system is unique in that $y \gg x$!
- ... but sensitivity to new physics is reduced

Theoretical estimates I

A. Short distance gives a tiny contribution, consider y as an example

m_c IS large !!!



$$y = \frac{1}{m_D \Gamma} \langle D^0 | T | \bar{D}^0 \rangle$$

... as can be seen from the straightforward computation...

$$\begin{aligned} \rightarrow y_{sd} = & \frac{N_C + 1}{2 \pi N_C \Gamma} X_D \frac{(m_s^2 - m_d^2)^2}{m_c^2} \frac{m_s^2 + m_d^2}{m_c^2} [C_2^2 + 2C_1 C_2 + C_1^2 N_C \\ & - \frac{2(2N_C - 1) B_D'}{1 + N_C} \frac{M_D^2 C_2^2}{B_D (m_c + m_u)^2} \left(1 + \left(N_C \frac{C_1^2}{C_2^2} + 2 \frac{C_1}{C_2} \right) \frac{2 - N_C}{2N_C - 1} \right) \end{aligned}$$

with $\langle D^0 | \bar{u} \Gamma_\mu c \bar{u} \Gamma^\mu c | D^0 \rangle = \frac{1 + N_C}{N_C} \frac{4 F_D^2 m_D^2}{2 m_D} B_D, \text{ etc.}$

4 unknown matrix elements

similar for x (trust me!)

Theoretical estimates I

A. Short distance + “subleading corrections” (in $1/m_c$ expansion):

$$y_{sd}^{(6)} \propto \frac{(m_s^2 - m_d^2)^2}{m_c^2} \frac{m_s^2 + m_d^2}{m_c^2} \mu_{had}^{-2} \propto m_s^6 \Lambda^{-6}$$

$$x_{sd}^{(6)} \propto \frac{(m_s^2 - m_d^2)^2}{m_c^2} \mu_{had}^{-2} \propto m_s^4 \Lambda^{-4}$$

4 unknown matrix elements

...subleading effects?

$$y_{sd}^{(9)} \propto m_s^3 \Lambda^{-3}$$

$$x_{sd}^{(9)} \propto m_s^3 \Lambda^{-3}$$

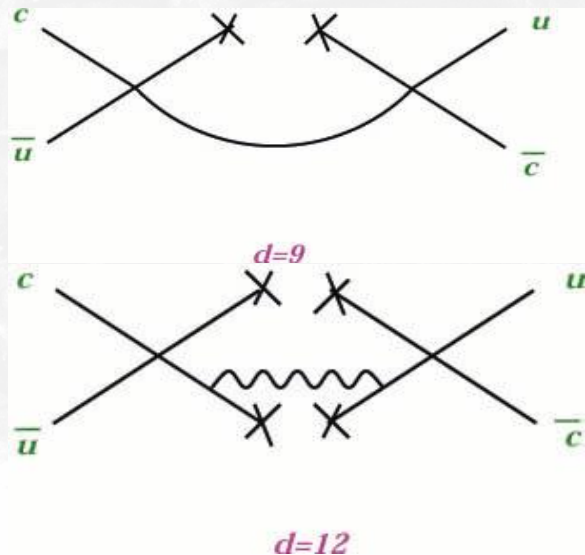
15 unknown matrix elements

Georgi, ...
Bigi, Uraltsev

$$y_{sd}^{(12)} \propto \beta_0 \alpha_s^2(\mu) m_s^2 \Lambda^{-2}$$

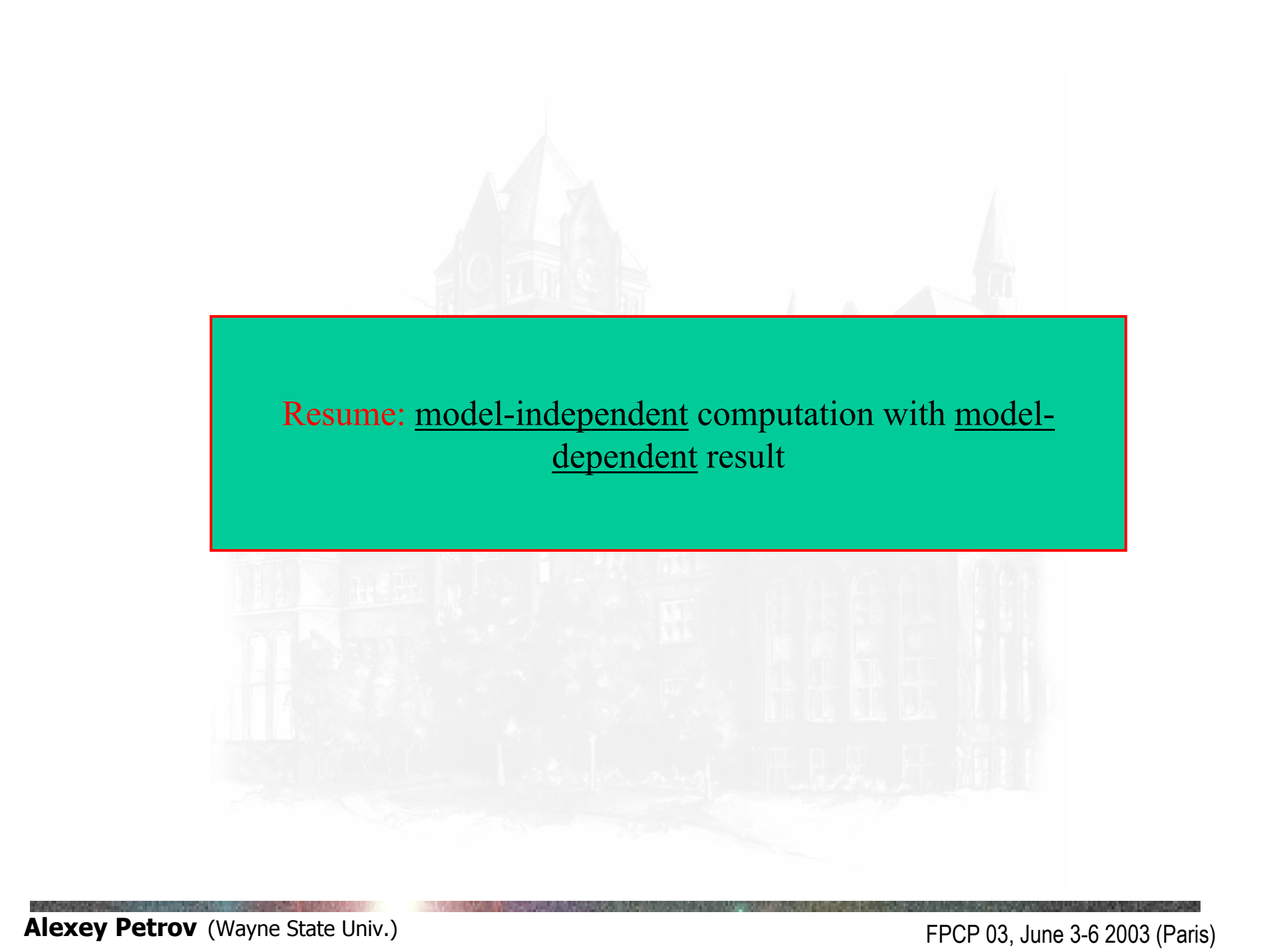
$$x_{sd}^{(12)} \propto \alpha_s(\mu) m_s^2 \Lambda^{-2}$$

Twenty-something unknown
matrix elements



Guestimate: $x \sim y \sim 10^{-3}$?

↳ **Leading contribution!!!**



Resume: model-independent computation with model-dependent result

Theoretical estimates II

B. Long distance physics dominates the dynamics...

m_c is NOT large !!!

$$y = \frac{1}{2\Gamma} \sum_n \rho_n \left[\langle D^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | \bar{D}^0 \rangle + \langle \bar{D}^0 | H_W^{\Delta C=1} | n \rangle \langle n | H_W^{\Delta C=1} | D^0 \rangle \right]$$

... with n being all states to which D^0 and \bar{D}^0 can decay. Consider $\pi\pi$, πK , KK intermediate states as an example...

$$y_2 = Br(D^0 \rightarrow K^+ K^-) + Br(D^0 \rightarrow \pi^+ \pi^-) - 2 \cos \delta \sqrt{Br(D^0 \rightarrow K^+ \pi^-) Br(D^0 \rightarrow \pi^+ K^-)}$$

Donoghue et. al.
Colangelo et. al.

cancellation expected!

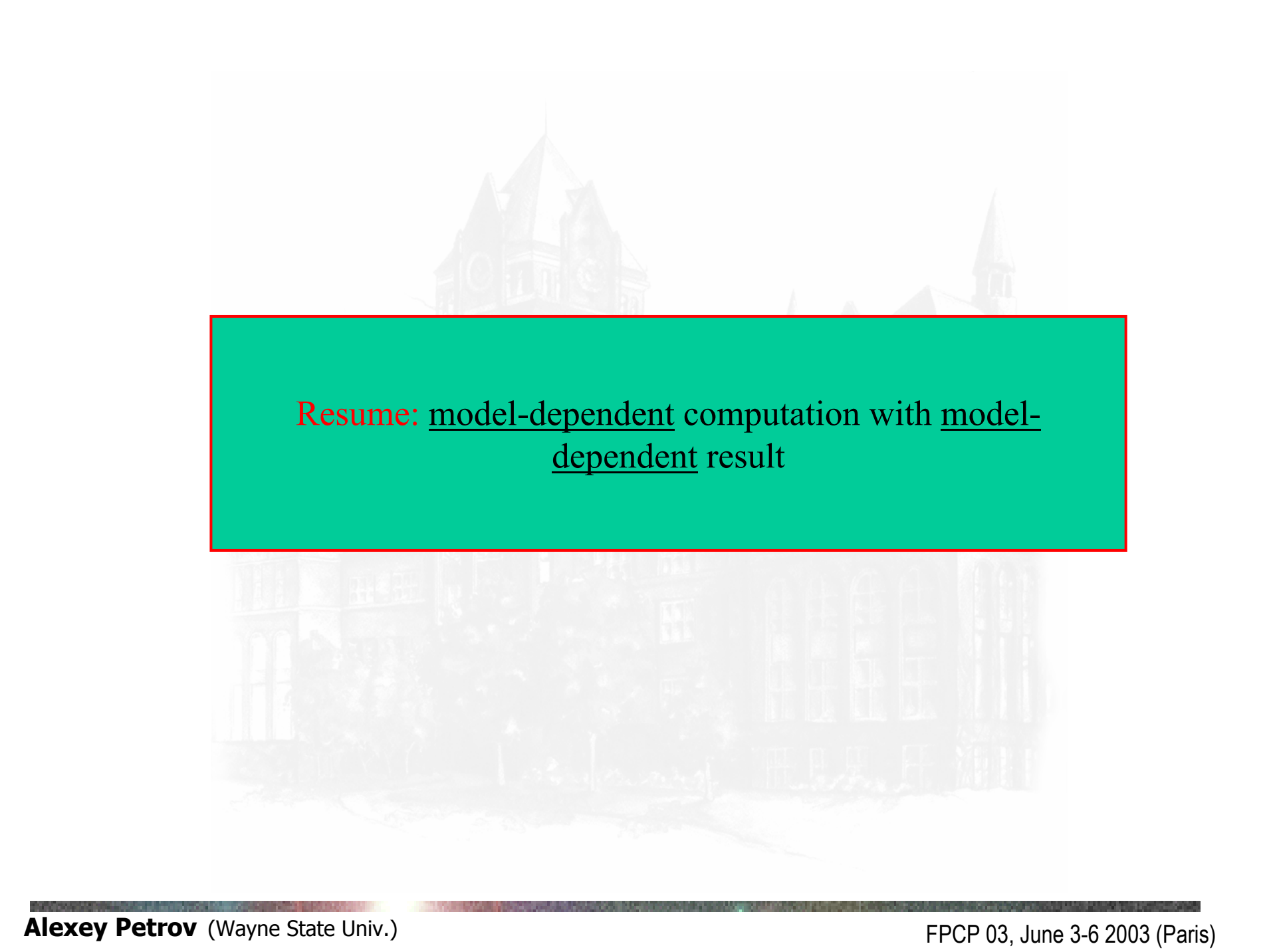
If every Br is known up to $O(1\%)$ \rightarrow the result is expected to be $O(1\%)!$

The result here is a series of large numbers with alternating signs, SU(3) forces 0

$x = ?$ Extremely hard...



need to restructure the calculation...



Resume: model-dependent computation with model-dependent result

Theoretical expectations: SU(3) breaking

- Neglecting the third generation, mixing arises at **second order** in SU(3) breaking

$$x, y \sim \sin^2 \theta_C \varepsilon_{SU(3)}^2$$

- Known counter-example:

Does not work if there is a **very narrow** light quark resonance with $m_R \sim m_D$

$$x, y \sim \frac{g_{DR}^2}{m_D^2 - m_R^2} \sim \frac{g_{DR}^2}{m_D^2 - m_0^2 - 2m_0\delta_R}$$

Most probably don't exist...

see E. Golowich and A.A.P.
Phys.Lett. B427, 172, 1998

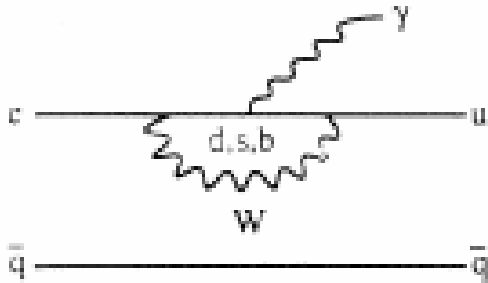
- What happens if part of the multiplet is **kinematically forbidden**?

Example: both $D^0 \rightarrow 4\pi$ and $D^0 \rightarrow 4K$ are from the same multiplet, but the latter is **kinematically forbidden**

Mixing is dominated by 4-body intermediate state contribution, incomplete cancellations naturally imply that $y \sim 1\%$

see A.F., Y.G., Z.L., and A.A.P.
Phys.Rev. D65, 054034, 2002

FCNC in charm: why do we care?



Rare charm decays	Rare beauty decays
<ul style="list-style-type: none"> • intermediate down-type quarks • SM: b-quark contribution is very small due to V_{ub} • $rate \propto f(m_s) - f(m_d)$ (zero in the SU(3) limit) 	<ul style="list-style-type: none"> • intermediate up-type quarks • SM: t-quark contribution is dominant • $rate \propto f(m_t^2)$ (expected to be large)
<ol style="list-style-type: none"> 1. Sensitive to long distance QCD 2. Sensitive to New Physics! 	<ol style="list-style-type: none"> 1. Computable in QCD (*) 2. Large in the SM: CKM!

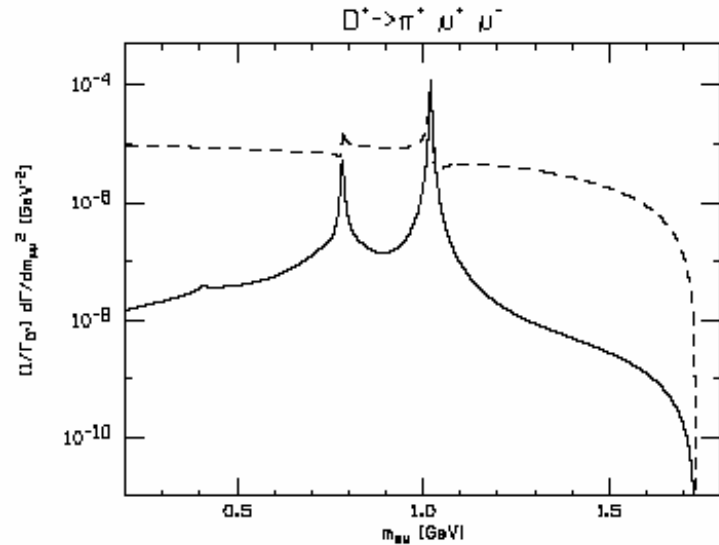
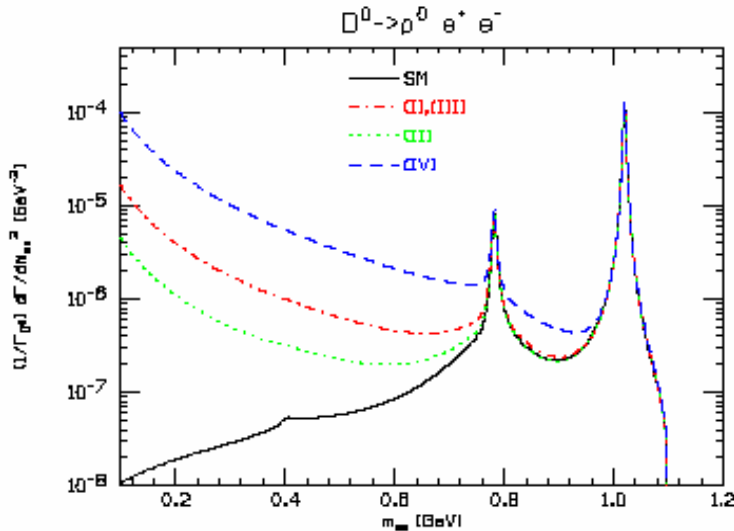
(*) depending on the process: OPE, factorization, ...

FCNC charm decays

1. In many cases NP contribution “gives a larger contribution” than the Standard Model

2. Example: \mathcal{R} SUSY

$$\delta H = -\frac{\tilde{\lambda}'_{i2k} \tilde{\lambda}'_{i1k}}{2m_{\tilde{d}_R}^2} (\bar{u}_L \gamma_\mu c_L) (\bar{l}_L \gamma^\mu l_L)$$



...or MSSM for different values of squark masses

see Burdman, Golowich, Hewett and Pakvasa
Phys.Rev. D66, 014009, 2002

5. Conclusions

➤ Did not talk about:

- Lifetimes and inclusive semileptonic decays
 - applications of $1/m$ techniques
- Charmed baryons and double-charmed baryons
 - issues in double-charmed baryon production
- Exclusive nonleptonic charm decays
 - direct CP violation
- Charmonium production and polarization
 - J/ψ production in e^+e^- collisions
- ...

Conclusions

- Spectroscopy: what are the new D_{sJ}^* states?
 - low mass triggers many possible explanations
- Leptonic and semileptonic decays
 - important inputs to B-physics/CKM extractions
- Charm mixing:
 - $x, y = 0$ in the SU(3) limit (as $V_{cb}^* V_{ub}$ is very small)
 - it is a **second** order effect
 - it is quite possible that $y \sim 1\%$ with $x < y$
 - expect new data from BaBar/Belle/CLEO/CLEO-c/CDF
- Observation of CP-violation or FCNC transitions in the current round of experiments are still “smoking gun” signals for New Physics



Additional Slides

Questions:

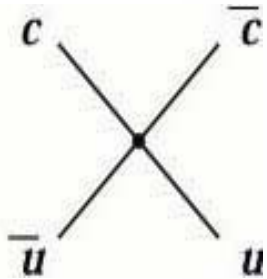
1. Can any model-independent statements be made for x or y ?

What is the order of SU(3) breaking?
i.e. if $x, y \propto m_s^n$ what is n?

2. Can one claim that $y \sim 1\%$ is natural?

Theoretical expectations

At which order in $SU(3)_F$ breaking does the effect occur? Group theory?



$$\langle D^0 | H_W H_W | \bar{D}^0 \rangle \Rightarrow \langle 0 | D H_W H_W D | 0 \rangle$$

is a singlet with $D \rightarrow D_i$ that belongs to $\mathbf{3}$ of $SU(3)_F$ (one light quark)

The $\Delta C=1$ part of H_W is $(\bar{q}_i c)(\bar{q}_j q_k)$, i.e. $3 \times \bar{3} \times \bar{3} = \bar{15} + 6 + \bar{3} + \bar{3} \Rightarrow H_k^{ij}$

$$O_{\bar{15}} = (\bar{s}d)(\bar{u}d) + (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) + s_1(\bar{u}c)(\bar{d}d) \\ - s_1(\bar{s}c)(\bar{u}s) - s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) - s_1^2(\bar{u}c)(\bar{d}s)$$

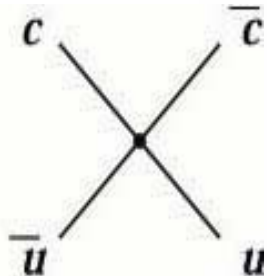


$$O_6 = (\bar{s}d)(\bar{u}d) - (\bar{u}c)(\bar{s}d) + s_1(\bar{d}c)(\bar{u}d) - s_1(\bar{u}c)(\bar{d}d) \\ - s_1(\bar{s}c)(\bar{u}s) + s_1(\bar{u}c)(\bar{s}s) - s_1^2(\bar{d}c)(\bar{u}s) + s_1^2(\bar{u}c)(\bar{d}s)$$

Introduce $SU(3)$ breaking via the quark mass operator $M_j^i = \text{diag}(m_u, m_d, m_s)$

All nonzero matrix elements built of D_i, H_k^{ij}, M_j^i must be $SU(3)$ singlets

Theoretical expectations



note that $D_i D_j$ is symmetric \Rightarrow belongs to $\mathbf{6}$ of $SU(3)_F$

$$\langle D^0 | H_W H_W | \bar{D}^0 \rangle \Rightarrow \langle 0 | \bar{D} H_W H_W D | 0 \rangle$$

Explicitly,

$$DD \Rightarrow D_6$$

$$H_W H_W \Rightarrow O_{\bar{6}_0} + O_{42} + O_{15'}$$

1. No $\bar{6}$ in the decomposition of $H_W H_W \Rightarrow$ **no** $SU(3)$ singlet can be formed

\Rightarrow **D mixing is prohibited by $SU(3)$ symmetry**

2. Consider a single insertion of $M_j^i \Rightarrow D_6 M$ transforms as $6 \times 8 = 24 + \bar{15} + 6 + \bar{3} \Rightarrow$ still **no** $SU(3)$ singlet can be formed

\Rightarrow **NO D mixing at first order in $SU(3)$ breaking**

3. Consider double insertion of $M \Rightarrow DMM : 6 \times (8 \times 8)_S = (60 + \bar{42}) + 24 + \bar{15} + \bar{15} + 6 + (24 + 15 + 6 + \bar{3}) + 6$

\Rightarrow **D mixing occurs only at the second order in $SU(3)$ breaking**

A.F., Y.G., Z.L., and A.A.P.
Phys.Rev. D65, 054034, 2002

Theoretical expectations: SU(3) breaking

- Two major sources of SU(3) breaking

1. phase space

$$m_K \neq m_\pi \neq m_\eta \dots$$

2a. matrix elements (absolute value)

$$f_K \neq f_\pi \dots$$

2b. matrix elements (phases a.k.a. FSI)

$$\text{Im} \frac{A(D^0 \rightarrow K^+ \pi^-)}{A(\bar{D}^0 \rightarrow K^+ \pi^-)} \neq 0$$

Take into account only the first source...

SU(3) and phase space

- “Repackage” the analysis: look at the complete multiplet contribution

$$y = \sum_{F_R} y_{F,R} Br(D^0 \rightarrow F_R) \sim \sum_{F_R} y_{F,R} \frac{1}{\Gamma} \sum_{n \in F_R} \Gamma(D^0 \rightarrow n)$$

y for each SU(3) multiplet

Each is **0** in SU(3)

- Does it help? If only phase space is taken into account: no (mild) model dependence

$$y_{F,R} = \frac{\sum_{n \in F_R} \langle \bar{D}^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \langle D^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}$$

$$= \frac{\sum_{n \in F_R} \langle \bar{D}^0 | H_W | n \rangle \rho_n \langle n | H_W | D^0 \rangle}{\sum_{n \in F_R} \Gamma(D^0 \rightarrow n)}$$

if CP is conserved

Can consistently compute

Example: PP intermediate states

- $n=PP$ transforms as $(8 \times 8)_S = 27 + 8 + 1$, take 8 as an example:

Numerator:

$$A_{N,8} = |A_0|^2 s_1^2 \left[\frac{1}{2} \Phi(\eta, \eta) + \frac{1}{2} \Phi(\pi^0, \pi^0) + \frac{1}{3} \Phi(\eta, \pi^0) + \Phi(\pi^+, \pi^-) - \Phi(\bar{K}^0, \pi^0) \right. \\ \left. + \Phi(K^+, K^-) - \frac{1}{6} \Phi(\eta, K^0) - \frac{1}{6} \Phi(\eta, \bar{K}^0) - \Phi(K^+, \pi^-) - \Phi(K^-, \pi^+) \right]$$

Denominator:

$$A_{D,8} = |A_0|^2 \left[\frac{1}{6} \Phi(\eta, K^0) + \Phi(K^+, \pi^-) + \frac{1}{2} \Phi(K^0, \pi^0) + O(s_1^2) \right]$$

phase space function

- This gives a calculable effect!

$$y_{2,8} = \frac{A_{N,8}}{A_{D,8}} = -0.038 s_1^2 = -1.8 \times 10^{-4}$$

1. Repeat for other states
2. Multiply by Br_{FR} to get y

Results

Final state representation	$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
<i>PP</i>	8	-0.0038
	27	-0.00071
<i>PV</i>	8 _S	0.031
	8 _A	0.032
	10	0.020
	$\overline{10}$	0.016
	27	0.040
<i>(VV)</i> _{s-wave}	8	-0.081
	27	-0.061
<i>(VV)</i> _{p-wave}	8	-0.10
	27	-0.14
<i>(VV)</i> _{d-wave}	8	0.51
	27	0.57

Final state representation	$y_{F,R}/s_1^2$	$y_{F,R}$ (%)
<i>(3P)</i> _{s-wave}	8	-0.48
	27	-0.11
<i>(3P)</i> _{p-wave}	8	-1.13
	27	-0.07
<i>(3P)</i> _{form-factor}	8	-0.44
	27	-0.13
<i>4P</i>	8	3.3
	27	2.2
	27'	1.9

- Product is naturally O(1%)
- No (symmetry-enforced) cancellations
- Does NOT occur for x

naturally implies that $y \sim 1\%$ and $x < y$!

Final state	fraction
<i>PP</i>	5%
<i>PV</i>	10%
<i>(VV)</i> _{s-wave}	5%
<i>(VV)</i> _{d-wave}	5%
<i>3P</i>	5%
<i>4P</i>	10%