# The phenomenology of rare and semileptonic B decays 

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## Outline

- Exclusive B decays can test the SM and probe for New Physics
- Theoretical methods:
- HQET
- Heavy Hadron $\chi P T$
- Soft-collinear effective theory (SCET)
- Recent progress at small and large recoil
- Outlook

Disclaimer: no discussion of inclusive decays, see the talk by Z. Ligeti

## Why are these decays interesting?

- Semileptonic $B \rightarrow M \ell \nu$ decays can be used to determine CKM matrix elements

$$
\Gamma\left(B \rightarrow D^{(*)} e \nu\right) \sim\left|V_{c b}\right|^{2}, \quad \Gamma(B \rightarrow \pi / \rho e \nu) \sim\left|V_{u b}\right|^{2}
$$

- Radiative decays give information about $\left|V_{t d}\right|$

$$
\frac{\mathcal{B}(B \rightarrow \rho \gamma)}{\mathcal{B}\left(B \rightarrow K^{*} \gamma\right)}=\left|\frac{V_{t d}}{V_{t s}}\right|^{2} R\left(1+\varepsilon_{A}\right)
$$

- The rare decays $B \rightarrow K^{*} \gamma$ and $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$are sensitive to New Physics effects through their total rate and differential distributions (e.g. forward-backward asymmetry)
- The leptonic radiative decay $B \rightarrow \gamma e \nu$ can give information about the hadronic structure of the $B$ meson (light-cone wave function)
- Exclusive decay amplitudes depend on more hadronic details than the inclusive modes: normalization + shape
- In $b \rightarrow c$ transitions heavy quark symmetry gives the normalization of the form factor (+ Luke's theorem)

Isgur, Wise (1990)

- No such luck for heavy-to-light decays $\rightarrow$ need to think harder...
- Exclusive decay amplitudes depend on more hadronic details than the inclusive modes: normalization + shape
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- No such luck for heavy-to-light decays $\rightarrow$ need to think harder...


## Good news:

In certain regions of the phase space, a model-independent description becomes possible

## Experimental status - semileptonic

$$
\mathcal{B}\left(B^{0} \rightarrow \rho^{-} e^{+} \nu\right)=(3.39 \pm 0.44 \pm 0.52 \pm 0.60) \times 10^{-4}
$$

Babar, hep-ex/0207080

$$
\begin{aligned}
\mathcal{B}\left(B^{0} \rightarrow \pi^{-} \ell^{+} \nu\right) & =(1.33 \pm 0.18 \pm 0.11 \pm 0.01 \pm 0.07) \times 10^{-4} \\
\mathcal{B}\left(B^{0} \rightarrow \rho^{-} \ell^{+} \nu\right) & =\left(2.17 \pm 0.34_{-0.54}^{+0.47} \pm 0.41 \pm 0.01\right) \times 10^{-4} \\
\mathcal{B}\left(B^{+} \rightarrow \eta \ell^{+} \nu\right) & =(0.84 \pm 0.31 \pm 0.16 \pm 0.09) \times 10^{-4}
\end{aligned}
$$

CLEO, hep-ex/0304019

Decay distributions are also being probed ( $B \rightarrow \rho e \nu$ )

CLEO, hep-ex/0304019


FPCP $2003-\mathrm{p.5}$

## Experimental status - radiative

1. Radiative decays

| Experiment | Lumi $\left(\mathrm{fb}^{-1}\right)$ | $\mathcal{B}\left(B^{0} \rightarrow K^{* 0} \gamma\right) \times 10^{5}$ | $\mathcal{B}\left(B^{ \pm} \rightarrow K^{* \pm} \gamma\right) \times 10^{5}$ |
| :---: | :---: | :---: | :---: |
| CLEO | 9 | $4.55_{-0.68}^{+0.72} \pm 0.34$ | $3.76_{-0.83}^{+0.89} \pm 0.28$ |
| Belle | 60 | $3.91 \pm 0.23 \pm 0.25$ | $4.21 \pm 0.35 \pm 0.31$ |
| Babar | 20.7 | $4.23 \pm 0.40 \pm 0.22$ | $3.83 \pm 0.62 \pm 0.22$ |

More precise data coming soon...
2. First observations of the rare radiative decays $B \rightarrow K^{(*)} \ell^{+} \ell^{-}$

$$
\begin{aligned}
\mathcal{B}\left(B \rightarrow K \ell^{+} \ell^{-}\right) & =\left(0.78_{-0.20-0.18}^{+0.24+0.11}\right) \times 10^{-6} \\
\mathcal{B}\left(B \rightarrow K^{*} \ell^{+} \ell^{-}\right) & =\left(1.68_{-0.58}^{+0.68} \pm 0.28\right) \times 10^{-6}
\end{aligned}
$$

Babar, hep-ex/0207082

$$
\begin{array}{rll}
\mathcal{B}\left(B \rightarrow K \ell^{+} \ell^{-}\right) & =\left(0.75_{-0.21}^{+0.25} \pm 0.09\right) \times 10^{-6} \\
\mathcal{B}\left(B \rightarrow K^{*} \ell^{+} \ell^{-}\right) & <1.4 \times 10^{-6} \quad \text { Belle, hep-ex/0107072 }
\end{array}
$$

## Hadronic uncertainty

- The hadronic form factors describing $B \rightarrow M$ exclusive transitions are computed in models, QCD sum rules, lattice QCD, etc...
- Large spread of predictions $\rightarrow$ theoretical uncertainties


$$
B \rightarrow \pi e \nu
$$

$B \rightarrow \rho e \nu$
In certain regions of phase space, a model-independent description becomes possible

Example: $B \rightarrow \pi \ell \nu$


Two regions where QCD simplifies $\rightarrow$ two effective theories:
$q^{2} \sim q_{\text {max }}^{2}$ - small recoil
QCD $\rightarrow$ HQET
$q^{2} \sim 0$ - the large energy region
QCD $\rightarrow$ SCET
Bauer, Fleming, DP, Stewart, 2001 (see talk by S. Fleming)

## Factorization

In the large energy region $E_{\pi} \gg \Lambda$, the heavy-light form factors satisfy a factorization theorem

$$
f_{B \rightarrow P}\left(q^{2}\right)=C(\mu) \zeta\left(E_{\pi}, \mu\right)+\int_{0}^{1} d x d k_{+} C_{i}(\mu, z) J_{i}\left(x, z, k_{+}, \mu\right) \phi_{B}^{+}\left(k_{+}\right) \phi_{\pi}(x)
$$

"nonfactorizable"
"factorizable"


## Factorization

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Bauer, DP, Stewart

$$
\begin{aligned}
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& \text { "nonfactorizable" }
\end{aligned}
$$

Ingredients: • Nonperturbative matrix elements (soft physics)
$\zeta\left(E_{\pi}, \mu\right)$ are matrix elements in the SCET
$\phi_{B}\left(k_{+}\right)$and $\phi_{\pi}(x)$ are light-cone wave functions

- Perturbative quantities - calculable

Wilson coefficients $C_{i}(\mu)=1+O\left(\alpha_{s}(Q)\right)$
Jet functions $J\left(x, z, k_{+}, \mu\right)=O\left(\alpha_{s}(\sqrt{\Lambda Q})\right)$

## Factorization

In the large energy region $E_{\pi} \gg \Lambda$, the heavy-light form factors satisfy a factorization theorem

Bauer, DP, Stewart
$f_{B \rightarrow P}\left(q^{2}\right)=C(\mu) \zeta\left(E_{\pi}, \mu\right)+\int_{0}^{1} d x d k_{+} C_{i}(\mu, z) J_{i}\left(x, z, k_{+}, \mu\right) \phi_{B}^{+}\left(k_{+}\right) \phi_{\pi}(x)$ "nonfactorizable"
"factorizable"

Comments: • The Wilson coefficients $C(\mu)$ contain Sudakov logs: sometimes it is assumed that the nonfactorizable term is suppressed as $m_{b} \rightarrow \infty$

- In the absence of the factorizable term, there are many symmetry relations among form factors

Charles et al, 1999

- However, both terms are of the same order in $\Lambda / Q$


## Model-independent approach

Measure as many independent form factors as possible, and extract the unknown nonperturbative matrix elements $\zeta(Q, \mu), \phi_{B}\left(k_{+}\right), \phi_{\pi}(x)$
E.g., at tree level in matching at the hard scale $Q$, the $B \rightarrow \pi / \rho$ form factors contain only 3 unknown matrix elements

$$
\zeta_{P}(Q, \mu), \quad \zeta_{\perp}(Q, \mu), \quad\left\langle k_{+}^{-1}\right\rangle_{B}=\int d k_{+} \frac{\phi_{B}\left(k_{+}\right)}{k_{+}}
$$

The $B$ wave function moment can be also extracted from the shape of the photon spectrum in $B \rightarrow$ रeע Korchemsky, DP, Yan, 1999

Working at tree level in $\alpha_{s}(Q)$ and $\alpha_{s}(\Lambda Q)$, there is one remaining symmetry relation for $B \rightarrow P \quad$ Beneke, Chapovsky, Diehl, Feldmann

$$
f_{+}\left(q^{2}\right)-f_{0}\left(q^{2}\right)=\frac{q^{2}}{m_{B}^{2}} f_{T}\left(q^{2}\right)+O(\Lambda / Q)
$$

## $B \rightarrow K^{*} \gamma$ and $B \rightarrow K^{*} e^{+} e^{-}$

Additional contributions from matrix elements of 4-quark operators $\rightarrow$ have to be included in the factorization relation
Bosch, Buchalla; Beneke, Feldmann, Seidel; Ali, Parkhomenko

(a)

(b)

(c)

(d)

(e)

Accounting for these effects, the observed $\mathcal{B}\left(B \rightarrow K^{*} \gamma\right)$ gives
$\left.T_{1}(0)\right|_{\mu=m_{b}}=0.27 \pm 0.04 \quad[$ vs. $0.38 \pm 0.06$ (LC-QCDSR)] Ball, 1995
Close to new lattice QCD \# $T_{1}(0)=0.25(5)(2)$ S.P.QCD R. (2002)

## Progress at zero recoil

- Normalization is not fixed from a symmetry
- Heavy quark symmetry determines the scaling of the form factors + symmetry relations among the tensor $T_{1,2}\left(q^{2}\right)$, vector $V\left(q^{2}\right)$ and axial $A\left(q^{2}\right)$ form factors Isgur, Wise; Burdman, Donoghue, 1991
$T_{1}\left(q^{2}\right)-\frac{m_{B}^{2}-m_{V}^{2}}{q^{2}} T_{2}\left(q^{2}\right)=\frac{2 m_{B}}{m_{B}+m_{V}} V\left(q^{2}\right)+O\left(m_{b}^{-1 / 2}\right)$
$T_{1}\left(q^{2}\right)+\frac{m_{B}^{2}-m_{V}^{2}}{q^{2}} T_{2}\left(q^{2}\right)=-\frac{m_{B}^{2}+m_{V}^{2}-q^{2}}{m_{B}\left(m_{B}+m_{V}\right)} V\left(q^{2}\right)+\frac{m_{B}+m_{V}}{m_{B}} A_{1}\left(q^{2}\right)+O\left(m_{b}^{-\frac{3}{2}}\right.$
Extract $T_{1}\left(q^{2}\right)$ by combining them, which requires knowledge of the $O\left(m_{b}^{-1 / 2}\right)$ correction in the first relation

Recently computed
Grinstein, DP, 2002

$$
T_{1}\left(q^{2}\right)=\frac{m_{B}-\bar{\Lambda}}{m_{B}+m_{V}} V\left(q^{2}\right)-\mathcal{D}\left(q^{2}\right)+O\left(m_{b}^{-3 / 2}\right)
$$

The correction depends only on the local matrix element of a dimension-4 operator

$$
\left\langle V\left(p^{\prime}, \varepsilon\right)\right| \bar{q} i D_{\mu} b|B(v)\rangle=-2 i \mathcal{D}\left(q^{2}\right) \epsilon_{\mu \nu \lambda \sigma} \varepsilon_{\nu}^{*} p_{\lambda} p_{\sigma}^{\prime}
$$

$\mathcal{D}\left(q^{2}\right)$ vanishes exactly in the quark model $\rightarrow$ likely to be small Application:

- Predict the tensor form factor $T_{1}\left(q^{2}\right)$ (relevant for rare decays $D \rightarrow K^{*} e^{+} e^{-}$) from the measured $D \rightarrow K^{*} e \nu$ form factors

$$
T_{1}(1)=0.74 \pm 0.06
$$

using $V(1)=1.35 \pm 0.11$
E791 Collaboration

## Conclusions and outlook

- Significant recent progress in the theory of exclusive semileptonic and radiative $B$ decays, with input from the soft-collinear effective theory (SCET)
- SCET separates the contributions of the physics on different scales, resulting into a factorization relation for the $B \rightarrow M$ form factor
- Clean separation into factorizable and nonfactorizable pieces
- Together with lattice QCD, the SCET provides a model-independent approach for the study of exclusive $B$ decays

More work to do:

- Resum all Sudakov logs, potential large numerical impact
- Investigate the structure of the power corrections $\sim \Lambda / Q$

