The Large Energy Expansion for B Decays: Soft Collinear Effective Theory

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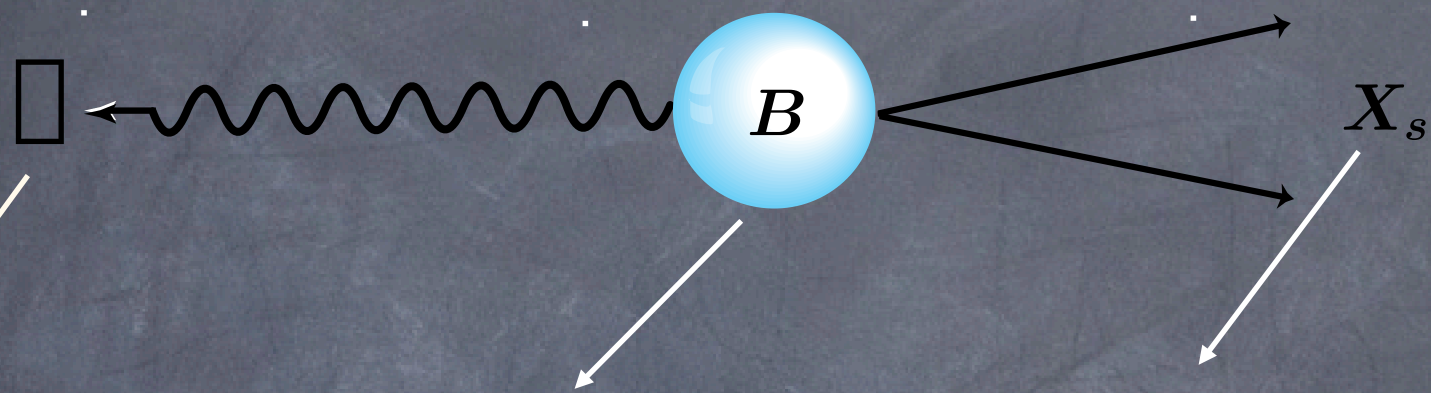
Outline

- Motivate and Elucidate the Soft-Collinear Effective Theory by Explaining the Decay Rate for $B \longrightarrow X_s\gamma$ near the Endpoint of the Photon Energy Spectrum $E_\gamma \rightarrow \frac{M}{2}$

Scaled Energy: $z = \frac{2E_\gamma}{M} \rightarrow 1$

Kinematics for $B \to X_s \gamma$



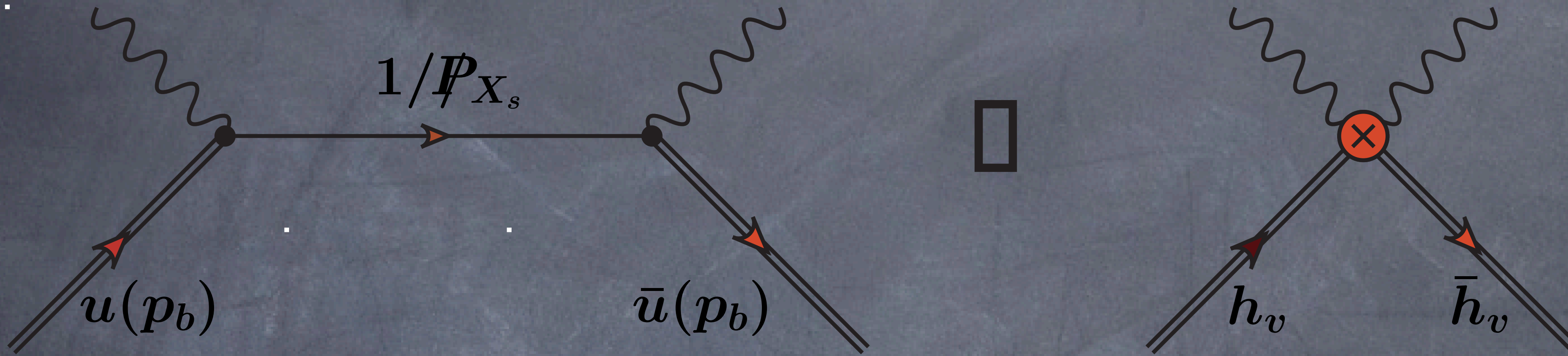
$\frac{1}{2} M z \bar{n}^\mu$ $\quad$ $M v^\mu + k^\mu$ $\quad$ $\frac{1}{2} M n^\mu + \frac{1}{2} M (1-z) \bar{n}^\mu + k^\mu$

$\bar{n}^\mu = (1,0,0,1)$ $\quad$ $v^\mu = (1,0,0,0)$ $\quad$ $n^\mu = (1,0,0,-1)$

$P_{X_s}^2 \approx M^2(1-z) + M n \cdot k$ $\quad$ $k^\mu \sim \Lambda_{QCD}$

for $1 - z \sim O(1)$ drop $n \cdot k$ $\longrightarrow$ $P_{X_s}^2 \sim M^2$

Operator Product Expansion



$$\frac{M}{p_{X_s}^2} \approx \frac{1}{M(1-z)+n\cdot k} \approx \frac{1}{M(1-z)} + O(\Lambda_{QCD})$$

$$\frac{d\Gamma}{dz} = \Gamma_0\, \delta(1-z)\, \langle B|\bar{h}_v h_v|B\rangle$$

→ 1

- To subleading order in $\frac{1}{M^2}$ and α_s

$$\frac{d\Gamma}{dz} = \Gamma_0 \left\{ \left[1 - \frac{\alpha_s C_F}{4\pi} \left(2 \log \frac{\mu^2}{M^2} + 5 + \frac{4}{3}\pi^2 \right) \right] \delta(1-z) + \frac{\alpha_s C_F}{4\pi} \left[7 + z - 2z^2 - 2(1+z)\log(1-z) - \left(4 \frac{\log(1-z)}{1-z} + \frac{7}{1-z} \right)_+ \right] + \frac{1}{2M^2} \left[(\lambda_1 - 9\lambda_2)\delta(1-z) - (\lambda_1 + 3\lambda_2)\delta'(1-z) - \frac{\lambda_1}{3}\delta''(1-z) \right] \right\} + O(\alpha_s^2, 1/M^3)$$

Near the endpoint z -> 1:

1) the delta-function terms destroy the non-perturbative expansion

2) the plus-function terms destroy the perturbativbe expansion

Summing Singular Terms

- Resum the most singular non-perturbative terms: non-perturbative shape function of width Λ_{QCD}/M

M. Neubert, Phys. Rev. **D49**, 3392 (1994); **D49**, 4623 (1994); I.~I.~Bigi et al., Int. J. Mod. Phys. **A9**, 2467 (1994).

- The series of singular plus-functions exponentiates taming the divergence

G. P. Korchemsky and G. Sterman Phys. Lett. **B340**, 96 (1994); R. Akhoury and I. Z. Rothstein, Phys. Rev. **D54**, 2349 (1996).

Endpoint kinematics

- Recall $P_X^2 \approx M^2(1-z) + M n\cdot k \sim M\Lambda_{QCD}$
- If $1-z \sim \Lambda_{QCD}/M$ we can not drop $n\cdot k$

The strange quark **jet** has momentum

$$P_X^\mu = \frac{1}{2}M n^\mu + \frac{1}{2}M(1-z)\bar{n}^\mu + k^\mu$$

offshellness is parametrically smaller than the lightcone momentum component:

$$\sqrt{P_{X_s}^2} \approx M\sqrt{\Lambda_{QCD}/M} \ll M$$

New Degree of Freedom

- Need to include modes with scaling

$$p = (p^+, p^-, p_\perp) = (n \cdot p, \bar{n} \cdot p, p_\perp) \sim M(\lambda^2, 1, \lambda)$$

with

$$\sqrt{p^2} \sim M\lambda$$

Collinear Momentum

inclusive decays $\sqrt{p^2_{X_s}} \approx M\sqrt{\lambda_{QCD}/M}$

so $\lambda \sim \sqrt{\frac{\Lambda_{QCD}}{M}}$

SCET

Bauer, Fleming, Luke, Phys. Rev. D 63: 014006, 2001
Bauer, Fleming, Pirjol, Stewart, Phys. Rev. D 63: 114020, 2001
Bauer & Stewart, Phys. Lett. B 516: 134, 2001
Bauer, Pirjol, Stewart, Phys. Rev. D 65: 054022, 2002

Soft Collinear Effective Theory

Massless energetic particles interacting with a background of soft quanta

Analogous to HQET:

Heavy particles interacting with a soft background

Momentum Decomposition

$\sim Q$ $\qquad$ $\sim \Lambda_{QCD}$

HQET: $$p^\mu \longrightarrow m_b v^\mu + k^\mu$$

Labels

Residual Momentum

SCET: $$p^\mu \longrightarrow \frac{1}{2}\bar{n}\cdot p\, n^\mu + p_\perp^\mu + k^\mu$$

$\sim Q$	$\sim \sqrt{Q\Lambda_{QCD}}$	$\sim \Lambda_{QCD}$
$\sim Q$	$\sim Q\lambda$	$\sim Q\lambda^2$

$$\lambda = \sqrt{\frac{\Lambda_{QCD}}{Q}}$$

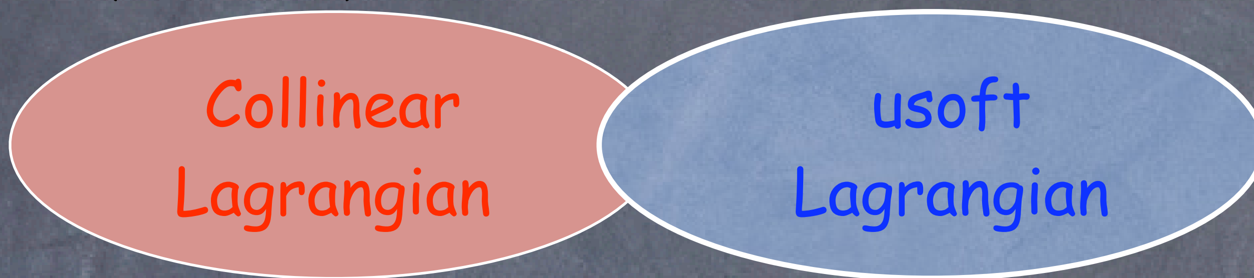
(Semi-Inclusive)

SCET Lagrangian
(hybrid formulation)

Bauer, Fleming, Luke, Phys. Rev. D 63: 014006, 2001
Bauer, Fleming, Pirjol, Stewart, Phys. Rev. D 63: 114020, 2001
Bauer & Stewart, Phys. Lett. B 516: 134, 2001
Bauer, Pirjol, Stewart, Phys. Rev. D 65: 054022, 2002

Beneke and Feldmann, Phys. Lett. B 553, 267 2003

$p \sim M(\lambda^2, 1, \lambda)$ $\qquad$ $k \sim M(\lambda^2, \lambda^2, \lambda^2)$



$$L = \bar{\xi}_{n,p'} \left\{ g n \cdot A_{n,q} + (\not{P}_\perp + g \not{A}^\perp_{n,q}) \frac{1}{\bar{n} \cdot P + g \bar{n} \cdot A_{n,q'}} (\not{P}_\perp + g \not{A}^\perp_{n,q''}) + i n \cdot \partial + i n \cdot A_{us} \right\} \frac{\not{\bar{n}}}{2} \xi_{n,p}$$

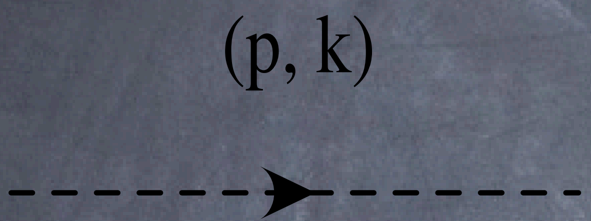
$$L = \bar{\psi} i \not{D}_{us} \psi$$

- Collinear: QCD on Light-Cone
- Ultra-Soft: QCD

} coupled through $i n \cdot A_{us}$ term

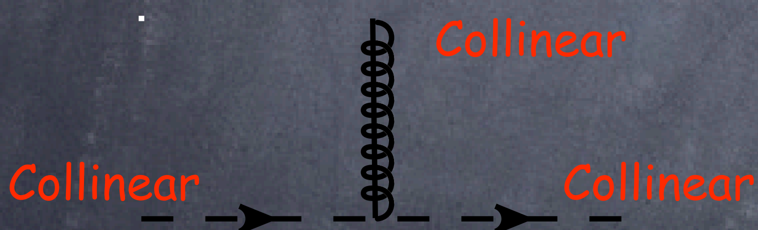
Feynman Rules

- Propagators

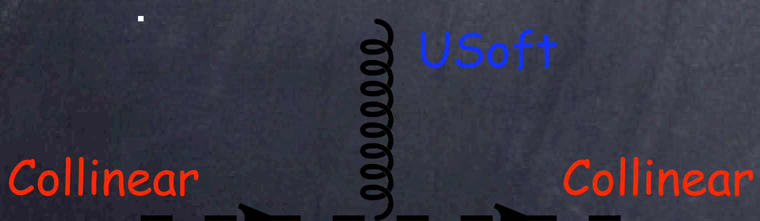


$$i\frac{\not n}{2}\frac{\bar n\cdot p}{n\cdot k\,\bar n\cdot p + p_\perp^2 + i\epsilon}$$

- Verticies



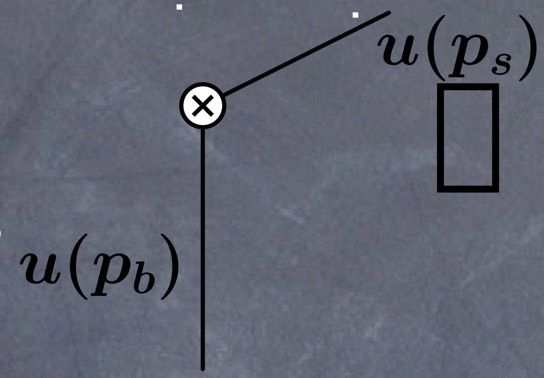
$$igT^A\left[n^\mu + \frac{\gamma_\mu^\perp \not p_\perp}{\bar n\cdot p} + \frac{\not p'_\perp \gamma_\mu^\perp}{\bar n\cdot p'} - \frac{\not p'_\perp \not p_\perp}{\bar n\cdot p\,\bar n\cdot p'}\bar n^\mu\right]\frac{\not{\bar n}}{2}$$

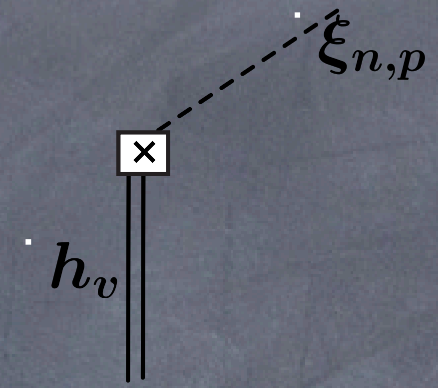


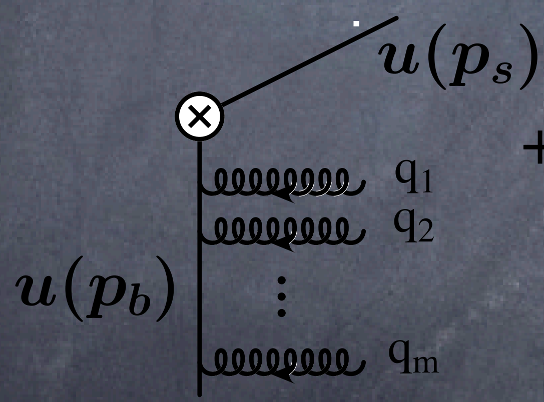
$$igT^A n^\mu \frac{\not{\bar n}}{2}$$

Heavy-Light Current

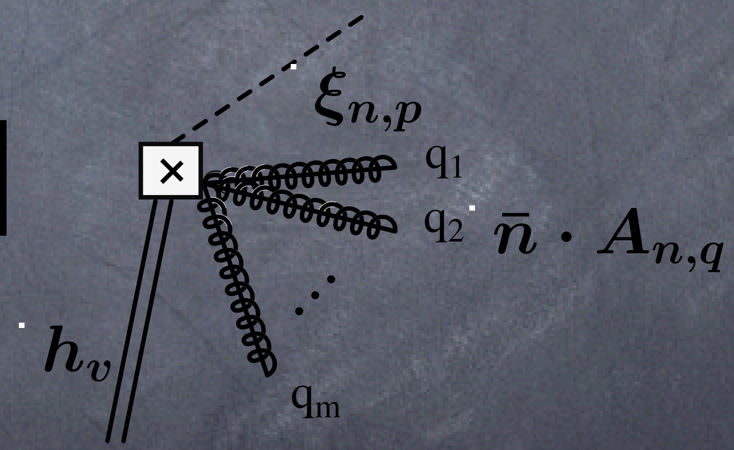
- Match QCD and SCET current (leading order in λ)







+ perms



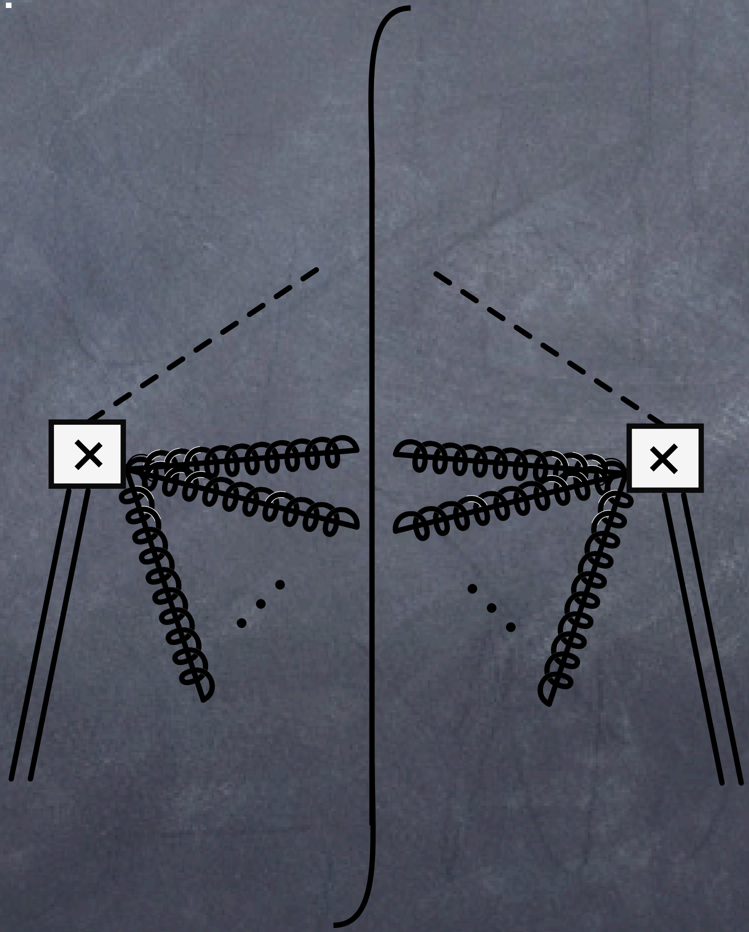
$$\bar{u}(p_s)\Gamma u(p_b) \quad \to \quad \bar{\xi}_{n,p} \exp\left(\frac{g\bar{n}\cdot A_{n,q}}{\bar{n}\cdot q}\right)\Gamma h_v$$

$W^\dagger$

$B \to X_s\gamma$ Decay Rate

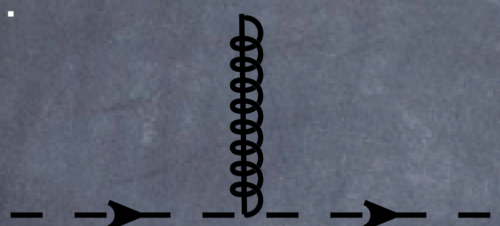
- Imaginary Part of Forward Scattering -> Decay Rate

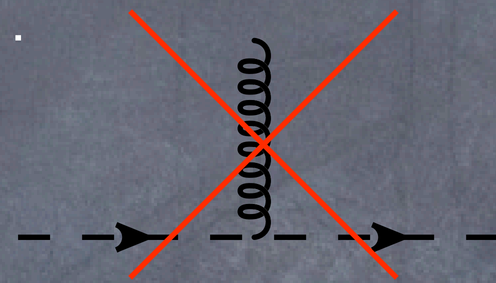
$$\frac{d\Gamma}{dz} =$$



Decoupling and Factorization of the Decay Rate in SCET

- Decouple USoft from Collinears in the Lagrangian through a field redefinition: $\xi_{n,p} = Y \xi_{n,p}^{(0)}$

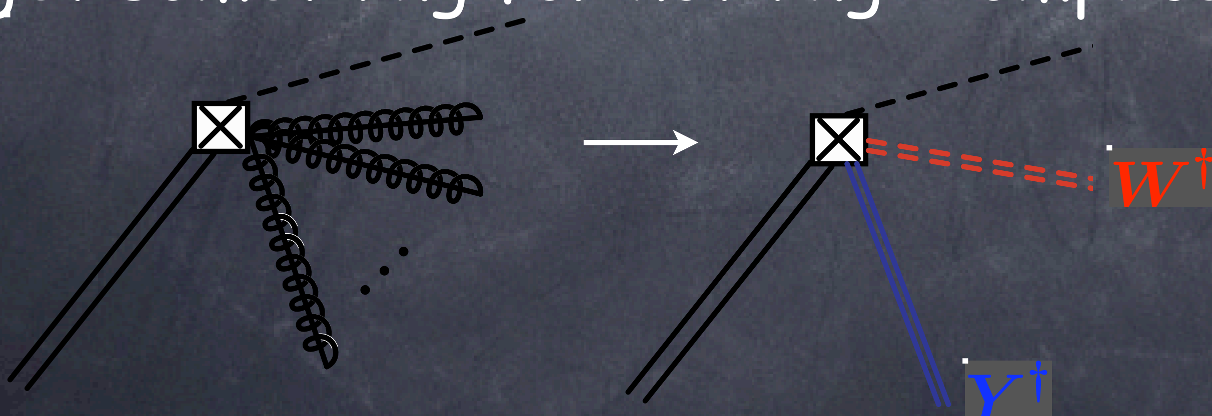




Introduce USoft Wilson Line

$$Y(x) = \mathrm{P}\exp\left(ig \int_{-\infty}^{x} ds\; n \cdot A_{us}(ns) \right)$$

- Don't get something for nothing: Complicates vertex

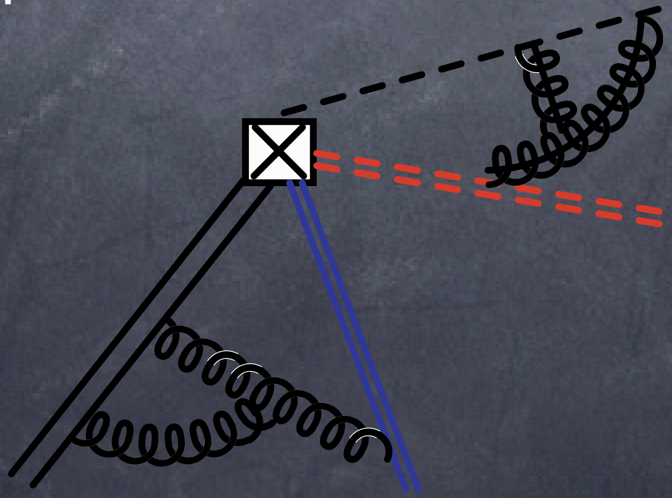


Decoupling and Factorization of the Decay Rate in SCET

- Heavy & USoft Wilson line interact
- Collinear & Collinear Wilson line interact

But

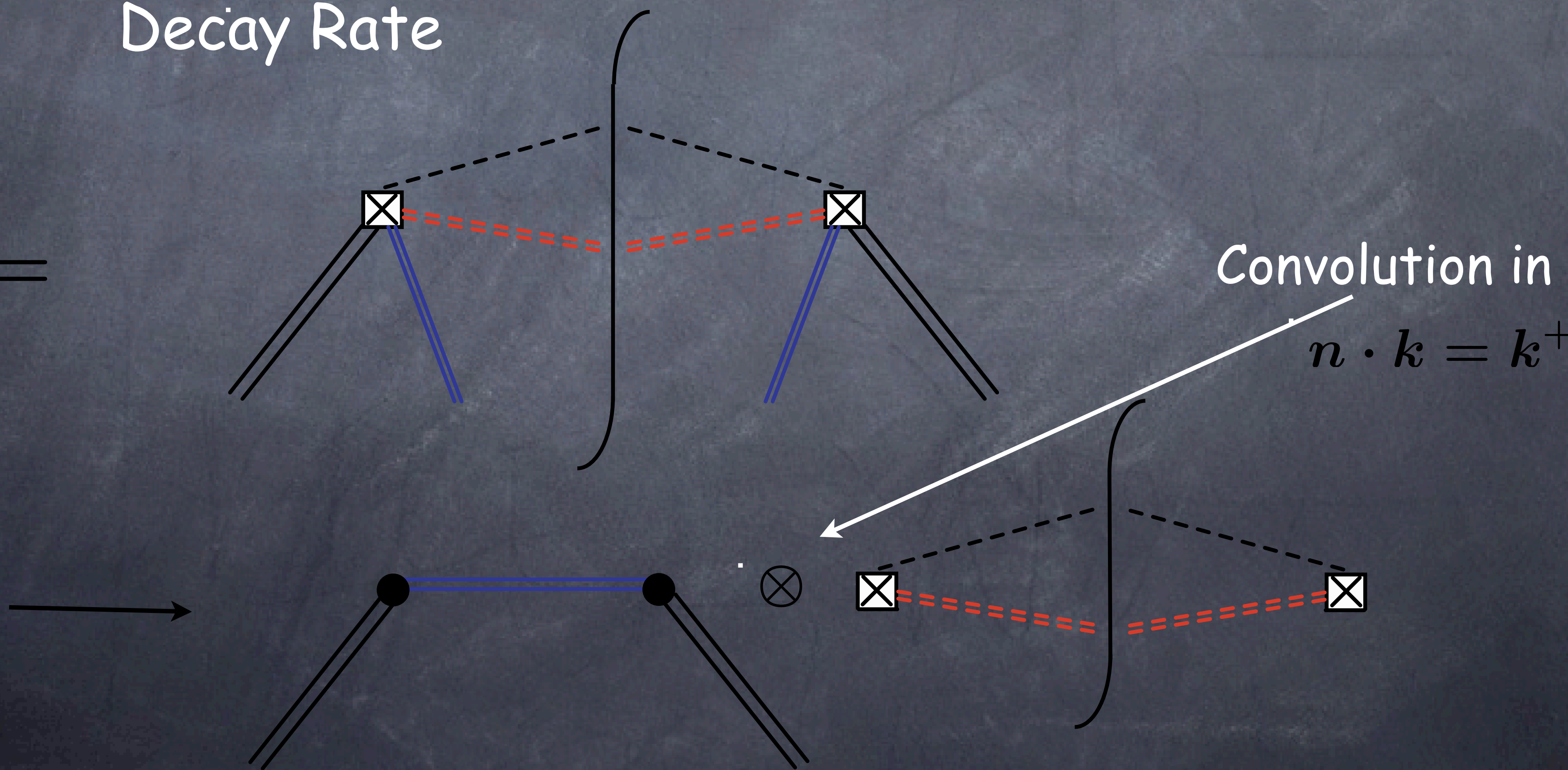
- Heavy/USoft do not interact with Collinear!



Decoupling and Factorization of the Decay Rate in SCET

- That is all we need for Factorization of the Decay Rate

$$\frac{d\Gamma}{dz} =$$



The Factored Decay Rate

Hard Coefficient

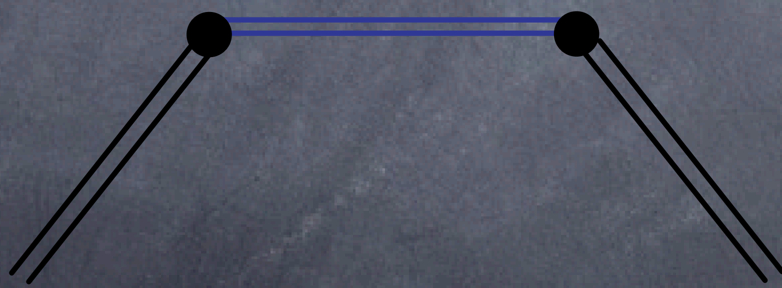
Shape Function

Jet Function

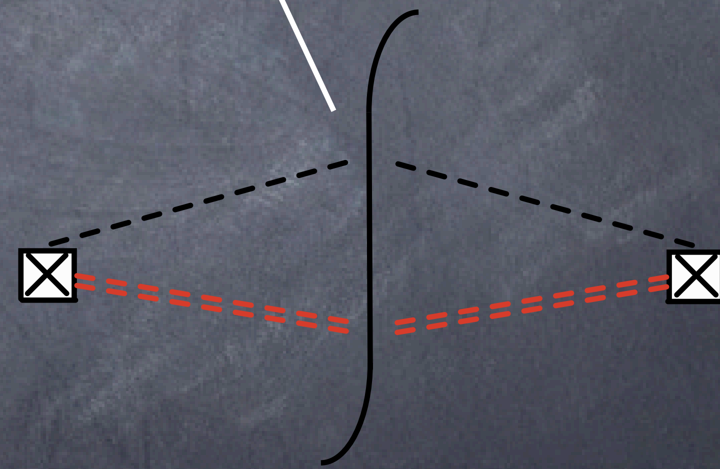
$$\frac{d\Gamma}{dz} = H(m_b, \mu) \int_z^1 d\eta \, S(\eta, \mu) J(\eta - z, \mu)$$

Expand in

$\alpha_s(m_b)$



Non-perturbative



Expand in

$\alpha_s(\sqrt{m_b \Lambda_{QCD}})$

What about the Plus-Functions?

- These can be Summed by Running using the RGE for the Jet Function and the RGE for the USoft funtion
  - Run the Jet function from m_b to $m_b\sqrt{1-z} \sim \sqrt{m_b \Lambda_{QCD}}$
  - Run the USoft function from m_b to $m_b(1-z) \sim \Lambda_{QCD}$

Conclusions

- Hopefully SCET is now a little less of a mystery (or at least you got a nap)
- There are many other application for SCET
  - Exclusive B Decays -- Dan Pirjol's talk