f_B, B_B . . . from Lattice QCD

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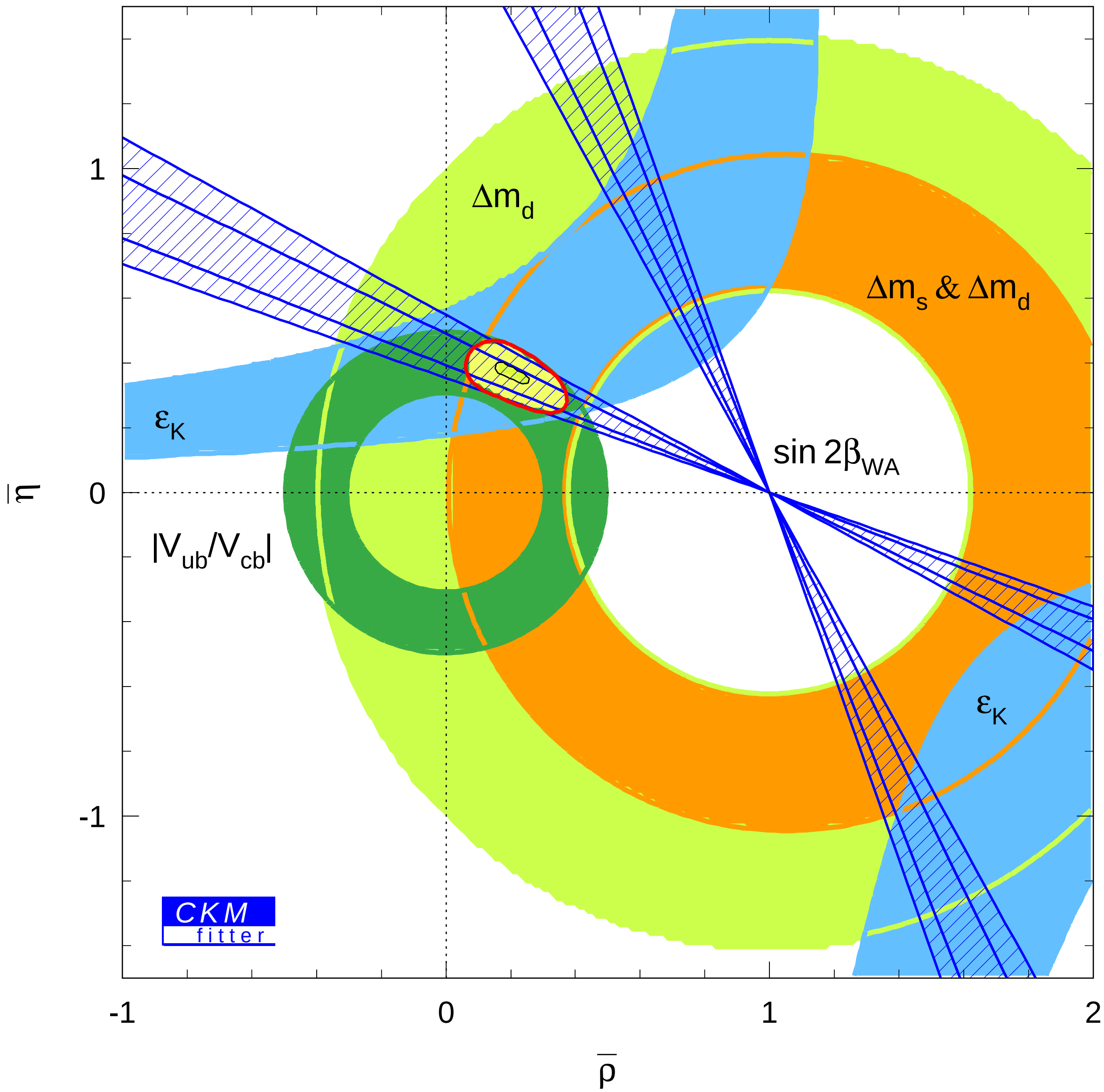
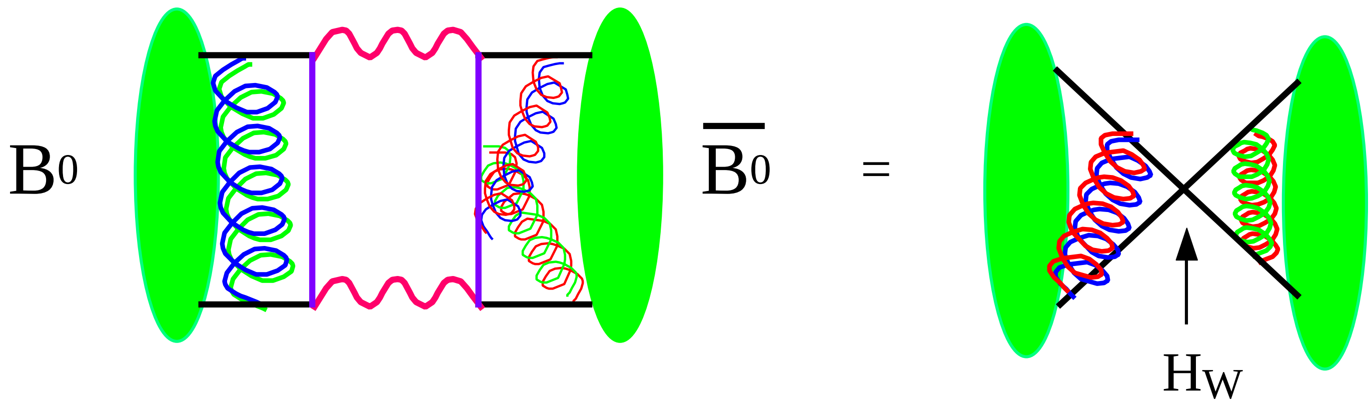
- Why lattice calculations?
- Current status - why is it taking so long?
- The future - recent breakthrough will enable us to reach light quark masses at last

The Unitarity triangle



Lattice QCD results for hadronic weak decay matrix elements needed as theory input for extraction of CKM parameters. Need small *reliable* lattice errors. Focus here on $B_{s/d}$ oscillations for V_{td}/V_{ts} (ratio reduces theory systematics).

B oscillations



Mixing of neutral B through box diagram gives rise to oscillations.

Frequency $\propto |V_{tq}|^2 \times$ matrix elmt of 4-quark op. between B mesons.

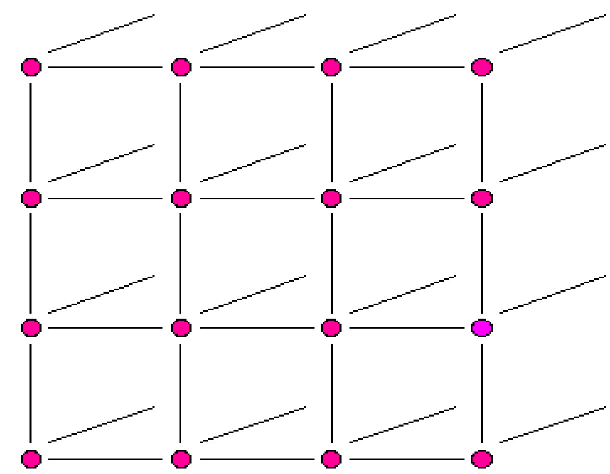
Include QCD effects fully $\rightarrow$ evaluate matrix elmt in lattice QCD.

$$\langle \overline{B}_q | (\overline{b}q)_{V-A} (\overline{b}q)_{V-A} | B_q \rangle = \frac{8}{3} M^2_{B_q} f^2_{B_q} B_{B_q}$$

$$\langle 0 | \overline{b} \gamma_0 \gamma_5 q | B_q \rangle = i M_{B_q} f_{B_q}$$

f_{B_q} is decay constant for purely leptonic decay. Also calculate this.

Lattice calculations



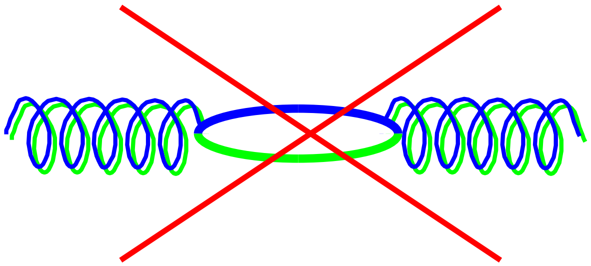
Are hard and time-consuming.

Progress (over 30 years!) through increase in computer power + physical insight.

Discretise spacetime and QCD d.o.f. onto a lattice, $a \geq$ 0.05 fm and $L \leq$ 3.0 fm. ($\equiv$ 4 GeV to 70 MeV). Integrate numerically.

Problems:

- Handling b quarks - use effective theories $\surd$
- Discretisation errors - use improved actions $\surd$
- Handling light quarks - very expensive to work with realistic u/d masses, especially as dynamical(sea) quarks. Results must be *chirally extrapolated*. Old results are *quenched*.



Quenched approximation is internally inconsistent.

$a \equiv \Lambda_{QCD}$ fixed from setting one lattice hadron mass $=$ expt. Answer depends on hadron used. Ditto quark masses.

Quenched results must be carefully defined.

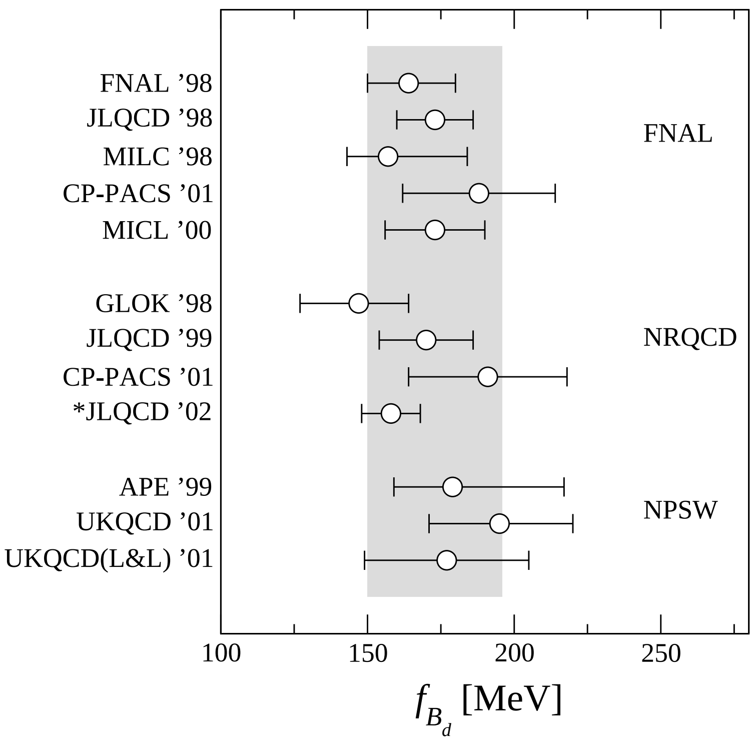
Chiral extrapolation from simulation u/d masses to physical ones uses chiral pert. th.

Small param. is $x = 2\mu m_q/(4\pi f_\pi)^2$, inc. $x \log x$ terms.

At m_s, $x_s^2 \approx 10\%$. $\Rightarrow$ need *all* $m_{u/d} < m_s/2$ before low/leading order formulae make sense.

Must test effect of higher orders in chiral expansion. Hard to fix chiral log. coeffs from lattice, especially on a finite volume.

Current status - f_B



Quenched results for f_B show consistency *but* result depends on how a is fixed and chiral extrapolations are poor. 'World averages' (Yamada, hep-lat/0120035):

$f_{B_d} = 173(23)$ MeV

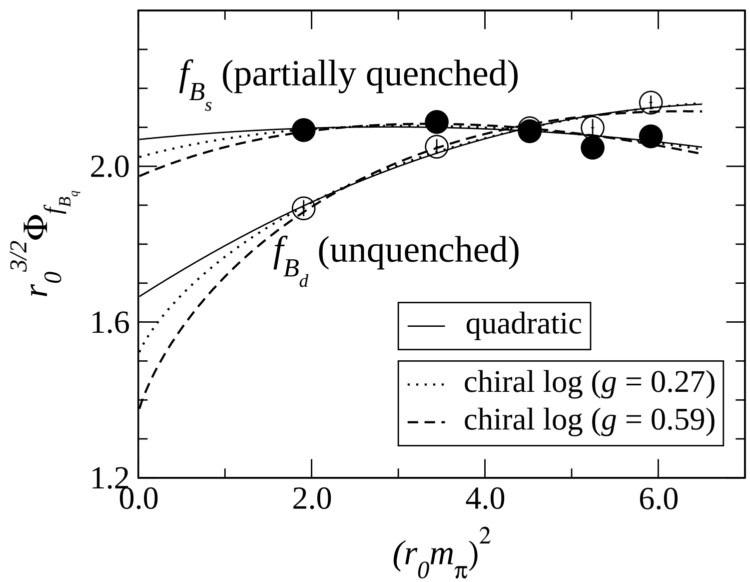
$f_{B_s} = 200(20)$ MeV

$f_{D_d} = 203(14)$ MeV

$f_{D_s} = 230(14)$ MeV

($f_{D_s} = 252(9)(20)$ MeV, Alpha collab. hep-lat/0302016)

Unquenched results for f_B



'Hot' issue is chiral extrapoln.

Expect log term

$$-0.75(1+3g^2)\frac{m_\pi^2}{(4\pi f_\pi)^2}\ln(m_\pi^2/\Lambda^2)$$

where $g = BB^*\pi$ coupling ≈ 0.35.

JLQCD data (Hashimoto, hep-lat/0209091) then gives large f_{B_s}/f_{B_d} in chiral limit. (*but* data not in chiral regime).

'World averages' (Yamada, hep/lat/0120035):

$f_{B_d}^{n_f=2}$ = 198(30)(+0 -34) MeV

$f_{B_s}^{n_f=2}$ = 230(30) MeV

$f_{B_s}^{n_f=2}/f_{B_d}^{n_f=2} \equiv r_b$ = 1.16(5)(+24 -0)

r_b/r_c = 1.02(2)

Current status - B_B

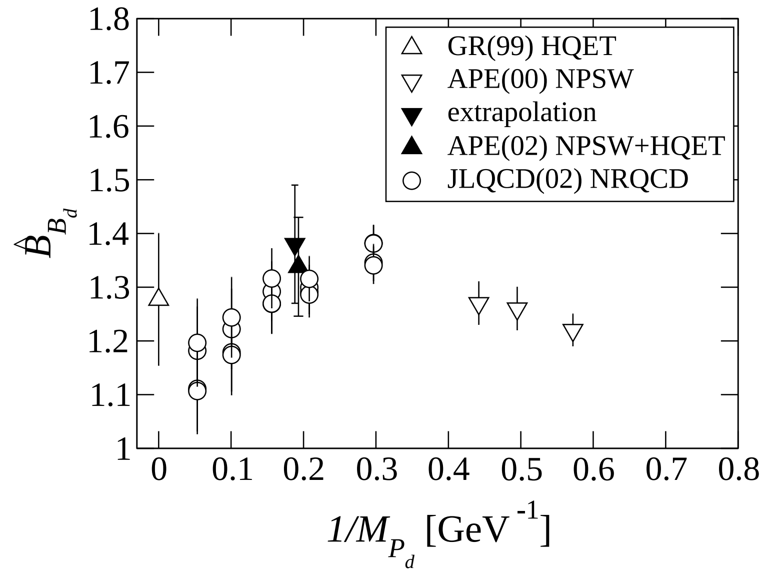
B_B is dimensionless and seems very insensitive to quenching or to light quark mass effects (far from chiral regime). M_B dependence still unclear.

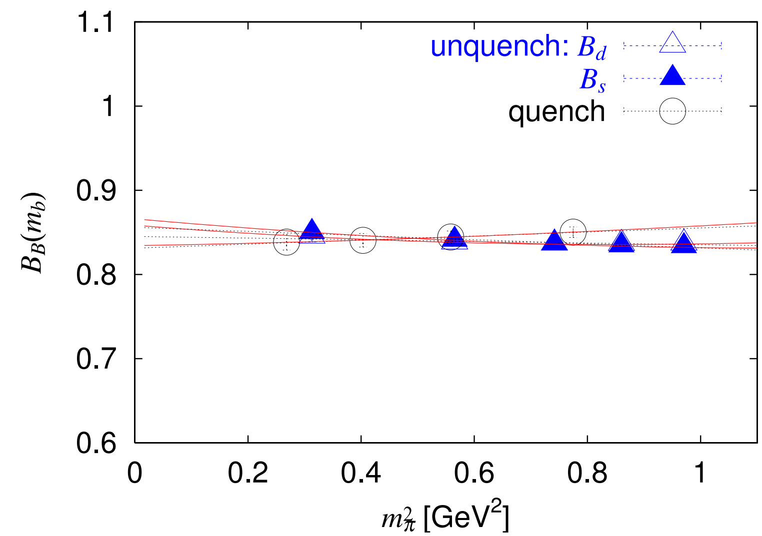
Only JLQCD has unquenched result:

$\hat{B}_{B_d}^{n_f=2} = 1.34913$

$\hat{B}_{B_s}^{n_f=2} / \hat{B}_{B_d}^{n_f=2} = 1.0(2)$

Yamada, hep-lat/0210035, Lellouch, hep-ph/0211359





Hopeful future

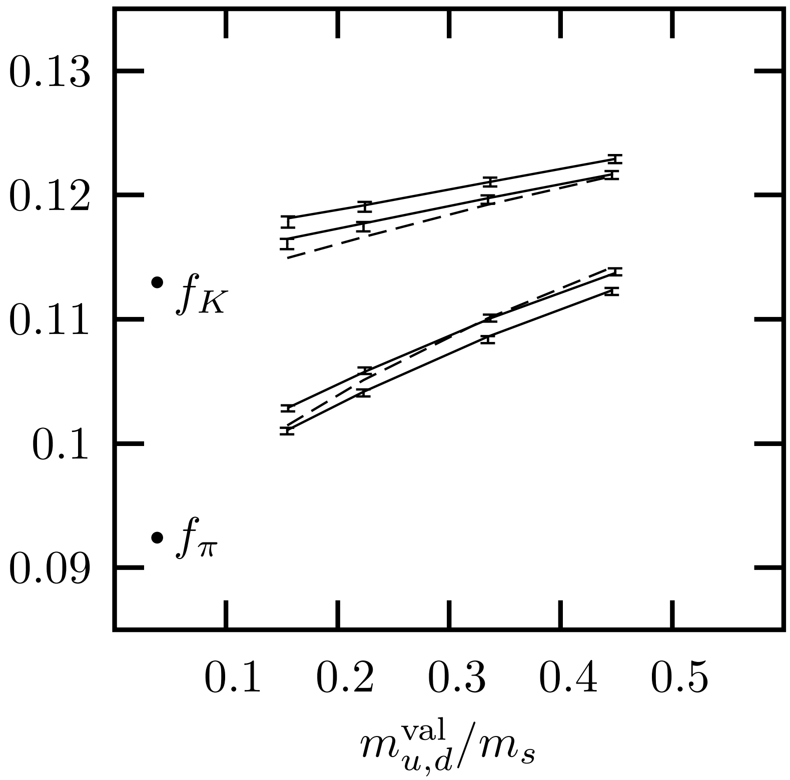
MILC collaboration have used new 'improved staggered' formalism to include dynamical u, d, and s quarks and with $m_{u/d}$ as low as $m_s/6$ i.e. close to the real world.

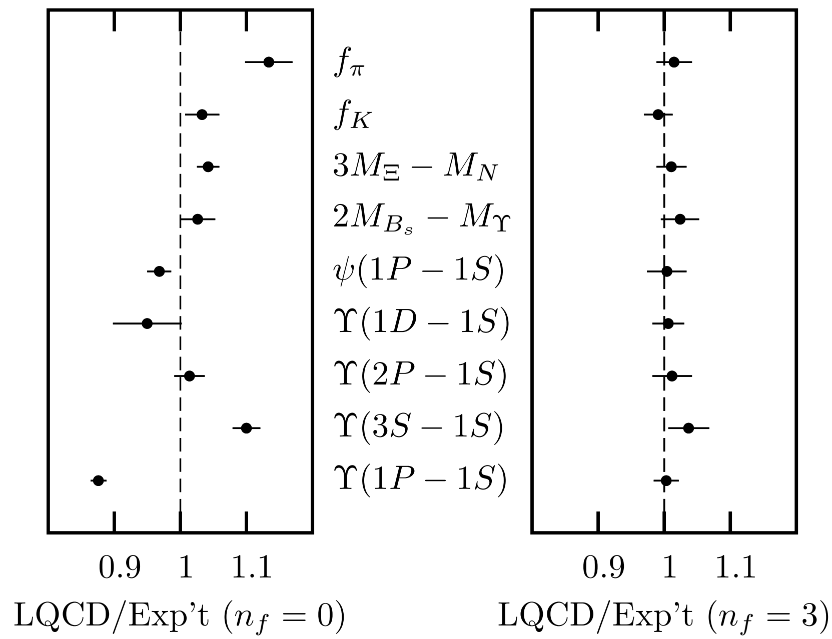
Theoretical caveat is that each flavour is quadrupled on lattice and need to divide by 4 in appropriate places.

Find (MILC/HPQCD/FNAL) that exptl answer is obtained for a wide range of simple quantities from heavy *and* light quark physics, i.e. a consistent theory at last!

Davies *et al*, hep-lat/0304004, Science, 16th May 2003

New results with 2+1 flavours of dynamical quarks





Chiral extraps use 2nd order chiral pert. th. and $m_{u/d} < m_s/2$.

Two different values for a agree (0.09fm and 0.13fm).

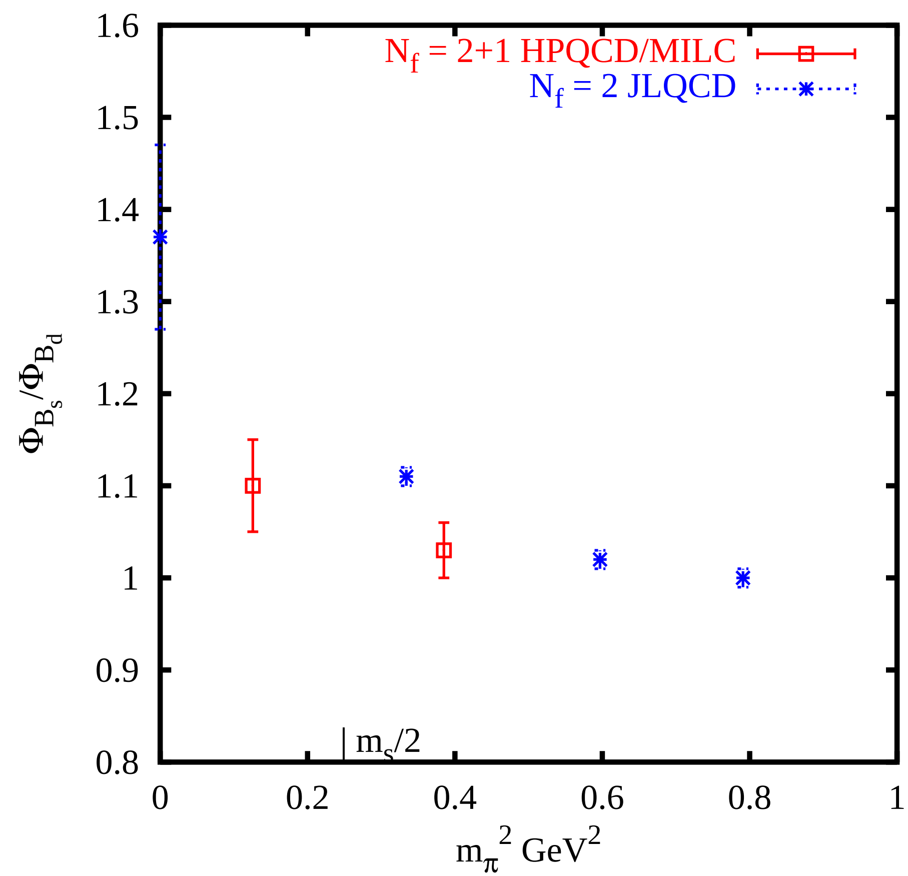
Further calcs will enable few % percent errors for HL matrix elements, *tested consistently* vs known Υ and D results (e.g. from CLEO).

Davies *et al*, hep-lat/0304004, Science, 16th May 2003

Preliminary 'real world' f_B results

Results obtained *below* $m_{u/d} = m_s/2$ indicate $f_{B_s}\sqrt{M_{B_s}}/f_{B_d}\sqrt{M_{B_d}}$ not as large as feared. m_s can be fixed consistently from m_K. Calculations continue.

(M. Wingate and J. Shigemitsu, HPQCD)



Conclusions

- f_B and B_B calculations making slow progress.
- Focus is now turning to unquenched calculations to remove inherent 10-20% error from lack of consistency in quenched approx.
- New results from improved staggered formalism allow calcs. at $m_{u/d} < m_s/2$ where chiral extrapolns make sense.
- These calcs can be tested vs Υ and D physics for added confidence.
- Watch this space! (as always).