Decays of heavy mesons and Lattice QCD

Damir Bećirević

FPCP-03

Paris, June 2003

$|V_{ub}|$ from $B \to \pi \ell \nu$

Belle, BaBar and also CLEO start measuring parts of the (large) q^2-spectrum

$$\underbrace{\frac{d\Gamma(B \to \pi e \nu)}{dq^2}}_{\text{measure}} = \frac{G_F |V_{ub}|^2}{192\pi^3 m_B^3} \lambda(q^2, m_B^2, m_\pi^2)^{3/2} \underbrace{|F_+(q^2)|^2}_{\text{compute}}$$

Definition:

$$\langle \pi^-(p)|\bar{b}\gamma_\mu u|B^0(p_B)\rangle = \left(p_B + p - q\frac{m_B^2 - m_\pi^2}{q^2}\right)_\mu F_+(q^2) + \frac{m_B^2 - m_\pi^2}{q^2} q_\mu F_0(q^2)$$

with $F_+(0) = F_0(0)$.

$B \to \rho \ell \nu$ may help too but it is harder

(i) for theorists (more form factors);

(ii) for experimenters (distinguish ρ from $\pi\pi$)

Hadronic form factors : Need a method that allows computing the nonperturbative QCD effects.

Lattice QCD

In the Euclidean metric

$$Z_{\text{QCD}} = \int \mathcal{D}A_\mu e^{-S_g} \prod_q \det(\not{D} + m_q)$$

∘ First principles:
The only parameters entering the computations are those which appear in the QCD lagrangian, namely

$$m_q \;\; (\kappa_q) \;\; \text{and} \;\; g_0^2 \;\; (\beta)$$

∘ Arbitrary accuracy:
Integral handled by using the Monte Carlo methods (the SU(3) gauge field configurations generated)

$$\Rightarrow \langle O \rangle = \frac{1}{Z_{\text{QCD}}} \int \mathcal{D}\mu \;\; e^{-S_{\text{QCD}}} O(x_1, \ldots, x_n) \simeq \frac{1}{N} \sum_i^N \{O_i\}$$

stat.errors $\propto 1/\sqrt{N}$ (central limit theorem)

Nowadays the statistical errors are at the level of a few % for almost all the quantities of phenomenological interest.

To make a problem solvable by a computer, significant approximations needed :-(

♠ Discretization effects: finite lattice spacing "a"

$$\mathcal{F}(a) = \mathcal{F}(0) + a\mathcal{F}'(0) + \ldots$$

(i) use OPE in "a" and improve the theory
(get rid of $\mathcal{O}(a^n)$ effects) *K.Symanzik 1983*

(ii)work at several (small) lattice spacings and go to $a \to 0$

♠ Matching and Renormalization:
"a" hard cut-off : renormalization perturbative and non-perturbative (NPR in RI/MOM and SF schemes)

G.Martinelli et al. 1995, M.Lüscher et al. 1996

♠ Quenching errors:
dynamical quark loops left out ($n_F = 0$):

$$\det(\not{D} + m_q) = \text{const.}$$

(nowadays we start probing physics with $n_F = 2$)

♠ Physical quark masses: current lattices not as fine as to resolve m_b, nor the lattice sizes are large enough to accomodate very light pseudoscalar mesons

All systematic uncertainties improvable: brute force and/or improving the lattice theory

⊗ TFlop computing resources: CP-PACS, QCDOC, APE-NeXt

⊗ Further improve the lattice QCD(?), algorithms(!)

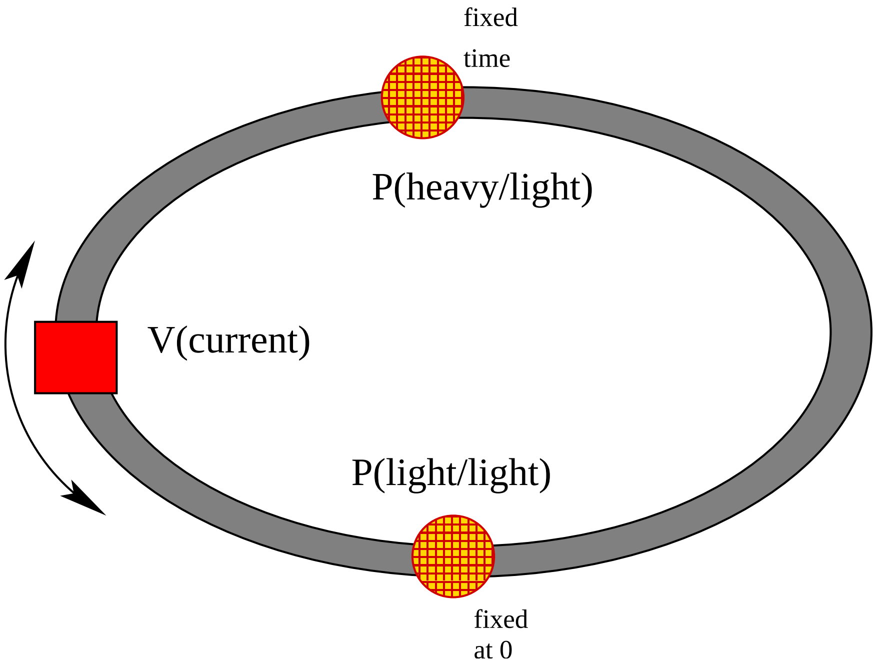
What do we compute?

3pt and 2pt correlation functions

$$C^{(3)}_{\mu}(t;\vec{q},\vec{p}_H) = \Big\langle \sum_{\vec{x},\vec{y}} e^{i(\vec{q}\vec{y}-i\vec{p}_H)\vec{x}} \, (\bar{q}\gamma_5 q)_0 \, (\bar{Q}\gamma_\mu q)_{\vec{y},t} \, (\bar{Q}\gamma_5 q)^{\dagger}_{\vec{x},t_F} \Big\rangle$$

$$C^{(2)}_{qq}(t;\vec{p}_H-\vec{q}) = \Big\langle \sum_{\vec{x}} e^{i(\vec{p}_H-\vec{q})\vec{x}} \, (\bar{q}\gamma_5 q)_0 \, (\bar{q}\gamma_5 q)^{\dagger}_{\vec{x},t} \Big\rangle$$

$$C^{(2)}_{Qq}(t;\vec{p}_H) = \Big\langle \sum_{\vec{x}} e^{i\vec{p}_H\vec{x}} \, (\bar{Q}\gamma_5 q)_0 \, (\bar{Q}\gamma_5 q)^{\dagger}_{\vec{x},t} \Big\rangle$$



Operators sufficiently separated!

`Matrix element ⇔ plateau of the ratio`

$$R_\mu(t) = \frac{C^{(3)}_{\mu}(t)}{C^{(2)}_{qq}(t)\,C^{(2)}_{Qq}(t_F-t)} \propto \langle P(p_H-q)|V_\mu|H(p_H)\rangle$$

Explore as many kinematical configurations $(\vec{p}_H,\vec{q})$ as possible!

$F_{+,0}(q^2)$ from the lattice

Very simple strategy

1. Generate an SU(3) gauge field configuration U (MC)
2. $\forall\ t \in [0,T)$, compute the correlation functions

$$C^{(3)}_\mu(t)_U \qquad C^{(2)}_{qq}(t)_U \qquad C^{(2)}_{Qq}(t)_U$$

3. Repeat 1. and 2. for $N_{\text{conf.}}$ independent U's and compute the ratio

$$R_\mu \overset{t_F \gg t \gg 0}{\Longrightarrow} \langle P(p_H - q)|V_\mu|H(p_H)\rangle$$

$\Rightarrow F_0(q^2), F_+(q^2)$ for $H_{Qq} \to P_{qq}$

4. Do 2. and 3. for several light quarks q and several heavy quarks Q, and for as many momentum injections as possible

However, currently

$$m_c \ll \pi/a \text{ , but } \quad m_b \not\ll \pi/a \quad \to \quad m_c \le m_Q < m_b$$

$$m_{P_{qq}} L \gtrsim 4 \quad \to \quad m_d < m_q \le m_s$$

Signals worsen rapidly as more momentum is given to H and/or P

How to reach $B \to \pi$ decay?

Problem 1: Heavy quark

Currently accessible lattices too coarse to accomodate m_b

4 ways out

- ♠ **QCD** with propagating quarks that are accessible: extrapolate to $1/m_B$ by using the heavy quark scaling laws (*APE, UKQCD*)
- ♠ **HQET** (static limit) $m_b \to \infty$: $\mathcal{L}_{\rm HQET} = Q^\dagger D_4 Q$
  - bad signal/noise : need huge statistics
  - non-perturbative renorm. devised (*Heitger et al., 2003*)

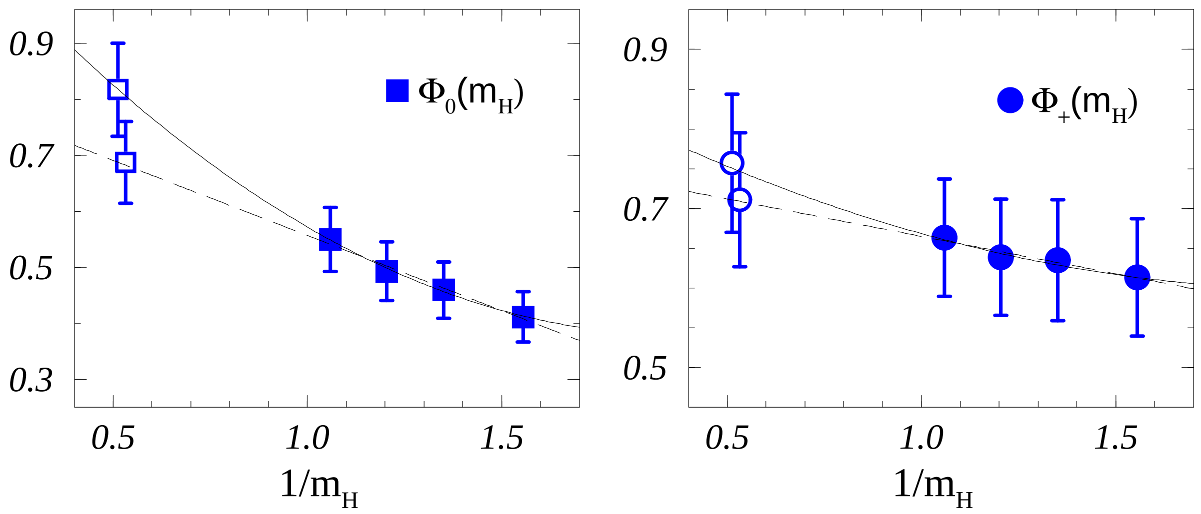
  (*SPQcdR*)
- ♠ **NRQCD** (static limit + $1/m_b$ terms which are cut-off as $m_Q v \ll m_Q$) : $\mathcal{L}_{\rm NRQCD} = Q^\dagger \left(D_4 - (\vec{D}^2 + \vec{\sigma}\cdot\vec{B})/2m_Q \right) Q$
  - expansion in $1/(am_Q) \Rightarrow$ no continuum limit
  - problems in including terms $\propto 1/m_Q$ in renormalisation

  (*JLQCD, HPQCD*)
- ♠ **FNAL approach**: use the full QCD Wilson action and go over the cut-off; redefine the mass and reinterpret the theory in terms of $1/m_Q$ expansion; in some cases in “renormalon shadows” (*FNAL*)

Problem 2: Accessible q^2's

♣ **QCD with propagating quarks**:
Directly accessed $F_{+,0}^{H\to P}$ around $q^2 \approx 0$.
Folklore: $q^2 \to vp = (m_H^2 + m_P^2 - q^2)/2m_H$
at fixed vp extrapolate $F_{+,0}^{H\to P}$ in $1/m_B$ by using the heavy quark scaling laws

$$\begin{aligned}\Phi_i(m_H) &= \{F_0(vp)m_H^{1/2}, F_+(vp)m_H^{-1/2}\} \\ &= a_i^{(0)} + a_i^{(1)}/m_H + a_i^{(2)}/m_H^2 \\ &\to \Phi_i(m_B)\end{aligned}$$



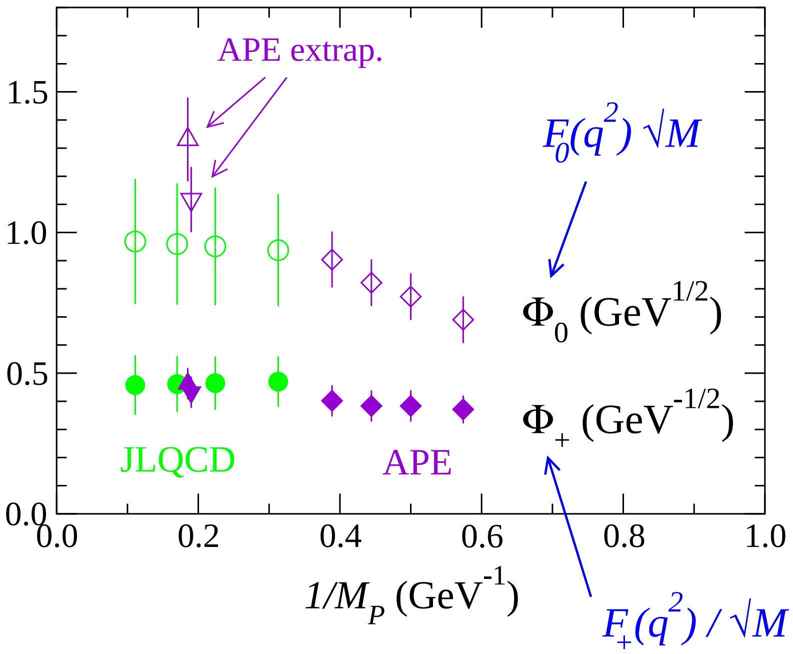
example for $vp \approx$ 0.9GeV

Reconstructed $q^2_{B\to P} = m_B^2 + m_P^2 - 2m_B vp$ close to q^2_{max}!

$15\ \mathrm{GeV}^2 \lesssim q^2 \lesssim 23\ \mathrm{GeV}^2$

Problem 2 (cont.)

- **NRQCD**: small momentum injections
  Directly accessed $F^{H \to P}_{+,0}$ very close to q^2_{max}.
  Getting to larger $1/(am_H)$



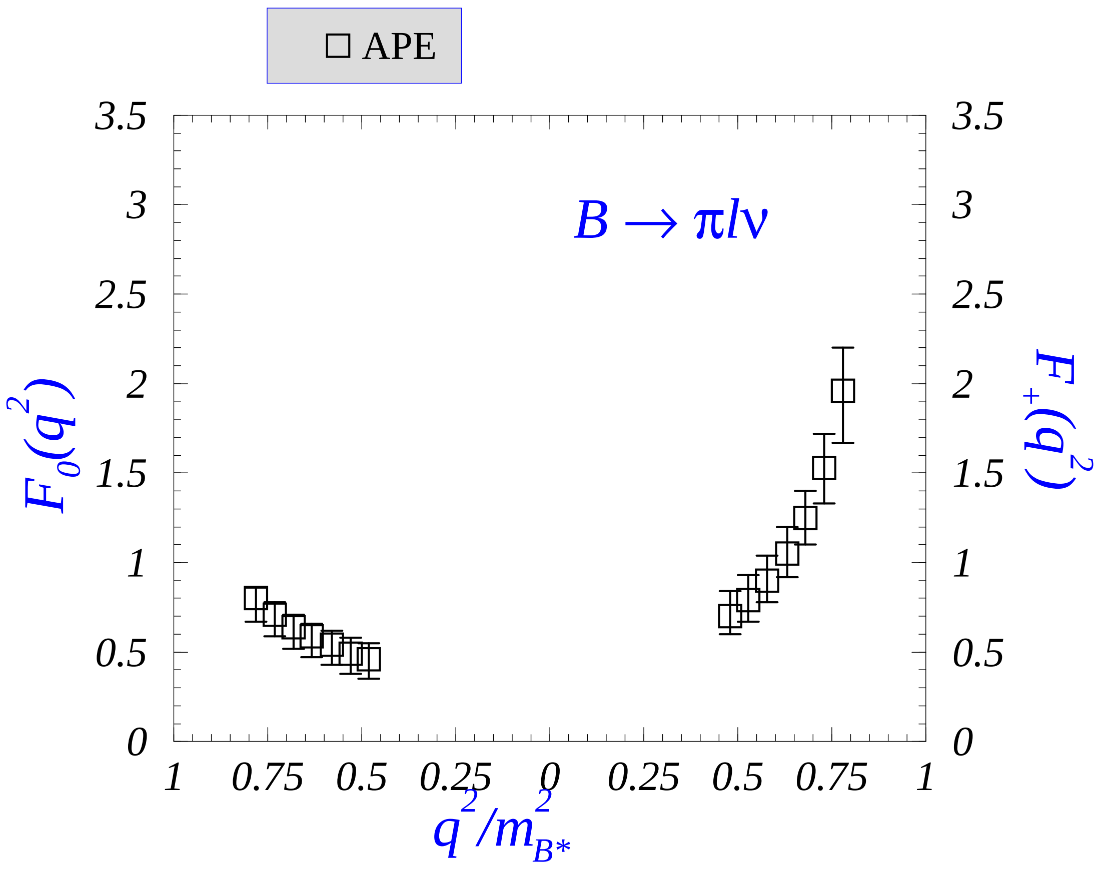
Reconstructed $q^2_{B \to P}$ can get somewhat away from q^2_{max}!

$16\ \mathrm{GeV}^2 \lesssim q^2 \lesssim 26\ \mathrm{GeV}^2$

- **FNAL**: similarly $17\ \mathrm{GeV}^2 \lesssim q^2 \lesssim 26\ \mathrm{GeV}^2$

RESULTS

After linearly/quadratically extrapolating $F_{+,0}$ in the light meson to a physical pion mass (will be back to that later)



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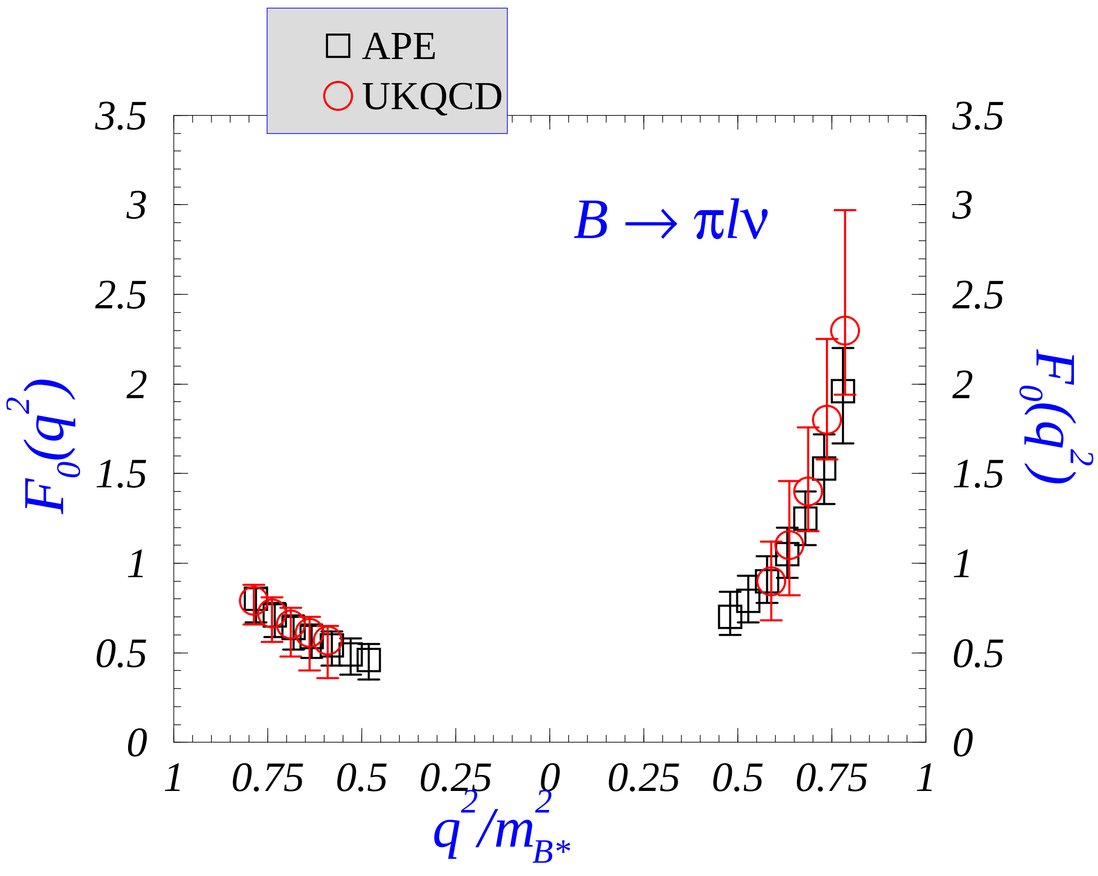

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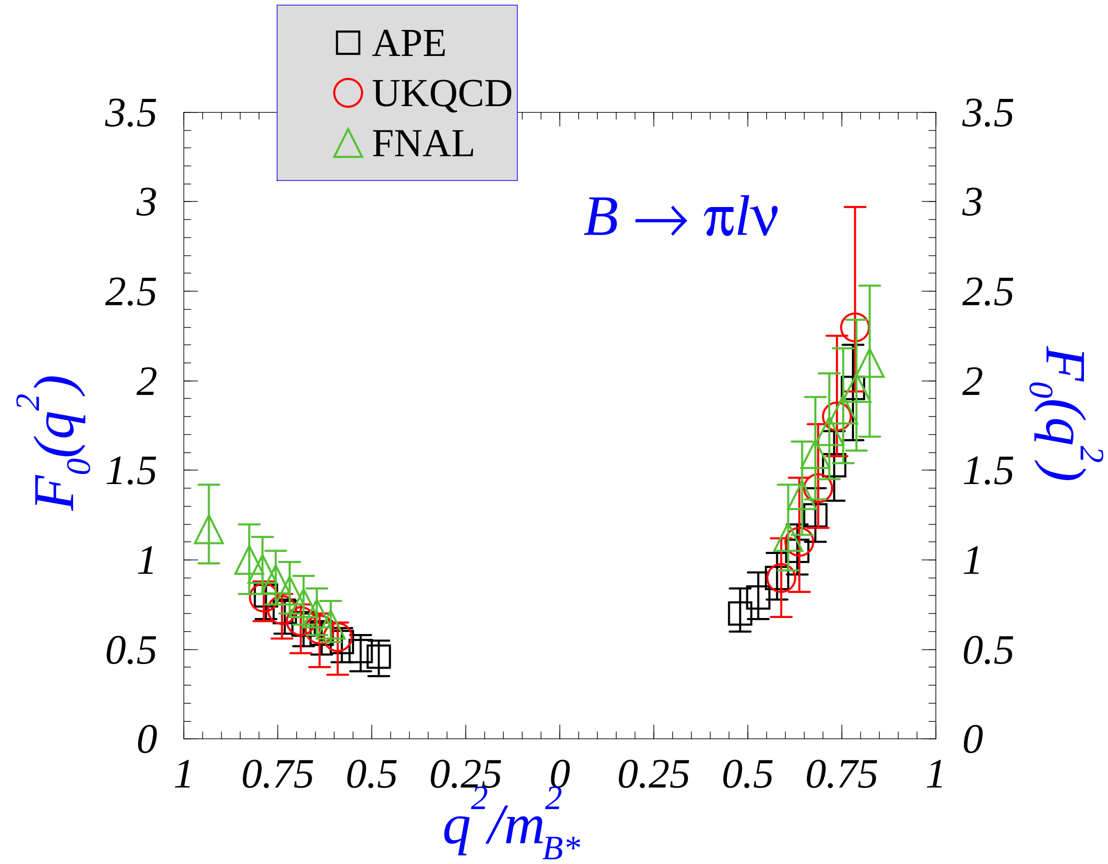

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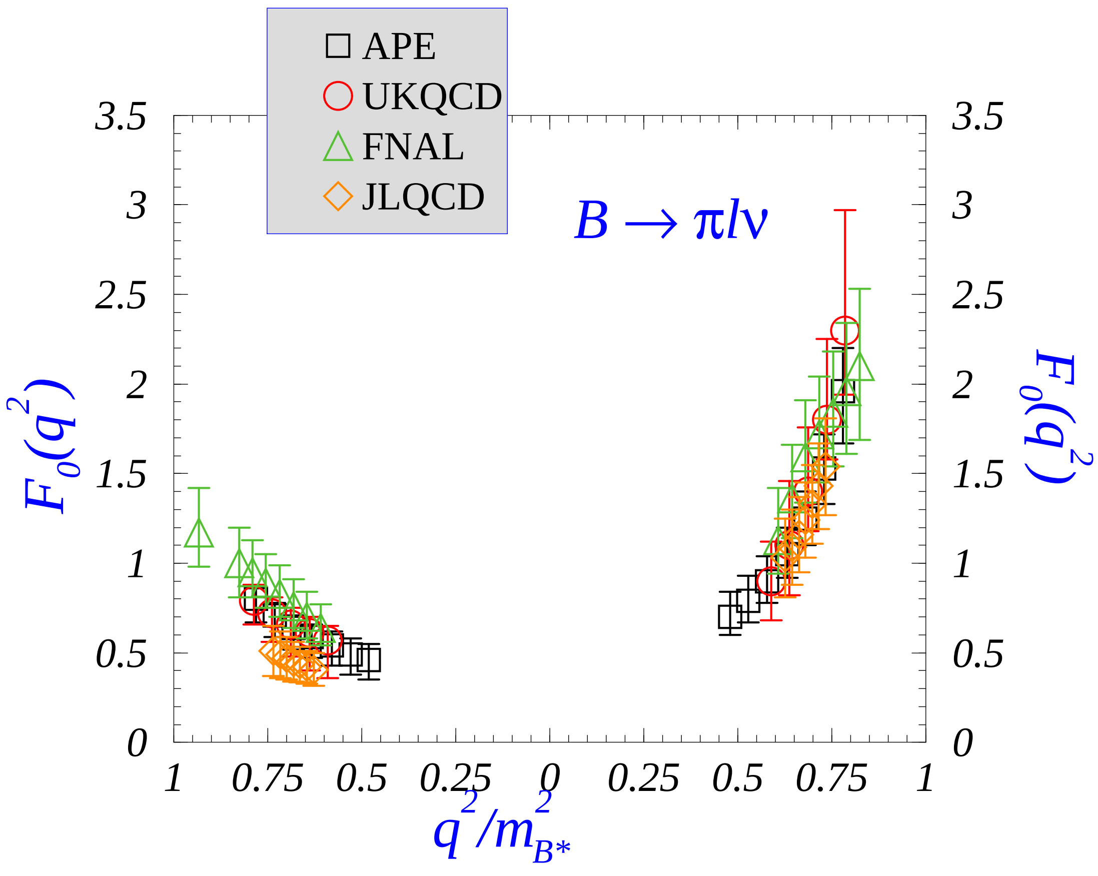

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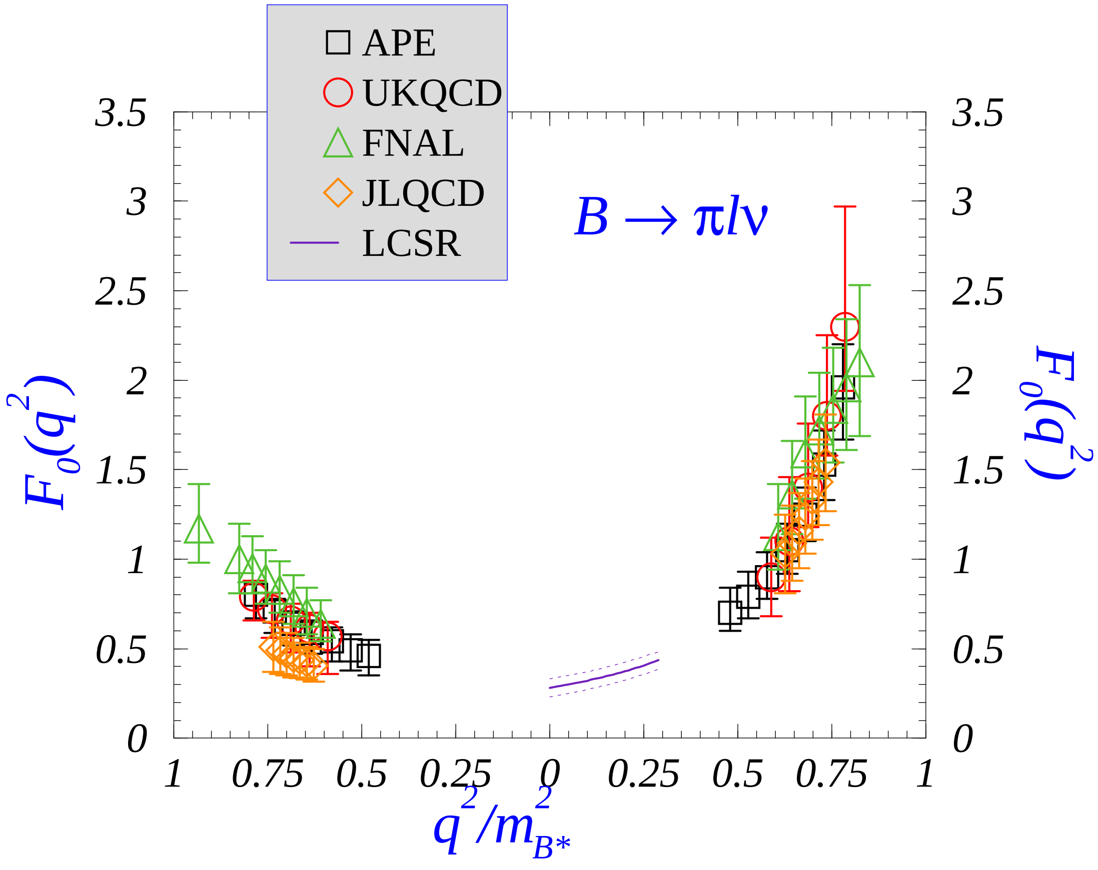


all results published in and after 2000!

To what q^2-form can we fit the data?

- Kinematical region large $[0 \leq q^2 \leq (m_B - m_\pi)^2]$. The nearest pole at $q^2 = m_{B^*}^2$ influences $F_+(q^2)$. Its position known [below cut @ $(m_B + m_\pi)^2$]! $F_0(q^2)$ couples to 0^+-states which are farther away from q^2_{max}.

- HQET (HQS) helps in heavy $\to$ light decays: scaling laws applicable for small recoils $q^2 \simeq q^2_{max}$ (N.Isgur, M.Wise, 1990)

$$F_+(q^2 \simeq q^2_{max}, m_H) \sim \sqrt{m_H} \qquad F_0(q^2 \simeq q^2_{max}, m_H) \sim 1/\sqrt{m_H}$$

- LEET: heavy $\to$ light form factors can be expressed in terms of the universal function (J.Charles et al, 1999)

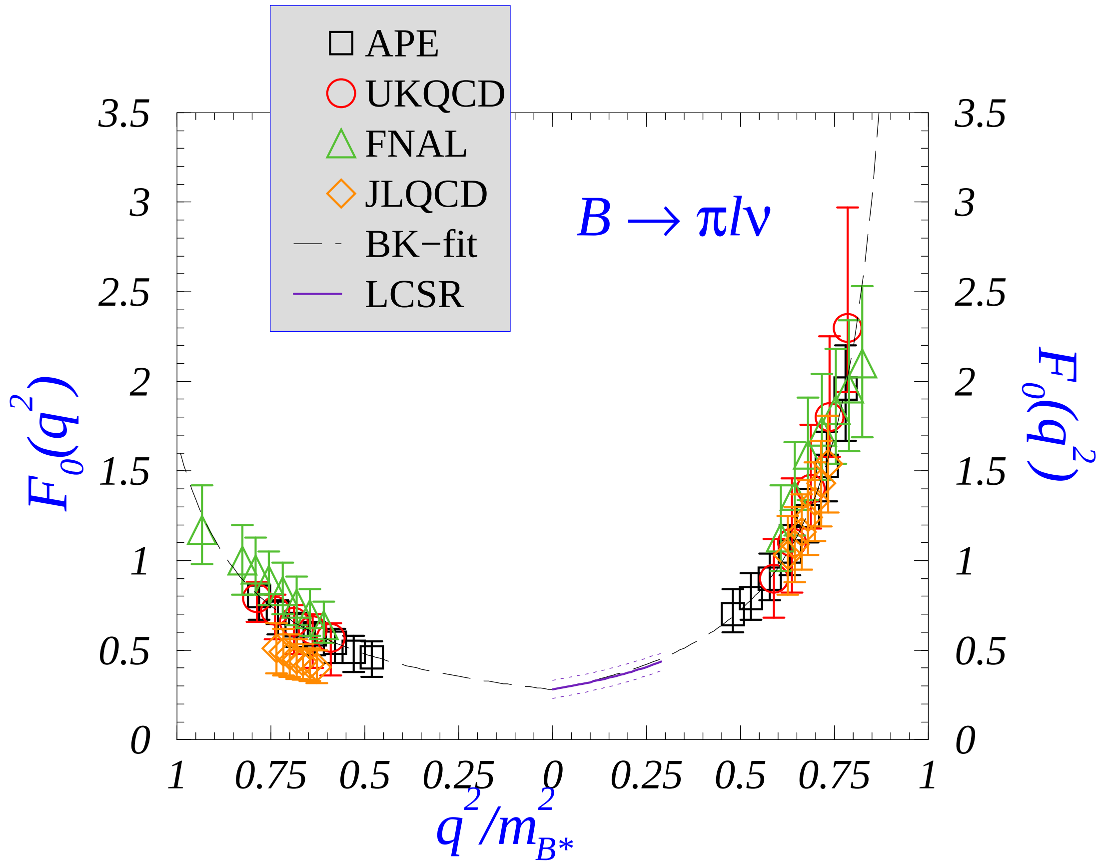
$$F_+(q^2) = \zeta_P(m_H, E) \qquad F_0(q^2) = \frac{2E}{m_H}\zeta_P(m_H, E)$$

$$F_{+,0}(q^2 \approx 0) \sim \sqrt{E}/m_H^2 \sim m_H^{-3/2}$$

applicable for $q^2 \simeq 0$ [explicitely verified by LCSR and in fact anticipated by (Chernyak, Zhitnitsky 1990)].
Sickness of LEET : different IR properties from QCD
$\Rightarrow$ SCET (Ch.Bauer et al. 2000), in which the Charles et al. relations remain valid.

$$F_+(q^2) = \frac{C(1-\alpha)}{(1-q^2/m_{B^*}^2)(1-\alpha q^2/m_{B^*}^2)}$$
$$F_0(q^2) = \frac{C(1-\alpha)}{1-\beta q^2/m_{B^*}^2}$$

(D.Becirevic, A.Kaidalov, 2000)



Agreement among various lattice approaches and with the LCSR results quite impressive.

(LCSR: A. Khodjamirian et al, 2000; P.Ball, R.Zwicky, 2002)

$F_{+,0}(0)$	α	β	ref.
$0.30(^{+6}_{-5})(^{+4}_{-9})$	$0.46(^{+09}_{-10})(^{+37}_{-05})$	$1.27(^{+14}_{-11})(^{+04}_{-12})$	UKQCD
0.26(5)(4)	0.40(15)(9)	$1.22(14)(^{+15}_{-00})$	APE-I
0.28(6)(5)	$0.45(17)(^{+06}_{-13})$	$1.20(13)(^{+15}_{-00})$	APE-II
$0.38(^{+4}_{-6})(^{+3}_{-7})$	$0.21(^{+13}_{-04})(^{+34}_{-00})$	$1.42(^{+06}_{-17})(^{+05}_{-19})$	FNAL
$0.23(^{+4}_{-3})$	$0.58(^{+12}_{-09})$	$1.28(^{+12}_{-20})$	JLQCD
0.28(5)	$0.32(^{+21}_{-07})$	–	KRWWY

`Warning: All studies quenched!` $F_0^{\text{QChPT}} < F_0^{\text{ChPT}}$

(D.B, S.Prelovsek, J.Zupan, 2002).

Problem 2: Remedy?!

- **QCD with propagating quarks**:
  Directly accessed $F_{+,0}^{H\to P}$ around $q^2 \approx 0$.
  Use the $F_{+,0}^{H\to P}(0)m_H^{3/2}$ scaling law.
  Compare APE Vs. LCSR

$$F_{+,0}^{\mathrm{latt.}}(0) = \frac{3.1(5)\ \mathrm{GeV}^{3/2}}{m_H^{3/2}}\left[1 - \frac{0.98(9)\ \mathrm{GeV}}{m_H}\right]$$

$$F_{+,0}^{\mathrm{lcsr}}(0) = \frac{3.2\ \mathrm{GeV}^{3/2}}{m_H^{3/2}}\left[1 - \frac{1.3\ \mathrm{GeV}}{m_H}\right]$$

  $\mathcal{O}(1/m_H)$-corrections large – similar in magnitude to the ones that appear in the calculation of f_B!
- **mNRQCD**:
  Define NRQCD in the frame in which the low q^2's are reached by sharing the three momenta between the "pion" and B-meson. Very nice idea - *(LCSR: Hashimoto et al, 1996)* has not been tested. Renormalisation of the operators by including $1/(am_b)$ terms is even more challenging than in the standard NRQCD. (wait for HPQCD - work in progress....)

Problem 3: Chiral extrapolation

- light quark accessible from the lattice $r = m_q/m_s^{phys.}$

$$1/2 \lesssim r \lesssim 3/2$$

need to extrapolate to $r_\pi^{phys.} \simeq 0.04$ (*H.Leutwyler 1996*)

- within the range of r, directly accessed from the current simulations, one observes a good linear/quadratic dependence of $F_{+,0} = \alpha + \beta \cdot r + \gamma \cdot r^2$

- **In unquenched studies**: Worry about the chiral logs (*A.Falk, B.Grinstein 1994; D.B.,Prelovsek,Zupan, 2002*):
(i) include them in extrapolation, or
(ii) get rid of them by forming suitable ratios with measured quantities [similar to what has been done for $(f_{B_s}/f_{B_d})/(f_K/f_\pi)$].
- **In partially unquenched studies ($n_F = 2$)**:
$r_{\text{sea}} \neq r_{\text{val.}}$ (*D.B.,Prelovsek,Zupan, 2003*)

$$\begin{aligned}
\delta F_+^{\chi-Loop} &= \frac{1}{(4\pi f)^2}\Big[\Big(-2g^2\frac{M_S^2}{(vp)^2} + 1 + 3g^2\Big) M_V^2 \ln(M_V^2) \\
&\quad - \frac{1+3g^2}{2} M_S^2 \ln(M_V^2) - 4\pi g^2 \frac{M_S^2}{vp} M_V\Big] + C_0'^p + C_2'^p M_V^2 + \dots\,, \\
\delta F_0^{\chi-Loop} &= \frac{1}{(4\pi f)^2}\frac{1+9g^2}{6}(2M_V^2 - M_S^2)\ln(M_V^2) + C_0'^v + C_2'^v M_V^2 + \dots\,,
\end{aligned}$$

where $M_S = 2B_0 m_s^{phys.} r_{\text{sea}}$, $M_V = 2B_0 m_s^{phys.} r_{\text{val.}}$, $C'^{p,v}_{0,2}$ functions of vp and M_S.

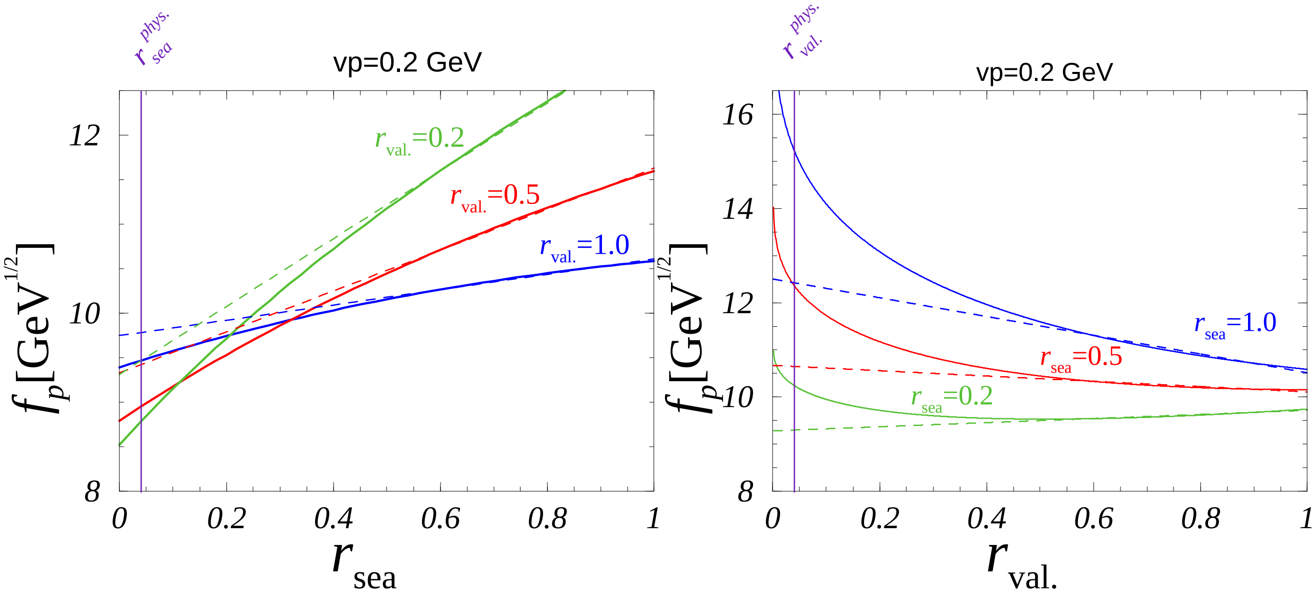
Problem 3: cont.

♣ In partially unquenched studies ($n_F = 2$):

$$\delta F_+^{\chi-Loop} = \frac{1}{(4\pi f)^2}\Big[\Big(-2g^2\frac{M_S^2}{(vp)^2} + 1 + 3g^2\Big) M_V^2 \ln(M_V^2) - \frac{1+3g^2}{2} M_S^2 \ln(M_V^2) - 4\pi g^2 \frac{M_S^2}{vp} M_V\Big] + C_0'^p + C_2'^p M_V^2 + \dots ,$$

$$\delta F_0^{\chi-Loop} = \frac{1}{(4\pi f)^2}\frac{1+9g^2}{6}(2M_V^2 - M_S^2)\ln(M_V^2) + C_0'^v + C_2'^v M_V^2 + \dots ,$$

$$f_p = 2F_+/\sqrt{m_B}$$

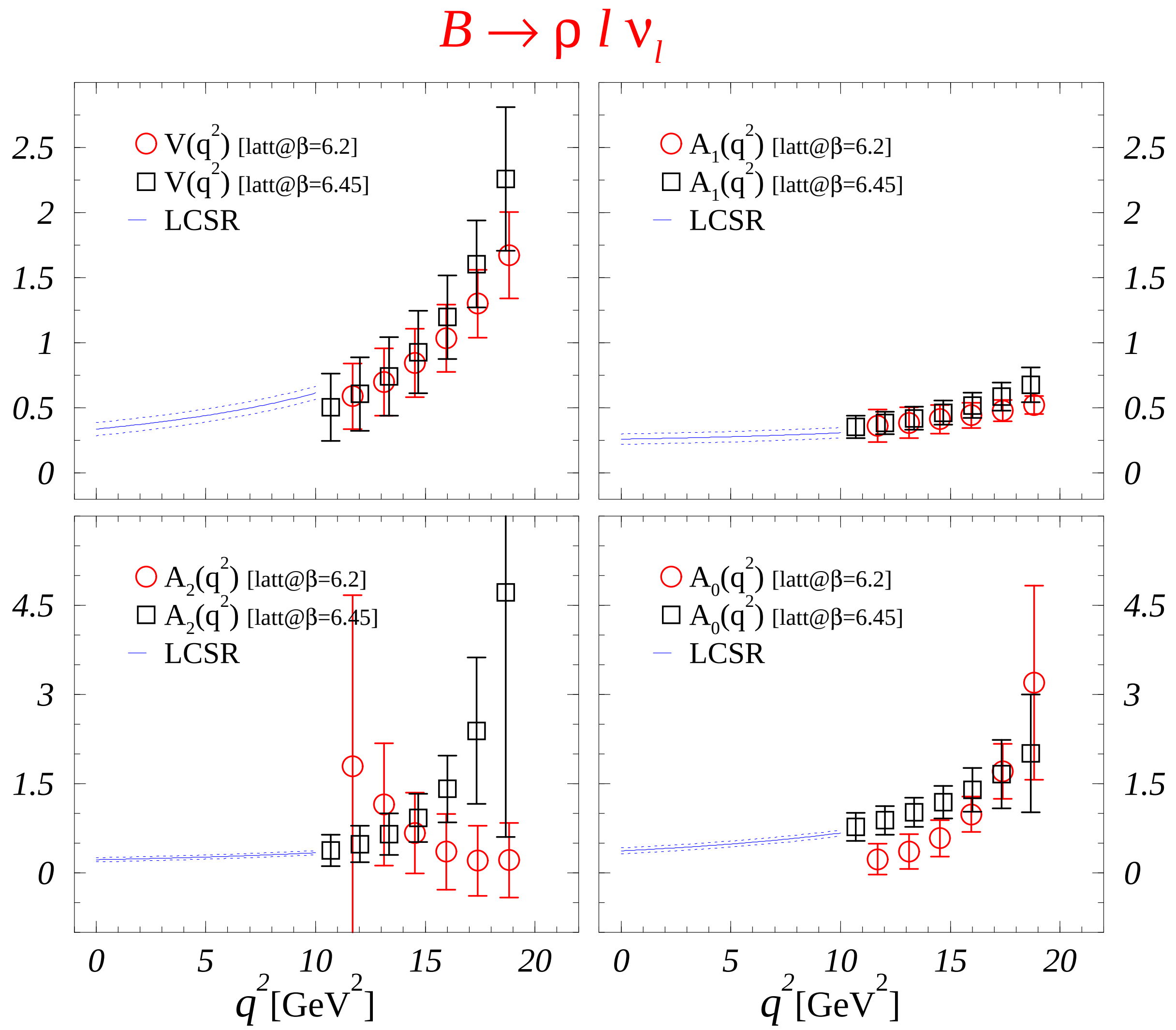


Small extrapolation errors from accessible quark masses is possible IFF the extrapolation is made first in $r_{\text{sea}} \to r_\pi^{phys.}$, and then in $r_{\text{val}} \to r_\pi^{phys.}$!

Numerically costly but within the reach for currently available computing architectures.

$B \to \rho\ell\nu$ form factors

harder :
more form factors, less constraints on the shapes



SPQcdR [preliminary!] Results this summer...

Combine lattice data at large q^2 with LCSR results at low q^2's (*P.Ball, V.Braun, 1998*)

On $B \to K^*\gamma$

$b \to s\gamma$ theory Vs. experiment

Inclusive decays:

$\otimes$ `experimentally difficult`

$$BR_{exp.}(B \to X_s\gamma) = \begin{cases} (3.21 \pm 0.43^{+0.32}_{-0.29}) \cdot 10^{-4} & \text{CLEO, 2001} \\ (3.36 \pm 0.53 \pm 0.68) \cdot 10^{-4} & \text{Belle, 2001} \end{cases}$$

$\otimes$ `theoretically rather clean`

(QCD calculation at NLO completed in *A.Buras et al,2002*)

$$BR_{th.}(B \to X_s\gamma)_{E_\gamma > 1.6\text{GeV}} = (3.57 \pm 0.30) \cdot 10^{-4}$$

Exclusive decays:

$\otimes$ `experimentally easier`

$$BR_{exp.}(B^+ \to K^{*+}\gamma) = \begin{cases} (3.79 \pm 0.86 \pm 0.28) \cdot 10^{-5} & \text{CLEO, 2000} \\ (3.83 \pm 0.62 \pm 0.22) \cdot 10^{-5} & \text{BaBar, 2002} \\ (4.97 \pm 0.56 \pm 0.38) \cdot 10^{-5} & \text{Belle, 2002} \end{cases}$$

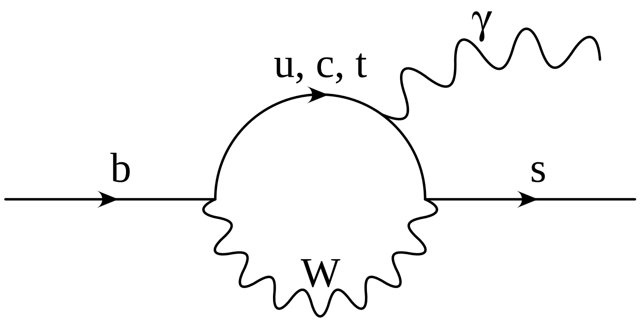
Large data samples from BaBar and Belle arriving $\Rightarrow$ Errors will go down!

$\otimes$ `theoretically not clean`

Large hadronic uncertainties:

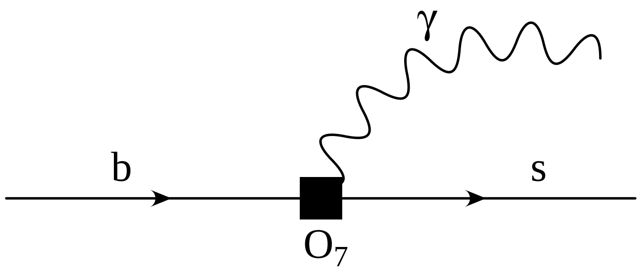
Lattice QCD may help!

Theoretical expression for $B \to K^*\gamma$ is derived by applying the OPE (expansion in $1/m_W^2$)



$\rightarrow$

$C_7 \times$



$+ \mathcal{O}(1/m_W^2)$

$$\boxed{\mathcal{H}_{eff}^{b\to s\gamma} = -\frac{G_F}{\sqrt{2}} V_{tb}^* V_{ts} C_7(\mu) \mathcal{O}_7(\mu)}$$

- $C_7(\mu)$ **Wilson coefficient**

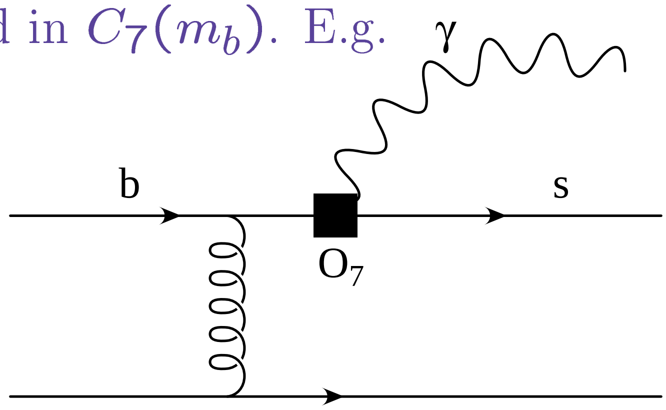
information on the short distance physics (stuff in the loops) [use perturbative QCD]

- $\mathcal{O}_7(\mu)$ **EM penguin operator**

$$\mathcal{O}_7 = -\frac{e m_b}{8\pi^2} \bar{s} \sigma_{\mu\nu} (1+\gamma_5) b F^{\mu\nu}$$

soft physics in $\langle K^* | \mathcal{O}_7 | B \rangle$ [need non-perturbative QCD]
(N.B. $\mu \sim m_b$)

★ Recently the hard spectator effects have been included in $C_7(m_b)$. E.g.



$$\boxed{|C_7(m_b)|^2 = 0.17(2)}$$

M. Beneke, T. Feldmann, D. Seidel (2001),
S. Bosch, G. Buchala (2001),
A. Ali, A. Ya. Parkhomenko (2001)

★ $B \to K^*\gamma$ **matrix element**

$$\langle K^*(p', e_\lambda)|\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b|B(p)\rangle = c^{(1)}_{\mu\nu}T_1(q^2) + c^{(2)}_{\mu\nu}T_2(q^2) + c^{(3)}_{\mu\nu}T_3(q^2)$$

$c^{(1,2,3)}$-known functions of the kinematical variables

$$(p, p', e_\lambda, m_{K^*}, m_B)$$

$T_{1,2,3}(q^2)$ - unknown form factors relevant for

$$B \to K^*\ell^+\ell^-$$

For the on-shell photon ($q^2 = 0$): $c^{(3)} = 0$ and $T_1(0) = T_2(0)$

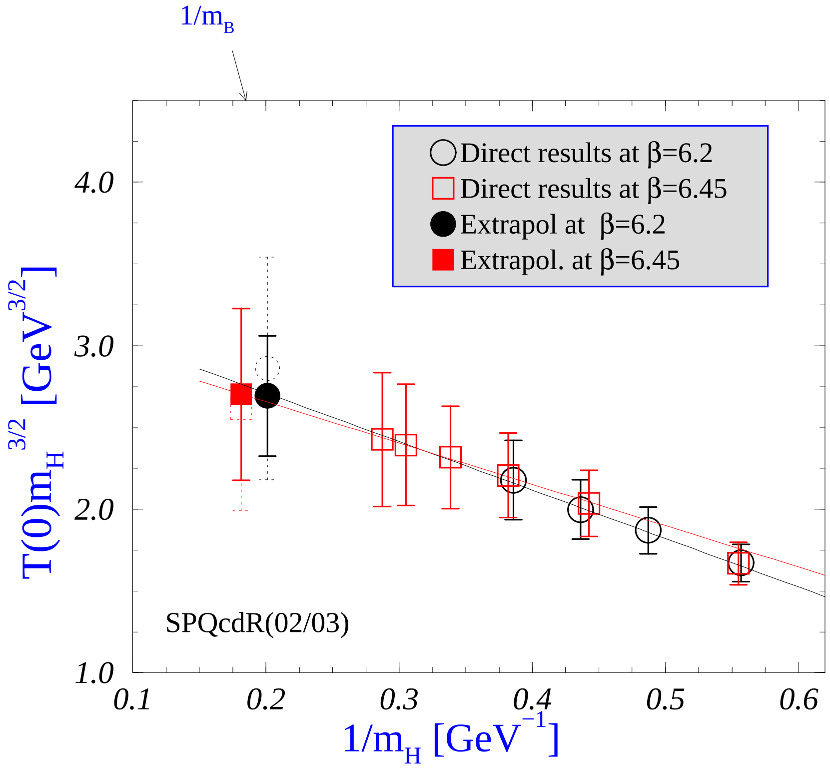
Situation similar to the $B \to \pi$ case. In the HQL/LEnL

$$T_1(q^2) = \zeta_\perp(m_H, E) \qquad T_2(q^2) = \frac{2E}{m_H}\zeta_\perp(m_H, E)$$

$$T_{1,2}(q^2 \approx 0) \simeq T_2(q^2 \approx 0) \sim \sqrt{E}/m_H^2 \sim m_H^{-3/2}$$

applicable for $q^2 \to 0$ (again explicitely verified by LCSR (P.Ball, V.Braun, 1998));

$$\boxed{T(0, m_P)m_P^{3/2} = a_0 + a_1/m_H + a_2/m_H^2}$$



@ $\mu = m_b$

$$T_{1,2}^{\text{latt.}}(0)_{\text{lin.}} = \frac{3.3\ \text{GeV}^{3/2}}{m_H^{3/2}}\left[1 - \frac{0.9\ \text{GeV}}{m_H}\right]$$

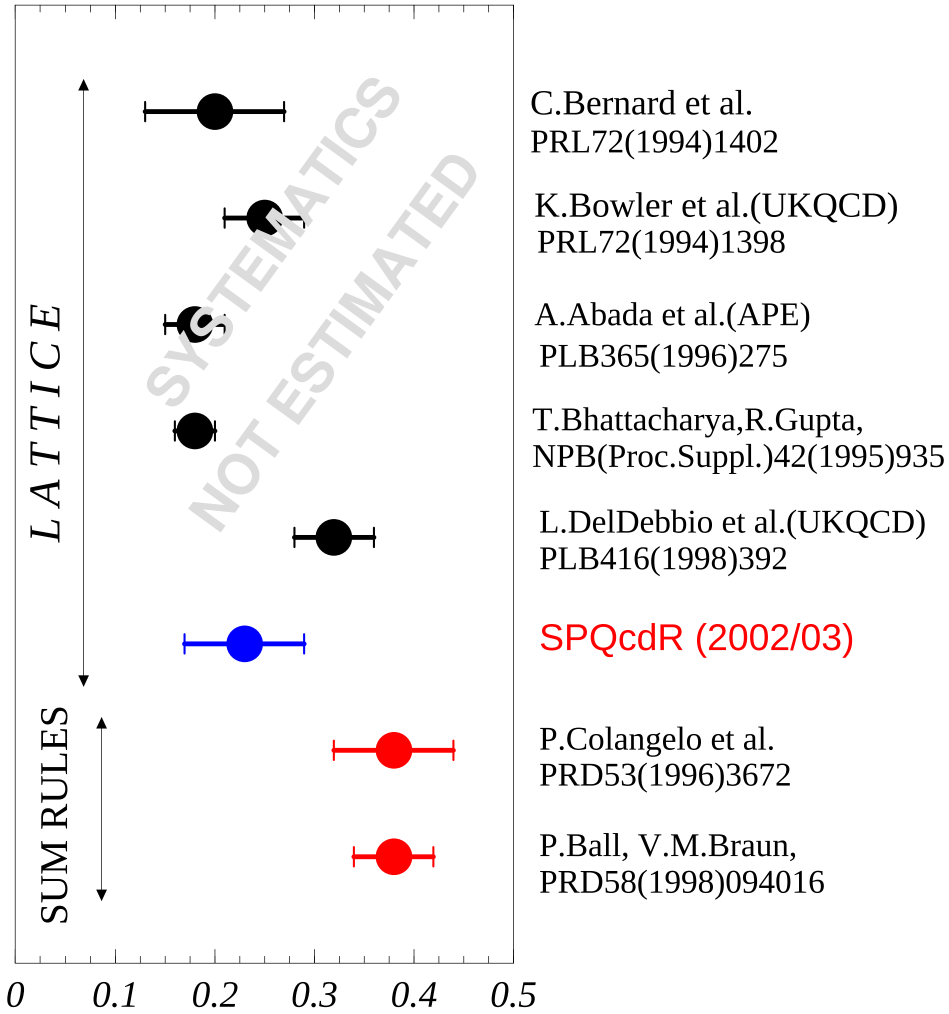
$$T_{1,2}^{\text{latt.}}(0)_{\text{quad.}} = \frac{3.8\ \text{GeV}^{3/2}}{m_H^{3/2}}\left[1 - \frac{1.4\ \text{GeV}}{m_H} + \frac{0.6\ \text{GeV}^2}{m_H^2}\right]$$

Result [still preliminary!]:

$$T^{B\to K^*}(0) = 0.25(5)(2), \quad \frac{T^{B\to K^*}(0)}{T^{B\to\rho}(0)} = 1.1(1)$$

Compared to the LCSR values, these results are much smaller:

$$T^{B\to K^*}(0) = 0.38(6), \quad \frac{T^{B\to K^*}(0)}{T^{B\to\rho}(0)} = 1.31(7)$$



Summary:

★ $B \rightarrow \pi(\rho)$ form factors essential ingredient for extracting $|V_{ub}|$ from experiment, and in understanding the non-factorisable effects in $B \rightarrow h\pi$ decays; Many recent lattice calculations are published – various ways to treat heavy quarks have been employed and overall agreement is quite pleasant; Moreover, there is a nice agreement with LCSR results

★ All results so far obtained in quenched approximation. Guesstimate of quenching errors from QChPT Vs ChPT (in the HQL) indicates that those errors might be large; Procedure for a safe chiral extrapolations in partially (un)quenched QCD is available – simulations will be performed soon.

★ At present, lattice data for $B \rightarrow \rho\ell\nu$ are combined with LCSR results; Lattice QCD helps LCSR in computing some of the hadronic parameters [$f_\rho^T(\mu)/f_\rho$ (D.B.,Lubicz,Mescia, Tarantino,2003, and V.Braun et al, in preparation]; Unquenching ρ is a challenge for the lattice community. Need to understand the nature of resonances $\Rightarrow$ $B \rightarrow \pi$ is safer!

★ $B \rightarrow K^*\gamma$: several simulations by SPQcdR – all pointing towards the value of the form factor that is smaller than the LCSR prediction. $T^{B\rightarrow\rho}(0)$ is however in good agreement with LCSR $\Rightarrow$ SU(3) breaking from the lattice $\approx 10\%$ instead of $\approx 30\%$ as suggested by LCSR!

★ Perspective: much effort in unquenching the $B \rightarrow \pi$ form factors; First results to be expected next year. Present lattice results are quenched and they should be taken with the grain of salt at least until the partially unquenched data become available.