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ANALYTICAL METHODS IN HEAVY QUARK PHYSICS

I Introduction

II General view on exact methods and dynamical methods

III Specific problems of dynamical methods (QCDSR and quark models)

IV Confrontation of experiment, dynamical methods and sum rules
Example of ρ^2, $\tau_{1/2}$, $\mu_\pi^2 (\lambda_1)$, ...

V Conclusions

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II a) Results … (suite)

– Finally, in exceptional cases, absolute predictions

For $b \to c$ $\quad \xi(1) = 1 \quad (w=1)$

Ex. $\quad \Gamma_{B \to X_c \ell \nu} = \Gamma_{b \to c \ell \nu}$ perturbative (quarks + gluons)

(NB absolute with knowledge of $m_{b,c}$ and α_s, but these are basic parameters of QCD)

b) Finite mass corrections; $1/m_Q$ expansions

Now, to obtain physical results, one must nevertheless know what happens at finite masses m_b, m_c. $1/m_Q$ exp.

Among simplest results

Ex: $w=1 \qquad h_{A_1}(1) = 1 + O\left(\frac{1}{m_Q^2}\right)$ + rad. corr.

Ex $\quad \Gamma_{B \to X_c \ell \nu} = \Gamma_{b \to c \ell \nu}\left(1 + O\left(\frac{1}{m_b^2}\right)\right)$

In both cases, the «power» corrections have coefficients controlled by heavy meson - heavy meson matrix elements of simple operators $\quad \frac{1}{m_b^2} \quad \mu_\pi^2\ (\text{or } -\lambda_1),\ \mu_G^2\ (\text{or } -\lambda_2)$

Here local $\qquad \frac{1}{m_b^3} \quad \rho_D^3, \ldots \qquad$ but not all local

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IV c) Experimental sources

1) $(\rho^2)_{A_1}$ not less than 1,2; several notably higher values (note the role of a curvature, bounded from below, in enhancing ρ^2) although it seems a rather difficult measure, with large errors.

$+ \ (\rho^2)^{SR}(1\,\mathrm{GeV})$ must not be very far from $\rho_{A_1}^2$ (taking into account various rad. corr.), with $\pm$ 0,2 of unknown $1/m_Q$ corrections $\Longrightarrow (\rho^2)^{SR} > 1$

2) $\tau_{1/2}(1)$ very crucial. CLEO, and then BELLE with more statistics: measurement of $B^- \to D^{**}(0^+, 1^+_{1/2})\pi^-$ with $D^{**}_{1/2}$ well identified. Fixes $\tau_{1/2}$ ($q^2 = 0$ or $w \sim 1,3$) assuming FACTORISATION which works well in this class?.
+ small finite mass corrections (should not be large, otherwise one would observe a strong difference between 0^+ and $1^+_{1/2}$) ?

$$\boxed{\tau_{1/2}(1) > \tau_{1/2}(q^2=0) \sim 0,4}$$

consistently from 0^+ and $1^+_{1/2}$

$\tau_{3/2}$ quite consistent with $B \to D^{**}(1^+_{3/2})\ell\nu$ ($\sim 0,3$ at $q^2=0$)

3) $\bar{\Lambda}, \mu_\pi^2, \rho_D^3$ from fits to moment of $b \to s\gamma$, $b \to c\ell\nu$

central values large errors

$$\begin{cases} (\bar{\Lambda})^{SR}(1\,\mathrm{GeV}) \sim 0,6 \text{ to } 0,72\,\mathrm{GeV} \ (\text{converted from } \bar{\Lambda}_{HQET}) \\ \mu_\pi^2 \sim 0,3 \text{ to } 0,45\,\mathrm{GeV}^2 \\ \rho_D^3 \sim 0,05 \end{cases}$$

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IV d) Consequences to be drawn from experiment

1) direct problems for dynamical calculations

– $\rho^2_{A_1}$ seems too large for QCDSR

Yet: Encouraging for BAKAMJIAN-THOMAS which predict naturally large ρ^2

– $\tau_{1/2}(q^2=0) \sim 0{,}4$ too large for QCDSR

decreasing form factor $\to 0{,}2$ at most at $q^2=0$

also too large for BAKAMJIAN THOMAS quark models

decreasing form factor $\to 0{,}2$ at $q^2=0$

discrepancy by a factor 2!

rad corr. ?

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2) with a conservative $\tau_{1/2}(w=1) \sim 0{,}4$, the bounds above give

$(\rho^2)^{SR} > 1{,}25$ confirms $\rho^2_{A_1}$ measure large

Sum rules
$$\begin{cases} \bar{\Lambda}^{SR}(1\,\text{GeV}) > 0{,}7 \\ \mu_\pi^2(1\,\text{GeV}) > 0{,}53 \\ \rho_D^3(1\,\text{GeV}) > 0{,}18 \end{cases}$$

μ_G^2 from B^*-B: $0{,}35\ \text{GeV}^2$

$\Rightarrow$ QCDSR estimates for $\bar{\Lambda}$ and μ_π^2 are in good agreement with these bounds, but not for ρ^2 once more.

$\Rightarrow$ Central values of fits from moments seem below these bounds, especially for μ_π^2. (Δ conservatively small?) all

Yet, compatible within large present errors

V CONCLUSIONS

a) Heavy quark theory has managed to establish a series of very strong relations between apparently remote quantities

e.g. strong bounds on $\left\{\begin{matrix}\mu_\pi^2\\ \rho^2\end{matrix}\right.$ from $B \to D^{**}_{1/2}\pi$, $D^{**}-D$, D^*-D. Rather high!

b) Improving the experimental determination of $B \to D^{**}(j=1/2)\pi$ and estimating the finite mass corrections is very important.

c) Up to now, dynamical methods (analytical) are not very predictive. There is need to improve considerably QCDSR, as well as to formulate new quark models for form factors, including the advantages of BAKAMJIAN-THOMAS and DIRAC equation.

d) Some lattice calculations are strongly needed to remove the uncertainty on crucial quantities like $\tau_{1/2}(1)$. A good precedent has been $\hat{g}$

Note added to page 18 after the talk:

In this estimate of $\tau_{1/2}$, we are also neglecting the color suppressed diagram in $B \to D^{**}\pi$. We thank BELLE collaborators (A. BANDAR) for underlining that.

II c) Effective theories

One can exhibit these heavy quark limit properties in a Lagrangian formalism. Very simple limit of QCD Lagr.

HQET $m_Q \to \infty$ $\mathcal{L} \sim i\, \bar{Q}_v v^\mu D_\mu Q_v$ (Q_v or h)
→ spin symmetry

LEET
$m_Q \to \infty$
$E_\pi \to \infty$ $\tilde{\mathcal{L}} \sim i\, \bar{q}_n \gamma (n^\mu D_\mu) q_n$

Q heavy quark, q light quark, v velocity of Q, n // light meson momentum

HQET allows to perform dynamical calculations or perturbative calculations in field theory in heavy quark limit. + $1/m_Q$

[NB However, one can also work in full QCD, with finite physical masses. This is not always more difficult. Yet heavy quark expansion, if converging sufficiently quickly, allows to concentrate on a limited set of parameters: ρ^2, μ_π^2, etc… ((almost) independent of quark masses)]

d) WILSON Operator product expansion (OPE)

Recall that OPE is central in heavy quark theory, since it lies behind the $1/m_Q$ expansion - for instance for effective theories. More generally: allows to develop GREEN functions in inverse powers of a large parameter, which may be also a large momentum (QCDSR at $m_Q = \infty$). The coefficients of the expansion are matrix elements of operators between hadrons or over vacuum, times perturbative factors.

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II B) Dynamical calculations for hadronic M.E.

- Need for dynamical calculations. Obviously, theory of heavy quark limit does not allow to avoid dyn. calc. except in exceptional cases: $B \to D^{(*)}$ at $w=1$. Inclusive $\Gamma(B \to X_c \ell \nu)$ at least requires $m_b, m_c (\bar{\Lambda})$. ρ^2, $\xi(w)$ remain unknown. $1/m_Q$ expansion requires new unknown parameters like μ_π^2 or ρ_D^3 .. to be calculated (or det. empirically as recently CLEO DELPHI)

- Most fundamental (entirely based on QCD Lagrangian) is lattice QCD method. Relies however on very heavy numerical calculations - and still rather approximate.

2 mainly analytical methods to calculate matrix element $\begin{cases} \text{QCDSR } (\neq \text{"Sum rules"}) \\ \text{quark models} \end{cases}$

Those latter methods both involve approximations to QCD which cannot be reduced systematically, neither can error be safely estimated. "Pas de voie royale pour QCD"

1°) QCDSR OPE expansion of Green functions. Large use of perturbative QCD + limited non perturbative ingredient through vacuum condensate.

This is the well founded side. Not so for the other side.

(similar to inclusive decays or SV sum rules)

II B) Dynamical calculations (suite)

QCDSR (suite). The other side is to extract the ground state properties from the calculated GREEN function. This is not possible by a systematic algorithm; rather a matter of art, with a phenomenological model and with various parameters to fix: continuum threshold, fiducial range of M^2, stability criteria... The resulting accuracy is then moderate and variable according to the problem under study.

2°) Quark models Useful in view of the uncertainties remaining in "fundamental" methods. Only qualitative relation to QCD. Main ingredient is the QCD inspired potential:

$$V(r) = \lambda r - \frac{4}{3}\frac{\alpha(r)}{r}$$

- The advantage of QM is to be founded on a direct intuition of bound state physics unlike QCDSR. Important since, after all, we deal really with bound states.
- However, bound state physics is something very complex beyond NR=non relativistic approximation.
- This complexity is reflected in the large variety of quark models. No "standard" quark model. One then gets quite often a very large range of predictions. The thing to do is then to understand what is behind each model. It requires time. Anyway, the accuracy in QM too will remain quite variable for possibly quite a long time.

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II C) Models as testing bench

[Before entering into more details about dynamical calculations]

Finally, one must tell a word about another important application of models.

Models may help to analyse certain features of QCD, or problems of QCD methods, in much simpler situations, where one can perform much more explicit calculations. This one may call a testing bench for QCD

Two important applications
- check of duality in inclusive decays
- checks of QCDSR approach.

Two types of models are useful
1°) t'HOOFT model (QCD_2, $N_c \to \infty$)
2°) Quark models

(Especially harmonic oscillator NR model satisfies exact duality.

[BAKAMJIAN-THOMAS models are useful to check leading order SV sum rules]

III SOME SPECIFIC PROBLEMS OF DYNAMICAL METHODS (analytical)

As we have said, a general problem of QCDSR and quark models is that, to a different extent, they are not <u>definite</u> <u>approximations</u> to QCD. However, in the present context of <u>heavy</u> quarks, they have more <u>specific</u> problems, useful to recall in view of the discrepancies observed with <u>experiment</u> (see IV)

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a) <u>QCDSR method</u>. Here, the main problem seems the standard treatment of <u>radial</u> <u>excitations</u> ("<u>continuum</u>" <u>model</u>) on the <u>hadronic</u>, or phenomenological side, or "L.H.S" in so called "three-point QCDSR", especially in HQET.

Let us indeed concentrate on the problem of <u>ground state</u> to <u>ground state</u> matrix elements (M.E.) of an operator $\mathcal{O}$
$\mathcal{O} = j^\mu$ or $j^{\mu 5}$ for ρ^2 and $\tau_{1/2}$; or $\vec{D}^2$ for μ_π^2, ...
[ground state = lowest state in a channel of definite J^P]

If one looks for $\langle H'|O|H\rangle$, one considers the <u>GREEN</u> <u>function</u>: $\langle 0|T(j^{H'}, \mathcal{O}, j^H)|0\rangle$

$j^{H,H'}$ are "<u>interpolating</u>" currents connecting the vacuum to H and H' with factors $f_{H,H'}$. Whence a contribution $\sim f_H f_{H'} \langle H'|\mathcal{O}|H\rangle$

III a) QCDSR (suite)

Knowing $f_{H,H'}$ from other QCDSR (two-point), and calculating the above GREEN function by OPE, one would thus manage to obtain

$$\langle H'|O|H\rangle \overset{?}{\sim} \frac{\langle 0|T(j^{H'}, O, j^{H}|0\rangle}{f_H f_{H'}}$$

Not so. $j^{H,H'}$ also connect the vacuum to all the radial excitations $H^{(n)} H'^{(n')}$. Then one must get rid of these radial excitations.

The standard way to do it is twofold:

→ 1°) One approximates the radial states by a continuum, itself calculated by OPE. This introduces a first basic parameter, the continuum threshold s_c.

→ 2°) This approximation introduces an error. One tries to reduce it by a BOREL transformation $\times \exp -\frac{s}{M^2} \exp -\frac{s'}{M'^2}$ which reduces the contribution of radial exc. if M^2, M'^2 sufficiently small. However M^2 must not be too small. Otherwise one has to calculate too much power corrections in the OPE calculation.

→ "Fiducial" window for $M^2; M'^2$: $M^2_{min} < M^2 < M^2_{max}$

Now, both steps 1°) and 2°) may fail somewhat, precluding good accuracy (Sometimes, very large error)

III a) QCDSR (suite)

- Particularly in three-point sum rules in HQET, (not only)
it may happen first that M^2 cannot be made sufficiently low to reduce enough the excitation contribution (rapidly rising spectral functions)
This seems to be the case for μ_π^2

- Second in addition, in three-point sum rules, it may happen that the representation of excitations by the OPE continuum is defective, particularly because one may have strong radial contributions with sign opposite to the ground state

[This seems to be the case for the $\hat{g} \sim g_{D^*D\pi}$ calculation. Including a negative radial excitation restores stability in M^2 and makes $\hat{g}$ twice larger, restoring agreement with experiment.]

According to a NRHO calculation, the problem could appear in ρ^2 QCDSR ?

Possible symptoms of problems are α) Lack of stability in M^2 ← μ_π^2
β) too critical dependence on s_c

But these symptoms may be absent.

III b) Quark models

Since there is a great variety of models we have to make a selection. Nevertheless, we will try to underline the physical problems leading to the various possible choices.

1°) Non relativist (NR) model

Basic one. Fully consistent and easy to use. Weaknesses: the ones of NR mechanics in a rather relativistic world,

To be specific α) NR kinematics may seem very rough for $b \to c$. Not absurd however at small recoil; but this is misleading if one calculates a derivative e.g. $\rho^2 = -\xi'(1)$ which is sensitive to relativistic boost effect.

β) Still more worrying, internal velocities are not small in heavy-light systems: Δ (excitation energy with respect to ground state) $\sim m \sim 0{,}35$ GeV. Therefore v/c expansion may be totally misleading $\longrightarrow$ unlike Υ

Ex 1 ρ^2 is too small $\rho^2 \sim 0{,}5$

Ex 2 $1/m_Q^2$ corrections to $h_{A_1}(1)$ (overlap effect in the NR approach) are much too small

2°) Relativistic models à la BAKAMJIAN-THOMAS

Include light-front quark models. Well adapted to $b \to c$ form factor calculations. Several qualities, starting from a much improved treatment of the center-of-mass motion, in a full Hamiltonian formalism.

A. Le Y.

III b) Quark models (suite)

BAKAMJIAN-THOMAS models present:

- realistic $\rho^2 \sim 1$; Corr. to $R_{A_1}(1)$ sensible (related to $\langle \vec{p}^2 \rangle$)
- Covariance in the $m_Q \to \infty$ limit as well as $E_\pi \to \infty$ in $B \to \pi$ (LEET limit)
- Symmetry relations of HQET and LEET
- relativistic form of BJORKEN and URALTSEV sum rules as well as higher derivative sum rules (curvature ....)

However they do not satisfy sum rules for higher moments like VOLOSHIN sum rule.

Another defect, which may become critical: approximation of free quark DIRAC spinors, essential to these models, may become too rough for light quark in $Q\bar{q}$

EX: corrections to $\hat{g}$ ($g_{D^*D\pi}$) are too large: $\hat{g}$ too low

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3°) DIRAC equation Another answer to defects of NR model.

light quark in the field of a static source (quark b or c)

Much improved treatment of DIRAC spinors, taking into account interaction ⟶ Restores agreement $\hat{g} \sim 0{,}6$

Convenient for $\begin{cases} \text{spectrum} \\ \text{elementary quantum emission from the light quark} \end{cases}$ $D^* \to D\pi$, π, γ

Not adapted to $b \to c$ (no treatment of CM motion)

$\exists$ extensions of DIRAC eq. to finite m_Q and to moving hadrons

However, no attempt to calculate form factors.

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III c) Decays of radial excitations

Finally, let us quote a problem for all dynamical calculations. Namely, we lack a satisfactory treatment of pionic decays of radial excitations D', B'..., like $D'^{(*)} \to D^{(*)}\pi$ Pionic decays

Indeed – standard QCDSR are precisely fitted to predict only the properties of the lowest state in each channel.

– Quark models are more suited to predict properties of radial excitations, since they give their wave functions.

However, as to strong decays, the elementary emission model does not seem satisfactory (ROPER $P_{11} \to N, \Delta\pi$ wrong by a factor 10)

The Quark pair creation model $^3P_0$ is better. Good for $\Upsilon' \to B\bar{B}$, $\psi' \to D\bar{D}$. But, since it is completely NR for both decay product, it cannot be quantitative for $D'^{(*)} \to D^{(*)}\pi$.

Since radial excitations are not either easily treated in lattice QCD, this is an interesting challenge.

A. Le-Y. (16)

IV Confrontation of experiment, dynamical calculations and S.V. sum rules example of ρ^2, $\tau_{1/2}$, μ_π^2 ($\bar\Lambda$, ρ_D^3).

a) Definitions – ρ^2 slope of $\xi(w)$

– $\tau_{1/2}(1)$ form factor for current transition from 0^- to $L=1, j=1/2$ excitations in the $m_Q \to \infty$ limit

controls $B \to D^{**} \ell \nu$, $B \to D^{**} \pi$

– μ_π^2 kinetic energy in B ($1/m_b^2$ corrections)

– ρ_D^3 controls the $1/m_b^3$ corrections

– $\bar\Lambda = m_B - m_b$ equivalent to m_b

b) Theoretical sources for these quantities

1°) ρ^2 and $\tau_{1/2}$ have been calculated in dynamical approaches

– in QCDSR ("three-point" sum rule)

$\rho^2 \sim 0{,}84$

$\overline{MS}$ at 1,4 GeV

rad. corr. included two loops

$\rho^{2\,SR}(1\,GeV)$ should not be far from this ρ^2

$\tau_{1/2}(1) = 0{,}22$

improved by radiative corrections $\to 0{,}35$

but very large uncertainty $0{,}2 - 0{,}4$

due to dependence on continuum thr.

IV b) Theoretical sources (suite)

- quark models BAKAMJIAN-THOMAS $\rho^2 \simeq 1$
  with GI wave eq
  $\tau_{1/2} = 0{,}225$
  $(\ll \tau_{3/2} = 0{,}54)$

2°) The parameters $\bar{\Lambda}$ and $\mu_\pi^2 (\lambda_1)$ can be calculated from QCDSR:

- $\bar{\Lambda}$ from m_B, very safe QCDSR (2 point) $\quad \bar{\Lambda}^{SR}(1\,\mathrm{GeV}) \sim 0{,}7$ GeV
- μ_π^2 3-points QCDSR (overestimation?) $\quad \mu_\pi^2 = 0{,}5\ \mathrm{GeV}^2 + 0{,}17\ \mathrm{GeV}^2 \sim 0{,}7\ \mathrm{GeV}^2$

3°) Bounds from sum rules (combinations of the basic SR)

ρ^2 and $\tau_{1/2}(1)$ are also related to each other by the bound

$$(\rho^2)^{SR}(1\,\mathrm{GeV}) > \frac{3}{4} + 3\,\tau_{1/2}^2(1)$$

At $\mu \sim 1$ GeV, estimated threshold of duality (~ 3 states)

Analogous bounds relate $\tau_{1/2}$ to $\bar{\Lambda}, \mu_\pi^2, \rho_D^3$

$$\mu_\pi^2 = \mu_G^2 + 9\,\Delta^2 \tau_{1/2}^2(1)$$
$$\bar{\Lambda}^{SR} > 2\Delta \left(\frac{1}{2} + 3\,\tau_{1/2}^2(1)\right)$$
$$\rho_D^3 > \Delta\, \mu_\pi^2$$

$\Delta \sim 0{,}35$ GeV excitation energy of $L=1$ $j=1/2$ with respect to $L=0$ at $m_Q = \infty$

ALL THESE BOUNDS ARE VERY SENSITIVE TO $\tau_{1/2}(1)$

A. Le-Y.

I INTRODUCTION

[As is well known, the study of heavy quark physics is stimulated particularly by the perspective of elucidating CP violation.]

Remarkable and unexpectedly strong results have been produced in a rather short time due to a very active and close cooperation of a community of theoreticians, and of experimentalists.

Although, at some places, recourse to heavy numerical methods of lattice QCD is necessary, what is nevertheless also remarkable is the richness of analytical results, and how far we can advance by the use of rather simple means.

We must be necessarily highly selective in this very short exposé. In particular, we choose to concentrate mainly on the problems of heavy to heavy hadron transitions; $b \to c$. ($\rho_1^2, \tau_{1/2}, \mu_\pi^2$)

Then, As a striking illustration, we will end by a discussion of the unexpectedly close connection which exists between the $\lambda_1(\mu_\pi^2)$ parameter and $B \to D^{**}\pi$ experiment

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Partial list of authors in the field covered by the talk

FALK, GRINSTEIN, BOYD, LIGETI, LEIBOVICH, NEUBERT URALTSEV, BIGI, BALL, BRAUN, BAGAN, SHIFMAN, BLOK, MANNEL, BRAUN, KHODJAMIRIAN, RUCKL, ROSNER, Q NEUBERT, SACHRAJDA, BURAS, BUCHALLA, COLANGELO, QUIGG DE FAZIO, PAVER, VAINSHTEIN, GOLOWICH, DONOGHUE, BURDMAN, MARTINELLI, HUANG, DAI, STECH, SOARES, GOITY, ROBERTS, EICHTEN, HILL, BARDEEN, MARTINELLI, ROSNER, QUIGG, LEBED, MELIKHOV, LUKE, ALI, GAMBINO, PHAM, NARDULLI, FLEMING, STEWART, JAUS, WYLER, PIRJOL,

→ { OLIVER, RAYNAL, LE YAOUANC, PÈNE, BECIREVIC, CHARLES, MORÉNAS }; + ORSAY lattice group.

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A) Exact results

II GENERAL VIEW

a) Results of infinite mass limit ($m_Q \to \infty$)

Concern both pure QCD (spectra, strong decays, ..) and hadron electroweak properties (form factors, ...)

— Symmetries Ex. $m_D = m_{D^*}$; universality of $B^{(*)} \to D^{(*)}$ form factors $\to \xi(w)$

— Sum rules (SV), leading to bounds. Recent ones on

SV Ex. $\rho^2 \geq 3/4$, bounds for curvature $\sigma^2 = 2c \geq \frac{15}{16}$ and higher derivatives (bounded from below)

NB also finite mass dispersive bounds on form factors, through t channel SR

— Further properties with inclusion of light mesons

1) Union of χPT (chiral pert. th.) and heavy quark limit for soft pions

Ex. VMD (exact at $1/m_Q$ included) in $B \to \pi l \nu$ at q^2_{max}

2) Hard mesons $E_\pi \to \infty$ (LEET) $B \to \pi l \nu$ with

- new symmetries Ex $f_0 \sim E_\pi f_+$
- asymptotic behaviour Ex $f_+(0) \sim E_\pi^{-2}$
- approx. factorisation of $B \to \pi\pi$ (BBNS)